

## Additional file 3. Precisions formulae when the candidate is related to reference individuals

### First approximation

Matrices **A** and **T** needed to apply formula {I} are the following:

The pedigree relationship matrix is  $\mathbf{A} = \begin{pmatrix} 1 & 1/2 & 0 & \dots & 0 \\ 1/2 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix}$  and

$$\mathbf{T} = \begin{pmatrix} a & b & 0 & \dots & 0 \\ b & a & 0 & \dots & 0 \\ 0 & 0 & 1/(1+\gamma) & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1/(1+\gamma) \end{pmatrix} \text{ with } \gamma = \frac{\sigma_q^2}{\sigma_e^2}, a = \frac{1+\gamma}{(1+\gamma)^2 - 1/4\gamma^2} \text{ and } b = -\frac{\gamma/2}{(1+\gamma)^2 - 1/4\gamma^2}$$

$$\{\mathbf{TATAT}\}_{cc} = a[(a + b/2)^2 + (b + a/2)^2] + b[2(a + b/2)(b + a/2)]$$

$$\{\mathbf{TE[G*TG*]T}\}_{cc} = a^2 h_{cc} + 2abh_{c1} + b^2 h_{11}, \text{ with } h = \mathbf{E}[\mathbf{G*TG*}]$$

$$\{\mathbf{E[G*TG*]}\}_{ij} = \sum_l \sum_k t_{kl} \left( \frac{1}{2} \tau \alpha_{ijkl}^{1111} - \frac{1}{4} \tau_2 \gamma_{ijkl}^{1111} + 4a_{ik} a_{jl} [\tau^2 - \tau_2] \right)$$

$$h_{cc} = \sum_l \sum_k t_{kl} \left( \frac{1}{2} \tau \alpha_{ckl}^{211} - \frac{1}{4} \tau_2 \gamma_{ckl}^{211} + 4a_{ck} a_{cl} [\tau^2 - \tau_2] \right)$$

Considering the structure of **T** matrix in this case:

$$h_{cc} = t_{cc} \left( \frac{1}{2} \tau \alpha_c^4 - \frac{1}{4} \tau_2 \gamma_c^4 + 4a_{cc}^2 [\tau^2 - \tau_2] \right) + t_{11} \left( \frac{1}{2} \tau \alpha_{c1}^{22} - \frac{1}{4} \tau_2 \gamma_{c1}^{22} + 4a_{c1}^2 [\tau^2 - \tau_2] \right) + \sum_{k \neq c \& 1} t_{kk} \left( \frac{1}{2} \tau \alpha_{ck}^{22} - \frac{1}{4} \tau_2 \gamma_{ck}^{22} + 4a_{ck}^2 [\tau^2 - \tau_2] \right) + 2t_{c1} \left( \frac{1}{2} \tau \alpha_{c1}^{31} - \frac{1}{4} \tau_2 \gamma_{c1}^{31} + 4a_{cc} a_{c1} [\tau^2 - \tau_2] \right)$$

The needed parameters, following table 1, are given in Supplementary table 3.

### Supplementary table 3

Parameters corresponding to the situation of a candidate son of a reference individuals ( $k \neq c \& 1$ )

$\alpha_c^4$	$\gamma_c^4$	$\alpha_{c1}^{22}$	$\gamma_{c1}^{22}$	$\alpha_{c1}^{31}$	$\gamma_{c1}^{31}$	$\alpha_{ck}^{22}$	$\gamma_{ck}^{22}$	$\alpha_{ck}^{31}$	$\gamma_{ck}^{31}$	$\alpha_{ckl}^{211}$	$\gamma_{ckl}^{211}$	$a_{cc}$	$a_{c1}$	$a_{ck}$
2	0	1	0	1	0	0	-4	0	0	0	-1	1/2	1/4	0

Thus  $h_{cc} = a(\tau + \tau^2 - \tau_2) + a \left( \frac{1}{2} \tau + \frac{1}{4} [\tau^2 - \tau_2] \right) + \frac{n_r - 1}{1 + \lambda} \tau_2 + 2b \left( \frac{1}{2} \tau + \frac{1}{2} [\tau^2 - \tau_2] \right)$ . I will note  $\tau + \tau^2 - \tau_2 = \gamma_1$  and  $\frac{1}{2} \tau + \frac{1}{4} [\tau^2 - \tau_2] = \gamma_2$

$$\Rightarrow h_{cc} = (a + b)(\tau + \tau^2 - \tau_2) + a \left( \frac{1}{2} \tau + \frac{1}{4} [\tau^2 - \tau_2] \right) + \frac{n_r - 1}{1 + \gamma} \tau_2$$

For symmetrical reasons, the diagonal term  $h_{11}$  is identical to  $h_{cc}$

$$h_{c1} = \sum_l \sum_k t_{kl} \left( \frac{1}{2} \tau \alpha_{c1kl}^{1111} - \frac{1}{4} \tau_2 \gamma_{c1kl}^{1111} + 4a_{ck} a_{1l} [\tau^2 - \tau_2] \right)$$

$$h_{c1} = t_{cc} \left( \frac{1}{2} \tau \alpha_{c1}^{31} - \frac{1}{4} \tau_2 \gamma_{c1}^{31} + 4a_{cc} a_{c1} [\tau^2 - \tau_2] \right) + t_{11} \left( \frac{1}{2} \tau \alpha_{1c}^{31} - \frac{1}{4} \tau_2 \gamma_{1c}^{31} + 4a_{11} a_{c1} [\tau^2 - \tau_2] \right) + \sum_{k \neq c \& 1} t_{kk} \left( \frac{1}{2} \tau \alpha_{kc1}^{211} - \frac{1}{4} \tau_2 \gamma_{kc1}^{211} + 4a_{ck} a_{1k} [\tau^2 - \tau_2] \right) + t_{1c} \left( \frac{1}{2} \tau \alpha_{1c}^{22} - \frac{1}{4} \tau_2 \gamma_{1c}^{22} + 4a_{c1}^2 [\tau^2 - \tau_2] \right) + t_{c1} \left( \frac{1}{2} \tau \alpha_{1c}^{22} - \frac{1}{4} \tau_2 \gamma_{1c}^{22} + 4a_{cc} a_{11} [\tau^2 - \tau_2] \right)$$

$$h_{c1} = a \left( \frac{1}{2} \tau + \frac{1}{2} [\tau^2 - \tau_2] \right) + a \left( \frac{1}{2} \tau + \frac{1}{2} [\tau^2 - \tau_2] \right) + \frac{n_r - 1}{1 + \gamma} \left( \frac{1}{4} \tau_2 \right) + b \left( \frac{1}{2} \tau + \frac{1}{4} [\tau^2 - \tau_2] \right) + b \left( \frac{1}{2} \tau + [\tau^2 - \tau_2] \right)$$

$$\Rightarrow h_{c1} = (a + b)(\tau + \tau^2 - \tau_2) + \frac{1}{4} b (\tau^2 - \tau_2) + \frac{n_r - 1}{1 + \gamma} \left( \frac{1}{4} \tau_2 \right)$$

For symmetrical reasons, the  $h_{1c}$  is identical to  $h_{c1}$

$$\{\mathbf{TE}[\mathbf{G}^* \mathbf{T} \mathbf{G}^*] \mathbf{T}\}_{cc} = a^2 h_{cc} + 2ab h_{c1} + b^2 h_{11} = (a + b)^3 (\tau + \tau^2 - \tau_2) + (a^2 + b^2) a \left( \frac{1}{2} \tau + \frac{1}{4} [\tau^2 - \tau_2] \right) + \frac{1}{2} ab^2 (\tau^2 - \tau_2) + \frac{n_r - 1}{1 + \gamma} \tau_2 \left[ a^2 + b^2 + \frac{1}{2} ab \right]$$

Using  $\sigma_q^2 E[\Lambda_{cc}] = \gamma \mathbf{T}_{cc} - \gamma^3 \{\mathbf{TATAT}\}_{cc} + \frac{\gamma^3}{\tau^2} \{\mathbf{TE}[\mathbf{G}^* \mathbf{T} \mathbf{G}^*] \mathbf{T}\}_{cc}$  and after some algebra the precision comes to be

$$\tilde{E}[r_{q_c, \hat{q}_c}^2] \sim \frac{1}{\gamma a + \gamma^3 \frac{\tau - \tau_2}{\tau^2} \left[ (a + b)^3 + (a^2 + b^2) \frac{1}{2} a \right] + \gamma^3 \frac{\tau_2}{\tau^2} \left[ \frac{1}{4} a (b^2 - a^2) + \frac{n_r - 1}{1 + \gamma} \left( a^2 + b^2 + \frac{1}{2} ab \right) \right]}{1} \quad [SM3.1]$$

## Second approximation

With the hypothesis of relatedness between reference individuals and absence of inbreeding, the averaged coefficients in formula  $\{III\}$  are  $\overline{a_{ci}^2} = \frac{1}{n_r} \frac{1}{16}$ ,  $\overline{a_{ci}^{22}} = \frac{1}{n_r}$  and  $\overline{\gamma_{ci}^{22}} = -4 \frac{n_r - 1}{n_r}$ .

$$\tilde{E}[r_{q_c, \hat{q}_c}^2] = 1 - \lambda_\beta \frac{(\sum_m \rho_m) + (\sum_m \rho_m^2) \left( 1 + \frac{1}{4n_r} - \frac{n_r - 1}{n_r} \right) - (\sum_m \rho_m)^2 \frac{1}{4n_r} - \left( \sum_m \frac{\rho_m^2}{\sigma_m^2} \right) \left( \frac{1}{2} \frac{1}{n_r} \right)}{n_r \tau}$$

Expectations of elements in  $\rho_m$  are given in table 2.

$$\tilde{E}[r_{q_c, \hat{q}_c}^2] = 1 - \lambda_\beta n_M \frac{\frac{k}{\omega} \theta + \frac{5}{4n_r} \frac{k}{\omega^2} \left[ \theta \left( \omega - \frac{2h}{\omega} \right) - 1 \right] - \frac{1}{4n_r} n_M \left( \frac{k}{\omega} \theta \right)^2 - \frac{1}{2n_r} \left( \frac{k}{2\omega^3} \{ 2\theta + \frac{\omega}{h} \} \right)}{\tau n_r}$$

Using  $\lambda_\beta = \tau / \gamma$ , and after some algebra,

$$\tilde{E}[r_{q_c, \hat{q}_c}^2] = 1 - \frac{n_M}{\gamma n_r^2} \left[ n_r \frac{k}{\omega} \theta + \frac{5}{4} \frac{k}{\omega^2} \left[ \theta \left( \omega - \frac{2h}{\omega} \right) - 1 \right] - \frac{1}{4} n_M \left( \frac{k}{\omega} \theta \right)^2 - \frac{k}{4\omega^3} \left\{ 2\theta + \frac{\omega}{h} \right\} \right]$$

$$\tilde{E}[r_{q_c, \hat{q}_c}^2] = 1 - \frac{n_M k \theta}{\gamma n_r \omega} - \frac{n_M k}{4\gamma n_r^2 \omega^2} \left( 5\theta \omega - \frac{10h\theta}{\omega} - \frac{2\theta}{\omega} - 5 - n_M k \theta^2 - \frac{1}{h} \right) \quad [SM3.2]$$