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# Computation of modes in embedded helical structures with the SAFE-PML method

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### Abstract

Helical multi-wire cables are widely used in bridge construction and can be degraded due to corrosion and fatigue. To reveal defects inside cable structures, elastic guided waves are of interest owing to their ability to propagate over long distances. In practice, cables are often buried into a solid matrix and can be considered as open waveguides. Waves can be leaky and strongly attenuate along the guide axis, which reduces the propagating distance. Maximizing this distance is necessary for non-destructive testing. The goal of this work is to propose a numerical method for computing modes in embedded helical structures, combining the so-called semi-analytical finite element (SAFE) method and a radial perfectly matched layer (PML) technique.

**Keywords:** waveguide, helical, leaky modes, finite element, perfectly matched layer

### 1 Introduction

The numerical modeling of cable waveguides encounters three difficulties : the helical nature of the geometry, the unbounded cross-section and the exponential transverse growth of leaky modes. The helical geometry can be represented in the twisting coordinate system proposed in [1]. In order to overcome the last two difficulties, a simple method consists in using absorbing layers of artificial growing viscoelasticity [2]. An alternative approach is to use a PML technique. Such a technique has already been developed for open straight waveguides [3]. The present work consists in extending the SAFE-PML method for embedded helical structures.

### 2 SAFE-PML formulation

The time harmonic dependence is chosen as  $e^{-i\omega t}$ . Acoustic sources and external forces are discarded

for computing eigenmodes. The 3D elastodynamic equations satisfied by the displacement vector  $\mathbf{U}$  are represented in Cartesian coordinates  $(X, Y, Z)$  as:

$$\nabla \cdot \sigma(\mathbf{U}) + \omega^2 \rho \mathbf{U} = 0 \quad (1)$$

where  $\rho$  is the material density,  $\sigma$  is the stress tensor related to the strain tensor  $\epsilon = (\nabla \mathbf{U} + (\nabla \mathbf{U})^T)/2$  by the relation  $\sigma = C : \epsilon$ ,  $C$  is the stiffness tensor and  $T$  is the matrix transpose. The tilde notation will be explained later.

For modeling of helical waveguides, Eq. (1) is rewritten in the twisting coordinate system  $(x, y, z)$  defined as  $x = X \cos(\tau Z) + Y \sin(\tau Z)$ ,  $y = -X \sin(\tau Z) + Y \cos(\tau Z)$ ,  $z = Z$  where  $\tau$  denotes the torsion of the  $(x, y)$  plane around the  $z$  axis. One considers a linearly elastic embedded helical waveguide  $S \times \mathbb{R}$  whose cross-section  $S$  and material properties in the transverse  $(x, y)$  plane are invariant along the  $z$  axis.

The SAFE method is applied, which consists in assuming the solutions of the form  $\mathbf{U}(x, y, z) = \mathbf{u}(x, y)e^{ikz}$ , where  $k$  is the axial wavenumber. The axial derivative  $\partial/\partial z$  is replaced with product by  $ik$ . We are led to a bidimensional problem satisfied by  $\mathbf{u}$  in the transverse directions  $(x, y)$  with the following variational formulation:

$$\int_S \delta \epsilon^T \sigma dx dy - \omega^2 \int_S \rho \delta \mathbf{u}^T \mathbf{u} dx dy = 0 \quad (2)$$

where  $\sigma$  and  $\epsilon$  denote the stress and strain vectors respectively.

In addition to the SAFE technique, the radial PML method will be implemented.

The formulation (2) is now transformed in cylindrical coordinates  $(r, \theta, z)$  defined from twisting coordinates  $(x, y, z)$  as  $x = x_{O'} + r \cos \theta$ ,  $y = y_{O'} + r \sin \theta$ . In the  $(x, y)$  plane, the point  $O' = (x_{O'}, y_{O'})$  is the center of this cylindrical

system.  $x_{O'}$  and  $y_{O'}$  are independent of the axial coordinate  $z$ .

The radial PML defines a complex radial coordinate  $r_c = \int_0^r \gamma(\xi) d\xi$  where  $\gamma(r) = 1$  for  $r \leq d$ ,  $\text{Im}(\gamma) > 0$  for  $r > d$ .  $d$  is the radius of the PML interface. The twisted radial PML is called centered if  $x_{O'} = y_{O'} = 0$  and off-centered if not. The change of variable  $r_c \mapsto r$  yields for any function  $f$  :

$$\frac{\partial f}{\partial r_c} = \frac{1}{\gamma} \frac{\partial f}{\partial r}, \quad dr_c = \gamma dr \quad (3)$$

The variational formulation (2) must be then transformed back to the coordinates  $(x, y)$ .

Finally, the FE discretization of the variation formulation (2) along the transverse directions  $(x, y)$  yields a matrix eigensystem of the following form:

$$\{\mathbf{K}_1 - \omega^2 \mathbf{M} + ik(\mathbf{K}_2 - \mathbf{K}_2^T) + k^2 \mathbf{K}_3\} \mathbf{U} = \mathbf{0} \quad (4)$$

where the column vector  $\mathbf{U}$  contains nodal displacements. Given  $\omega$  and finding  $k$ , this eigenproblem is quadratic. The linearization of this eigensystem is detailed in [3] and yields non hermitian matrices, which makes the numerical treatment of the eigensystem complicated.

### 3 Results

A Dirichlet condition is chosen at the exterior boundary of truncated domain. Finite elements are triangles with six nodes. Following [3], the PML layer is close to the core in order to reduce the effects of the transverse growth of leaky modes on numerical results. The PML function  $\gamma$  should be chosen as smooth as possible to minimize numerical reflection [3].  $\gamma$  is a parabolic function in this work.

The twisted SAFE-PML method is validated with a cylinder test case owing to the fact that any arbitrary twist can be applied (a twisted cylinder remains a cylinder). The centered PML is used. Figure 1 compares the numerical results computed in twisting and untwisting coordinate systems and yields the same physical modes. However, their axial wavenumbers are translated by  $\pm\tau m$  in the twisting system, where  $m$  denotes their circumferential order.

A first application consists of a steel helical wire buried in concrete. Since the computational domain with a centered PML is quite

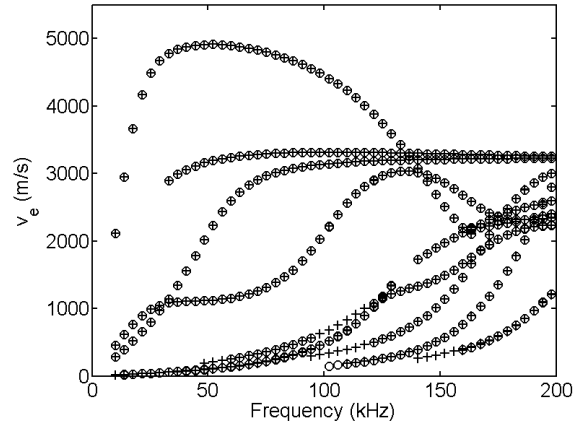


Figure 1: Energy velocity for an embedded cylindrical bar obtained by untwisted (circles) and twisted SAFE-PML (crosses) methods.

large, an off-centered PML should be preferred. The off-centered PML is validated by comparing the numerical results with those computed with the centered PML method, which has been checked in the previous test case. Results show that the twist of the helical geometry enhances the modal axial attenuation.

A second numerical example is given by a steel seven-wire buried in concrete. Compared with the results of a free strand, the modal behavior is strongly modified due to the introduction of the surrounding medium. Modes of lowest attenuation are identified, which may be of interest for inspecting cable structures.

### References

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