ONE-DIMENSIONAL ELASTIC COLLISIONS IN A PLATONIC QUADRIDIMENSIONAL SPACE

Alain Jégat

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CHOCS ÉLASTIQUES FRONTAUX DANS UN ESPACE QUADRIDIMENSIONNEL PLATONICIEN

Alain Jégat

Abstract

From the concepts presented in the articles entitled " Towards a modeling of De Broglie waves in a platonic quadridimensional space" and " An idea of the mass of a particle in a platonic quadridimensional space" is here developed the study, in this Platonic model, of the simple case of elastic collision of two relativistic particles during a frontal impact.

By proposing the introduction of some additional absolute concepts and their application to this study, this article also aims to enrich the toolbox available for analysis of relativistic phenomena in this geometrical framework.

Résumé

À partir des concepts exposés dans les articles intitulés « Vers une modélisation de l'onde de phase de De Broglie dans un espace quadridimensionnel platonicien» et «Une idée de la masse d'une particule dans un espace quadridimensionnel platonicien», est développée ici l'étude, dans le cadre du modèle platonicien, du cas simple de la collision élastique de deux particules relativistes lors d'un choc frontal.

En proposant l'introduction complémentaire de quelques notions absolues et leur application à cette étude, cet article vise aussi à enrichir la palette d'outils disponible pour l'analyse, dans ce cadre géométrique, des phénomènes relativistes.

1. The geometrical framework

This modeling is based on the Platonic space outlined in the following articles:

> « UN MODÈLE PLATONICIEN (EUCLIDIEN-PROJECTIF) POUR LA THÉORIE DE LA RELATIVITÉ RESTREINTE » (pré-publication hal-01081576, version 1).
>
> « A PLATONIC (EUCLIDEAN-PROJECTIVE) MODEL FOR THE SPECIAL THEORY OF RELATIVITY » (pre-publication hal-01165196, version 1).

$\left(O,\vec{i},\vec{j},\vec{k},\vec{h}\right)$ is a frame for the four-dimensional Euclidean space whose axes are denoted $\left(OX\right)$, $\left(OY\right)$, $\left(OZ\right)$, $\left(Ow\right)$; the direction of the projection is that of the vector $\vec{h}$.

Following the hal-01207447 v1 and hal-01213447 v1 articles, the notion of relativistic mass of a particle is described here as a result of its interaction with a stratification of the four-dimensional Platonic space by a sequence of hyperplanes $H^{(n)}$ which are orthogonal to the direction of the projection $\vec{h}$, regularly spaced by a distance $\Delta w_0 > 0$.

2. Introduction of some absolute concepts

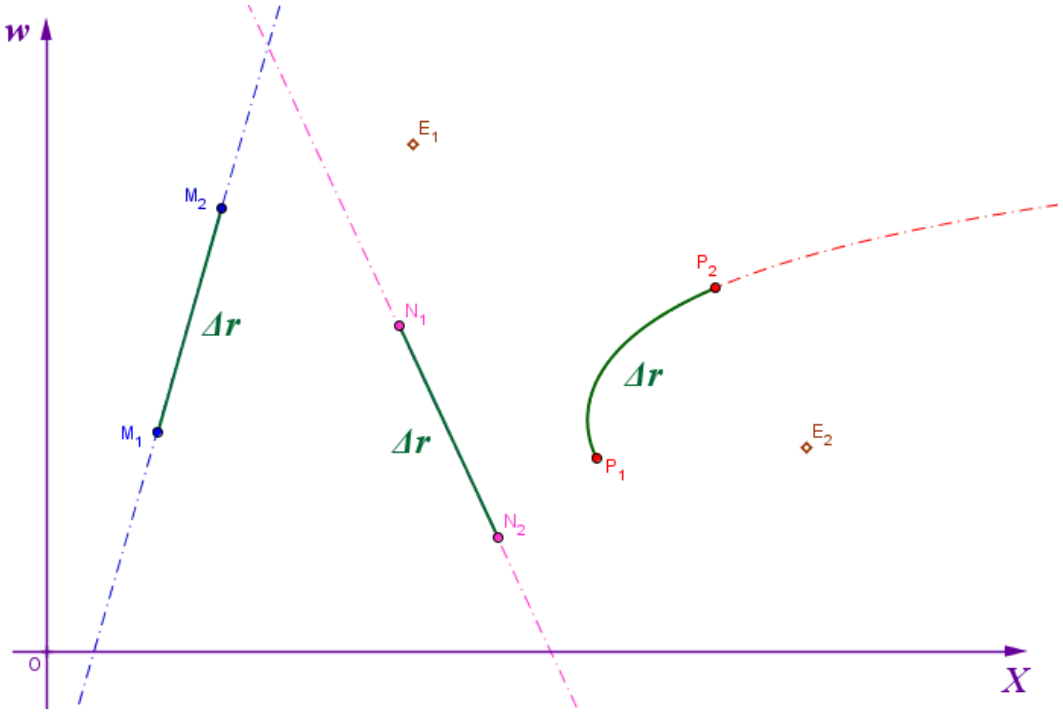
This section is intended to introduce some concepts, described as absolute, which are independent of any moving reference frame in the Platonic space.

The application of these absolute concepts will allow us to obtain the relations between the initial and final trajectories of two relativistic particles after a one-dimensional elastic collision.

2.1. Absolute time concept

Given the regular motion of the objects observed in the Platonic universe (« between two events, the distances traveled by all the observed objects are equal, regardless of their trajectory »), we can introduce a notion of absolute time in the following terms :

« ***Absolute time*** between two events is the distance traveled by (all) the mobiles between these two events .»

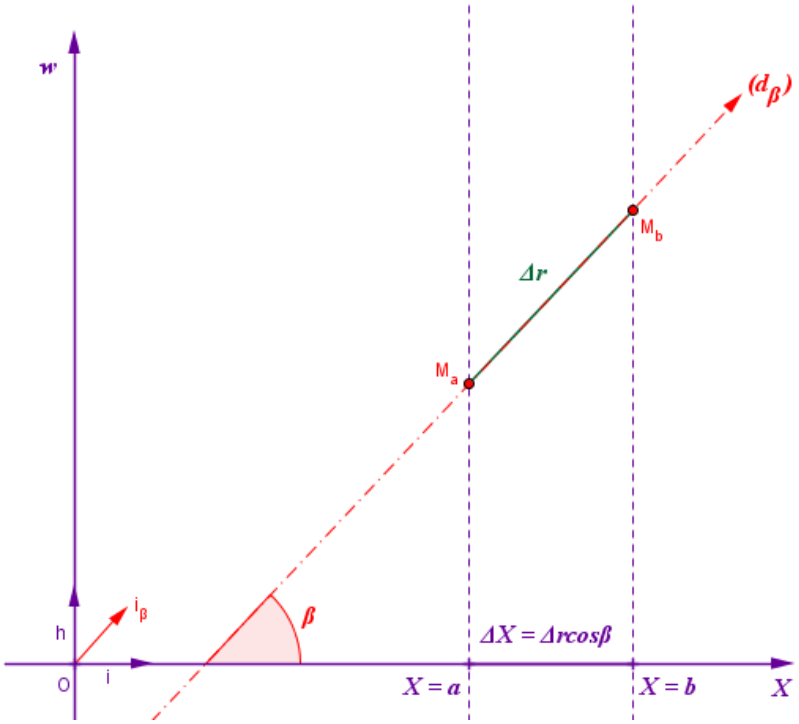


Thus, in the above diagram, if the event E_1 occurs while the mobiles M, N and P are respectively at M_1, N_1 and P_1 and if the event E_2 occurs while the mobiles M, N and P are respectively at M_2, N_2 and P_2, we can define the ***absolute time*** ΔT between these two events by the relation $\Delta T = \Delta r$.

Note that in this Platonic space, one can thus introduce a notion of absolute time which is measured in ***meters***.

2.2. Absolute velocity

Into the frame $\left(O,\vec{i},\vec{h}\right)$, let us consider a ponctual mobile M whose straight trajectory is directed by a vector $\vec{i_\beta}$, with $\left(\vec{i},\vec{i_\beta}\right)=\beta$ and two distinct lines $X=a$ and $X=b$.



With $\Delta X = b-a$, we can define the ***absolute velocity*** of M by :

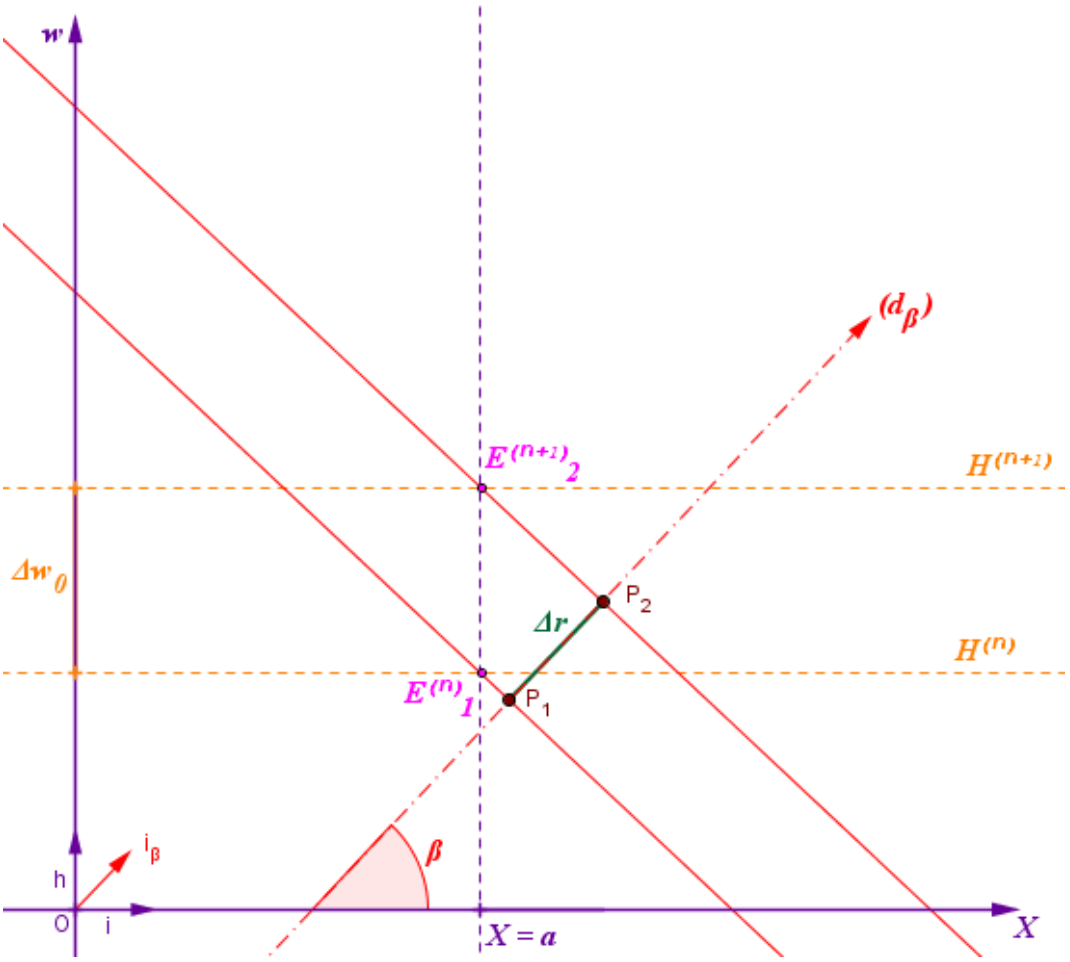
$$v_{abs}=\frac{\Delta X}{\Delta T}=\frac{\Delta r\cos\beta}{\Delta r}=\cos\beta\,.$$

Note that here the absolute velocity of M is a dimensionless quantity.

2.3. Absolute frequency of De Broglie waves

Let us consider a ponctual particle P moving along a straight line led by a vector $\vec{i_\beta}$, with $\left(\vec{i},\vec{i_\beta}\right)=\beta$, linked to a stratification of the Platonic space whose distance is Δw_0.

To this particle may be associated De Broglie waves (see the articles cited in reference).



Consider a line whose equation is $X=a$.

We can define the absolute frequency of these De Broglie waves considering two of its successive occurrences $E_1^{(n)}$ and $E_2^{(n+1)}$ at $X=a$ (see above diagram).

The absolute time separating these two occurrences being $\Delta T=\Delta r=\Delta w_0 \sin\beta$, the ***absolute frequency*** of the De Broglie waves can be given by : $\upsilon_{abs}=\dfrac{1}{\Delta T}=\dfrac{1}{\Delta w_0 \sin\beta}$.

Note that this absolute frequency is expressed in m^{-1}.

2.4. Absolute energy and absolute mass of a particle

① By adopting for ***absolute quantum of action*** the quantity $h_{abs} = hc$ (in $J.m$), associated to the absolute frequency of the De Broglie waves $\upsilon_{abs} = \dfrac{1}{\Delta w_0 \sin\beta}$ (in m^{-1}), we are led to define as ***absolute energy*** of the particle *P* (in *J*) the quantity :

$$E_{abs} = h_{abs}\upsilon_{abs} = \frac{hc}{\Delta w_0 \sin\beta} \ .$$

The link between the energy E_{abs} and the energy E of the particle measured in a reference frame R_α (where α is arbitrary, not a multiple of π) is given by :

$$E = E_{abs} \times \frac{1-\cos\alpha\cos\beta}{\sin\alpha} = \frac{E_{abs}\sin\beta}{\sqrt{1-\dfrac{v^2}{c^2}}} \ .$$

② From this result, we can define the notion of ***absolute mass*** (in kg):

$$m_{abs} = \frac{E_{abs}}{c^2} = \frac{h}{c\Delta w_0 \sin\beta} \qquad \left(= \frac{m_0}{\sin\beta}, \ \ with \ \ m_0 = \frac{h}{c\Delta w_0} \right).$$

(Note that the induced formula $\Delta w_0 = \dfrac{h}{cm_0}$ corresponds for Δw_0 to the ***Compton wavelength*** of a particle, m_0 being the rest mass of this particle *P*.)

③ Into the reference frame R_α, is so obtained the following coherence :

$$E = h\upsilon = \frac{hc}{\Delta w_0} \times \frac{1-\cos\alpha\cos\beta}{\sin\alpha\sin\beta} = mc^2 ,$$

$$\text{with} \quad \upsilon = \frac{\upsilon_{abs}\sin\beta}{\sqrt{1-\dfrac{v^2}{c^2}}} = \frac{c}{\Delta w_0} \times \frac{1-\cos\alpha\cos\beta}{\sin\alpha\sin\beta}$$

$$\text{and} \quad m = \frac{m_{abs}\sin\beta}{\sqrt{1-\dfrac{v^2}{c^2}}} = \frac{h}{c\Delta w_0} \times \frac{1-\cos\alpha\cos\beta}{\sin\alpha\sin\beta} .$$

2.5. Absolute linear momentum of a particle

By analogy with the classical notion of momentum of a particle, we define its ***absolute momentum*** as being equal to the product of its absolute velocity by its absolute mass,

$$i.e. \quad P_{abs} = m_{abs} v_{abs} = \frac{h}{c\Delta w_0 \tan\beta} \ .$$

(Note that this absolute momentum is expressed in kg.)

3. Application to the one-dimensional elastic collisions

3.1. Two hypotheses

Let us consider two ponctual particles P_1 and P_2 whose straight trajectories are directed respectively by the vectors $\overrightarrow{i_\beta}$ et $\overrightarrow{i_\gamma}$, associated to the stratifications whose distances are respectively Δw_{01} and Δw_{02}.

After the elastic collision of these two particles P_1 and P_2, if we call $\overrightarrow{i_{\beta'}}$ and $\overrightarrow{i_{\gamma'}}$ the direction vectors of their new trajectories , we will choose as hypotheses :

❶ the sum of absolute energy is conserved,

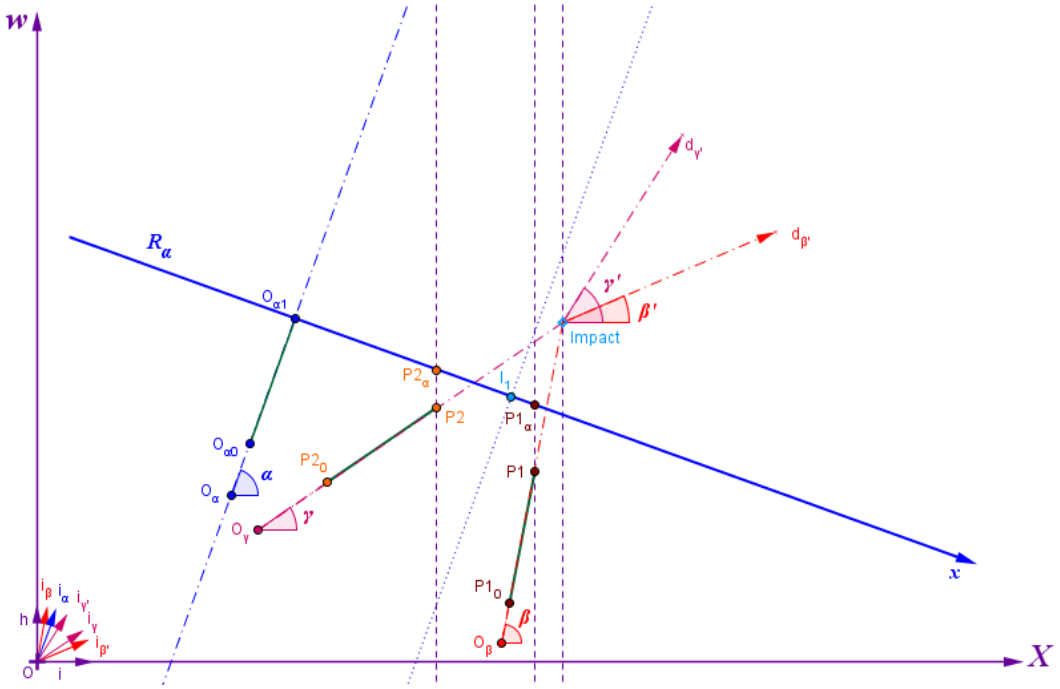
$$i.e. \quad \frac{hc}{\Delta w_{01}\sin\beta} + \frac{hc}{\Delta w_{02}\sin\gamma} = \frac{hc}{\Delta w_{01}\sin\beta'} + \frac{hc}{\Delta w_{02}\sin\gamma'} \ ;$$

❷ the sum of absolute momentum is conserved,

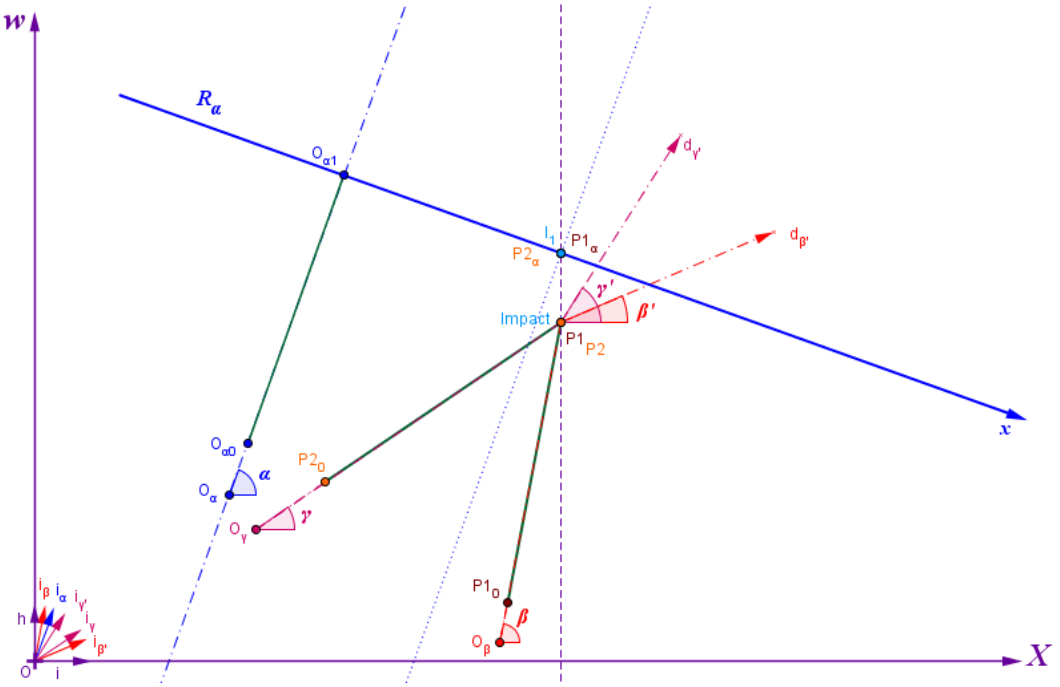
$$i.e. \quad \frac{h}{c\Delta w_{01}\tan\beta} + \frac{h}{c\Delta w_{02}\tan\gamma} = \frac{h}{c\Delta w_{01}\tan\beta'} + \frac{h}{c\Delta w_{02}\tan\gamma'} \ .$$

With the relations $m_{01} = \frac{h}{c\Delta w_{01}}$ and $m_{02} = \frac{h}{c\Delta w_{02}}$ (see articles cited by reference), these two assumptions will lead to the results described in the following paragraph.

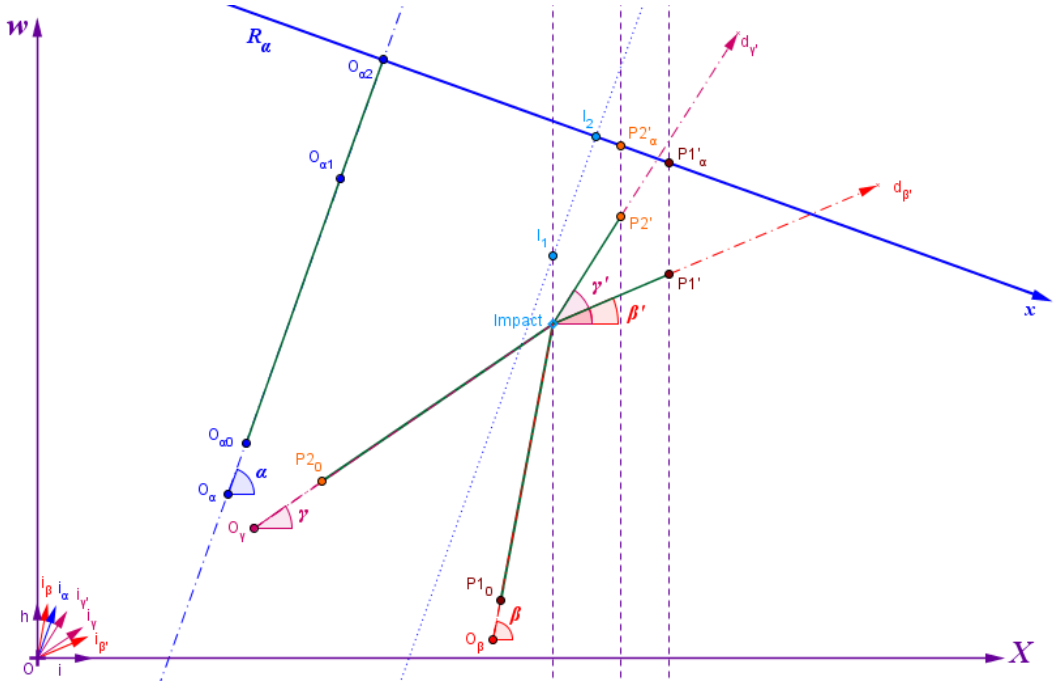
3.2. One-dimensional elastic collisions



- Diagram 1 -



- Diagram 2 -



- Diagram 3 -

Let us consider two particles P_1 and P_2 whose rest masses are respectivly m_{01} and m_{02} coming into elastic collision in the Platonic space at the point « *Impact* » (see Diagram 2).

This impact is perceived by the observer I_1 into the reference frame R_α ; the trajectory of this observer is represented by a blue dotted line.

(Note that in the Platonic space, this collision can be represented with distinct values of $w(P_1)$ and $w(P_2)$. Here , they are chosen equal in Diagram 2 by pure graphical convenience.)

Before the collision, in Diagram 1, the trajectory of P_1 is directed at an angle β , the trajectory of P_2 at an angle γ . The particles P_1 and P_2 are observed into R_α at $P_{1\alpha}$ and $P_{2\alpha}$.

After the collision, the trajectory of P_1 is directed at an angle β', that of P_2 at an angle γ'.

The particles P_1 and P_2 , designated by P'_1 et P'_2 , are observed into R_α at $P'_{1\alpha}$ and $P'_{2\alpha}$.

From the two assumptions made in the previous paragraph , these two angles are given, with $0 < \beta < \pi$ and $0 < \gamma < \pi$, by the following equations (cf. expressions with the rest masses at the end of this article):

$$\cos\beta' = \cos\gamma + \frac{\left(\Delta w_{01}{}^2 - \Delta w_{02}{}^2\right)\left(\cos\gamma - \cos\beta\right)\sin^2\gamma}{\Delta w_{01}{}^2\left(1 - 2\cos\beta\cos\gamma + \cos^2\gamma\right) + \Delta w_{02}{}^2\sin^2\gamma + 2\Delta w_{01}\Delta w_{02}\sin\beta\sin\gamma};$$

$$\cos\gamma' = \cos\beta + \frac{\left(\Delta w_{01}{}^2 - \Delta w_{02}{}^2\right)\left(\cos\gamma - \cos\beta\right)\sin^2\beta}{\Delta w_{02}{}^2\left(1 - 2\cos\beta\cos\gamma + \cos^2\beta\right) + \Delta w_{01}{}^2\sin^2\beta + 2\Delta w_{01}\Delta w_{02}\sin\beta\sin\gamma}.$$

It is noteworthy that on the one hand, these relations are ***absolute*** to the extent that they do not depend on the angle of α of the reference frame.

On the other hand, they lead, regardless of the value of the angle α of the reference frame, to the conservation in this reference frame of the global energy and global momentum, namely:

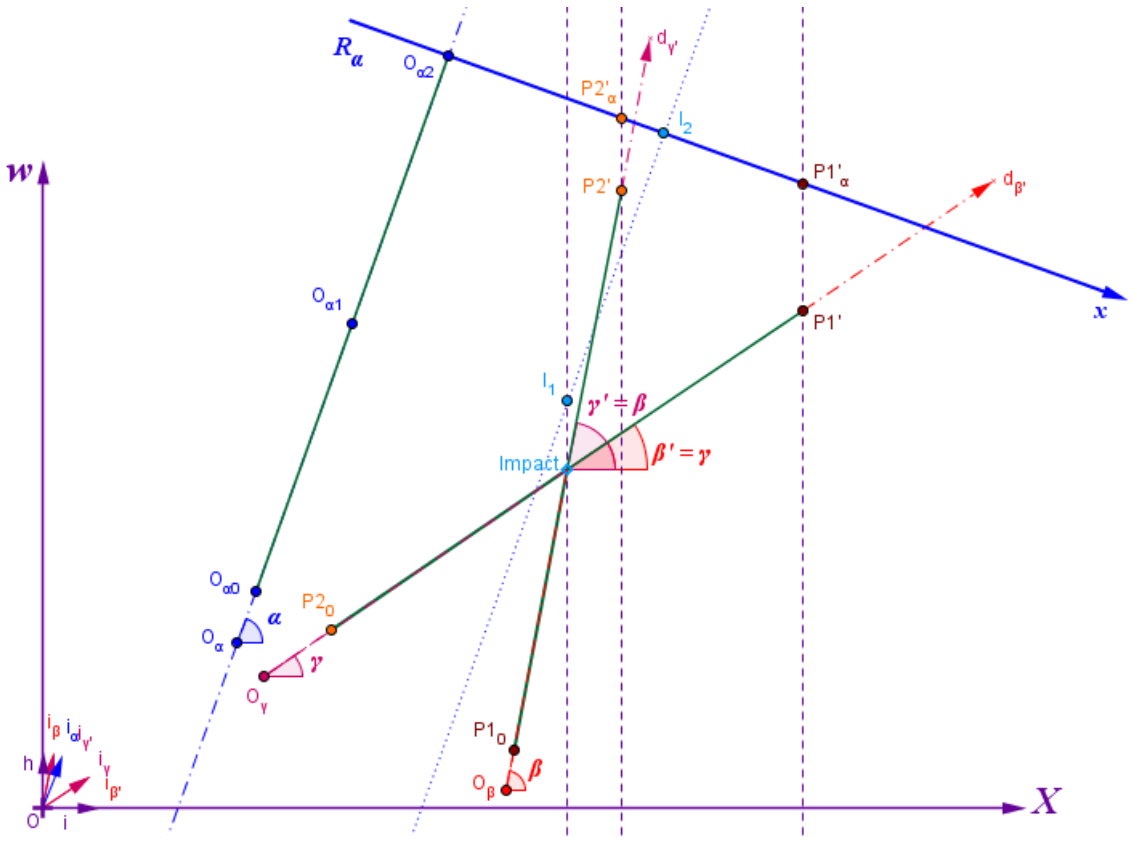
$$\begin{cases} \dfrac{m_{01}v_1}{\sqrt{1-\frac{v_1^2}{c^2}}}+\dfrac{m_{02}v_2}{\sqrt{1-\frac{v_2^2}{c^2}}}=\dfrac{m_{01}v_1'}{\sqrt{1-\frac{v_1'^2}{c^2}}}+\dfrac{m_{02}v_2'}{\sqrt{1-\frac{v_2'^2}{c^2}}} \\ \dfrac{m_{01}c^2}{\sqrt{1-\frac{v_1^2}{c^2}}}+\dfrac{m_{02}c^2}{\sqrt{1-\frac{v_2^2}{c^2}}}=\dfrac{m_{01}c^2}{\sqrt{1-\frac{v_1'^2}{c^2}}}+\dfrac{m_{02}c^2}{\sqrt{1-\frac{v_2'^2}{c^2}}} \end{cases}$$

i.e. $$\begin{cases} m_{01}\dfrac{\cos\beta-\cos\alpha}{\sin\alpha\sin\beta}+m_{02}\dfrac{\cos\gamma-\cos\alpha}{\sin\alpha\sin\gamma}=m_{01}\dfrac{\cos\beta'-\cos\alpha}{\sin\alpha\sin\beta'}+m_{02}\dfrac{\cos\gamma'-\cos\alpha}{\sin\alpha\sin\gamma'} \\ m_{01}\dfrac{1-\cos\beta\cos\alpha}{\sin\alpha\sin\beta}+m_{02}\dfrac{1-\cos\gamma\cos\alpha}{\sin\alpha\sin\gamma}=m_{01}\dfrac{1-\cos\beta'\cos\alpha}{\sin\alpha\sin\beta'}+m_{02}\dfrac{1-\cos\gamma'\cos\alpha}{\sin\alpha\sin\gamma'} \end{cases}.$$

3.3. Study of some particular cases

① First, if $\Delta w_{01}=\Delta w_{02}$ (i.e. $m_{01}=m_{02}$) we obtain, independently of the reference frame R_α, a geometrical ***symmetry*** whose simplicity is noteworthy .

Indeed, in this case, the above formulas lead to $\beta'=\gamma$ and $\gamma'=\beta$.

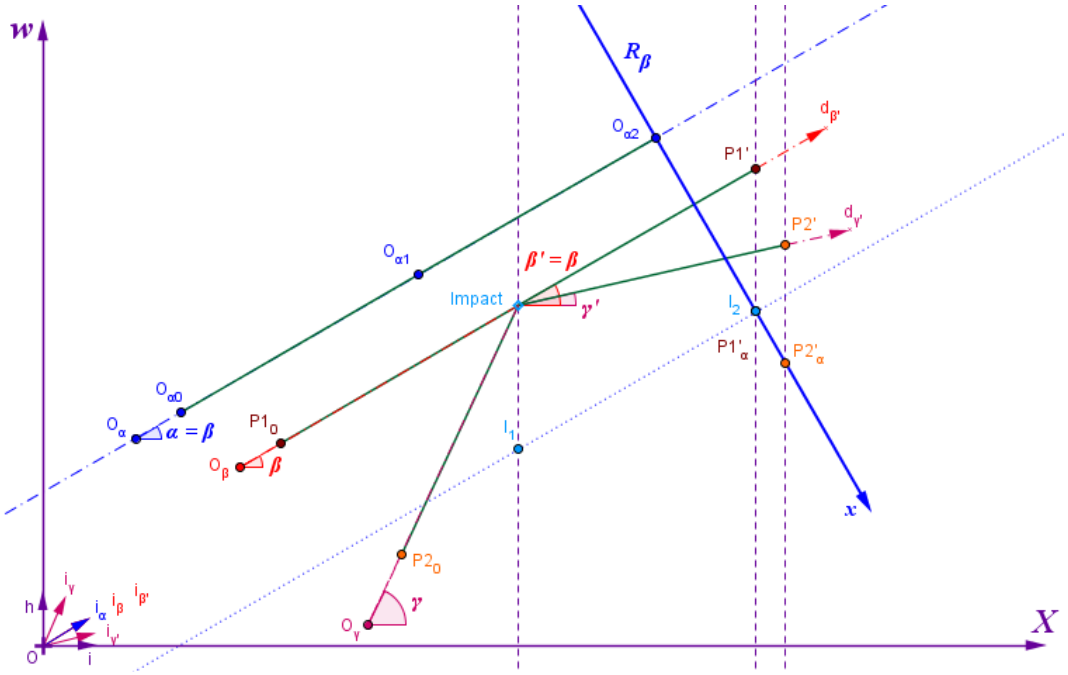


② If $\Delta w_{01} \to 0$ or if $\Delta w_{02} \to +\infty$, then $\beta' \to \beta$ and $\cos\gamma' \to \dfrac{2\cos\beta - \cos\gamma\left(1+\cos^2\beta\right)}{1+\cos^2\beta - 2\cos\beta\cos\gamma}$.

This last result, of abstruse appearance, leads just to the following fact: in any reference frame R_β where the particle P_1 is at rest, the speed of P_2 and P_2' are ***opposite***.

Indeed, in this reference frame, we have (cf. hal-01165196, v1) : $\dfrac{v_\beta(P_2)}{c} = \dfrac{\cos\gamma - \cos\beta}{1-\cos\beta\cos\gamma}$

and there is obtained, after simplification: $\dfrac{v_\beta(P_2')}{c} = \dfrac{\cos\gamma' - \cos\beta}{1-\cos\beta\cos\gamma'} = -\dfrac{\cos\gamma - \cos\beta}{1-\cos\beta\cos\gamma}$.



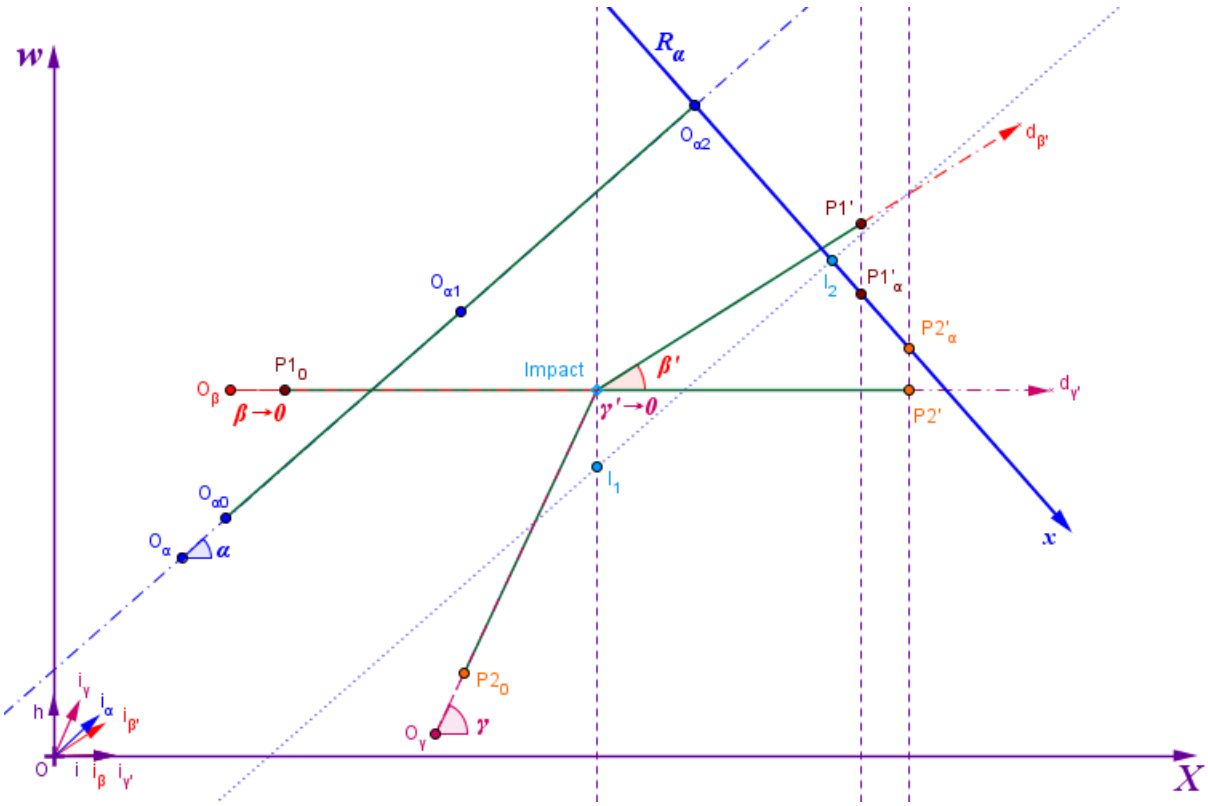
③ Symmetrically, if $\Delta w_{01} \to +\infty$ or if $\Delta w_{02} \to 0$, we obtain $\gamma' \to \gamma$ and $\cos\beta' \to \dfrac{2\cos\gamma - \cos\beta\left(1+\cos^2\gamma\right)}{1+\cos^2\gamma - 2\cos\beta\cos\gamma}$.

④ • If $\beta \to 0$ (i.e. if $v(P_1) \to c$), then $\cos\beta' \to \dfrac{\Delta w_{02}^2(1+\cos\gamma) - \Delta w_{01}^2(1-\cos\gamma)}{\Delta w_{02}^2(1+\cos\gamma) + \Delta w_{01}^2(1-\cos\gamma)}$

and $\gamma' \to 0$ (i.e. $v(P_2') \to c$).

• If $\beta \to \pi$ (i.e. if $v(P_1) \to -c$), then $\cos\beta' \to \dfrac{\Delta w_{01}^2(1+\cos\gamma) - \Delta w_{02}^2(1-\cos\gamma)}{\Delta w_{01}^2(1+\cos\gamma) + \Delta w_{02}^2(1-\cos\gamma)}$

and $\gamma' \to \pi$ (i.e. $v(P_2') \to -c$).



④ • If $\gamma \to 0$ (i.e. if $v(P_2) \to c$), then $\cos\gamma' \to \dfrac{\Delta w_{01}^2(1+\cos\beta) - \Delta w_{02}^2(1-\cos\beta)}{\Delta w_{01}^2(1+\cos\beta) + \Delta w_{02}^2(1-\cos\beta)}$

and $\beta' \to 0$ (i.e. $v(P_1') \to c$).

• If $\gamma \to \pi$ (i.e. if $v(P_2) \to -c$), then $\cos\gamma' \to \dfrac{\Delta w_{02}^2(1+\cos\beta) - \Delta w_{01}^2(1-\cos\beta)}{\Delta w_{02}^2(1+\cos\beta) + \Delta w_{01}^2(1-\cos\beta)}$

and $\beta' \to \pi$ (i.e. $v(P_1) \to -c$).

4. Expressions of cos(β') and cos(γ') based on the rest masses

$$\cos\beta' = \cos\gamma + \frac{\left(m_{02}{}^2 - m_{01}{}^2\right)(\cos\gamma - \cos\beta)\sin^2\gamma}{m_{02}{}^2\left(1 - 2\cos\beta\cos\gamma + \cos^2\gamma\right) + m_{01}{}^2\sin^2\gamma + 2m_{01}m_{02}\sin\beta\sin\gamma};$$

$$\cos\gamma' = \cos\beta + \frac{\left(m_{02}{}^2 - m_{01}{}^2\right)(\cos\gamma - \cos\beta)\sin^2\beta}{m_{01}{}^2\left(1 - 2\cos\beta\cos\gamma + \cos^2\beta\right) + m_{02}{}^2\sin^2\beta + 2m_{01}m_{02}\sin\beta\sin\gamma}.$$

5. Conclusion

Through this study, this article aims to show that the introduction of absolute concepts (energy, momentum , ...) can easily model relativistic phenomena in the Platonic framework .

From this, it seems promising to continue their application (eg to study the elastic two-dimensional relativistic collisions or inelastic collisions), or even to increase their number (eg by introducing the concept of absolute angular momentum) to solve the modeling of other problems, as the two-body problem .

And beyond is needed of course the search for deeper understanding of the links between these concepts of absolute energy, momentum, angular momentum, … and the geometrical structures proposed into the Platonic model.

6. References

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