

# Modèles multi-échelles de métamatériaux pour le contrôle des ondes élastiques

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Institut Fresnel, CNRS/Aix-Marseille Université

## Travail effectué en collaboration avec:

André Diatta, Mohamed Farhat et Stefan Enoch.  
Emmanuel Javelaud et Stéphane Brûlé (Ménard)  
Guillaume Dupont, Bernard Molin (IRPHE/AMU)  
Dimitri Komatitsch (LMA/AMU)

## Plan de l'exposé:

Introduction sur la réfraction négative et le cloaking  
Partie 1: Focalisation des vagues et des ondes de Lamb  
Partie 2: Capes à vagues, et d'ondes de Lamb dans les plaques  
Partie 3: Vers le contrôle des ondes sismiques  
Conclusion et perspectives

# Collaborations

- IRPHE & Laboratoire Mécanique Acoustique (CNRS/ Aix-Marseille Université/Ecole Centrale de Marseille)
- Institut Langevin (CNRS/ESPCI/UPMC/ Univ. Paris 7)
- Imperial College London (Royaume-Uni)
- Liverpool University (Royaume-Uni)
- Sydney University (Australie)
- Austin University (USA)
- Massachusetts Institute of Technology (USA)

# Financement

2011-2016 : ERC Starting Grant (Projet Anamorphism)



# Introduction: réfraction négative et cloaking pour les ondes optiques et acoustiques

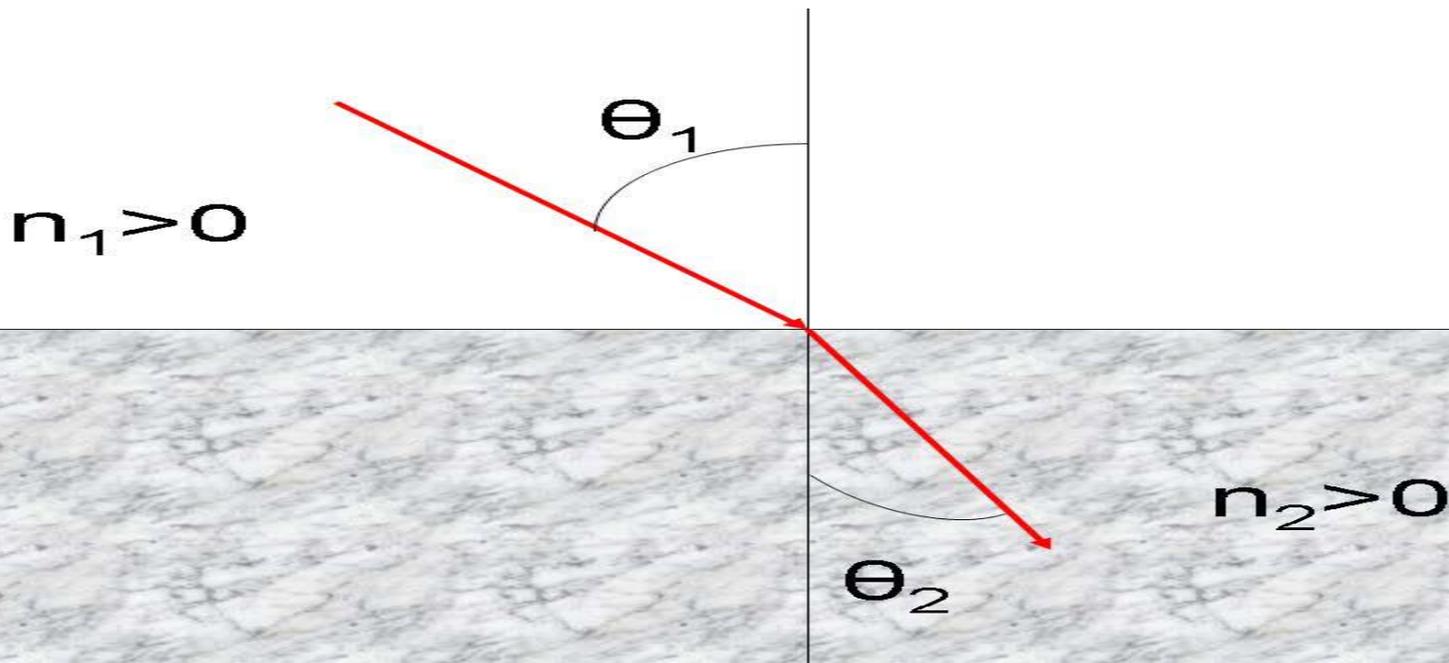




Loi de Snell (1621)  
-Descartes (1637)  
... Ibn Sahl (983)



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

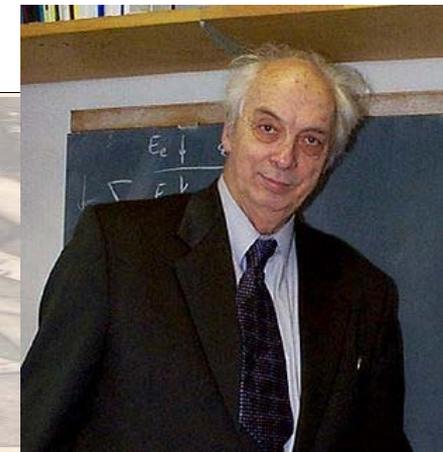
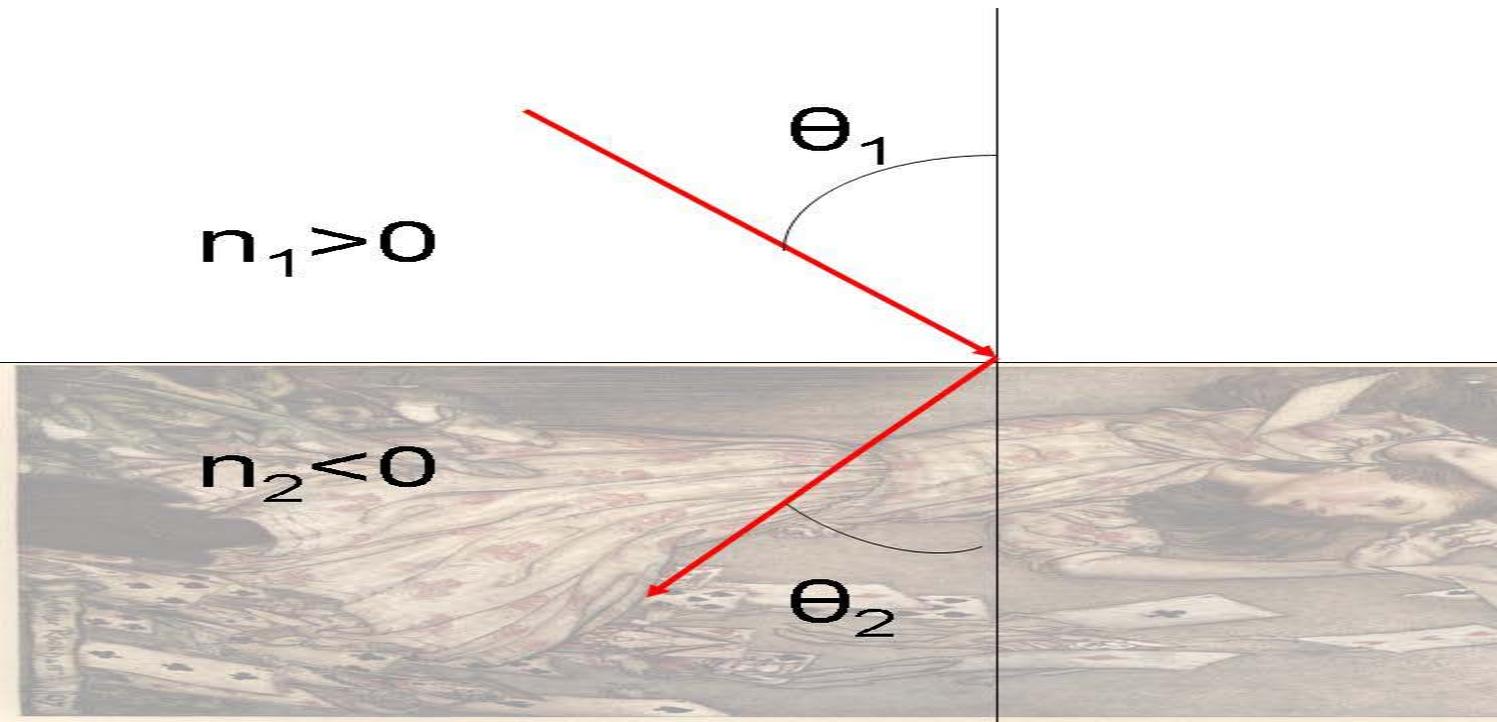




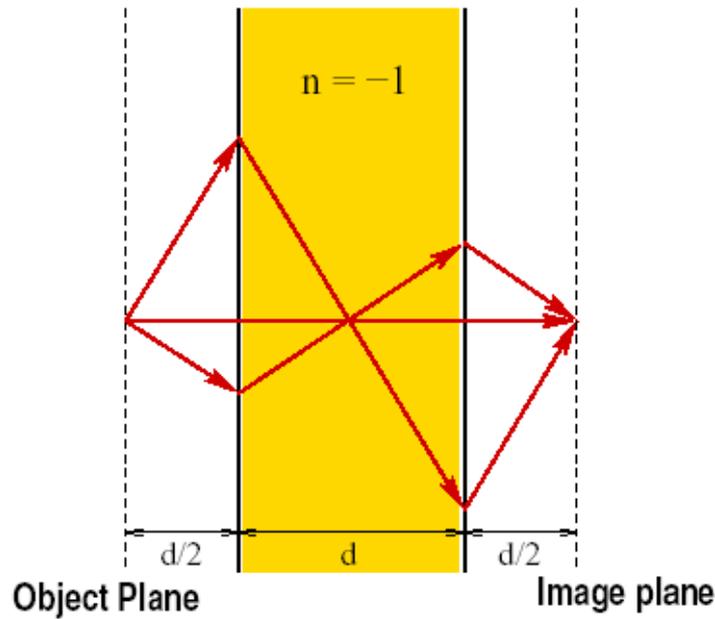
# Loi de Snell- Descartes inversée (Veselago, 1968)



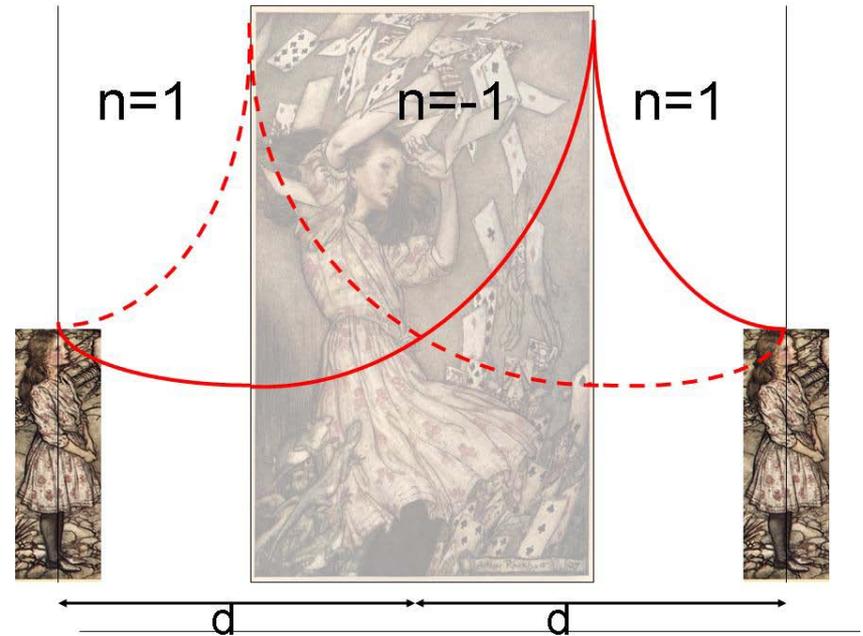
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



# Lentille plate « parfaite » par réfraction négative (Veselago, 1968 & Pendry, 2000)



Veselago, 1968

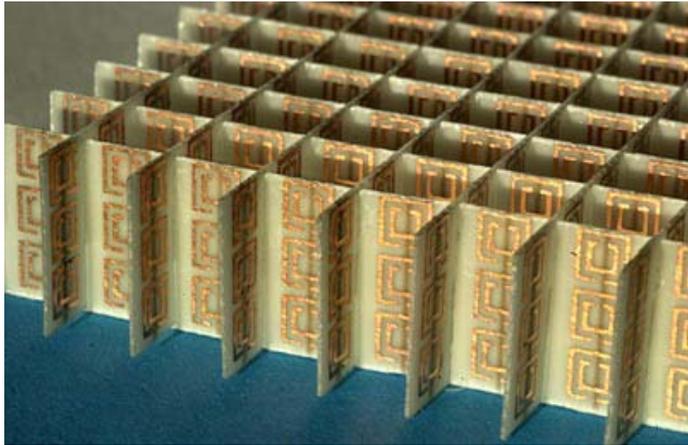


Pendry, 2000

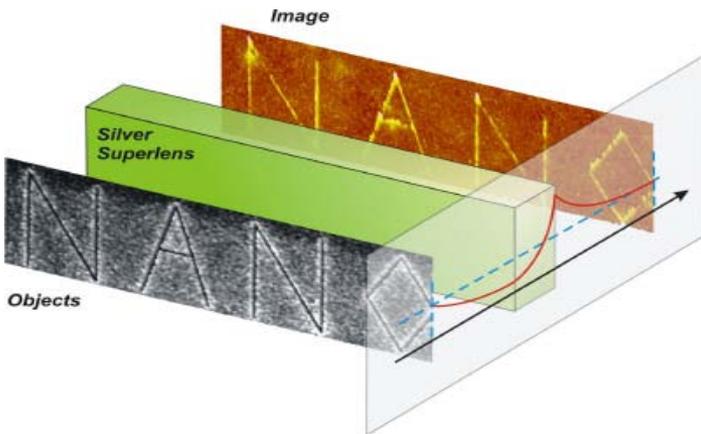
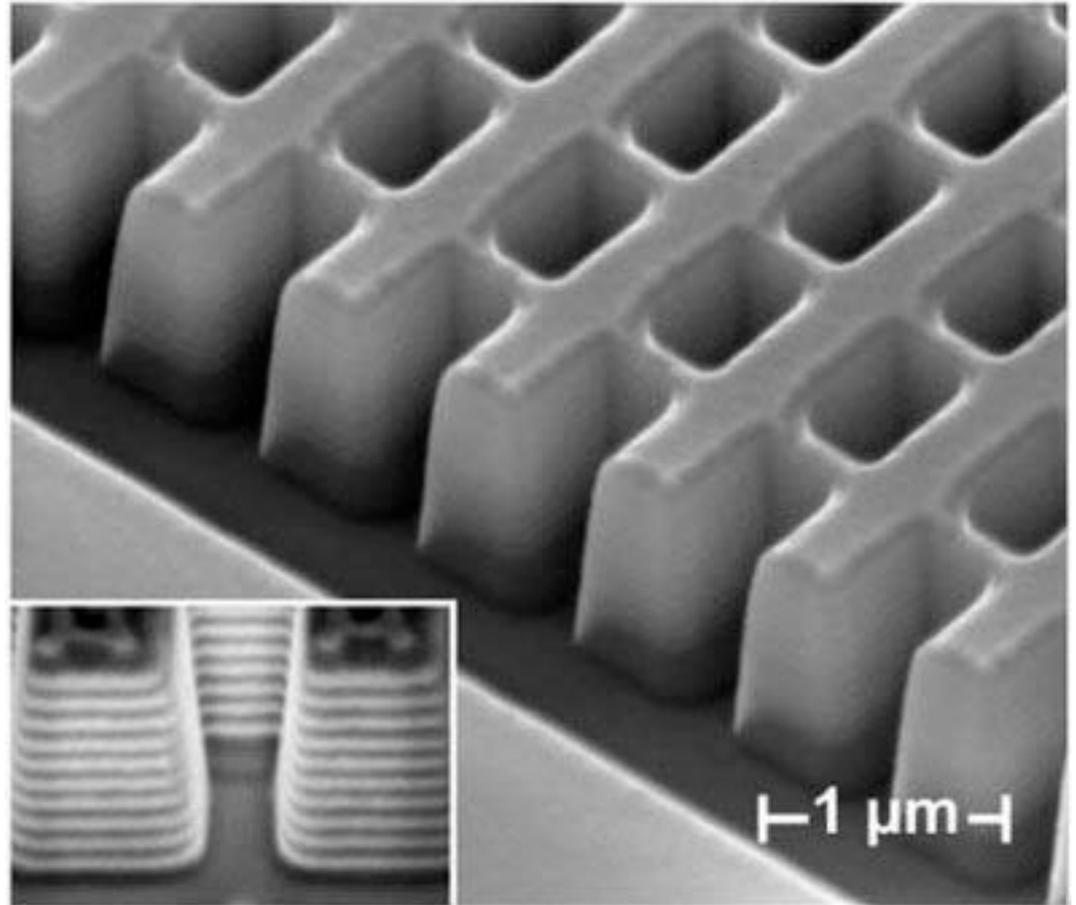


# Réalisations expérimentales de métamatériaux électromagnétiques

Smith, 2001



Zhang, 2008



Zhang, 2005

Tout cela vaut aussi pour les ondes de pression!!!

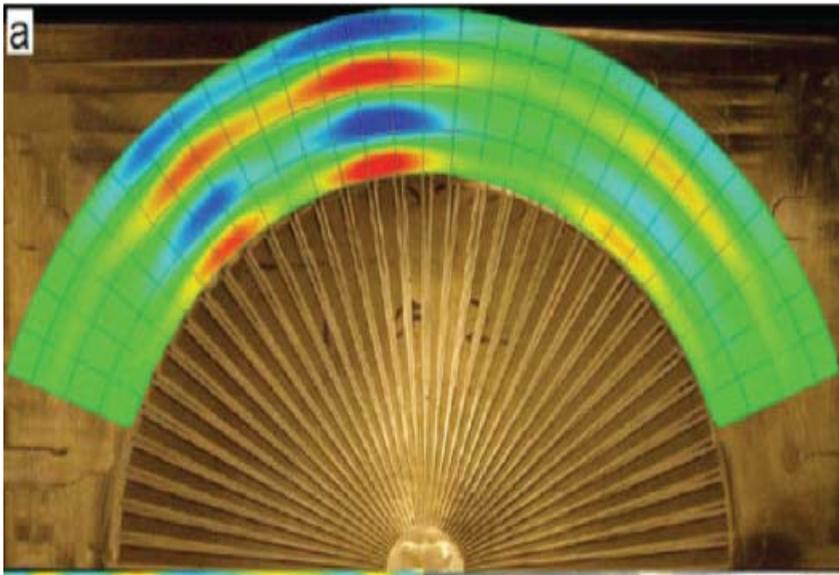
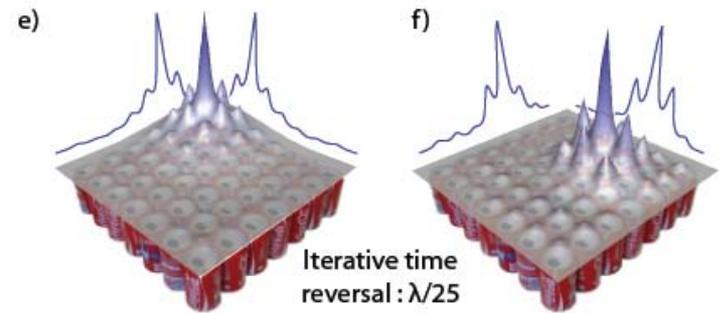
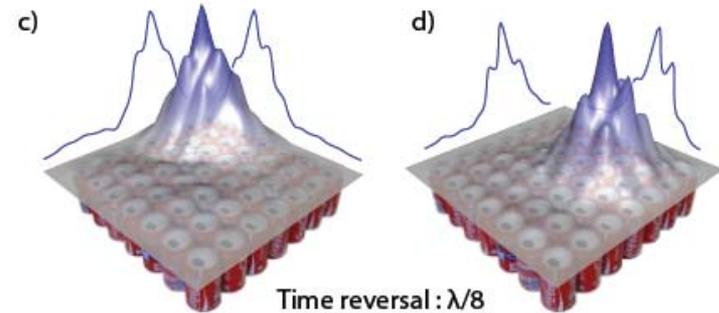
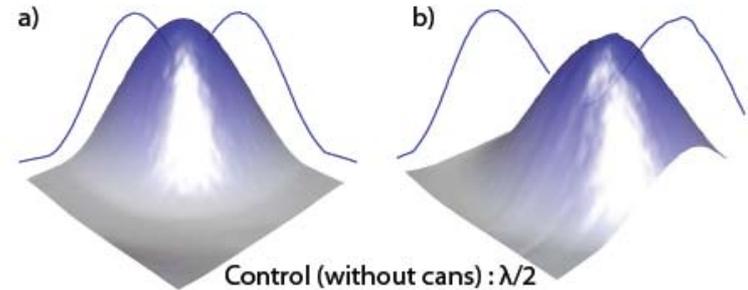
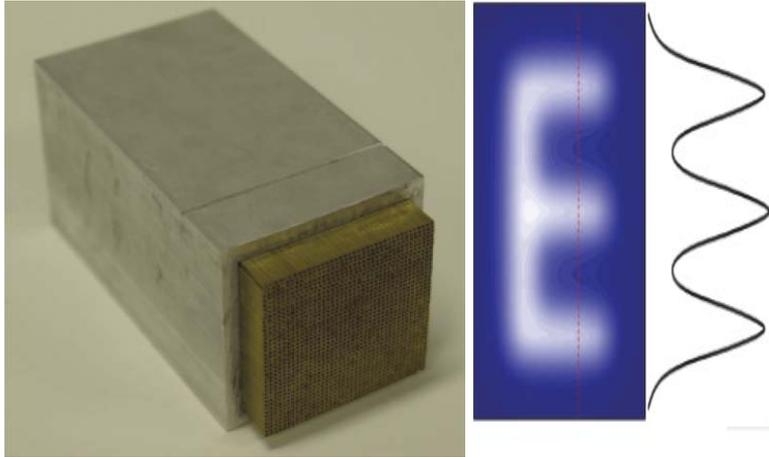
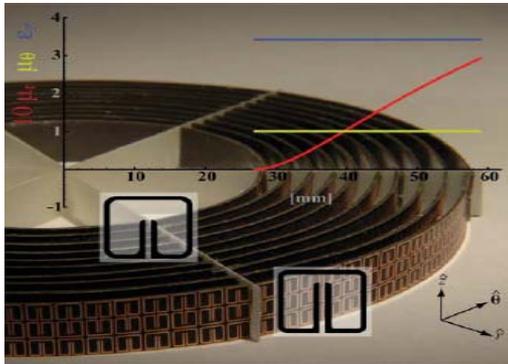


Image sub-wavelength avec transmission extraordinaire (haut et hyper-lens (bas) de Xiang Zhang et Jensen Li (Berkeley)

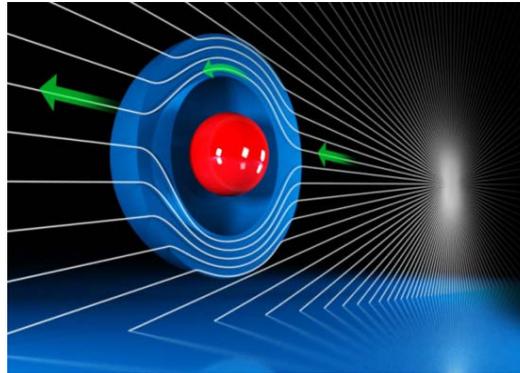
Super lensing avec les canettes de coca de Geoffroy Lerosey (ESPCI Paris)

# Les capes d'invisibilité: un nouveau paradigme pour les métamatériaux

Cape par anisotropie

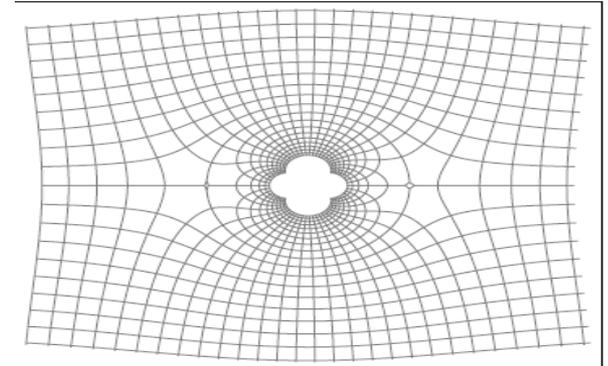


*Pendry, Schurig, Smith  
(Imperial College & Duke)*



Un objectif

Cape par gradient d'indice



*Leonhardt  
(Saint-Andrews)*

Optique transformationnelle

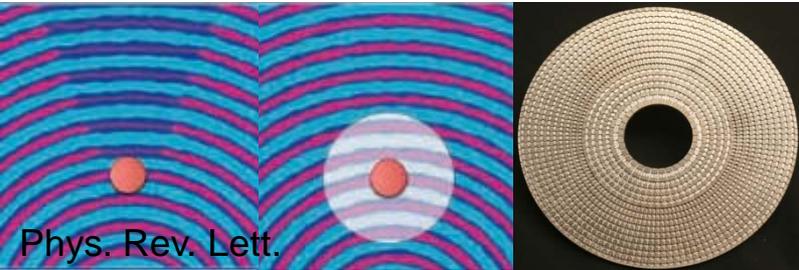
Optique conforme

Deux approches

Qu'en est-il de l'acoustique?

# Les capes de protection pour d'autres types d'ondes

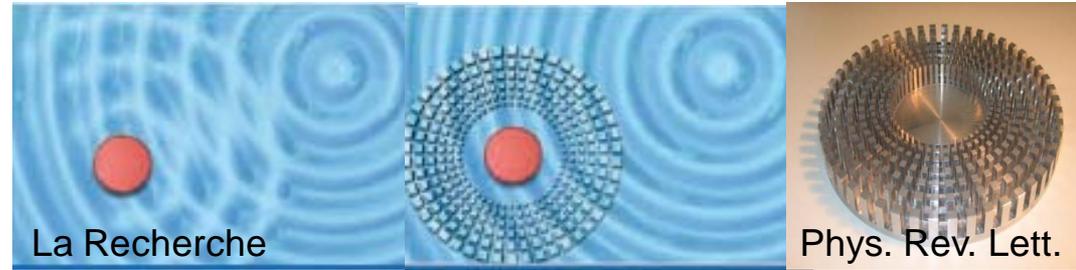
Ondes de pression (dans un fluide)



Phys. Rev. Lett.

validation expérimentale: Fang (MIT)  
d'après: Shurig, Cummer, Chan etc.

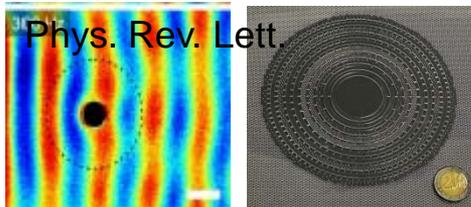
Vagues (surface d'un liquide)



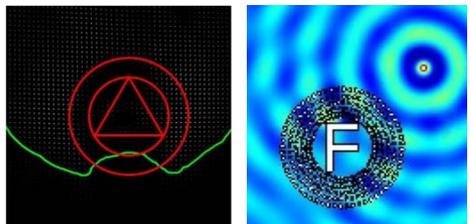
La Recherche

Phys. Rev. Lett.

Farhat, Guenneau, Enoch, Movchan  
(Institut Fresnel & Liverpool University)



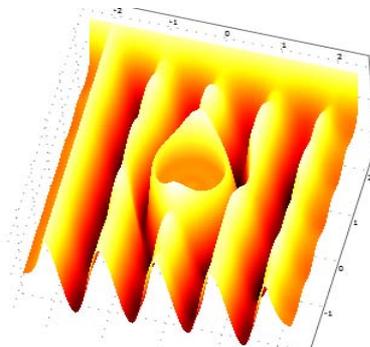
Phys. Rev. Lett.



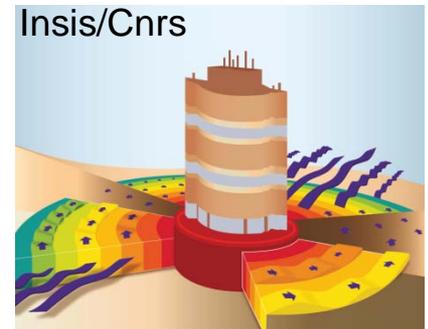
Wegener  
(Karlsruhe)

Expérience

Vandenberghé  
(Marseille)



Vibrations  
de plaques

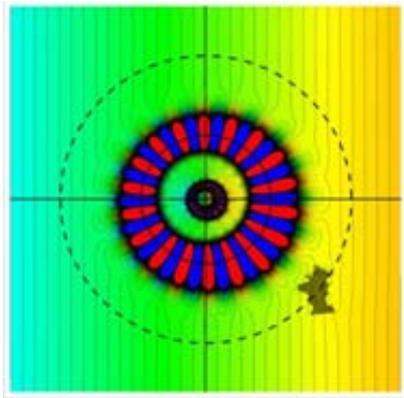


Insis/Cnrs

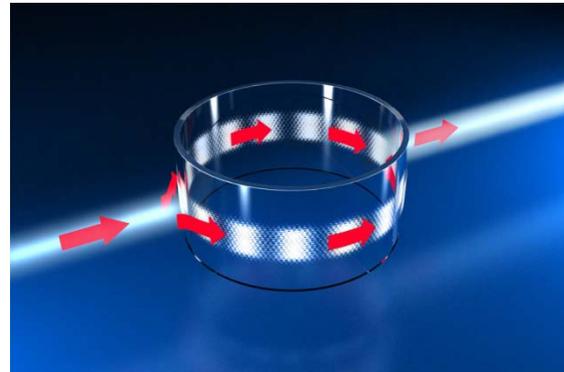
Ondes sismiques?

# Les capes plasmoniques: l'optique sens dessus dessous

Cape par réaction

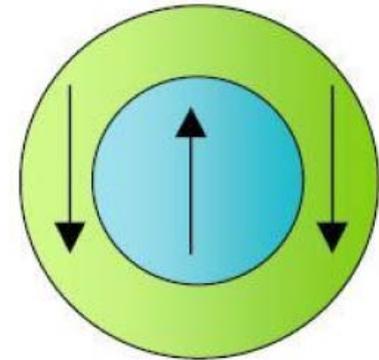


*Nicorovici, Milton;  
Mc Phedran, Botten:  
(Univ. Sydney & Utah)*



Un objectif

Cape par ultra-réfraction



*Alu, Engheta  
(Univ. Austin & Pensylvannie)*

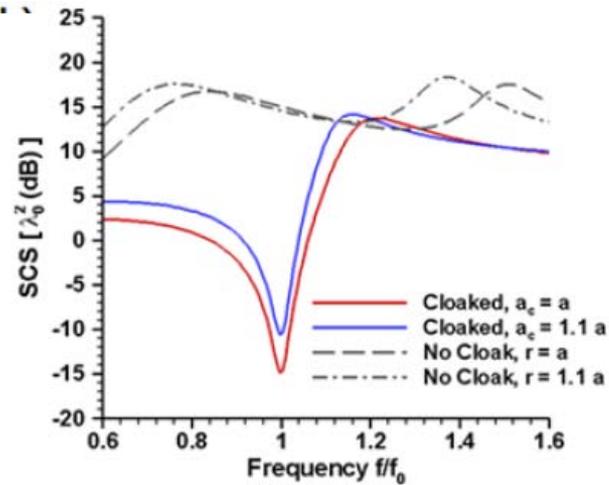
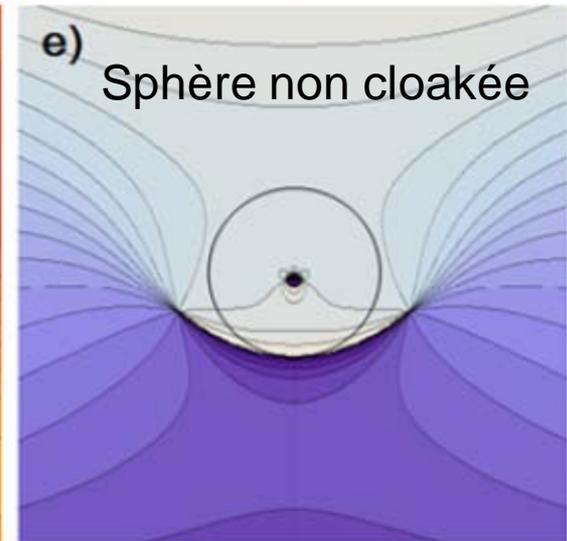
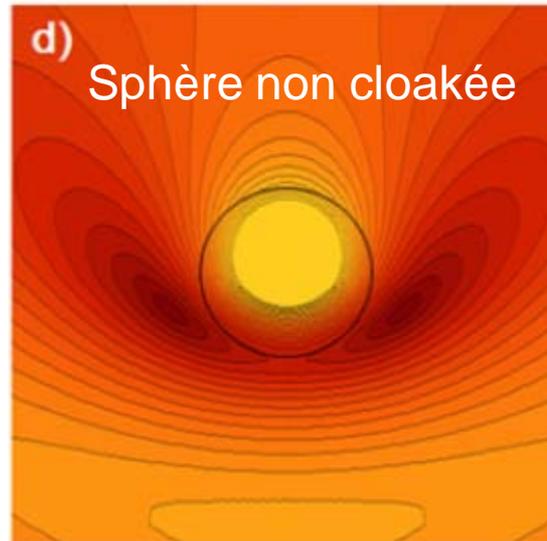
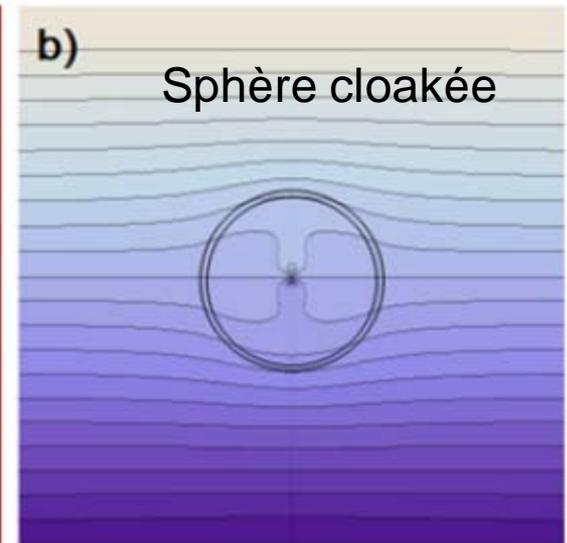
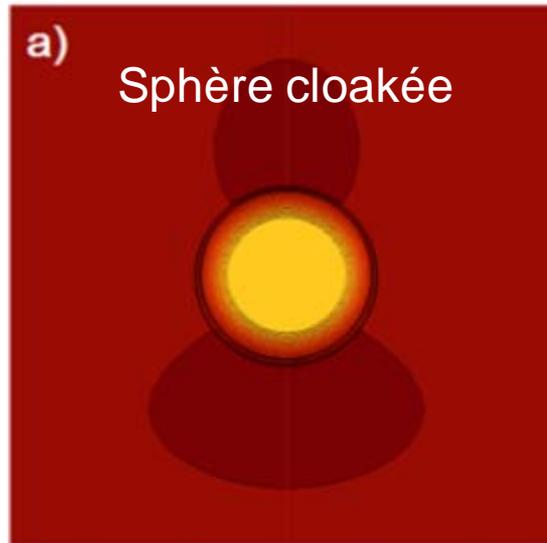
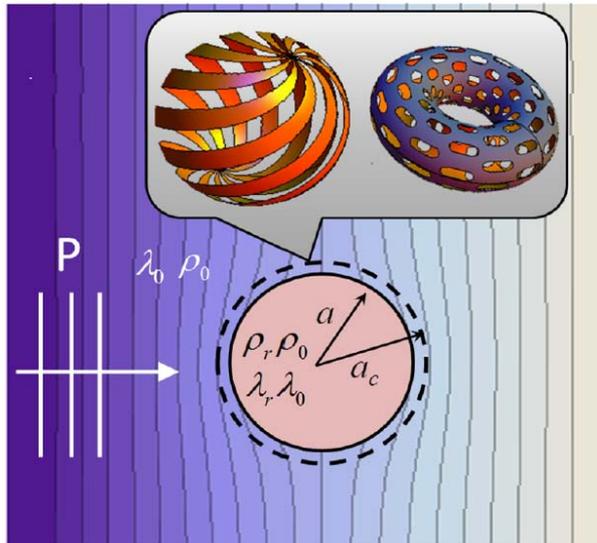
Cape d'indice négatif

Cape d'indice proche zéro

Deux approches

Qu'en est-il de l'acoustique?

# Cloaking acoustique par réaction



FSS invisibility for three dimensional objects

Radius  $R = \lambda/10$  (a,d) Amplitude; (d,e) Phase

# Partie 1: focalisation des vagues, et des ondes de Lamb dans les plaques



# Une lentille à vagues made in Marseille (réfraction négative)

Journal of Computational and Applied Mathematics 234 (2010) 2011–2019



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journal homepage: [www.elsevier.com/locate/cam](http://www.elsevier.com/locate/cam)



## Biblio:

X. Hu, Y. Schu, X. Liu, R. Fu, j. Zi,  
Superlensing effect in liquid surface waves,  
Phys. Rev. E 69 (2004) 030201–030204.

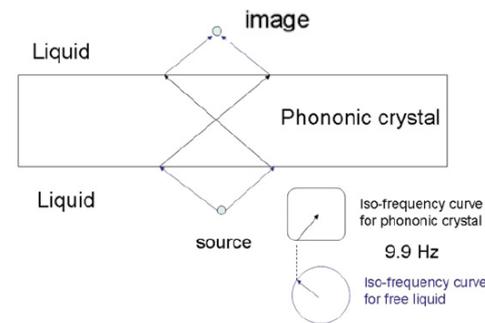
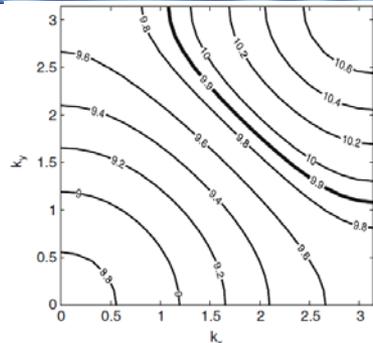
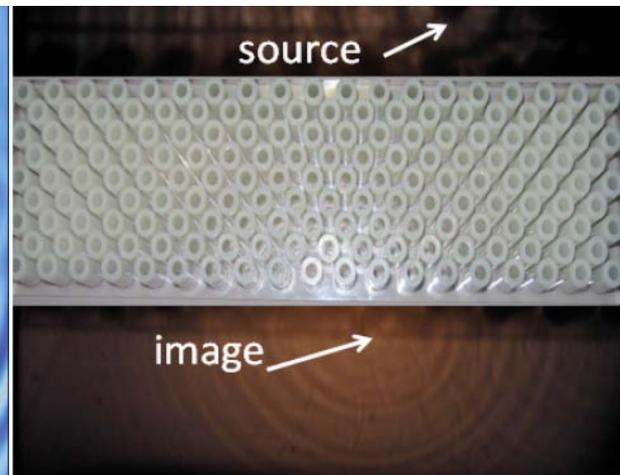
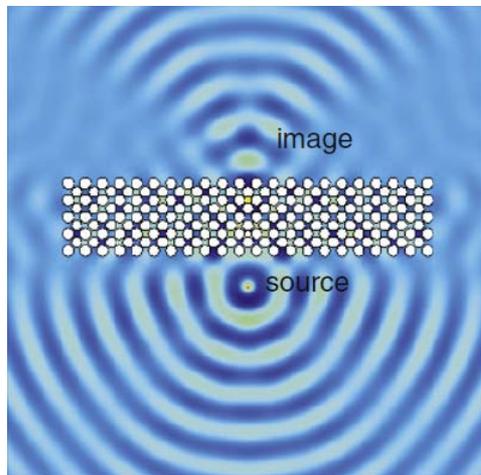
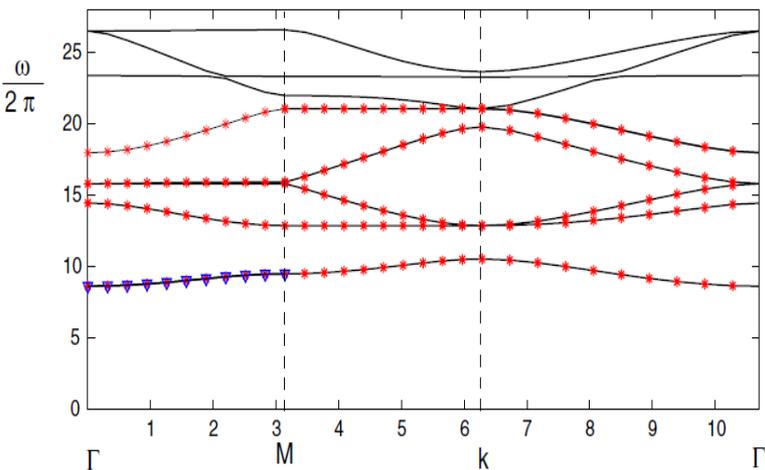
Farhat, Guenneau, Enoch, Movchan

All-angle-negative-refraction and ultra-refraction for liquid surface waves in 2D phononic crystals

$$(\nabla^2 + \kappa^2)\psi = 0. \quad \omega^2 = g\kappa \tanh \kappa h (1 + d_c^2 \kappa^2)$$

$$\mathbf{n} \cdot \nabla \psi + \alpha \psi = 0,$$

@9.9 Hz



$$\kappa = \sqrt{\Lambda} \left( 1 + \frac{|\mathbf{k}|^2}{2\Lambda} \right) + O(|\mathbf{k}|^4).$$

$$\Lambda^{-1} = \frac{d^2}{2\pi} \left[ \ln \left( \frac{a}{d} \right) + \beta \right]$$

$$\beta = -2\pi\gamma_1 - 2 \ln 2 + 1/\alpha \approx 1.31053292 + 1/\alpha.$$

# Focalisation des ondes de Lamb dans les plaques perforées

Concept théorique

APPLIED PHYSICS LETTERS 96, 081909 (2010)

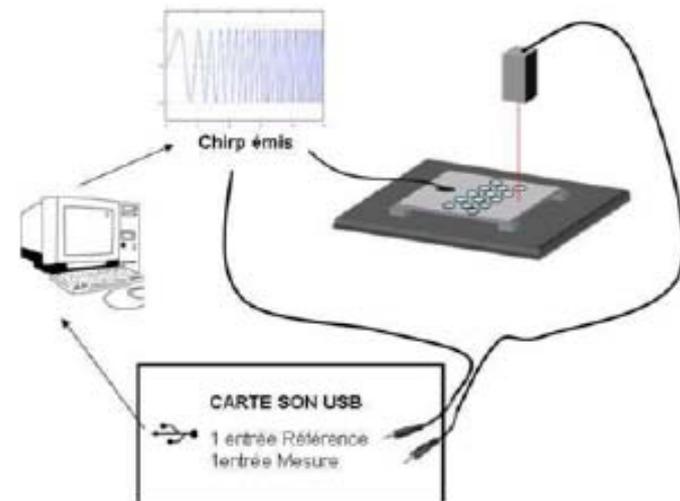
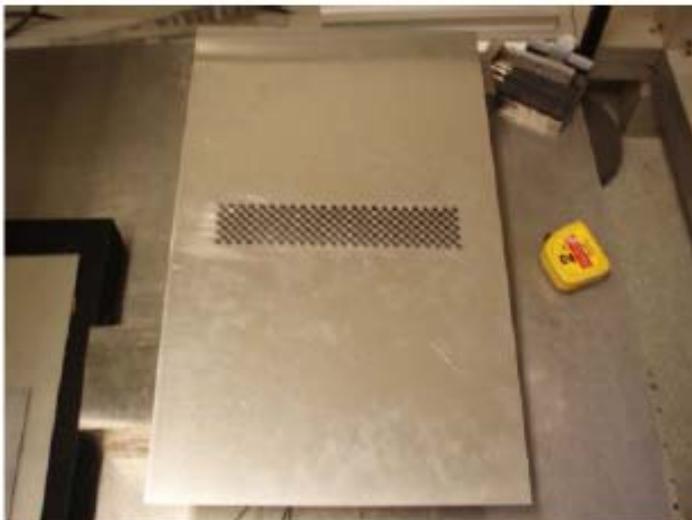
## Focussing bending waves via negative refraction in perforated thin plates

Mohamed Farhat,<sup>1,a)</sup> Sebastien Guenneau,<sup>2</sup> Stefan Enoch,<sup>1</sup> Alexander B. Movchan,<sup>2</sup> and Gunnar G. Petursson<sup>2</sup>

<sup>1</sup>*Institut Fresnel, CNRS, Aix-Marseille Université, Campus Universitaire de Saint-Jérôme, 13013 Marseille, France*

<sup>2</sup>*Department of Mathematical Sciences, Liverpool University, Peach Street, Liverpool L69 3BX, United Kingdom*

Réalisation expérimentale: Marc Dubois et Patrick Sebbah (Institut Langevin, 2012)



The out of plate displacement  $U$  satisfies :

$$\Delta^2 U - \frac{\rho h}{D} \omega^2 U = (\Delta + \beta^2)(\Delta - \beta^2)U = 0,$$

with  $\beta^2 = \omega \sqrt{\rho h / D}$ ,  $D$  the flexural rigidity,  $\rho$  its density and  $h$  its thickness. Here,  $D = Eh^3 / (12(1 - \nu^2))$ , where  $E$  and  $\nu$  are the Young modulus and the Poisson coefficient.

On the boundaries of holes, we set stress free conditions :

$$\begin{aligned} \frac{\partial^2 U}{\partial x^2} + \nu \frac{\partial^2 U}{\partial y^2} &= 0, \quad \text{on } y = \pm l, \\ \frac{\partial^3 U}{\partial y^3} + (2 - \nu) \frac{\partial^3 U}{\partial y \partial x^2} &= 0, \quad \text{on } y = \pm l \end{aligned}$$

and similarly on  $x = \pm l$  interchanging  $x$  and  $y$ .

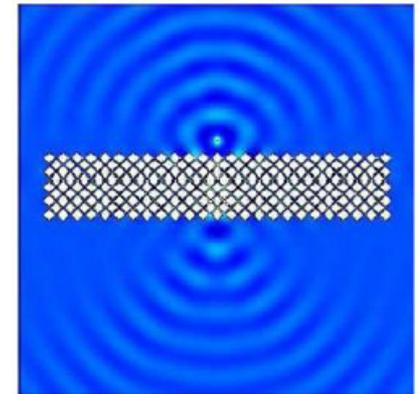
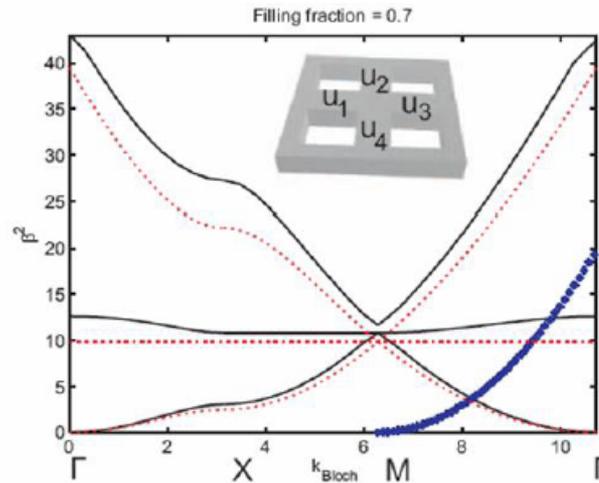
Floquet-Bloch conditions on the unit cell sides

$$U(\mathbf{x} + 2l\mathbf{n}) = U(\mathbf{x})e^{i2l\mathbf{n}\cdot\mathbf{k}},$$

# Comparaison modèle éléments finis 2D/ modèle FDTD 3D

Left : FEM computations of dispersion curves (continuous curves) and analytical estimate (red dotted curves) for Floquet-Bloch waves ; Right : Negative refraction occurs at  $\beta^2 = 3.68$ .

Phase

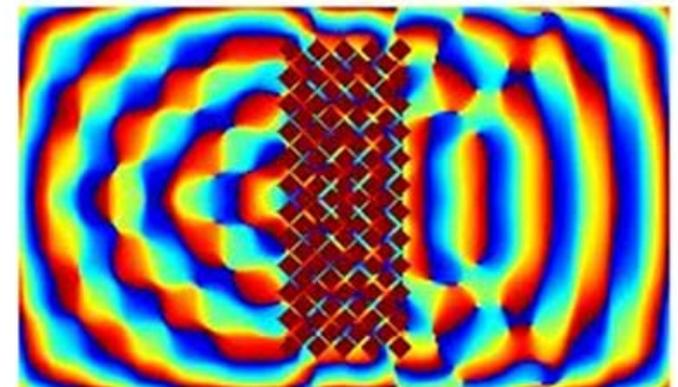
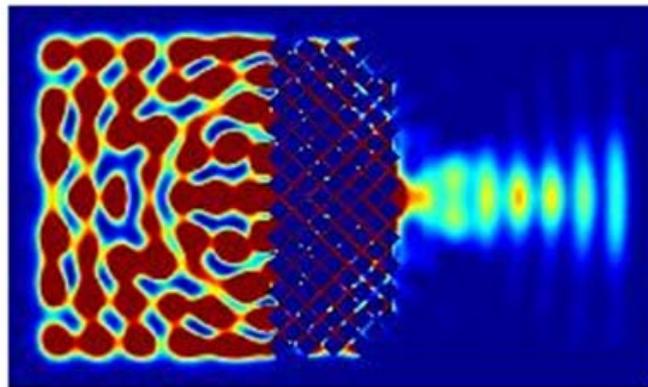


Intensité

Phase

Simulation FDTD  
3D (équation Navier)

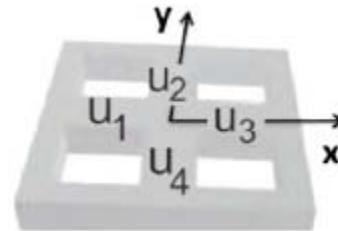
Emmanuel Bossy  
(Institut Langevin)



The out of plane displacement  $u_n$ ,  $n$  integer, satisfies for  $x \in ((2n - 1)l/2, (2n + 1)l/2)$  :

$$\frac{d^4 u_n}{dx^4} - \frac{\omega^2}{\Xi^4} u_n = 0 ,$$

where  $\Xi^4 = \frac{EI}{\rho A}$  and  $l$  is the length of the periodic cell. Here,  $l = bh^3/12$  and  $A = bh$  ( $b$  is the bars' thickness), are the inertial moment and the area of the bars' cross section,  $\rho$  is the density and  $E$  the Young modulus.



We seek solutions in the form

$$u_n(x) = a_n \sin\left(\frac{\sqrt{\omega}}{\Xi} x\right) + b_n \cos\left(\frac{\sqrt{\omega}}{\Xi} x\right) + c_n \sinh\left(\frac{\sqrt{\omega}}{\Xi} x\right) + d_n \cosh\left(\frac{\sqrt{\omega}}{\Xi} x\right),$$

Floquet-Bloch conditions can be differentiated as many times as we want, so that for  $j = 0, 1, 2, 3$

$$u_1^{(j)}(l) = e^{2ik_1 l} u_3^{(j)}(l), \quad u_2^{(j)}(l) = e^{-2ik_2 l} u_4^{(j)}(l),$$

with  $u_1(l) = U(-l, 0)$ ,  $u_2(l) = U(0, l)$ ,  $u_3(l) = U(l, 0)$  and  $u_4(l) = U(0, -l)$ ; Moreover,  $u_i^{(0)} = u_i$ ,  $u_i^{(j)} = \frac{d^j u_i}{dx^j}$  and  $\mathbf{k} = (k_1, k_2)$  is the Bloch vector. At the central node, displacement fields  $u_j$ ,  $j = 0, 1$  meet, and we assume their continuity and differentiability :

$$u_1^{(j)}(0) = u_3^{(j)}(0), \quad u_2^{(j)}(0) = u_4^{(j)}(0), \quad u_1'(0) = -u_4'(0).$$

Finally, the three laws of conservation of momentum (two angular momenta and a linear momentum) lead to :

$$\begin{aligned}EI(u_1''(0) - u_3''(0)) &= I_0\omega^2 u_4'(0) \\EI(u_2''(0) - u_4''(0)) &= I_0\omega^2 u_1'(0) \\EI(u_1'''(0) - u_3'''(0)) &= -M\omega^2 u_1(0),\end{aligned}$$

where  $I_0$  represents the rotational inertia of the elastic body corresponding to the intersection of bars, and  $M$  its mass.

This brings the dispersion relation

$$\begin{aligned}\sin(\kappa l) (\cos(k_1 l) + \cos(k_2 l) - 2 \cos(\kappa l)) \\(\cos(k_1 l) + \cos(k_2 l) - 2 \cosh(\kappa l)) = 0,\end{aligned}$$

where  $\kappa = \sqrt{\omega/\Xi}$ .

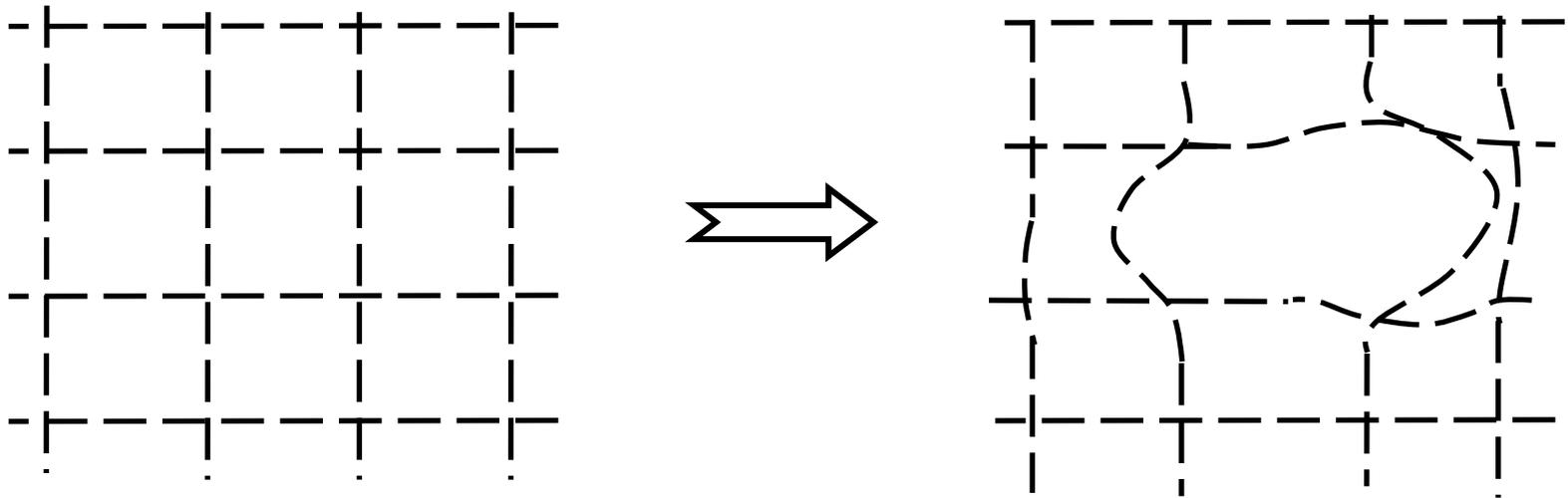
# Partie 2: Capes à vagues, et d'ondes de Lamb dans les plaques



# La philosophie de la physique de transformation

*Un simple changement de coordonnées...*

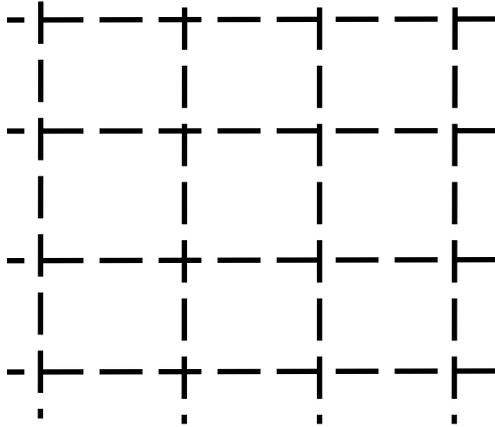
$(x,y,z)$   $\Rightarrow$   $x' = p(x,y,z,t)$   $y' = q(x,y,z,t)$   $z' = r(x,y,z,t)$



*...peut avoir des conséquences majeures!*

**Espace de départ:**

*équation aux dérivées partielles avec coefficient  $m$  scalaire (isotrope)*

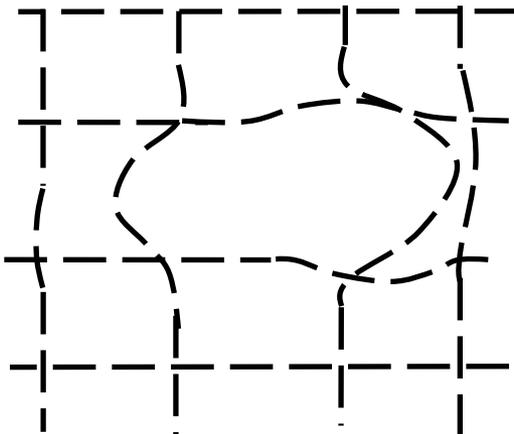


$$L[m(x, y, z, t); u] = S(x, y, z, t)$$

**Espace transformé:**

*équation non modifiée dans la zone requise*

*⇒ La solution est immédiatement donnée par la structuration  $M$  (matrice) de la matière*



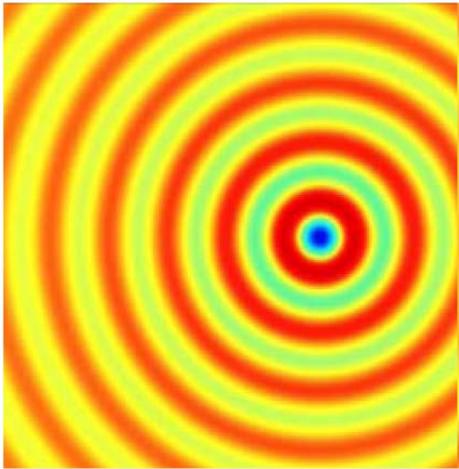
$$L[M(x', y', z', t); u] = S(x', y', z', t)$$

## Qu'est ce que l'invisibilité (multi-onde)?

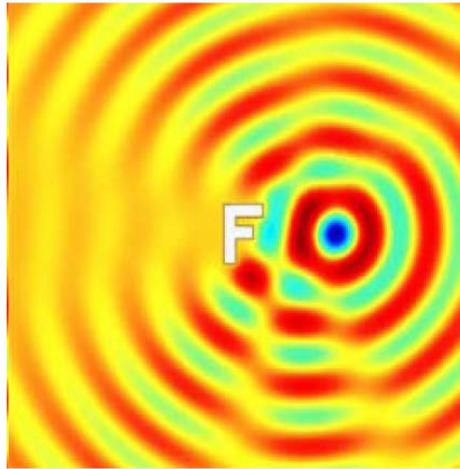
*Cette simulation numérique vaut pour:*

- les ondes électromagnétiques (TE/TM)*
- les vaguelettes à la surface de l'eau*
- les ondes de pression dans un fluide*
- les ondes de cisaillement hors-plan*

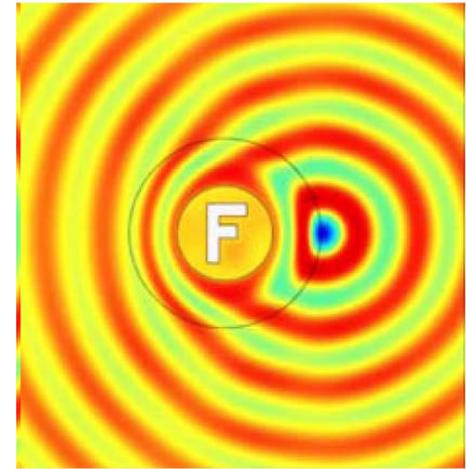
*Etc...*



*Point source en  
espace libre*



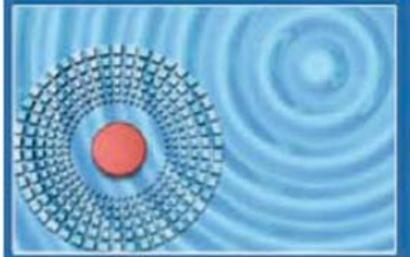
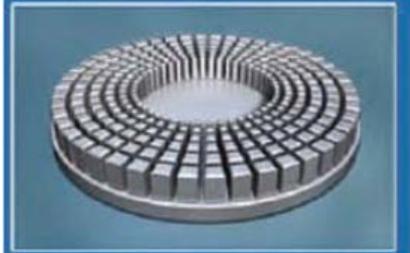
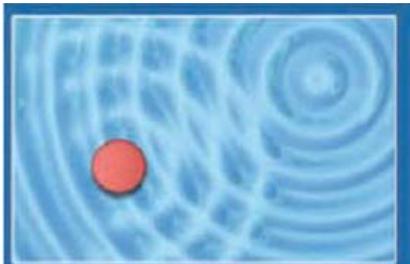
*Point source en  
présence d'un objet  
diffractant*



*Point source en  
présence d'une cape  
entourant un objet  
diffractant*

**Equation de Navier-Stokes (cas général non linéaire):**

$$\rho \left( \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) \mathbf{u} - \mu \nabla^2 \mathbf{u} = -\nabla p + \rho \mathbf{g}$$

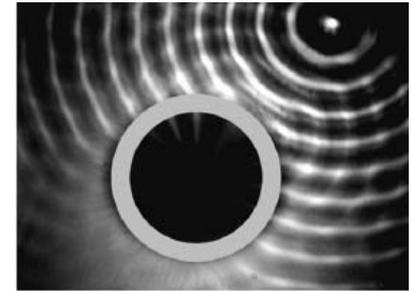
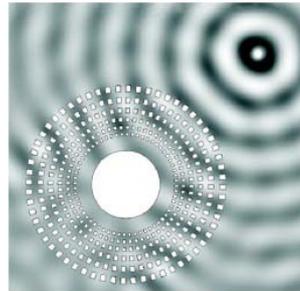


Infographie La Recherche 2012

**Equation de Helmholtz pour vaguelettes sur la surface libre:**

$$\nabla \cdot ([\mu_{\text{hom}}] \nabla \phi_{\text{hom}}(\mathbf{x})) = \kappa^2 \phi_{\text{hom}}(\mathbf{x})$$

avec relation de dispersion  $\omega^2 = g\kappa(1 + \kappa^2 d_c^2) \tanh(\kappa h)$ ,



Farhat, Enoch, Guenneau, Movchan, Phys. Rev. Lett. 101, 134501, 2008

**G. Dupont**

O. Kimmoun

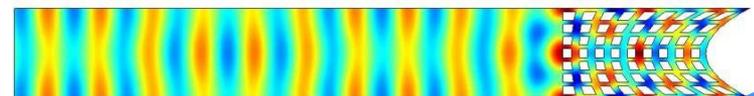
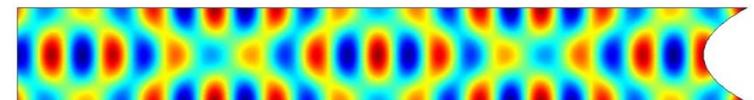
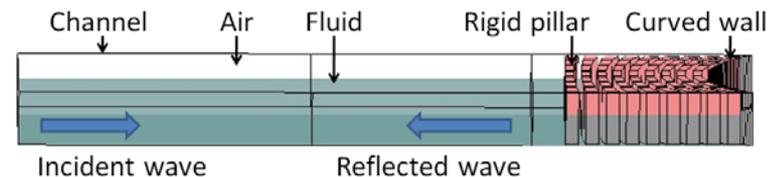
B. Molin

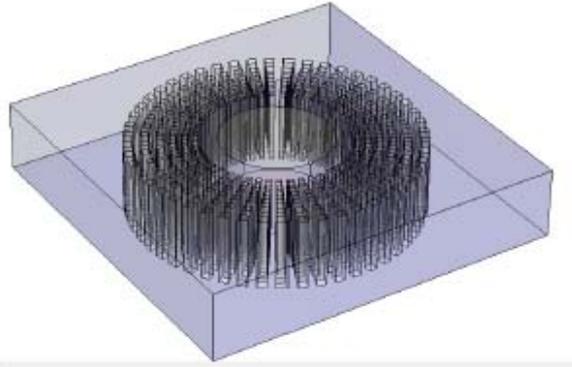
(IRPHE/ECM)

S. Enoch

S. Guenneau

Institut Fresnel

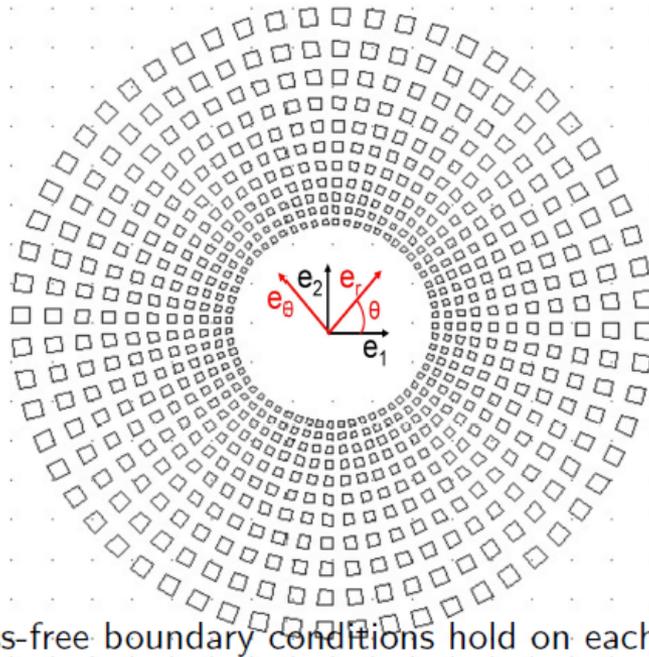




Vue 3D

**FIGURE:** Thin elastic plate with stress-free inclusions

Cross-section of the cloak and associated coordinate axes



Vue de dessus

**FIGURE:** Stress-free boundary conditions hold on each inclusion

# Vérification numérique (Institut Fresnel)



Carte de la phase de l'onde de Lamb

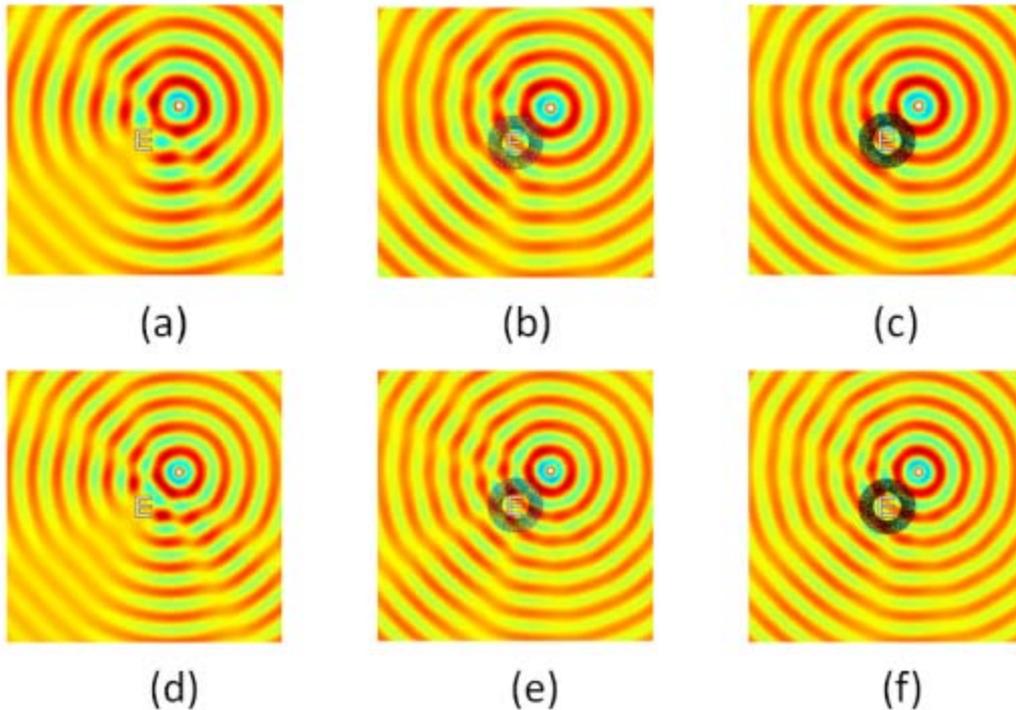
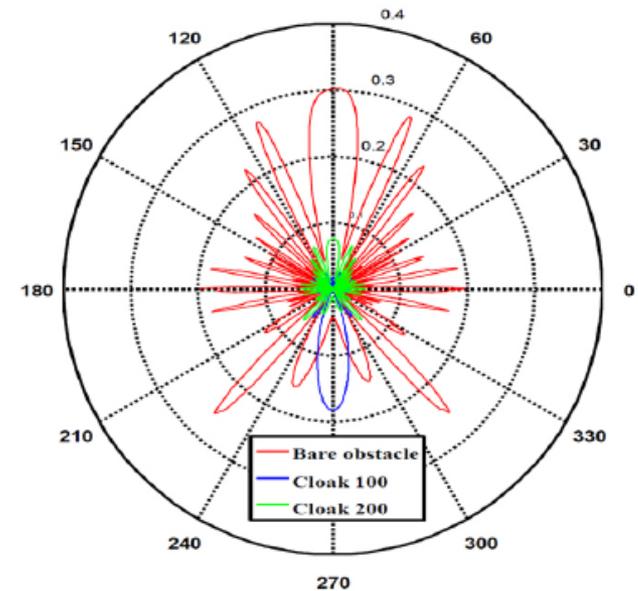


Diagramme de diffraction



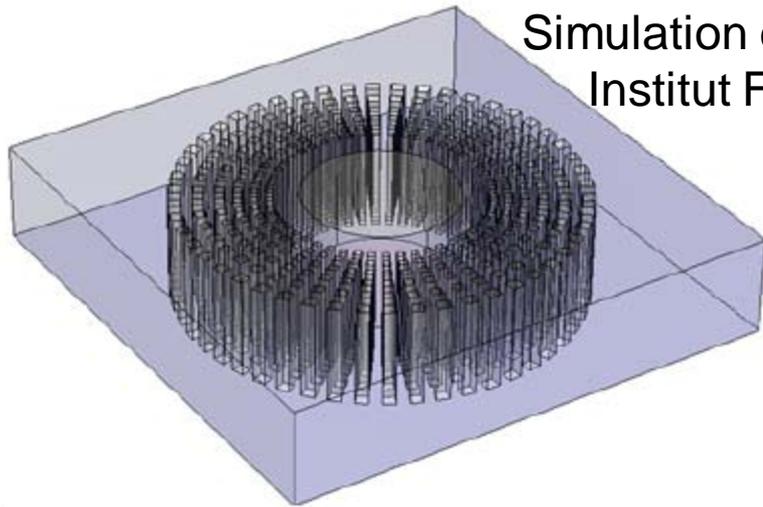
Cloaking à 200 et à 400 Hertz pour:

Plaque de plexiglas de 1mm d'épaisseur de dimension transverse infinie (PMLs)

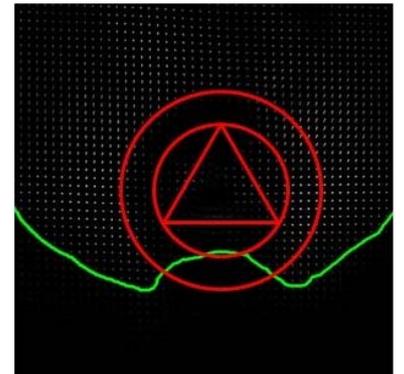
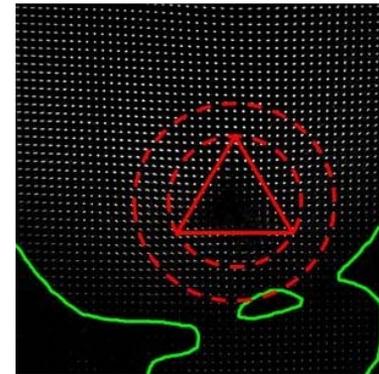
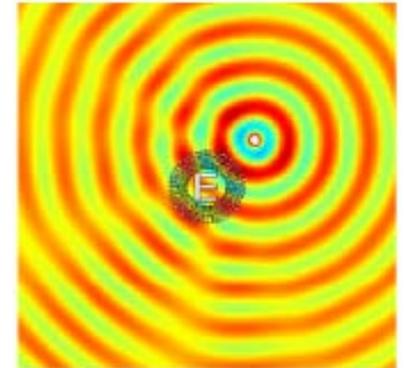
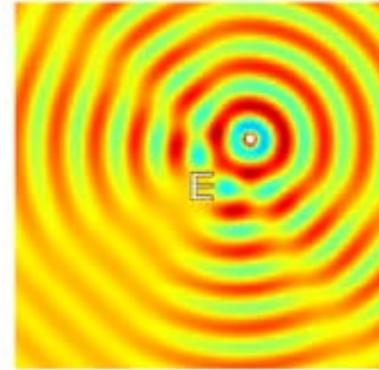
Cape de 10 cm de diamètre (avec ~100 puis ~200 perforations)

# Vérification expérimentale (IRPHE)

## Cape de diamètre 10cm, épaisseur 1mm



Simulation et design:  
Institut Fresnel



Validation expérimentale pour  
une centaine de perforations dans du plexiglas  
**Nicolas Vandenberghe** (IRPHE/AMU)  
**Alix Vaillant** (Institut Fresnel/IRPHE)

Cloaking en temporel et en  
champ proche à 400 Hertz

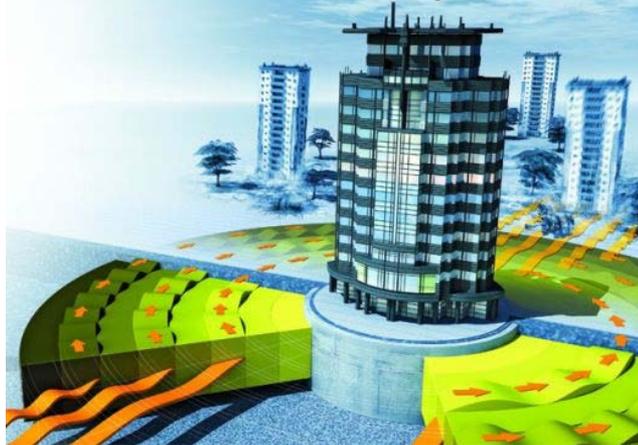
# Partie 3: vers le contrôle des ondes sismiques



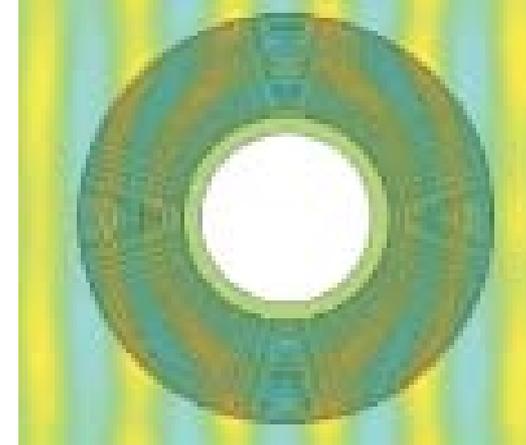
# Grande échelle (~100m): Capes parasismiques?



Popular Science Magazine 2009

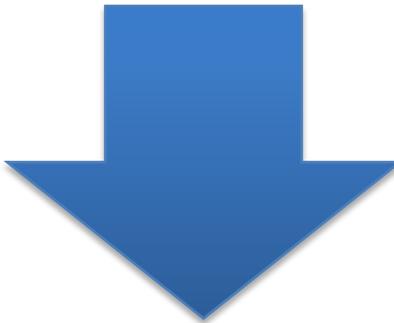


Institut Fresnel (PRL 2009)



Ebauche  
théorique (2009)

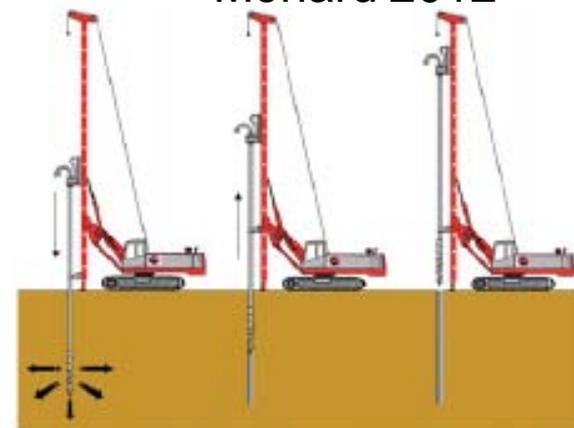
Enoch, Guenneau



La Recherche 2012



Ménard 2012



Design à tester  
sur site (2012)

Brûlé, Javelaud

# Partenaire industriel pour les métamatériaux sismiques: Ménard

Dakar (Sénégal) - 2009



# Conclusions et perspectives

Synthèse des résultats présentés:

Les métamatériaux acoustiques permettent d'envisager un contrôle accru:

- des ondes de pression dans un fluide
- des vagues à la surface d'un liquide,
- des ondes de flexion dans les plaques (Lamb)
- des ondes de surface pour les séismes (Rayleigh)

Perspectives:

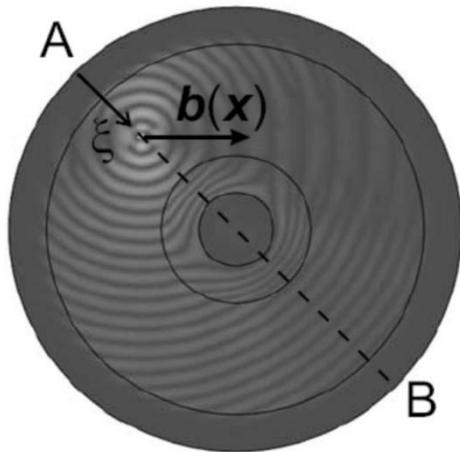
Peut-on envisager de contrôler d'autres types d'ondes?  
Qu'en est-il des phénomènes de diffusion?

# Vers des capes d'invisibilité pour ondes de pression et cisaillement couplées dans les solides (matériaux de type Cossérat)

Cas harmonique

Guenneau, Brun, Diatta, Movchan  
Institut Fresnel/Liv. Univ. (2009)

$$\nabla \cdot \mathbb{C}' : \nabla \mathbf{u} + \rho' \omega^2 \mathbf{u} = 0$$



*Densité hétérogène scalaire:*

$$\rho' = \frac{r-r_0}{r} \left( \frac{r_1}{r_1-r_0} \right)^2 \rho.$$

*Tenseur d'élasticité sans les symétries mineures:*

$$\mathbb{C}'_{rrrr} = \frac{r-r_0}{r} (\lambda + 2\mu), \quad \mathbb{C}'_{\theta\theta\theta\theta} = \frac{r}{r-r_0} (\lambda + 2\mu),$$

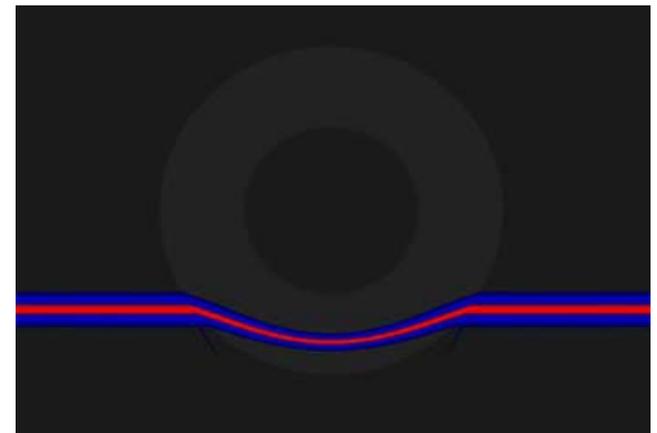
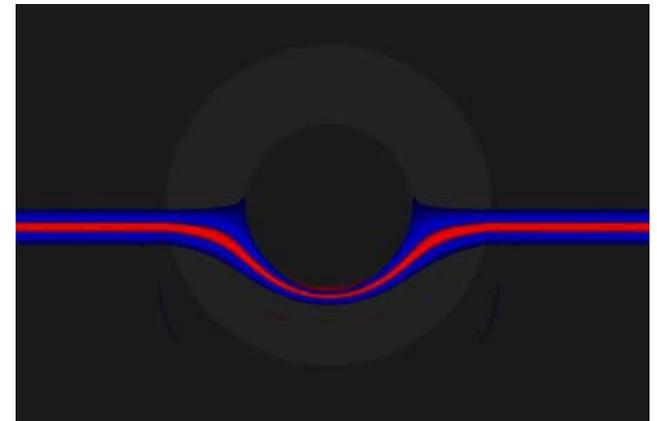
$$\mathbb{C}'_{rr\theta\theta} = \mathbb{C}'_{\theta\theta rr} = \lambda, \quad \mathbb{C}'_{r\theta\theta r} = \mathbb{C}'_{\theta rr\theta} = \mu,$$

$$\mathbb{C}'_{r\theta r\theta} = \frac{r-r_0}{r} \mu, \quad \mathbb{C}'_{\theta r\theta r} = \frac{r}{r-r_0} \mu,$$

Cas temporel

Komatitsch, Cristini, Xie (LMA/CNRS)  
Guenneau (Institut Fresnel)

$$\rho \partial_t^2 \mathbf{u} = \nabla \cdot \boldsymbol{\sigma} + \mathbf{f}$$



# Merci pour votre attention!



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