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The distinction of instability and turbulence between the Taylor-Couette flow and electrically driven flow in annular channel

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Abstract

The comparative study of turbulence in rotating flow under uniform axial magnetic field is considered. Two different systems of Taylor-Couette flow (TCF) and electrically driven flow (EDF)[1] are analyzed in the same annulus geometry applying fully-3D numerical simulations [2]. The approximation of low magnetic Reynolds number, corresponding to liquid metal flows, is used. In the case of TCF, the shear flow is driven by inner cylinder, and the EDF is driven by the mean Lorentz force, which is a product of applied field and electric currents injected in the radial direction. The latter mimics the case of pressure driven flows. Therefore, despite similar geometry, the mechanisms of generation of shear and turbulence are different, as well as potential routes to the suppression of eddies by magnetic field. The resemblances and differences of turbulent behaviors of both flows are revealed in present study.

Key words: Taylor-Couette flow, electrically driven flow, turbulence

Introduction

Taylor-Couette flow (TCF, hereafter) is the typical shear flow driven by rotating inner cylinder; analogous to the Dean flow, electrically driven flow (EDF, hereafter) is the typical Poiseuille flow in which the pressure gradient is replaced by the Lorentz force. For the both flows, the electrical conducting fluid is confined between two concentric cylinders with the same geometry, e.g. aspect ratio and radius ratio.

Apparently, the mechanisms of these two flows are distinct. However, unlike the plane Couette flow and the plane Poiseuille flow, due to the presence of the centrifugal instability in concentric cylinders, there will be more different and interesting turbulent behaviors. Meanwhile, comparing these two flows in the same geometry, there also will be some similar and dissimilar behaviors.

Via the comparative study of Taylor Couette flow and electrically conducting flow, we hope to reveal the differences of the turbulent behaviors.

We consider the flow of the incompressible viscous electrically conducting fluid with constant kinematic viscosity ν , electrical conductivity σ , magnetic diffusivity λ , magnetic permeability μ_0 , and density ρ , contained in the annulus between two concentric cylinders of radii r_1 and r_2 . The flow is subject to an axial uniform magnetic field $\vec{B} = B \cdot e_z$, where e_z is the unit vector in axial direction.

The important geometrical parameters are the radius ratio $\eta = R_1/R_2$ and $\Gamma = \text{height}/\text{gapwidth}$

For Taylor-Couette flow, we chose the velocity of inner cylinder as the velocity scale, $u_0 = \Omega R_1$. For electrically driven flow, the velocity scale is based on the difference of the voltage of two cylinders, $u_0 = \Delta\phi / (Bd)$. Reynolds number is

$Re = u_0 d / \nu = \Delta\phi / (B\nu)$, and Hartmann number is $Ha = Bd \sqrt{\sigma / \rho \nu}$.

Laboratory liquid metals usually have very small magnetic Reynolds number ($Re_m = \Omega R_1 d / \lambda$, where $\lambda = 1 / (\sigma \mu_0)$).

Thereafter, the governing equations written in dimensionless form are,

$$\frac{\partial \bar{u}}{\partial t} + (\bar{u} \cdot \nabla) \bar{u} = -\nabla p + \frac{1}{Re} (\nabla^2 \bar{u}) + \frac{Ha^2}{Re} (-\nabla \phi + \bar{u} \times \bar{e}) \times \bar{e} \quad (1)$$

$$\nabla \cdot \bar{u} = 0 \quad (2)$$

$$\nabla^2 \phi = \nabla(\bar{u} \times \bar{e}) \quad (3)$$

$$\bar{j} = (-\nabla \phi + \bar{u} \times \bar{e}) \quad (4)$$

where t is the dimensionless time, $\bar{u}(u, v, w)$ are the dimensionless velocities components in the azimuthal, axial and radial direction, respectively, p is the dimensionless pressure, $\bar{e}(e_\theta = 0, e_z = 1, e_r = 0)$ are the components of the magnetic field in three directions respectively, for cylindrical coordinates (θ, z, r) .

In the present study, we use periodical boundary condition in axial direction, and the dimensionless domain length is $\Gamma = 2\pi$. The radius ratios are $\eta = 0.8$ in the both flows.

In TCF, the potential of both cylinders are 0; and in EDF, the potential of inner and outer cylinder are 0 and 1 respectively. For velocity, no-slip boundary condition is applied for both cylinders.

Numerical method

The equation system (1-4) is solved numerically using the finite difference scheme via direct numerical simulation. The scheme is of the second order of approximation and is based on the spatial discretization, which is nearly fully conservative in regard of mass, momentum, kinetic energy, and electric charge conservation [3, 4]. The explicit Adams-Bashforth/Backward-Differentiation method of the second order is employed for the time discretization. The velocity and pressure are decoupled by the projection method. At every time step, the two Poisson equations—the projection method equation for pressure and the equation for potential (3) are solved using FFT in the azimuthal direction and cyclic reduction direct solver(a part of the software package Fishpack) in the meridional plane. The OpenMP interface for shared memory multiprocessors is used for the parallelized algorithm.

More details of the numerical method are described in [2].

Results

As the definitions of Re are not same. So, we use the similar Re_{mean} based on the mean azimuthal velocity as the standard for the comparison. The cases that we consider here are the TCF for $Re = 4000$ and $Ha = 0$, and the EDF for $Re = 50000$ and $Ha = 1$, which the corresponding $Re_{meanTCF} = 1780$ and $Re_{meanEDF} = 1810$ are similar.

It is possible for us to compare the two results. The velocity fluctuations are defined according to the average in time, in axial and azimuthal direction [5],

$$u' = u - \langle u \rangle_{\theta z t} \quad (5)$$

where u' is the velocity fluctuation due to the turbulence and mean Taylor vortex motion(or mean azimuthal vortex motion in EDF).

The velocity fluctuations are defined according to the average in time and azimuthal direction,

$$u'' = u - \langle u \rangle_{\theta t} \quad (6)$$

where u'' is the velocity fluctuation due to the turbulence.

We compare the root mean square velocity fluctuations in three directions and the kinetic energy of velocity fluctuations in two flows.

Kinetic energy of velocity fluctuations of the turbulence and mean Taylor vortex motion can be written as

$$E_{total} = 0.5 \langle u'^2 + v'^2 + w'^2 \rangle_{\theta z t} \quad (7)$$

Kinetic energy of velocity fluctuations of the turbulence is

$$E_{turbulence} = 0.5 \langle u''^2 + v''^2 + w''^2 \rangle_{\theta z t} \quad (8)$$

Then, we can obtain the contribution of kinetic energy of velocity fluctuations from the mean Taylor vortex motion(or mean azimuthal vortex motion in EDF),

$$E_{TV} = E_{total} - E_{turbulence} \text{ (for TCF),} \quad (9)$$

$$E_V = E_{total} - E_{turbulence} \text{ (for EDF)} \quad (10)$$

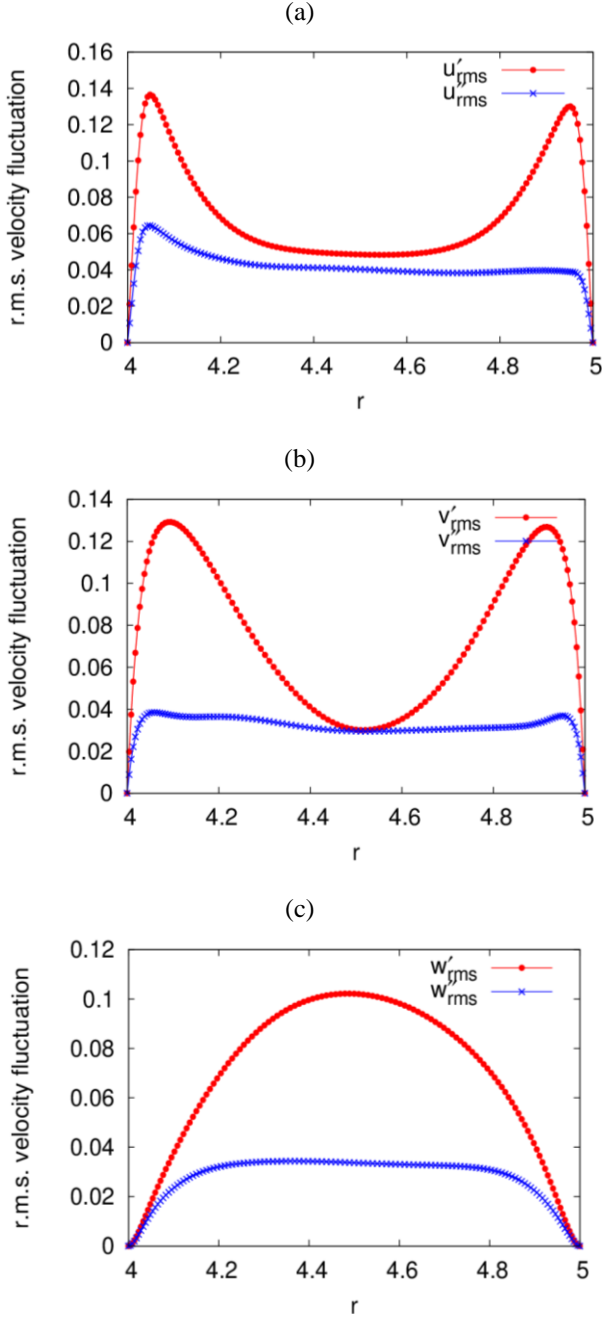


Fig. 1: r. m. s. velocity fluctuations in TCF.

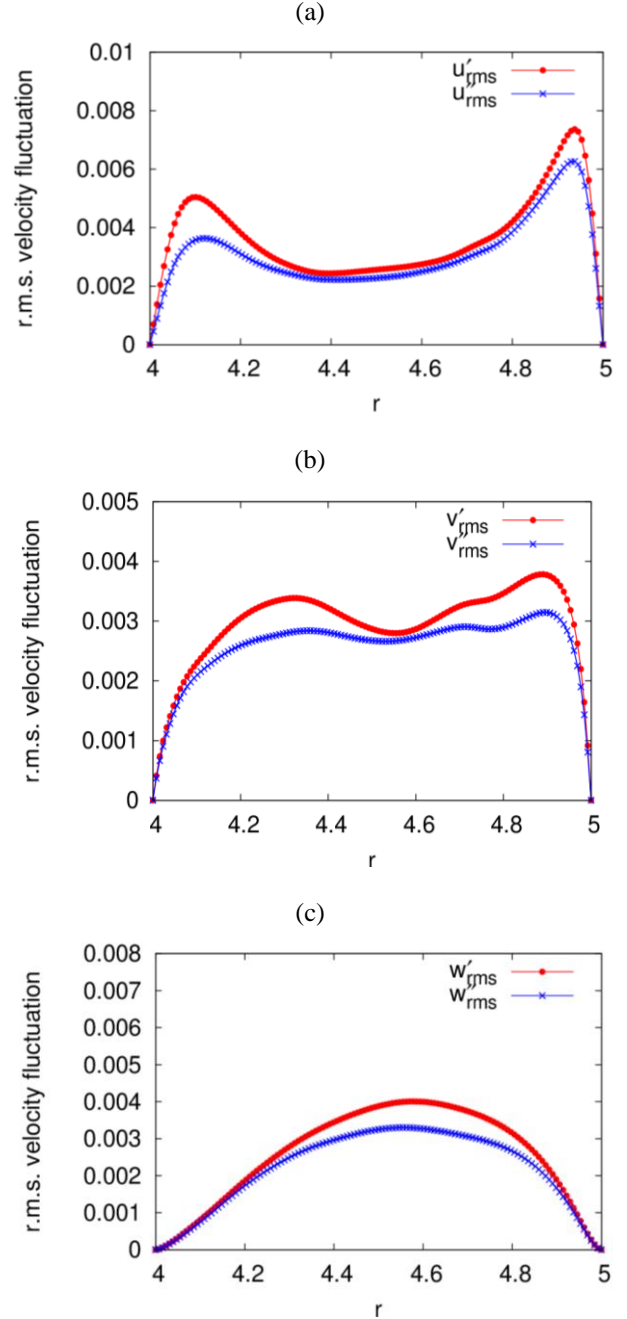


Fig. 2: r. m. s. velocity fluctuations in EDF.

In Fig. 1(a), for the azimuthal velocity fluctuations in TCF, near the both cylinders, the total fluctuations are almost the same and much larger than those in the middle part; the turbulent fluctuations near the inner cylinder are largest which means that the mean flow (Taylor vortex flow) contributes more near the outer cylinder. In EDF, as shown in Fig. 2(a), the total fluctuations near the both cylinders are also larger than those in the middle part, however, the strength of total and turbulent fluctuations near the outer cylinder is larger than the inner cylinder. And the contribution of mean flow is small.

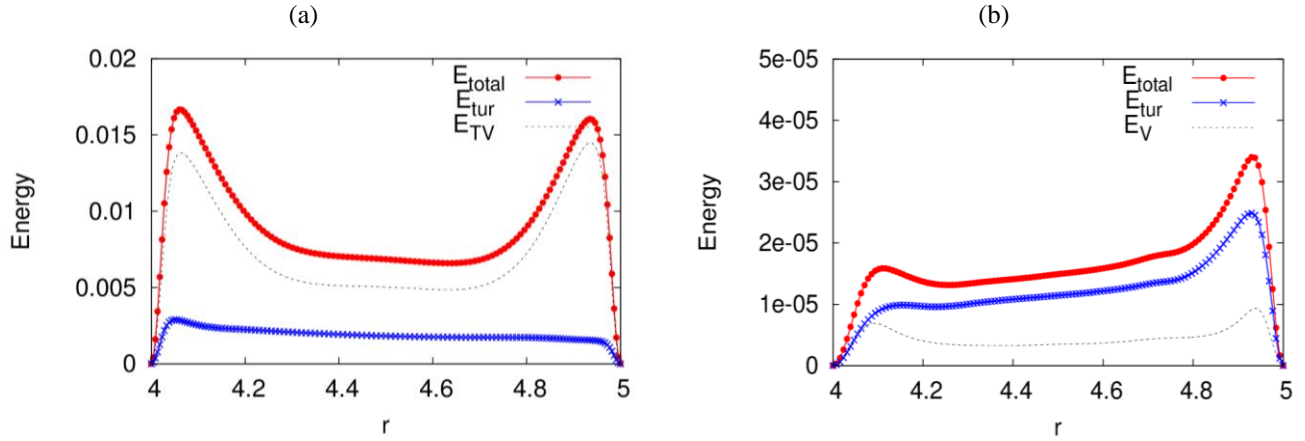


Fig. 3: Kinetic energy of velocity fluctuations, including total energy (turbulence and mean TV motion), turbulence alone and Taylor vortex alone. (a) in TCF, (b) in EDF.

For the axial velocity fluctuations (in Fig. 1(b) and Fig. 2(b)), there are two peaks adjacent to both cylinders in TCF respectively, which are not obvious in EDF.

For the radial velocity fluctuations (in Fig. 1(c) and Fig. 2(c)), in TCF more fluctuations are from mean flow and these fluctuations locate at the middle part of the gap. In EDF, the fluctuations locate at the middle gap, but much closer to the outer cylinder.

Fig. 3 are two plots of kinetic energy of fluctuations. In TCF, the turbulent intensities in two shear layers, near the two cylinders are largest, and the most part of the kinetic energy of total fluctuations are the contributions from the mean azimuthal motion (Taylor vortex flow), especially near the outer cylinder. But, for the EDF, the situation is opposite. The turbulent fluctuations supply the most part of the total fluctuations, so the effect of mean azimuthal flow is weak. And the turbulent intensities near the outer cylinder are much larger than those near the inner cylinder in EDF.

Conclusions

In the present study, with the radius ratio $\eta = 0.8$ and $Re_{mean} \approx 1800$, we compare the root mean square velocity fluctuations and kinetic energy of fluctuations in Taylor-Couette flow and electrically driven flow. The contributions from the mean TV motion (or vortex flow in EDF) play important roles in these two turbulent flows, and the contributions of mean flow in TCF are much larger than that in EDF. The mean motion in EDF has less effect on the turbulence. For the both flows, the velocity fluctuations of the azimuthal and axial velocity near the wall are stronger than those in the middle region. For the EDF the turbulent fluctuations near the outer cylinder are larger than that near the inner cylinders; for the TCF, turbulent fluctuations near the cylinders are same.

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