A Reliability Assessment Framework for Systems With Degradation Dependency by Combining Binary Decision Diagrams and Monte Carlo Simulation

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***Abstract*—Components are often subject to multiple competing degradation processes. This paper presents a reliability assessment framework for multi-component systems whose component degradation processes are modeled by multi-state and physics-based models with limited statistical degradation/failure data. The piecewise-deterministic Markov process modeling approach is employed to treat dependencies between the degradation processes within one component or/and among components. A computational method combining binary decision diagrams (BDDs) and Monte Carlo simulation (MCS) is developed to solve the model. A BDD is used to encode the fault tree of the system and obtain all the paths leading to system failure or operation. MCS is used to generate random realizations of the model and compute the system reliability. A case study is presented, with reference to one branch of the residual heat removal system of a nuclear power plant.**

***Index Terms*—System reliability analysis, degradation dependency, piecewise-deterministic Markov process, binary decision diagrams, Monte Carlo simulation.**

ACRONYMS

PBMs	Physics-based models
MSMs	Multi-state models
FTA	Fault tree analysis
CCFs	Common cause failures
BDDs	Binary decision diagrams
MCS	Monte Carlo simulation
RHRS	Residual heat removal system
WDFLM	Weighting depth-first left-most
DFLM	Depth-first left-most
ite	if-then-else

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Y.H.Lin and Y.F.Li are with the Chair System Science and the Energy Challenge, Fondation Electricité de France (EDF), CentraleSupélec, Université Paris-Saclay, Grande Voie des Vignes, 92290 Chatenay-Malabry, France (e-mail: yanhui.lin@centralesupelec.fr; yanfu.li@centralesupelec.fr)

E. Zio is with the Chair System Science and the Energy Challenge, Fondation Electricité de France (EDF), CentraleSupélec, Université Paris-Saclay, Grande Voie des Vignes, 92290 Chatenay-Malabry, France, and also with the Politecnico di Milano, 20133 Milano, Italy (e-mail: enrico.zio@centralesupelec.fr; enrico.zio@polimi.it)

NOTATIONS

C	Number of components in the system
$\boldsymbol{L}$	Group of degradation processes modeled by PBMs
$\boldsymbol{K}$	Group of degradation processes modeled by MSMs
$\boldsymbol{D}_{O_c}$	Degradation state of component O_c
$\overrightarrow{X_{L_m}}(t)$	Time-dependent continuous variables of degradation process L_m
$\overrightarrow{X_{L_m}^{\boldsymbol{D}}}(t)$	Non-decreasing degradation variables vector
$\overrightarrow{X_{L_m}^{\boldsymbol{P}}}(t)$	Physical variables vector
$\boldsymbol{\mathcal{F}}_{L_m}$	Set of failure states of degradation process L_m
$Y_{K_n}(t)$	State variable of degradation process K_n
$\boldsymbol{S}_{K_n}$	Finite state set of degradation process K_n
$\boldsymbol{\mathcal{F}}_{K_n}$	Set of failure states of degradation process K_n
$\vec{Z}(t)$	Degradation state of the system
$\boldsymbol{\theta}_{K_n}$	Environmental and operational factors in K_n
$\lambda_i(j\|\boldsymbol{\theta}_{K_n})$	Transition rate from state i to j
$\boldsymbol{\theta}_{L_m}$	Environmental and operational factors in L_m
$\overrightarrow{f_{L_m}}(\cdot,\cdot\|\boldsymbol{\theta}_{L_m})$	Physics equations of degradation process L_m
$\overrightarrow{Z_{p,q}}(t)$	Stochastic process of one group of interdependent degradation processes
$N(\cdot,\cdot,\cdot\|\boldsymbol{\theta}_{K_n})$	Semi-Markov kernel

I. Introduction

Most components undergo degradation processes before failure. A number of degradation models have been proposed in the field of reliability engineering based on the available information/data, which can be mainly classified into the following groups: statistical distributions (e.g. Bernstein distribution [1]), stochastic processes (e.g. Gamma process [2]), multi-state models (MSMs) (e.g. semi-Markov model [3]) and physics-based models (PBMs) (e.g. probabilistic superposition model [4]). Among the existing degradation models, PBMs [5-7] and MSMs [8-10] can be used to describe the evolution of degradation in structures, systems and components, for which statistical degradation/failure data are insufficient, e.g. the highly reliable devices in the nuclear and aerospace industries. A PBM gives an integrated mechanistic description of the component life consistent with the underlying real degradation mechanisms (e.g. wear, corrosion, fatigue, etc.) by using physics knowledge and equations [4], whereas a MSM describes the degradation process in a discrete way, supported by material science knowledge, degradation and/or failure data from historical field collection or degradation tests [11, 12].

In reality, components are often subject to multiple competing degradation processes. The dependencies among these processes within one component (e.g. the wear of rubbing surfaces influenced by the environmental stress shock within a micro-engine [13]), or/and among different components (e.g. the degradation of the pre-filtrations stations leading to a lower performance level of the sand filter in a water treatment plant [14]) need to be considered. Components can be dependent due to functional dependence, where the failure of a trigger component causes other components to become inaccessible or unusable [15, 16]. Competing failure propagation and failure isolation effects have been studied in [17, 18], where a failure not only causes outage to the component from which the failure originates, but also propagates through all other system components causing the entire system failure and failure isolation occurs when the failure of one component causes other components within the same system to become isolated from the system.

Recently, the authors have employed the piecewise-deterministic Markov process (PDMP) modeling framework to integrate PBMs and MSMs for treating the dependencies among degradation processes [19] for a system with a small number of components, where the whole system is modeled by one PDMP. For systems of larger size, the high dimension of its PDMP can lead to very heavy computational burdens, because solving the PDMP of a small system is already time consuming due to the combinatorial nature of MSMs and the need to simulate the trajectory between any two system states [19]. In addition, the dependencies may only exist within certain groups of components and leave different groups being independent [20], and the causes of systems failure are not easy to be identified.

Fault tree analysis (FTA) [21] is typically used to identify the combinations of events leading to system failure and compute its probability by using minimal cut sets found from the fault tree structure. For real systems, this can be computationally intensive, when the tree structure is large and, especially, if it contains repeated basic events [22]. In addition, all basic events are usually assumed statistically independent.

Common cause failures (CCFs) of components have been considered in [23-25]: implicit and explicit methods have been developed to evaluate the system reliability. In binary-state systems, components failures with dependent propagation effects have been studied in [26], within a dynamic FTA framework. The statistical dependence of component states across different phases of phased-mission systems has been treated by using multiple-valued decision diagrams to encode fault trees in [27, 28].

On the contrary, the dependencies of the degradation processes leading to failure of different components need to be considered which render certain basic events under different gates being dependent. To the knowledge of the authors, there is no published research work to tackle this problem, of practical reference [29].

To take into account such dependencies at a relatively low computational cost for systems of larger size, a system reliability assessment method is proposed combining binary decision diagrams (BDDs) [30] and Monte Carlo simulation (MCS) [31]. Instead of modeling the degradation of the whole system by one PDMP as in [19], the proposed method can identify the groups of components being dependent and decompose the original PDMP into a group of smaller ones which are independent from each other and easier to be solved. Besides, the states of these PDMPs leading to the systems failure can be easily obtained. Firstly, a fault tree is transformed to a BDD from which all paths leading to the system failure or operation can be efficiently obtained. BDDs [30] are directed acyclic graphs, encoding Shannon’s decomposition of a formula, and have been implemented in many domains; they possess the feature of sharing equivalent subgraphs and hence can reduce the computational time and memory requirements [32]. An algorithm based on BDD has been developed for reliability analysis of phased-mission systems with multimode failures in [33] to improve the efficiency and reduce the computational complexity. BDD has also been employed for network reliability and sensitivity analysis in [34]. Secondly, MCS is used to estimate the probability of each path to compute the system reliability taking into account the dependencies between basic events, since analytically solving the PDMPs is difficult, if not impossible, due to the large size and complex behavior of the system [35].

The rest of this paper is organized as follows. Section 2 provides the assumptions and model descriptions. The proposed reliability assessment method is presented in Section 3. Section 4 presents one case study on one branch of a residual heat removal system (RHRS) of a nuclear power plant. Section 5 concludes the work.

II. Assumption and Model Description

A. General Assumptions

We consider a multi-component system, made of C components denoted by $\boldsymbol{O} = \{O_1, O_2, \dots, O_C\}$.

The following assumptions are made:

- The fault tree of the system is available and contains Q basic events denoted by $\boldsymbol{e} = \{e_1, e_2, \dots, e_Q\}$ which include the failures of components and other events such as erroneous operation caused by human errors. The component-failure type of events are determined by their underlying degradation processes.
- Each component may be affected by multiple degradation processes, possibly dependent. The degradation processes can be separated into two groups: (1) $\boldsymbol{L} = \{L_1, L_2, \dots, L_M\}$ modeled by M PBMs; (2) $\boldsymbol{K} = \{K_1, K_2, \dots, K_N\}$ modeled by N MSMs, where $L_m, m = 1, 2, \dots, M$ and $K_n, n = 1, 2, \dots, N$ are the indexes of the degradation processes. The degradation state of a component $O_c \in \boldsymbol{O}, c = 1, 2, \dots, C$, is determined by its degradation processes $\boldsymbol{D}_{O_c} \subseteq \boldsymbol{L} \cup \boldsymbol{K}$ and the component fails when its degradation processes enter its failure state space (see the two bullets below for its definition).
- A degradation process $L_m \in \boldsymbol{L}$ in the first group is described by d_{L_m} time-dependent continuous variables $\overrightarrow{X_{L_m}}(t) = \left(\overrightarrow{X_{L_m}^{\boldsymbol{D}}}(t), \overrightarrow{X_{L_m}^{\boldsymbol{P}}}(t)\right) \in \mathbb{R}^{d_{L_m}}$ in terms of: (1) the non-decreasing degradation variables vector $\overrightarrow{\mathrm{X}_{L_m}^{\boldsymbol{D}}}(\mathrm{t})$ (e.g. crack length) representing the component degradation condition; (2) the physical variables $\overrightarrow{X_{L_m}^{\boldsymbol{P}}}(t)$ (e.g. velocity) influencing $\overrightarrow{\mathrm{X}_{L_m}^{\boldsymbol{D}}}(\mathrm{t})$ and vice versa.d_{L_m} is the number of non-decreasing degradation variables and physical variables for a degradation process L_m. Their evolution is characterized by a system of first-order differential equations $\overrightarrow{\dot{X}_{L_m}}(t) = \overrightarrow{f_{L_m}}\left(\overrightarrow{X_{L_m}}(t), t \middle| \boldsymbol{\theta}_{L_m}\right)$, i.e. physics equations, where $\boldsymbol{\theta}_{L_m}$ represents the environmental factors to L_m (e.g. temperature and pressure) and the parameters used in $\overrightarrow{f_{L_m}}$. The evolution of physical variables can be characterized by physics equations. The environmental factors are the parameters of the physics equations and their evolution is notcharacterized by physics equations. If any environmental or operational factoris modeled by physics equations and influencing the degradation variables, then, it is considered as one physical variable.L_m fails when one $x_{L_m}^i(t) \in \overrightarrow{X_{L_m}^{\boldsymbol{D}}}(t)$ reaches or exceeds its corresponding failure threshold denoted by $x_{L_m}^{i\ *}$. The failure state set of L_m is denoted by $\boldsymbol{\mathcal{F}}_{L_m}$. An example of L_1 is shown in Fig. 1.
- A degradation process $K_n \in \boldsymbol{K}$ in the second group is described by the state variable $Y_{K_n}(t)$, which takes values from a finite state set $\boldsymbol{S}_{K_n} = \{0_{K_n}, 1_{K_n}, \dots, d_{K_n}\}$, where '$d_{K_n}$' is the perfect functioning state and '0_{K_n}' is the complete failure state. All intermediate states are functioning or partially functioning. The transition rates $\lambda_i(j \mid \boldsymbol{\theta}_{K_n}), \forall\, i, j \in \boldsymbol{S}_{K_n}, i > j$ characterize the degradation transition probabilities from state i to state j, where $\boldsymbol{\theta}_{K_n}$ represents the environmental factors to K_n and the related coefficients of λ_{K_n}. The failure state set of K_n is denoted by $\boldsymbol{\mathcal{F}}_{K_n} = \{0_{K_n}\}$. An example of K_1 is shown in Fig. 2.

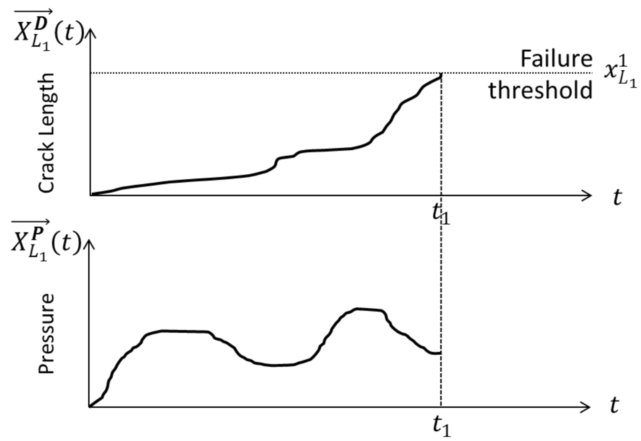

Fig. 1. An illustration of L_1.

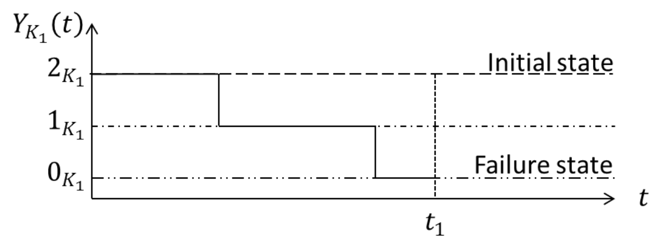

Fig. 2. An illustration of K_1.

Dependencies between degradation processes may exist both within and across groups $\boldsymbol{L}$ and $\boldsymbol{K}$. The degradation levels of the components in the first group may influence the transition times and transition directions of the degradation processes of the second group and the degradation states of the second group may influence the evolution trajectories of the continuous variables in the first group [19]. PDMPs are employed to model this dependency, the detailed formulations are shown in eqs. (1) and (2).

B. PDMPs for Dependent Degradation Processes

Let us consider one group of interdependent degradation processes $\boldsymbol{L}_p = \{L_{p_1}, \dots, L_{p_n}\}$ and $\boldsymbol{K}_q = \{K_{q_1}, \dots, K_{q_m}\}$, which have no dependencies with the other degradation processes. Their degradation states are represented by

$$\overrightarrow{Z_{p,q}}(t) = \begin{pmatrix} \begin{pmatrix} \overrightarrow{X_{L_{p_1}}}(t) \\ \vdots \\ \overrightarrow{X_{L_{p_n}}}(t)\end{pmatrix} = \overrightarrow{X_p}(t) \\ \begin{pmatrix} Y_{q_1}(t)\\ \vdots \\ Y_{q_m}(t)\end{pmatrix} = \overrightarrow{Y_q}(t)\end{pmatrix} \in \boldsymbol{E}_{p,q} = \mathbb{R}^{d_{\boldsymbol{L}_p}} \times \boldsymbol{S}_{\boldsymbol{K}_q}, \forall t \geq 0 \quad (1)$$

where $\boldsymbol{E}_{p,q}$ is the space combining $\mathbb{R}^{d_{\boldsymbol{L}_p}}$ ($d_{\boldsymbol{L}_p} = \sum_{k=1}^{n} d_{L_{p_k}}$) and $\boldsymbol{S}_{\boldsymbol{K}_q} = \{0, 1, \dots, d_{\boldsymbol{K}_q}\}$ denotes the state set of process $\overrightarrow{Y_q}(t)$.

The evolution of the vector of degradation states $\overrightarrow{Z_{p,q}}(t)$ involves (1) the stochastic transition process of $\overrightarrow{Y_q}(t)$ and (2) the deterministic progression of $\overrightarrow{X_p}(t)$, between successive transitions of $\overrightarrow{Y_q}(t)$, given $\overrightarrow{Y_q}(t)$. The first process is governed by the transition rates of $\overrightarrow{Y_q}(t)$, which depend on the degradation levels of the components in the first group, as follows:

$$\lim_{\Delta t \to 0} P\left(\overrightarrow{Y_q}(t+\Delta t) = \vec{j} \middle| \overrightarrow{Z_{p,q}}(t) = (\overrightarrow{X_p}(t), \overrightarrow{Y_q}(t) = \vec{\imath})^T, \boldsymbol{\theta}_{\boldsymbol{K}_q}\right) = \lambda_{\vec{\imath}}^{q}\left(\vec{j} | \overrightarrow{X_p}(t), \boldsymbol{\theta}_{\boldsymbol{K}_q}\right) \Delta t, \forall \vec{\imath}, \vec{j} \in \boldsymbol{S}_{\boldsymbol{K}_q}, \vec{\imath} \neq \vec{j} \quad (2)$$

where the parameter vector $\boldsymbol{\theta}_{\boldsymbol{K}_q}$ represents environmental and operational factors influencing the degradation processes in $\boldsymbol{K}_q$. The second evolution process is described by the deterministic physics equations which depend on the degradation states of the second group as follows:

$$\dot{\overrightarrow{X_p}}(t) = \begin{pmatrix} \dot{\overrightarrow{X_{L_{p_1}}}}(t) \\ \vdots \\ \dot{\overrightarrow{X_{L_{p_n}}}}(t)\end{pmatrix} = \begin{pmatrix} \overrightarrow{f_{L_{p_1}}}\left(\overrightarrow{Z_{p,q}}(t), t \middle| \boldsymbol{\theta}_{L_{p_1}}\right) \\ \vdots \\ \overrightarrow{f_{L_{p_n}}}\left(\overrightarrow{Z_{p,q}}(t), t \middle| \boldsymbol{\theta}_{L_{p_n}}\right)\end{pmatrix} = \overrightarrow{f_{\boldsymbol{L}_p}}\left(\overrightarrow{Z_{p,q}}(t), t \middle| \boldsymbol{\theta}_{\boldsymbol{L}_p} = \left(\boldsymbol{\theta}_{L_{p_1}}, \dots, \boldsymbol{\theta}_{L_{p_n}}\right)\right) \quad (3)$$

where the parameter vector $\boldsymbol{\theta}_{L_{p_k}}, k = 1, 2, \dots, n$ represents environmental and operational factors influencing the degradation processes in L_{p_k}. It should be noted that the evolution of one degradation process in $\overrightarrow{Z_{p,q}}(t)$ depends on the states of all the degradation processes in $\overrightarrow{Z_{p,q}}(t)$.

III. METHODOLOGY

In this section, a computational method combining BDDs and MCS is proposed.

A. BDDs

A BDD is a directed acyclic graph encoding Shannon's decomposition of a formula. A BDD has two terminal vertices labeled 1 and 0 to indicate the failure and operation of the system, respectively. Each non-terminal vertex is labeled with a variable and has two outgoing edges: 1-edge and 0-edge which indicate the occurrence and non-occurrence of the corresponding basic event, respectively.

A BDD is employed to encode the fault tree of the system according to the given ordering of the indicator variable $X_i$whichdenotes the occurrence or non-occurrence of the basic event i ($X_i = 1$ indicating the occurrence of the basic event i and $X_i = 0$ indicating the opposite). The size of the BDD largely depends on the given ordering and the problem of finding the global optimal ordering is an intractable task [36, 37]. Several ordering heuristics have been developed, whose performances may vary on different problems. In this work, we employ the weighting depth-first left-most (WDFLM) ordering technique proposed in [38], which leads to satisfactory results according to the tests in [39, 40]. WDFLM first assigns weight 1 to each basic event. Then, it traverses the fault tree bottom-up to calculate the weight of each gate by adding the weights of all its inputs, i.e. gates and basic events. Fig. 3 shows an example of a fault tree where the weights of the gates are obtained through WDFLM.

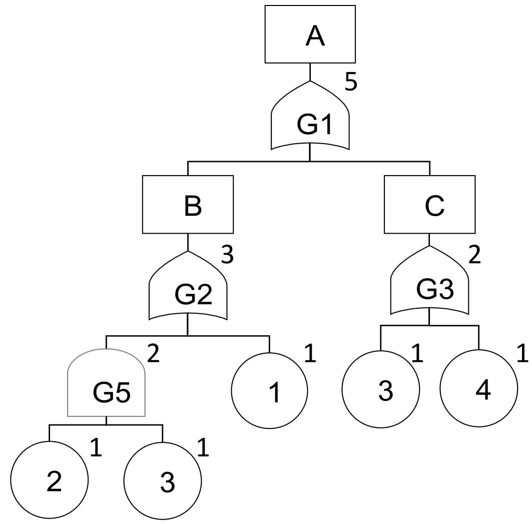

Fig. 3. An illustration of fault tree labeled with weights.

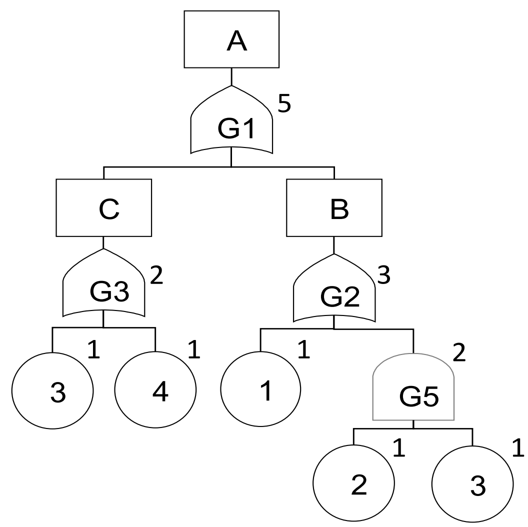

Fig. 4. An illustration of fault tree with rearranged inputs of gates.

Finally, the depth-first left-most (DFLM) ordering technique [41] is applied to the fault tree to get the variable ordering. In this technique, the basic events are placed in the

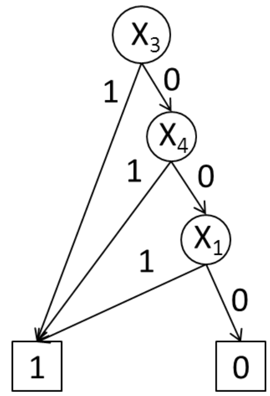

Fig. 5. BDD for fault tree in Fig. 3.

ordered list as soon as they are encountered during the DFLM traversal of the fault tree. Let $<$ be a total ordering of variables, for the fault tree in Fig. 3,it is $X_3 < X_4 < X_1 < X_2$.

Based on the variable ordering, the related BDD can be constructed using the bottom-up procedure. Firstly, all basic events $i, i \in \boldsymbol{e}$ are associated with the if-then-else (ite) structure [42] $ite(X_i, 1, 0)$, where $ite(X_i, f_1, f_2) = (X_i \wedge f_1) \vee (\neg X_i \wedge f_2)$which means if the basic event i occurs then consider function f_1 else consider function f_2. Then, work from the bottom to the top of the fault tree and obtain the ite structure for each gate by using the following principle: let us consider two variables $X_a < X_b$ and four functions f_1, f_2, f_3, f_4, let <> be any logic operation AND or OR, then:

$$ite(X_a, f_1, f_2) <> ite(X_a, f_3, f_4) = ite(X_a, f_1 <> f_3, f_2 <> f_4) \quad (4)$$

and

$$ite(X_a, f_1, f_2) <> ite(X_b, f_3, f_4) = ite\big(X_a, f_1 <> ite(X_b, f_3, f_4), f_2 <> ite(X_b, f_3, f_4)\big) \quad (5)$$

The ite structure of the top event of the fault tree in Fig. 3 can be obtained as $ite(X_3, 1, ite(X_4, 1, ite(X_1, 1, 0)))$. The associated BDD shown in Fig. 5 can be constructed by breaking down each ite structure into its left and right branches, and eliminating the vertexes that are not useful (a vertex is not useful when its two outgoing edges point to the same vertex or it is equivalent to another vertex) [43].

Finally, all the paths leading to system failure can be obtained as $(1) X_3 = 1, (2) X_3 = 0, X_4 = 1, (3) X_3 = 0, X_4 = 0, X_1 = 1$ and the path leading to system operation is $X_3 = 0, X_4 = 0, X_1 = 0$. The exact system reliability is equal to the sum of the probability of occurrence of each path leading to system operation or $1 -$ the sum of the probability of occurrence of each path leading to system failure.

B. MCS for PDMPs

To derive the probability of occurrence of one path, all the PDMPs containing the variables involved in that path need to be solved. Since the PDMPs are independent from each other, the product of the probabilities of PDMPs being in the states indicated by the path equals the probability of occurrence of that path. Analytically solving the PDMPs is a difficult task, whereas MCS is well suited.

We develop a MCS algorithm for solving the PDMPs. It consists of sampling the transition time and the arrival state for the MSMs and, then, calculating the behavior of the PBMs within the transition times using the physics equation.

Refer to one PDMP presented in Section 2.2. Let $\overrightarrow{Z^k_{p,q}} = \overrightarrow{Z_{p,q}}(T^k) = \begin{pmatrix} \overrightarrow{X_p}(T^k) \\ \overrightarrow{Y^k_q} \end{pmatrix} \in \boldsymbol{E}_{p,q}, k \in \mathbb{N}$, where $\overrightarrow{Y^k_q} \in S_{K_q}, k \in \mathbb{N}$ denotes the state of $\overrightarrow{Y_q}(t)$ after k transitions from the beginning (a transition occurs as long as any one of the elements in $\overrightarrow{Y_q}(t)$ changes its state) and T^k denotes the time of arrival at state $\overrightarrow{Y^k_q}$. Then, $\left\{\overrightarrow{Z^k_{p,q}}, T^k\right\}_{k \geq 0}$ is a Markov renewal process defined on the space$\boldsymbol{E}_{p,q} \times \mathbb{R}^+$ [44]. We can obtain that

$$P\left[\overrightarrow{Z^{n+1}_{p,q}} \in B, T^{n+1} \in [T^n, T^n + \Delta t] | \overrightarrow{Z^n_{p,q}} = \vec{\imath}, \boldsymbol{\theta}_{K_q}\right] = \iint_{B*[0,\Delta t]} N\left(\vec{\imath}, \overrightarrow{dz}, ds | \boldsymbol{\theta}_{K_q}\right), \quad \forall\, n \geq 0, \Delta t \geq 0, \vec{\imath} \in \boldsymbol{E}_{p,q}, B \in \varepsilon \quad (6)$$

where $\varepsilon$is a σ-algebra of $\boldsymbol{E}_{p,q}$ and $N\left(\vec{\imath}, \overrightarrow{dz}, ds | \boldsymbol{\theta}_{K_q}\right)$ is a semi-Markov kernel on $\boldsymbol{E}_{p,q}$, which verifies that $\iint_{\boldsymbol{E}_{p,q}*[0,\Delta t]} N\left(\vec{\imath}, \overrightarrow{dz}, ds | \boldsymbol{\theta}_{K_q}\right) \leq 1, \forall\, \Delta t \geq 0, \vec{\imath} \in \boldsymbol{E}_{p,q}$. It can be further developed as:

$$N\left(\vec{\imath}, \overrightarrow{dz}, ds | \boldsymbol{\theta}_{K_q}\right) = dF_{\vec{\imath}}\left(s | \boldsymbol{\theta}_{K_q}\right) \beta\left(\vec{\imath}, \overrightarrow{dz} | s, \boldsymbol{\theta}_{K_q}\right) \quad (7)$$

where

$$dF_{\vec{\imath}}\left(s | \boldsymbol{\theta}_{K_q}\right) \quad (8)$$

is the probability density function of $T^{n+1} - T^n$ given $\overrightarrow{Z^n_{p,q}} = \vec{\imath}$ and

$$\beta\left(\vec{\imath}, \overrightarrow{dz} | s, \boldsymbol{\theta}_{K_q}\right) \quad (9)$$

is the conditional probability of state $\overrightarrow{Z^{n+1}_{p,q}}$ given $T^{n+1} - T^n = s$.

The simulation procedure consists of sampling the transition time from (8) and the arrival state from (9) for $\overrightarrow{Y_q}(t)$, then, calculating$\overrightarrow{X_p}(t)$ within the transition times, by using the physics equation eq. (3)until the time of system evolution reaches a certain mission time T_{miss}.

To calculate the probability of occurrence of one path (let $\overrightarrow{\boldsymbol{Z}^*_{p,q}}$ indicate the state space, which contains all the states of $\overrightarrow{Z_{p,q}}(t)$ that are consistent with the state of the path), the procedure of the MCS is presented as follows:

SetN_{max} (the maximum number of replications) and $k = 0$ (index of replication)
Set$k' = 0$ (number of trials that end in the state indicated by the path)
While$k < N_{max}$

Initialize the system by setting $\overrightarrow{Z_{p,q}}(0) = \begin{pmatrix} \overrightarrow{X_p}(0) \\ \overrightarrow{Y_q} \end{pmatrix}$ (initial state), and the time $T = 0$ (initial system time)

Set $t' = 0$ (state holding time)
While $T \leq T_{miss}$
Sample a t' by using (8)
Sample an arrival state $\overrightarrow{Y_q'''}$ for stochastic process $\overrightarrow{Y_q}(t)$ from all the possible states by using (9)
Calculate $\overrightarrow{X_p}(s), \forall s \in [T, T + t']$ by using eq. (3)
Set $\overrightarrow{Z_{p,q}}(s) = \begin{pmatrix} \overrightarrow{X_p}(s) \\ \overrightarrow{Y_q'} \end{pmatrix}, \forall s \in [T, T + t'[$
Set $T = T + t'$, $\overrightarrow{Z_{p,q}}(T) = \begin{pmatrix} \overrightarrow{X_p}(T) \\ \overrightarrow{Y_q'''} \end{pmatrix}$ and $\overrightarrow{Y_q'} = \overrightarrow{Y_q'''}$
End While
If $\overrightarrow{Z_{p,q}}(T_{miss}) \in \overrightarrow{\boldsymbol{Z}^*_{p,q}}$
Set $k' = k' + 1$
End if
Set $k = k + 1$
End While □

The estimated probability of occurrence of one path at time T_{miss} can be obtained by

$$\hat{P}(T_{miss}) = 1 - k'/N_{max} \quad (10)$$

with the sample variance [45] as follows:

$$var_{\hat{P}(T_{miss})} = \hat{P}(T_{miss})(1 - \hat{P}(T_{miss}))/(N_{max} - 1) \quad (11)$$

C. MCS for PDMPs

The flowchart of the whole proposed computational method combining BDDs and MCS is shown in Fig. 6.

IV. Case Study

The illustrative case refers to one branch of the RHRS [46] of a nuclear power plant shown in Fig. 7. The fault tree is shown in Fig. 8. The definitions of the basic events are presented in Table I.

By knowledge and experience of the field experts, the degradation dependency is described as follows: the degradation of the pump can lead it to vibrate [47], which will, in turn, cause the vibration of the other neighboring components (e.g. the valve) and therefore aggravate the degradation process of the latter [48]. The dependency exists between basic events 1,2,3,4 and 6, as indicated in Fig. 6. The component degradation models provided by the expert colleagues of Electricité de France are presented below. Some degradation processes are modeled by PBMs if their degradation data is unavailable and, thus, the physics equations have to be used, whereas the others are modeled by MSMs supported by the degradation and/or failure data from historical field collection.

The circuit breaker, motor and pump contactor each have one degradation processmodeled by MSMs K_1, K_2 and K_3 respectively, as shown in Fig. 9.

The pump has two degradation processes modeled by MSMs K_4 and K_5, as shown in Fig. 10. K_4 relates to the failure on demand and K_5 relates to the external leakage which can cause the pump to vibrate when $Y_{K_5}(t)$ reaches the state 1_{K_5}.

Closure due to human error follows one MSM K_6, as shown

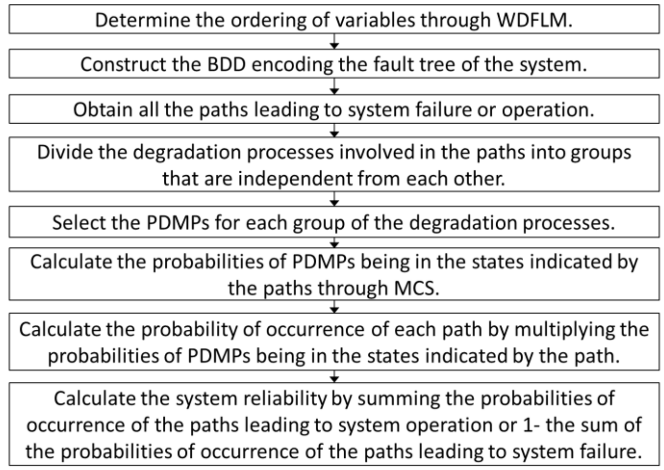

Fig. 6. The flowchart of the computational method.

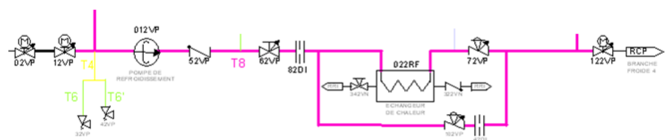

Fig. 7. The diagram of one branch of the RHRS.

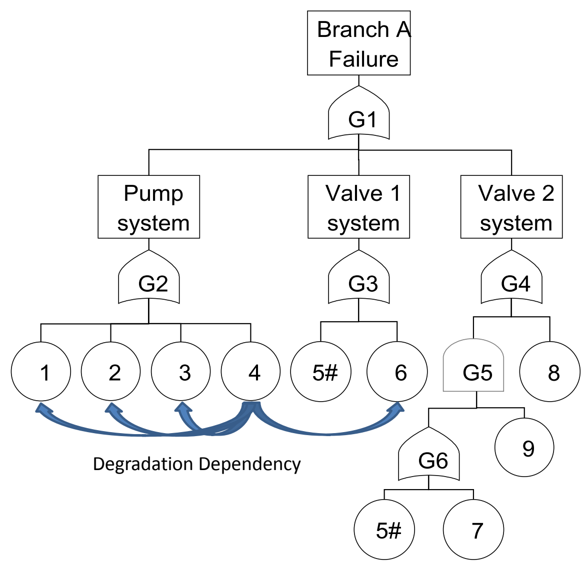

Fig. 8. The fault tree of one branch of the RHRS.

in Fig. 11.

The valve has one degradation process modeled by one PBM L_1 related to the crack propagation due to manufacturing defects. L_1 is based on a deterministic crack growth model, which follows Paris–Erdogan law [49]. For the phase of crack propagation, the threshold is defined as the number of cycles calculated as follows,

$$N_c = \frac{1/(\frac{m}{2}-1) * (1/a_0^{(\frac{m}{2}-1)} - 1/a_c^{(\frac{m}{2}-1)})}{C(f(R)_{Max}\, Y_{Max} \sqrt{\pi} \Delta\sigma_{Max})^m} \quad (12)$$

where the definition of the parameters can be found in [50].

TABLE I
DEFINITIONS OF THE BASIC EVENTS

Basic Event	Definition
1	Failure of the circuit breaker
2	Failure of the motor
3	Failure of the pump contactor
4	Failure of the pump
5#	Closure due to human error
6	Failure of the valve
7	Failure of the diaphragm
8	Failure of the pneumatic valve VP1
9	Failure of the pneumatic valve VP2

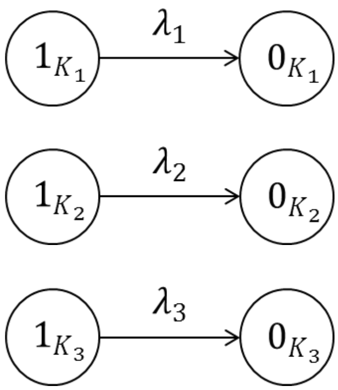

Fig.9. The representation of the degradation processes of the circuit breaker, motor and pump contactor.

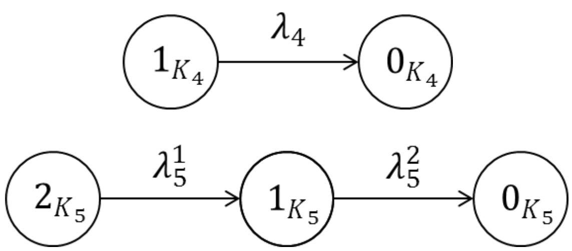

Fig.10. The representation of the degradation processes of the pump.

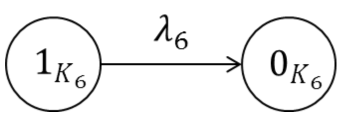

Fig.11. The process of closure due to human error.

The valve fails when the number of solicitation exceeds N_c. The equivalent number of solicitations executed per year is assumed to be constant and equal to d_c.

The diaphragm has one degradation process modeled by one PBM L_2 related to the cavitation erosion mechanism, which can cause the thickness loss. The threshold is defined as the thickness required to ensure pressure resistance, which is calculated as follows,

$$t_m = PD_0/2(S + yP) \quad (13)$$

where P is the estimated pressure for RHRS, D_0 is the outside diameter of the pipe, y is a coefficient and S is the allowable stress in the pipe. The diaphragm fails when the thickness loss exceeds t_m. The annual loss of thickness is assumed to be constant and equal to d_m.

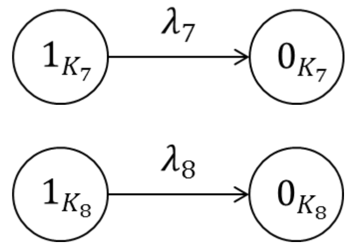

Fig.12. The representation of the degradation processes of the pneumatic valves.

TABLE II
PARAMETER VALUES

Parameter	Value
λ_1	6.65e-8 /h
λ_2	1.8e-6 /h
λ_3	4.4e-7 /h
λ_4	1.3e-5 /h
λ_5^1	4.7e-5 /h
λ_5^2	1.3e-5 /h
λ_6	1.5e-5 /h
λ_7	1.95e-8 /h
λ_8	1.95e-8 /h
m	4 S.U.
a_0	3.6 mm
a_c	9.3 mm
C	1.8e-12 S.U.
$f(R)_{Max}$	2 S.U.
Y_{Max}	1.18 S.U.
$\Delta\sigma_{Max}$	0 MPa
d_c	10 /yr
P	41 b
D_0	273 mm
S	101 Mpa
y	0.4 S.U.
d_m	7 mm /yr
λ_1'	9.31e-8 /h
λ_2'	2.52e-6 /h
λ_3'	6.16e-7 /h
λ_4'	1.82e-5 /h
d_c'	15 /yr

The pneumatic valves VP1and VP2 each have one degradation processmodeled by MSMs K_7 and K_8 respectively, as shown in Fig. 12.

K_5 has impacts on K_1, K_2, K_3, K_4 and L_1. When $Y_{K_5}(t)$ reaches the state 1_{K_5} the transition rates of K_1, K_2, K_3 and K_4 will increase toλ_1', λ_2', λ_3' and λ_4', respectively, and d_c in L_1 will change to d_c' . All the parameter values in the degradation models are presented in Table II. For confidentiality, we use artificially scaled values; they are set in a way to simulate the system under accelerated aging conditions.

Applying the WDFLM ordering heuristic [38], the variable ordering obtained is $X_{5\#} < X_6 < X_1 < X_2 < X_3 < X_4 < X_8 < X_9 < X_7$. The corresponding BDD is shown in Fig. 13. There are two paths leading to system operation: (1) $X_{5\#} = 0, X_6 = 0, X_1 = 0, X_2 = 0, X_3 = 0, X_4 = 0, X_8 = 0, X_9 = 0$ and (2) $X_{5\#} = 0, X_6 = 0, X_1 = 0, X_2 = 0, X_3 = 0, X_4 = 0, X_8 = 0, X_9 = 1, X_7 = 0$.

The degradation processes are divided into five groups: $\{K_6\}, \{L_2\}, \{K_7\}, \{K_8\}$ and $\{K_1, K_2, K_3, K_4, K_5, L_1\}$. Each of the

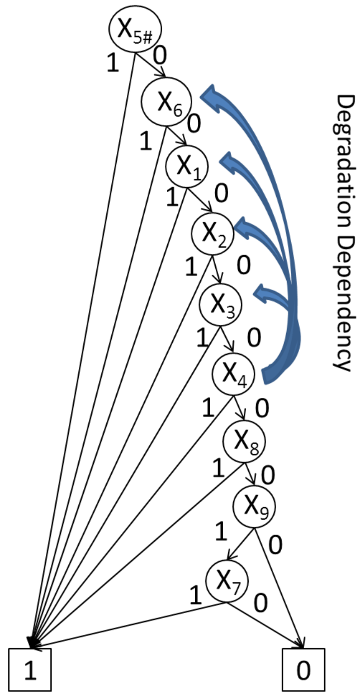

Fig. 13. The BDD corresponding to the fault tree shown in Fig. 8.

first four groups has only one degradation model. The PDMP related to the last group is presented as follows,

$$\overrightarrow{Z_{1,5}}(t) = \begin{pmatrix} N(t) \\ Y_{K_1}(t) \\ Y_{K_2}(t) \\ Y_{K_3}(t) \\ Y_{K_4}(t) \\ Y_{K_5}(t) \end{pmatrix} \in \boldsymbol{E}_{1,5} = \mathbb{R} \times \boldsymbol{S}_{\boldsymbol{K}_1} \times \ldots \times \boldsymbol{S}_{\boldsymbol{K}_5}, \forall t \geq 0 \tag{14}$$

where $N(t)$ denotes the number of solicitations applied till t, $\dot{N}(t) = \begin{cases} d_c, if Y_{K_5}(t) = 2_{K_5} \\ d_c', if Y_{K_5}(t) = 1_{K_5} \end{cases}$ and $Y_{K_q}(t), q = 1,2,\ldots,5$ are characterized by the related transition rates.

MCS over a time horizon of 8 years has been run 10^6 times to solve the PDMPs and, then, estimate the probability of occurrence of each path. The numericalexperimentsare carried out in MATLAB on a PC with an Intel Core 2 Duo CPU at 3.06 GHz and a RAM of 3.07 GB. The estimated system reliability with and without dependency throughout the time horizon, under accelerated conditions, is shown in Fig. 14. The average computation time is 34.3 s. We can see from the Figure that neglecting dependency can lead to overestimation of the system reliability. The system reliability with dependency has experienced one rapid decrease after around 6.2 year (point A), which is due tothe valve failure in some simulation trials caused by the vibration of the pump. This sharp decrease in system reliability relates to the sharp increase in the system failure time density function, as shown in Fig. 15.

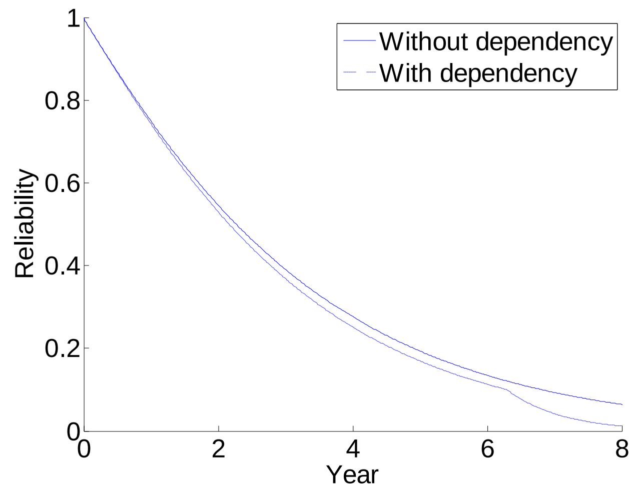

Fig. 14. The estimated system reliability with/without dependency.

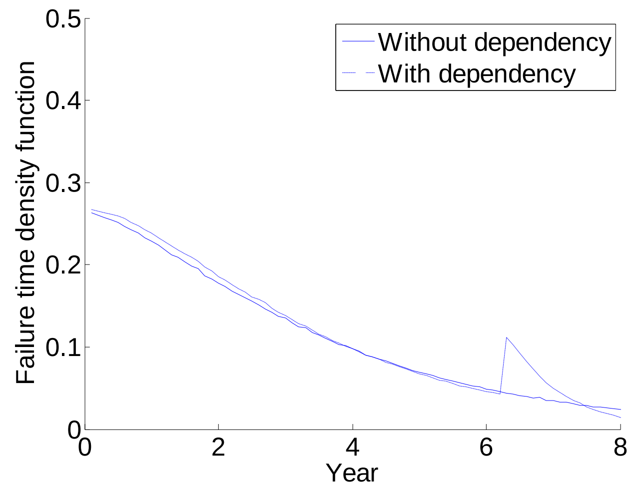

Fig. 15. The system failure time density function with/without dependency.

V. Conclusion

In this paper, we have proposed a framework for the reliability assessment of systems whose components have dependent competing degradation processes. The modeling framework rests on MSMs and PBMs, and the PDMP modeling approach is employed to treat dependencies between the degradation processes within one component or/and among components. The numerical solution involves the translation of the system fault tree into a BDD, and the estimation of the probabilities of the paths of events occurrences by MCS. The case study demonstrates the relevance of degradation process dependencies for the system reliability.

It is interesting to include failure isolation as future research in our proposed model. Failure detection and isolation can be used to mitigate degradation dependency by performing corresponding maintenance tasks or failure isolation actions.

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Yan-Hui Linhas been a doctoral student at Chair System Science and the Energy Challenge, Fondation Electricité de France (EDF), CentraleSupélec, Université Paris-Saclay, France since August 2012. He received the B.Sc. degree in Applied Mathematics from Beihang University, China, the M.Sc. degree in Applied Mathematics from Ecole Centrale Paris, France and the M.Sc. degree in Aircraft Design from Beihang University, China, in 2010, 2012 and 2013, respectively. His research interests are in reliability, degradation and maintenance modeling, Monte Carlo simulation, and optimization under uncertainty.

Yan-Fu Liis an Assistant Professor at Chair System Science and the Energy Challenge, Fondation Electricité de France (EDF), CentraleSupélec, Université Paris-Saclay, France. Dr. Li completed his PhD research in 2009 at National University of Singapore, and went to the University of Tennessee as a research associate. His current research interests include reliability modeling, uncertainty analysis, evolutionary computing, and Monte Carlo simulation. He is the author of more than 30 publications, all in refereed international journals, conferences, and books.

Enrico Zio received the Ph.D. degree in nuclear engineering from Politecnico di Milano and MIT in 1995 and 1998, respectively. He is currently Director of the Chair System Science and the Energy Challenge, Fondation Electricité de France (EDF), CentraleSupélec, Université Paris-Saclay, France, and full professor at Politecnico di Milano, Italy. His research focuses on the characterization and modeling of the failure/repair/maintenance behavior of components, complex systems and their reliability, maintainability, prognostics, safety, vulnerability and security, Monte Carlo simulation methods, soft computing techniques, and optimization heuristics.