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# Complementary writing of maximum and least material requirements, with an extension to complex surfaces

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## Abstract

Maximum and least material requirements describe the strictly necessary fitability and accuracy functional requirements for an assembly involving connections with clearances. In the ISO 2692:2014 [1] dimensioning and tolerancing standard, the writing of this requirement violates the principle of independency and is limited to features of size. This paper proposes two complementary writings and several explanations for the application of the concepts. In order to make the definitions consistent with those of ISO 1101:2012 [2] standard, the requirements are defined by means of unilateral tolerance zones. For features of size, the dimension of the tolerance zone for the specified surface and for the reference is written directly between brackets in the specification. For all complex surfaces, the tolerance zone is defined by an offset surface of the nominal surface. The offset value is written between braces. The definitions of form, location and orientation specifications with these modifiers are given for simple elements and for a pattern of holes. Composite specifications, which associate orientation and location tolerance zones with respect to the same nominal, are defined. An example with flutter on a primary reference shows that it is no longer possible to use all the degrees of freedom to associate the subsequent references. The use of an orientation plane to deal with unidirectional chains of dimensions is defined. In terms of metrology, the characteristic to evaluate is the margin between the actual surface and the limit surface of the tolerance zone when the tolerance zone on the references is respected. This margin enables one, for example, to determine a capability.

Three applications present an assembly of a mechanism with clearances, a connection with a complex surface and a 3D chain of dimensions at least material which requires a composite specification.

*Keywords:* ISO Standard, maximum/least material condition, functional tolerancing, composite specification,

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## 1. Introduction

### 1.1. The context

In functional dimensioning, maximum or least material specifications should be used systematically for all connections with clearances because they define the strictly necessary requirements.

In practice, many designers are not familiar with the use of these specifications and many metrology computer programs are still unable to verify these specifications in perfect agreement with the definitions. One reason is probably the complexity of ISO 2692:2014 standard [1], which presents the concepts through the notion of virtual boundary along with very complex rules due to the violation of the principle of independency. Under very strict criteria, the dimension of the virtual boundary depends on the nominal diameter, the dimensional tolerance, the type of surface, the modifier and, possibly, the tolerance on the element of reference. Such information is very difficult to obtain in the numerical continuity context, especially for the calculation of 3D chains of dimensions and for metrology.

Figure 1, borrowed from figure A.13 of 2692:2014 standard, gives a schematic view of the rules for calculating the diameters of the virtual states on the toleranced element and the datum. Six values must be collected in four specifications.

The definition is limited to features of size, whereas many connections with clearances are created from more complex features.

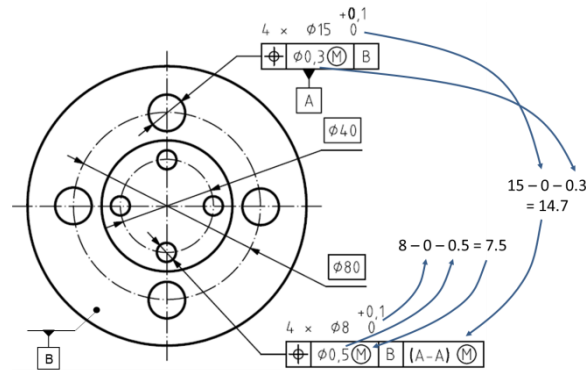


Figure 1 – Dimensions of the maximum material virtual states

Very often it is unnecessary to verify the local dimensions when the maximum or least material specifications are met.

This paper develops a more direct and complementary type of writing which satisfies the principle of independency and sets some rules for the application of the specifications.

### 1.2. The scientific context

The principle of maximum or least material specifications by virtual boundary are defined in ISO standards 2692 (1988) and in ASME standard (1994). This virtual boundaries are used as virtual gauges since the early 1990's by [3,4]. Robinson uses maximum material parts among assembly specifications, tolerance specifications and assembly tolerance analysis [5].

Some mathematical models can use the condition of the virtual boundary as T-maps [6], domains [7], analysis lines [8] and polytopes [9]. These different methods are applied to the tolerance analysis. Some papers confront these different methods [10,11]. Muthy uses simplex method for metrology [12]

Pairel and al. have exposed a conceptual model of “virtual fitting gauges” [13,14]. In this model, they can exploit maximum and least material requirement. They developed some algorithms implemented in a conventional software package for metrology. In [15,16], a presentation of the usability of this software is done in a pattern of holes.

Dantan and al. define the gauge with internal mobilities to limit the geometrical variations of the part [17]. The permissible geometrical variations are compared to the worst geometry of its environment. [18] use virtual gauges with internal mobilities to verify the maximum material and least material requirements.

Anselmetti uses these principles for functional tolerancing in assembling and for 3D tolerance stack-up [19].

## 2. Definition of the $\text{M}$ and $\text{L}$ tolerance zones

### 2.1. The tolerance zone for a cylinder

The new writing is simply to place in the specification the diameter of the virtual state of the specified surface and of datum. The meaning is exactly the same as classical writing Figure 1. This writing is new for ISO standard but is described in ASME Y14.5 2009 standard (section 4.1)

For a dimension, the diameter D of the tolerance zone is followed by  $\text{M}$  or  $\text{L}$ . For a specification, the diameter is placed between brackets after a  $\text{Ø}$  symbol. The unilateral tolerance zone is bounded by a cylinder of diameter D.

The maximum material tolerance zones are shown in figure 2:

- For a shaft, the tolerance zone is inside the cylinder of diameter D.
- For a hole, the tolerance zone is outside the cylinder of diameter D.

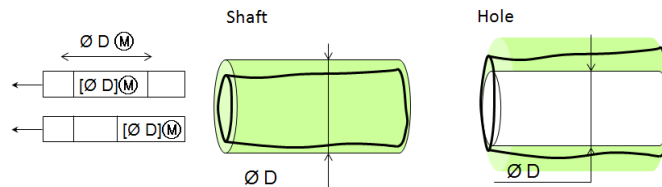


Figure 2 - The maximum material tolerance zone for a cylinder

The least material tolerance zones are shown in figure 3:

- For a shaft, the tolerance zone is outside the cylinder of diameter D.
- For a hole, the tolerance zone is inside the cylinder of diameter D.

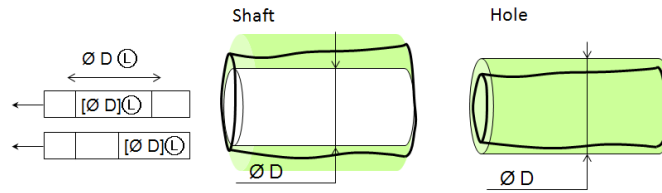


Figure 3 - The least material tolerance zone for a cylinder

## 2.2. Example

The maximum material is a means to guarantee the fittability of two parts by specifying a boundary between the elements (figure 4). The values of  $D$  and  $d$  must be chosen, depending on the desired minimum clearance, such that  $\text{Min. clearance} = D - d$

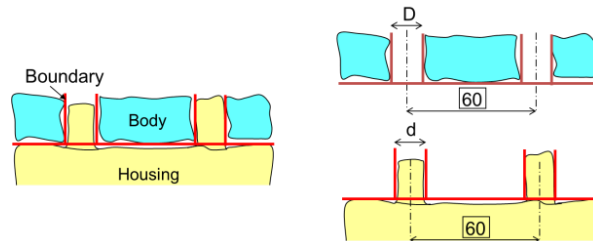


Figure 4 - Fittability with two pins

Using modifier  $\text{M}$ , the writing on the drawing is immediate (figure 5).

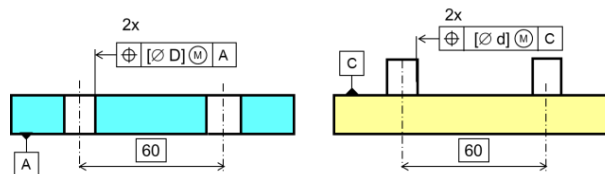


Figure 5 - Maximum material dimensioning

## 2.3. The tolerance zone for two parallel planes facing each other

Modifiers  $\text{M}$  or  $\text{L}$  indicate that the specified element is the set of the two lateral sides which are identified by the arrows. For a dimension, the size  $D$  of the tolerance zone is followed by the modifier. For a specification, the dimension is placed between brackets without a  $\text{Ø}$  symbol. The tolerance zone is bounded by two planes at a distance of  $D$ .

The maximum material tolerance zones are shown in figure 6:

- For a tenon (a solid part), the tolerance zone is inside the two planes at a distance of  $D$ .
- For a slot (a hollow part), the tolerance zone is outside of the two planes at a distance of  $D$ .

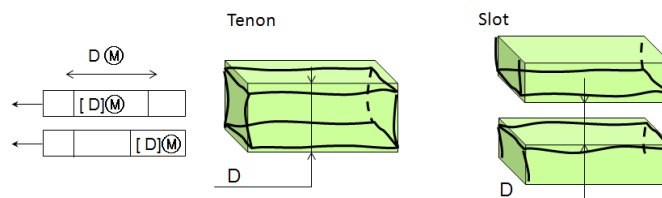


Figure 6 - The maximum material tolerance zone for two planes

The least material tolerance zones are shown in figure 7:

- For a tenon (a solid part), the tolerance zone is outside of the two planes at a distance of  $D$ .
- For a slot (a hollow part), the tolerance zone is inside the two planes.

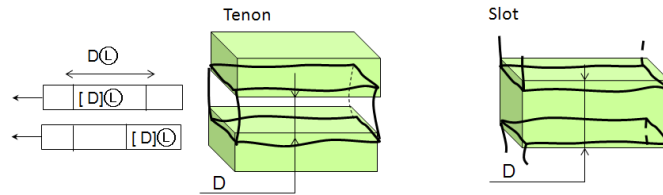


Figure 7 - The least material tolerance zone for two planes

## 2.4. The tolerance zone for a surface

The modifier  $\textcircled{M}$  or  $\textcircled{L}$  indicates that the toleranced element is the surface. The normal  $\mathbf{n}$  to the surface points outside the material. The value of the offset deviation  $d$  is placed between braces without a  $\textcircled{\varnothing}$  symbol. The tolerance zone is bounded by an offset surface with a deviation  $d$  (figure 8):

- At maximum material condition, the direction of the offset  $d$  follows the normal  $\mathbf{n}$  to the surface. The surface at maximum material condition must be outside of the material.
- At least material condition, the direction of the offset  $d$  is the opposite of the normal  $\mathbf{n}$  to the surface. The surface at least material condition must be inside the material.

In figure 8, the offsets  $d$  are positive. A negative offset is designated, for example, as  $\{-0,02\} \textcircled{M}$ .

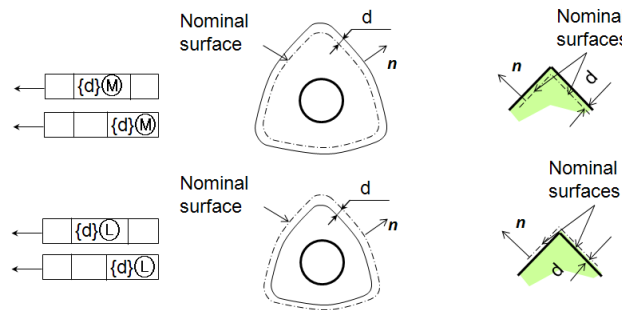


Figure 8 - The maximum/least material tolerance zone for a surface

The superposition of two specifications with different offsets at  $\textcircled{M}$  and at  $\textcircled{L}$  defines a tolerance zone which is not centered on the nominal surface.

## 3. Complementary definitions

### 3.1. Specification of a pattern

A pattern is identified by the indication  $Nx$  above the tolerance frame. The pattern is a set of  $N$  surfaces with  $N$  tolerance zones. For a pattern made up of identical surfaces, a single arrow connects the tolerance frame to one of the surfaces. If the surfaces in the pattern are different, one arrow is needed per type of surface. Each element of the pattern has its own tolerance zone (figure 9): There are  $N$  tolerance zones. The modifier CZ is not implicit.

- (a) Location: each zone is centred on the nominal surface.
- (b) Orientation: each zone is defined centred on the nominal surface, then translated independently of the other zones.
- (c) Orientation in common zone: each zone is defined centred on the nominal surface, then translated as one common zone.
- (d) Form: each zone is free with respect to its nominal surface and independent of the other zones.

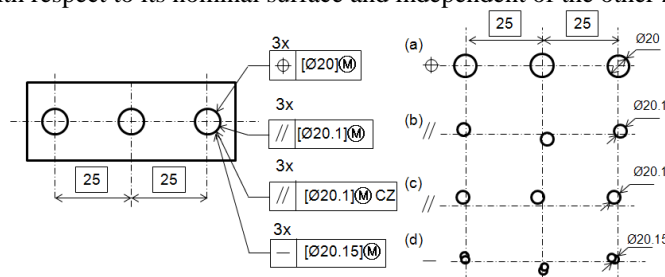


Figure 9 - Specification of a pattern of holes

### 3.2. Composite specification

A composite specification imposes a double condition on the specified surfaces by defining the location and orientation tolerance zones with respect to the same nominal surfaces. Where applicable, the nominal model is positioned with respect to the datum system in such a way that, if possible, the actual surfaces lie within all the tolerance zones simultaneously. One composite specification is represented on two lines, with no separator between the specification symbols and with a single datum system where applicable.

A composite specification restrains the orientation defects of a surface to within the location tolerance zone, even if the datum system leaves residual freedoms. The objective is to control the influence of the overhangs in calculating the resultant of a three-dimensional chain of dimensions.

Figure 10 contains a composite specification and a form specification. The composite specification requires that the three actual cylinders be simultaneously in the  $\varnothing 20.2 \text{ (L)}$  location tolerance zones and in the  $\varnothing 20.1 \text{ (L)}$  orientation tolerance zones with respect to the same nominal model. The straightness specification is independent of the composite specification and imposes three independent tolerance zones  $\varnothing 20.02 \text{ (M)}$ .

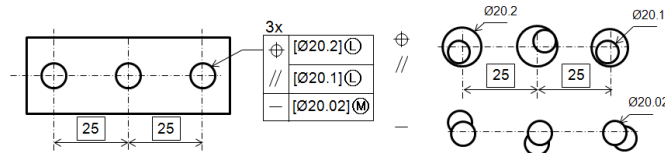


Figure 10 - A composite specification for a pattern of holes

### 3.3. Datum at maximum material condition

A datum at maximum material condition is identified by  $\text{(M)}$  at the right of the datum. The real tolerated surface and the real datum surface must be simultaneously inside their tolerance zone. The specification is not consistent if this condition cannot be met, included on datum surface.

Figure 11 corresponds to inspection of specification (1) figure 17. The sizes of the part holder are directly defined by tolerancing. Real part must be placed inside the part holder.

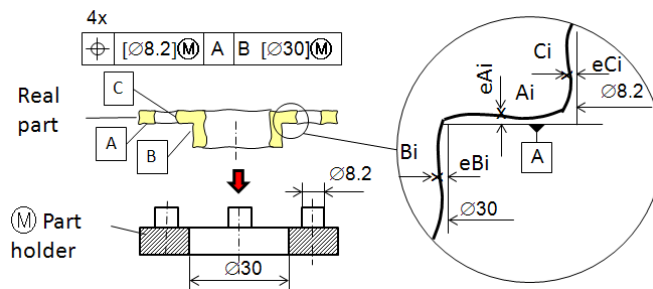


Figure 11 – Inspection with maximum material condition

The corresponding association method is obtained in two steps: ( $e_i$  is negative if the point is inside the boundary)

Step 1: For each point  $A_i \in A$ ,  $e_{A_i} \leq 0$ ,  $\max(|e_{A_i}|)$  minimal.

Step 2: For each point  $B_i \in B$ ,  $e_{B_i} \leq 0$  and for each point  $C_i \in C$ ,  $e_{C_i} \leq 0$ ,  $\max(e_{C_i})$  minimal.

### 3.4. Datum at least material condition

Figure 12, corresponds to inspection of specification (4) in figure 17 with a functional approach. A part holder simulates the maximum material boundary corresponding to the surface B. A gauge simulates the part at least material sizes.

A probe is set up to “0” on the gauge in contact with part holder. This probe demonstrates that both least material conditions on B and on D are met with real for all orientations of part into the part holder. This method is equivalent to an extended tolerance zone  $\varnothing 20.19$  for the hole.

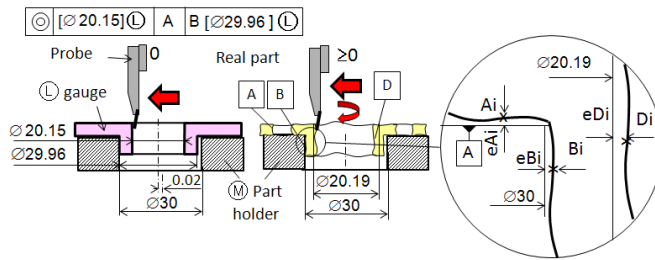


Figure 12 - Inspection with least material condition

This method is equivalent to a tolerance zone extended in each point with the floating of the gauge in this point. In this simple case, the tolerance zone of the hole becomes  $\varnothing 20.19$ .

The association method is obtained in two steps:

Step 1: For each point  $A_i \in A$ ,  $eA_i \leq 0$ ,  $\max(|eA_i|)$  minimal

Step 2: For each point  $B_i \in B$ ,  $eB_i \leq 0$ . For each point  $D_i \in D$ ,  $eD_i$  is minimized and the condition is  $eD_i \geq 0$ .

### 3.5. Specification with an orientation plane

For a unidirectional chain of dimensions going through a cylindrical connection, the specification must contain an orientation plane which is perpendicular to the direction of the chain of dimensions. Specification (2) of figure 13 imposes a  $\varnothing 18 \text{ L}$  cylindrical tolerance zone at least material condition. Orientation plane P is defined in the nominal model. It indicates that it is sufficient for the tolerance zone to be respected on the two generatrices of the cylinder in the plane perpendicular to the orientation plane.

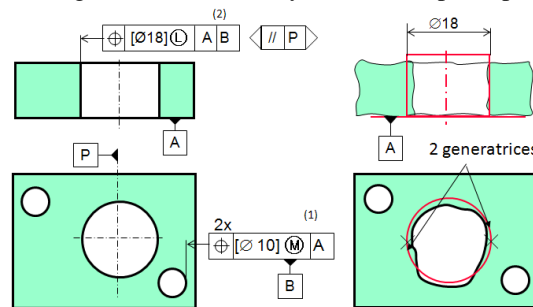


Figure 13 - A tolerance zone with an orientation plane

### 3.6. The margin on the maximum material specification

At maximum or least material condition, the specification is met if the tolerance zone is respected. The margin is obtained by maximizing the smallest deviation between the actual specified surface and the limit of the tolerance zone. The specification is met if the margin is positive.

For a specification with a datum at maximum or least material condition, the margin on the datum must also be positive or zero. If the condition on the datum fails, the defect can be characterized by the margin on the datum and the margin on the tolerated element, ignoring the modifier on the datum.

Figure 14a contains two specifications. The margin on location (1) is  $m_1$  between the actual holes and the limit of the  $\varnothing 10.1 \text{ M}$  tolerance zones (figure 14b). The margin on location (2) is defined when the holes B respect the tolerance zone on the datum. Margin  $m_2$  is defined between the actual hole and the limit of the  $\varnothing 29.9 \text{ L}$  tolerance zone (figure 14c).

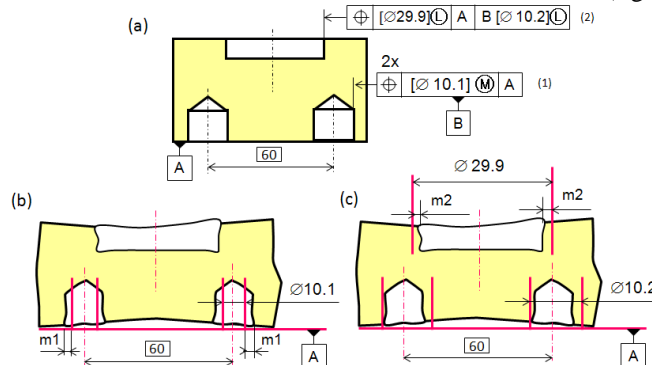


Figure 14 - The margin at maximum/least material condition

When measuring a set of parts produced on one machine tool, if  $\bar{m}$  and  $\sigma$  are respectively the average and the standard deviation of the margins, the capability of the process is:

$$C_{pk} = \frac{\bar{m}}{3\sigma} \quad (1)$$

#### 4. Applications

##### 4.1. Dimensioning of a mechanism

The mechanism shown in figure 15 imposes a height requirement X at the end of the shaft with respect to the datum A of the body. The cylindrical connections have some clearance.

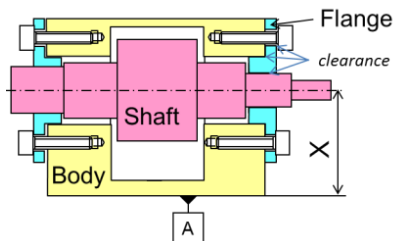


Figure 15 - The mechanism considered

Figure 16 shows the dimensioning of the shaft. Specification (1) concerns the composed surface A. In order to ensure fittability, the two actual cylinders must be positioned, if possible, within two coaxial cylinders  $\text{Ø}30\text{M}$  and  $\text{Ø}20\text{M}$ . To meet requirement X, coaxiality (2) imposes on the axis of the actual cylinder a tolerance zone  $\text{Ø}0.2$ , centered on two coaxial cylinders  $\text{Ø}29.9\text{L}$  and  $\text{Ø}19.9\text{L}$  which must lie within the material of the actual cylinders A.

The general tolerancing is prescribed by specification (3), which must contain no modifier.

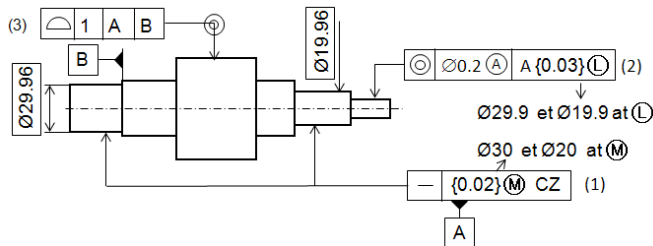


Figure 16 – Tolerancing of the shaft

The dimensioning of the flange is shown in figure 17. Position (1) guarantees the fittability of the flange on the body under two conditions: the actual cylinder B must respect the tolerance zone  $\text{Ø}30\text{M}$  perpendicular to plane A; the 4 holes must respect the 4 tolerance zones  $\text{Ø}8,2\text{M}$  in nominal location with respect to A|B to allow the penetration of the 4 screws. Diameter specification (2)  $\text{Ø}8.5\text{L}$  limits the contact pressure under the heads of the four screws.

Coaxiality (3) sets a tolerance zone  $\text{Ø}20.02\text{M}$  to ensure the fittability of the shaft within the flange.

Coaxiality (4) sets a tolerance zone  $\text{Ø}20.15\text{L}$  to guarantee the accuracy of the assembly in order to meet requirement X.

For requirement X to be met, the maximum clearance between the flange and the body is limited by specifications (3) and (4) of the flange, which set the same tolerance  $\text{Ø}29.96\text{L}$ . The maximum clearance between the flange and the shaft is limited by specification (2) of the shaft and specification (4) of the flange.



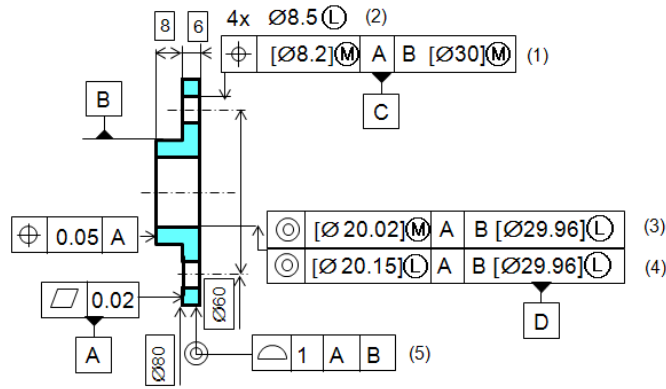


Figure 17 - Tolerancing of the flange

#### 4.2. An assembly with a complex surface

Figure 18 shows the dimensioning principle for a complex connection made with a conical triangular geometry. The primary connection is provided by plane A, while the centering and orientation are given by surface B. Flatness (1) yields the quality of the contact. The position at maximum material condition (2) provides the fitability of the connection. Diameter (3) provides the fitability in hole C. Coaxiality (4) guarantees the accuracy of the assembly by setting a least material tolerance zone for surface B.

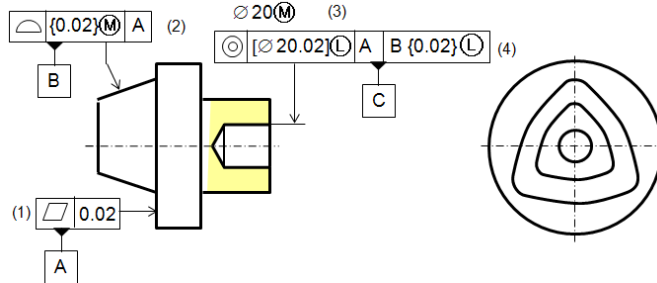


Figure 18 - Assembly with a complex surface

#### 4.3. Calculation of a 3D tolerance stack-up

The mechanism shown in figure 19a includes a slide which is supposed to move along the two shafts. One must determine the minimum clearance between the shafts and the slide. The dimensioning of the body is shown in figure 19b. The deviation  $e$  is maximum when the holes respect the orientation tolerance zone  $\text{Ø}20.1\text{Ⓢ}$  while reaching the limit of the position tolerance zone  $\text{Ø}20.2\text{Ⓢ}$ . Without composite specification, orientation tolerance zones are not parallel to position tolerance zone.

The worst-case deviation is:

$$e = (DL - ds)/2 + (Do - ds).L/E$$

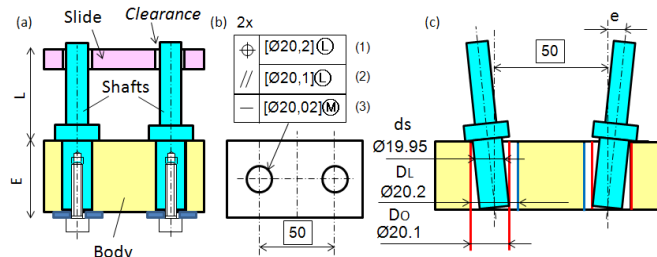


Figure 19 - 3D tolerance stack-up

### 5. Conclusion

The specifications illustrated in figure 16 and figure 17 correspond to exactly the same definitions as in the ISO 2692:2014 standard, except that the dimensions of the tolerance zones are written directly without requiring the local dimensions to be controlled, which is unnecessary because these are already verified through the specifications at  $\text{Ⓜ}$  and  $\text{Ⓢ}$ .

Thus, this new writing is complementary to the current writing and does not generate any conflict. The additional information concerning the management of the primary flutter and the use of orientation planes can be applied to classical specifications. This

writing simplifies the dimensioning process considerably. It improves its readability and facilitates its decoding by analysis, tolerancing and metrology software.

The complete dimensioning requires a maximum material specification to guarantee the fittability of each interface and a least material specification for each link between two interfaces. The extension to complex surfaces opens multiple possibilities.

This proposal could lead to an amendment to ISO 2692 standard which would not alter the body of the current norm.

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