Observer design for a class of nonlinear systems under a persistent excitation

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June 19, 2016

Abstract

The problem of state reconstruction from input and output measurements for nonlinear time delay systems remain open in many cases. In this paper we propose an adaptive observer to solve this problem for a class of unknown variable time-delay nonlinear systems where the state matrix depends on the input persistency excitation. To achieve this we combine the use of a Kalman-like observer with a suitable choice for the Lyapunov-Krasovskii functional. This is done under a sufficient number of hypothesis to guarantee the convergence of the observer inside a sphere depending of the delay upper bound. The proposed strategy is tested in simulation by considering a mixed piece-wise and sinusoid time delay function and its efficiency when the problem of persistency excitation occurs.

Keywords: Nonlinearity, Robust Observers, Time-Delay, Lyapunov Function.

1 Introduction

Time delay systems are widely used in many applications areas since time delay tends to be considered as an inherent property of many systems. This led to the investigation of an observer design for such systems in the recent years, delay can be present in the state, in the input or the output and can be constant ([9]) or time-varying ([18] , [2]). The methods used to solve this problem consist in different observation approaches from an asymptotic approach to sliding mode, as well as many others, for both linear and non-linear systems, but many of those methods concern only a particular case. In the perspective to be more general, the delay can be considered unknown as in [14] and [15] .

Recently, this become an important center of interest (see [16] , [6], [20] , [7], [13]). In [11] an observer for nonlinear systems in triangular form with variable

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and bounded state delay is described. The approach is an extension of known techniques ([5]) for time-varying delays, where delay is considered as a disturbance and a robust observer is developed ([8] , [19] , [12], [17] , [2]). Notice that in this approach only the state is estimated whereas the delay plays the role of an unknown disturbance. In other cases a robust observer might still be used, but its improvement is the use of the time delay which is identified separately. Among all the topics of time-delayed dynamical models, identification of time delays has practical importance, analogous to the significant role of parameters estimation for dynamical systems described by ordinary differential equations. Nevertheless, identification of time delays is no easy work, because models with time delays generally fall into the class of functional differential equations with infinite dimensions ([1], [3] , [4], [22], [21]). The need for robust observer in the case of non-linear time-delay systems is still present since several methods using the delay identification rely on the existence of such an observer.

In the present work, we propose a robust observer inspired by the work in [10] but for a more general case we consider a class of non-linear time-delay systems with bounded state and delay with the triangular matrix in the state equation depending nonlinearly of the input. The first contribution of this paper is the conception of an observer with guaranty practical stability (which is proven by using the proposed Lyapunov-Krasovskii functional), the observation error converges to a ball depending on the upper bound of the chosen variable delay. The second main contribution leads to propose a way to solve the problem of singularity on the input signal. More precisely there is a condition of persistency on one of the variable present in the state equation which can led to a loss of observability on some of the state variables. The proposed solution is to switch to an estimator when the condition is not met to ensure a rapid convergence when the observability is regained.

The paper is organized as follows. The system description is presented in Section II. In Section III, the observer design and its convergence analysis for a class of time-delay nonlinear systems in triangular form are given. Following in Section IV, simulation results highlight the performances of the proposed observer and the problem of singularities on the input are solved. Finally, in Section VI, some concluding remarks are given.

2   System description

The class of the chosen system consists in a time-delay non-linear system in strictly triangular form:

$$\Sigma_{\tau(t)} : \begin{cases} \dot{x}(t) = A(u(t))x(t) + \Psi(x(t), x_{\tau(t)}, u(t), u_{\tau(t)}), \\ t \geq 0 \\ y(t) = Cx(t), \\ x(s) = \varphi(s), \quad \forall s \in [-\tau^*, 0] \end{cases} \tag{1}$$

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where $x_{\tau(t)} = x(t - \tau(t))$ and $u_{\tau(t)} = u(t - \tau(t))$ are respectively the delayed state and input, $x(t) \in R^n$ is the state of the system, $u(t) \in R^m$ is the input, $y(t) \in R$ represents the output of the system and

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}, \quad x_{\tau(t)} = \begin{pmatrix} x_{1,\tau(t)} \\ \vdots \\ x_{n,\tau(t)} \end{pmatrix},$$

$$A(u(t)) = \begin{pmatrix} 0 & F(u(t)) \\ 0 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & \cdots & 0 \end{pmatrix},$$

where $x_{i,\tau(t)} = x_i(t - \tau(t))$, for $i = 1, ..., n$ and $F(u(t)) = diag(f_1(u(t)), ..., f_{n-1}(u(t)))$

$\tau(t)$ is a positive and real-value function representing the unknown, variable time delay, affecting both the state and the input of the system which admits τ^* as an upper bound, and $x(s) = \varphi(s), \forall s \in [-\tau^*, 0]$ is an unknown continuous bounded initial function.

The vector function $\Psi(x, x_{\tau(t)}, u, u_{\tau(t)})$ is given by

$$\Psi(x, x_{\tau(t)}, u, u_{\tau(t)}) = \begin{pmatrix} \Psi_1(x_1, x_{1,\tau(t)}, u, u_{\tau(t)}) \\ \Psi_2(x_1, x_{1,\tau(t)}, x_2, x_{2,\tau(t)}, u, u_{\tau(t)}) \\ \vdots \\ \Psi_n(x, x_{\tau(t)}, u, u_{\tau(t)}) \end{pmatrix}$$

where the nonlinearities $\Psi_i(x_1, x_{1,\tau(t)}, ..., x_i, x_{i,\tau(t)}, u, u_{\tau(t)})$ have a triangular structure with respect to $x_1, ..., x_i$ and $x_{1,\tau(t)}, ..., x_{i,\tau(t)}$, for $i = 1, ..., n$.

(A,C) is on observable canonical form and Ψ is triangular inferior with respect to x and x_τ therefore, the system $\Sigma_{\tau(t)}$ (1) is uniformly observable for any input and time-delayed input.

To complete the description of system $\Sigma_{\tau(t)}$ (1), the following assumptions are considered for a delay unknown and variable.

   A1. The state and the input are considered bounded[1], that is $x(t) \in \chi \subset R^n$ (that is a compact subset of R^n) and $u(t) \in U \subset R^m$ (that is a subset of R^m).
A2. The function $\Psi(x, x_{\tau(t)}, u, u_{\tau(t)})$ is globally Lipschitz (on χ) w.r.t x, $x_{\tau(t)}$ and $u_{\tau(t)}$, uniformly w.r.t. u.
A3. The time-varying delay satisfies the following properties:
i) $\exists \tau^* > 0$, such that $\sup(\tau(t))_{t \geq 0} \leq \tau^*$.
ii) $\exists \beta > 0$, such that $1 - \dot{\tau}(t) \geq \beta$.

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[1]The boundedness of the state excludes implicitly all initial conditions that generate unbounded state.

3 Observer design

Consider system (1), then an observer for the class of systems of the form (1) is given by

$$O_{\tau^*} : \begin{cases} \dot{z}(t) & = & A(u(t))z(t) + \Psi(z(t), z_{\tau^*}, u(t), u_{\tau^*}) \\ & & -S^{-1}C^T\{\hat{y}(t) - y(t)\} \\ \hat{y}(t) & = & Cz(t) \end{cases} \tag{2}$$

where S is symmetric, positive definite and verify the following equation:

$$-\rho S - A^T S - SA + C^T C = \dot{S}. \tag{3}$$

Proposition 3.1 *For seek of simplicity, hereinafter we have chosen the arbitrarily fixed time-delay observer (2) equal to τ^*. We also chose to have the notation $A = A(u(t))$ to facilitate the reading.*

Let us now define $e = z - x$ the observation error, whose dynamics is

$$\begin{aligned} \dot{e} & = & \{A - S^{-1}C^T C\}e \\ & & +\Psi(z, z_{\tau^*}, u, u_{\tau^*}) - \Psi(x, x_{\tau(t)}, u, u_{\tau(t)}) \end{aligned} \tag{4}$$

Theorem 3.1 *Suppose that assumptions A1-A3 are fulfilled and $\|\varepsilon(s)\| < \delta_1$ for any bounded $\delta_1 > 0$ and $\forall\ s \in [-\tau^*, 0]$. Then, $\exists\ \rho_0 \geq 1$ such that the observation error dynamics (4) is δ_2-practically stable[2] for all $\rho \geq \rho_0$ and for some bounded $\delta_2 > 0$.*

Proof 3.1 *In order to invoke assumptions A1 and A2, the term $\{\Psi(z, z_{\tau^*}, u, u_{\tau^*}) - \Psi(x, x_{\tau(t)}, u, u_{\tau(t)})\}$ is rewritten as follows by adding and subtracting $\Psi(x, x_{\tau^*}, u, u_{\tau^*})$*

$$\begin{aligned} \Psi(z, z_{\tau^*}, u, u_{\tau^*}) - \Psi(x, x_{\tau(t)}, u, u_{\tau(t)}) = & \Psi(z, z_{\tau^*}, u, u_{\tau^*}) \\ & -\Psi(x, x_{\tau^*}, u, u_{\tau^*}) \\ +\bar{\Psi}(x, x_{\tau^*}, u, u_{\tau^*}, & x_{\tau(t)}, u_{\tau(t)}) \end{aligned}$$

where

$$\begin{aligned} \bar{\Psi}(x, x_{\tau^*}, u, u_{\tau^*}, x_{\tau(t)}, u_{\tau(t)}) : & = & \Psi(x, x_{\tau^*}, u, u_{\tau^*}) \\ & - & \Psi(x, x_{\tau(t)}, u, u_{\tau(t)}) \end{aligned} \tag{5}$$

characterizes the difference between the term that depends on the upper bound of the unknown delay and the term which depends on the unknown delay. Define the Lyapunov-Krasovskii candidate functional

$$V(e) = e^T S e + \int_{t-\tau(t)}^{t} exp^{-\frac{\alpha}{2\tau^*}(t-\sigma)} e^T(\sigma)e(\sigma)d\sigma \tag{6}$$

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[2]Roughly speaking, practical stability means that the observation error converges exponentially to a ball B_r with radius $r > 0$.

with α a positive constant defined thereafter. Taking the time derivative of (6) along the trajectories of system (4) and making use of (3), we have

$$
\begin{aligned}
\dot{V}(e) \quad + \quad & \frac{\alpha}{2\tau^*}V(e) \leq -(\rho - \frac{\alpha}{2\tau^*})e^T S e \\
& -(1 - \dot{\tau}(t))e_{\tau(t)}^T e_{\tau(t)} exp^{-\frac{\alpha\tau(t)}{2\tau^*}} \\
& +2e^T S\{\Psi(z, z_{\tau^*}, u, u_{\tau^*}) - \Psi(x, x_{\tau^*}, u, u_{\tau^*})\} \\
& +2e^T S\bar{\Psi}(x, x_{\tau^*}, u, u_{\tau^*}, x_{\tau(t)}, u_{\tau(t)})
\end{aligned}
\tag{7}
$$

The following inequalities hold globally (on χ) thanks to assumption A2

$$
\|\{\Psi(z, z_{\tau^*}, u, u_{\tau^*}) - \Psi(x, x_{\tau^*}, u, u_{\tau^*})\}\|
\tag{8}
$$
$$
\leq \nu\|(z - x)\| + \nu\|(z_{\tau^*} - x_{\tau^*})\| \leq \nu\|e\| + \nu\|e_{\tau^*}\|
$$

$$
\|\bar{\Psi}(x, x_{\tau^*}, u, u_{\tau^*}, x_{\tau(t)}, u_{\tau(t)})\|
\tag{9}
$$
$$
\leq \nu_0\|x_{\tau^*} - x_{\tau(t)}\| + \nu_0\|u_{\tau^*} - u_{\tau(t)}\|
$$

where ν is a Lipschitz constant in (8), and $\nu_0 > \nu_{\bar{\Psi}}$, with $\nu_{\bar{\Psi}}$ a Lipschitz constant of $\bar{\Psi}$, in (9).
From assumption A1, there exists a bounded constant $\nu_1 > \nu_0\nu_{xu}$ such that (9) can be written as

$$
\|\bar{\Psi}(x, x_{\tau^*}, u, u_{\tau^*}, x_{\tau(t)}, u_{\tau(t)})\| \leq \nu_1
\tag{10}
$$

where ν_{xu} is a positive constant which refers to the boundedness of $\|x_{\tau^*} - x_{\tau(t)}\| + \|u_{\tau^*} - u_{\tau(t)}\|$.
Next, writing (7) in terms of $\|e\|$ and $\|e_{\tau^*}\|$, we have the following inequality

$$
\lambda_1 e^T(t)e(t) \leq e^T(t)Se(t) \leq \lambda_2 e^T(t)e(t)
\tag{11}
$$

where $\lambda_{min}(S) := \lambda_1 > 0$ and $\lambda_{max}(S) := \lambda_2 > 0$ are respectively, the minimum and maximum eigenvalues[3] of S and $\|S\|_2$ is the 2-norm matrix of S satisfying $\|S\|_2 = \vartheta > 0$.
Taking into account (8), (10), (11) and using assumptions A3 i) and A3 ii), then (7) can be expressed as

$$
\begin{aligned}
\dot{V}(e) + \frac{\alpha}{2\tau^*}V(e) \quad \leq \quad & -\rho_1(\rho, \alpha)\|e\|^2 + \rho_2\|e\|\|e_{\tau^*}\| \\
& -\beta\|e_{\tau^*}\|^2 exp^{-\frac{\alpha}{2}} + \mu_1\|e\|
\end{aligned}
\tag{12}
$$

where $\rho_1(\rho, \alpha) = \lambda_1(\rho - \frac{\alpha}{2\tau^*}) - 2\lambda_2\nu$, $\rho_2 = 2\lambda_2\nu$, $\mu_1 = 2\nu_1\vartheta$.
Furthermore, using the following inequality $\mu_1\|e\| < \frac{\eta}{2}\|e\|^2 + \frac{1}{2\eta}\mu_1^2 < 0$ with $\eta \in (0, 1)$, (12) can be expressed only in function of quadratic errors terms. It follows that

$$
\begin{aligned}
\dot{V}(e) + \frac{\alpha}{2\tau^*}V(e) - \frac{1}{2\eta}\mu_1^2 \quad \leq \quad & -(\rho_1(\rho, \alpha) - \frac{\eta}{2})\|e\|^2 \\
& + \quad \rho_2\|e\|\|e_{\tau^*}\| - \beta\|e_{\tau^*}\|^2 exp^{-\frac{\alpha}{2}}.
\end{aligned}
\tag{13}
$$

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[3]λ_1 and λ_2 are obtained directly by solving equation (3), as A and C are given explicitly depending on the dimension n of the system.

Now, the right side of the above inequality can be rewritten as follows

$$- \quad (\rho_1(\rho, \alpha) - \frac{\eta}{2} - \frac{\rho_2^2}{4\beta exp^{-\frac{\alpha}{2}}})\|e\|^2 - \frac{\rho_2^2}{4\beta exp^{-\frac{\alpha}{2}}}\|e\|^2$$

$$+ \quad \rho_2\|e\|\|e_{\tau^*}\| - \beta\|e_{\tau^*}\|^2 exp^{-\frac{\alpha}{2}}$$

$$= \quad -(\rho_1(\rho, \alpha) - \frac{\eta}{2} - \frac{\rho_2^2}{4\beta} exp^{\frac{\alpha}{2}})\|e\|^2$$

$$-\{\frac{\rho_2}{2\sqrt{\beta} exp^{-\frac{\alpha}{4}}}\|e\| - \sqrt{\beta}\|e_{\tau^*}\| \ exp^{-\frac{\alpha}{4}}\}^2$$

To satisfy inequality (13), all we need to do is to choose α and ρ such that $(\rho_1(\rho, \alpha) - \frac{\eta}{2} - \frac{\rho_2^2}{4\beta} exp^{\frac{\alpha}{2}}) > 0$. Set $\alpha = \frac{2}{q} \ln \rho$. Then, $\exists \rho_0 \geq 1$ such that the following inequality is verified

$$\rho - \frac{1}{q\tau^*} \ln \rho - \frac{\nu^2 \lambda_2^2}{\lambda_1 \beta} \sqrt[q]{\rho} - \frac{\eta}{2\lambda_1} - \frac{2\lambda_2 \nu}{\lambda_1} > 0 \tag{14}$$

where $\rho > \rho_0$ and $q \geq 2$. Thereafter, (13) becomes $\dot{V}(e) \leq -\frac{\ln\rho}{q\tau^*} V(e) + \frac{\mu_1^2}{2\eta}$, which is equivalent to

$$V(e(t)) \quad \leq \quad exp^{-\frac{\alpha}{2\tau^*}t} V(e(0)) + \frac{2\tau^*\Omega}{\alpha}\{1 - exp^{-\frac{\alpha}{2\tau^*}t}\}$$

$$\leq \quad exp^{-\frac{\alpha}{2\tau^*}t} V(e(0)) + \frac{2\tau^*\Omega}{\alpha} \tag{15}$$

where $\Omega = \frac{\mu_1^2}{2\eta}$.

Now, the objective is to prove the uniform practical stability of (4). For that, (15) should be rewritten in terms of the observation error norm. Then, from [10], the following inequality is obtained

$$V(e(t)) \quad \leq \quad \lambda_2\|e(t)\|^2 + \delta_M(\alpha, \tau^*) \max_{s\in[t-\tau^*,t]} \|e(s)\|^2$$

$$\leq \quad \delta_M^*(\alpha, \tau^*) \max_{s\in[t-\tau^*,t]} \|e(s)\|^2 \tag{16}$$

where $\delta_M^*(\alpha, \tau^*) = \lambda_2 + \delta_M(\alpha, \tau^*)$ with $\delta_M(\alpha, \tau^*) = \frac{2\tau^*(1-exp^{-\frac{\alpha}{2}})}{\alpha}$.

By using (11) and (16), it follows that

$$\lambda_1\|e(t)\|^2 \quad \leq \quad V(e(t)) \leq exp^{-\frac{\alpha}{2\tau^*}t} V(e(0)) + \frac{2\tau^*\Omega}{\alpha}$$

$$\leq \quad \delta_M^*(\alpha, \tau^*) exp^{-\frac{\alpha}{2\tau^*}t} \max_{s\in[-\tau^*,0]} \|e(s)\|^2$$

$$+ \quad \frac{2\tau^*\Omega}{\alpha} \tag{17}$$

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*where an upper bound $V(e(0))$ comes from (16) by setting $t = 0$.
Consequently, (17) can be written in terms of $\|e(t)\|$, as follows*

$$\|e(t)\| \quad \leq \quad K(\alpha, \tau^*) exp^{-\frac{\alpha}{4\tau^*}t} \max_{s \in [-\tau_c, 0]} \|e(s)\| + \Gamma \tag{18}$$

*with $K(\alpha, \tau^*) = \sqrt{\frac{\delta_M^*(\alpha, \tau^*)}{\lambda_1}}$ and $\Gamma = \sqrt{\frac{2\tau^*\Omega}{\alpha\lambda_1}}$.
Then, there exist $\delta_1 > 0$, $\delta_2 > 0$ and $T_0 > 0$, such that*

$$\|e(t)\| \leq K(\alpha, \tau^*) exp^{-\frac{\alpha}{4\tau^*}T_0} \delta_1 + \Gamma \leq \delta_2, \ \forall t \geq T_0$$

where $\max_{s \in [-\tau^*, 0]} \|e(s)\| \leq \delta_1$. Finally, from the change of variable $e = \Delta_\rho \varepsilon$, the observation error $\varepsilon(t)$ satisfies

$$\|\varepsilon(t)\| \leq \rho^{n-1} \|e(t)\| \leq \rho^{n-1} \delta_2 \tag{19}$$

where δ_2 corresponds to parameter ζ in the ζ-practical stability.

Then, we can conclude that observation error (19) is globally (on χ) δ_2-practically stable. This ends the proof of Theorem 1.

4   Illustrating example

Let us consider the unknown time-variable delay nonlinear system

$$\Sigma_{\tau(t)} : \begin{cases} \dot{x}_1 &=& -\gamma_1 x_{1,\tau(t)}^2 + u x_2 \\ \dot{x}_2 &=& -x_1 u_{\tau(t)} - \gamma_2 x_1 x_{2,\tau(t)} \\ y &=& x_1 \end{cases} \tag{20}$$

where

$$A \quad = \quad \begin{pmatrix} 0 & u \\ 0 & 0 \end{pmatrix},$$

$$\Psi(x, x_{\tau(t)}, u, u_{\tau(t)}) \quad = \quad \begin{pmatrix} -\gamma_1 x_{1,\tau(t)}^2 \\ -x_1 u_{\tau(t)} - \gamma_2 x_1 x_{2,\tau(t)} \end{pmatrix},$$

with $\gamma_1 = \gamma_2 = 0.01$. From this definition of Ψ, $A2$ assumption is satisfied. The input $u = sin(2\pi f t)$ with $f = 50Hz$. It is bounded and from figures (1) and (2) the states are bounded, which makes assumption A1 holds. The function $\tau(t)$ is defined as follows:

$$\tau(t) = (sin(\frac{2\pi t}{50}) + 1)f(t) \tag{21}$$

$$f(t) = \begin{cases} 0 & if & t \in [0, 10] \\ 1 & if & t \in [10, 35] \\ 0 & if & t \in [35, 50] \end{cases} \tag{22}$$

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where the assumption A3 is verified with $\tau^* = 2\ s$. For system (20), an observer O_{τ^*} is designed as (2), with

$$z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix},$$

$$\Psi(z, z_{\tau^*}, u, u_{\tau^*}) = \begin{pmatrix} -\gamma_1 z_{1,\tau^*}^2 \\ -z_1 u_{\tau^*} - \gamma_2 z_1 z_{2,\tau^*} \end{pmatrix}.$$

The initial conditions for the system are $x(0) = [2, 1]^T$, for the observer $z(0) = [1, 2]^T$. $\rho = 10$.

If the time delay is constant and known, that is $\tau(t) = \tau^* = \tau_c$, then assumption $H1$ is satisfied.

Fig. 1 and Fig. 2 show the obtained simulation results. According to the practical stability enhanced in the proof of Theorem 1, it can be seen that the observation error (Fig. 1 and Fig. 2) of the observer converges to a ball with a radius $r > 0$ depending on two parameters: the size of the instantaneous state dynamic variation and the delay difference between the observer and the system. When the delay is equal to zero, the observer behaves like a normal observer in the case of a non-linear system.

On Fig 3, a new input is chosen which arises a problem of persistency, indeed when $A(u(t))$ is not persistent, we lose the observability on x_2 and the observer diverges. The solution proposed is to switch to an estimator when $A(u(t)) = 0$ this is done by cutting the correction term $-S^{-1} C^T \{\hat{y}(t) - y(t)\}$ in equation 2.

5   Conclusion

In this paper, we propose an observer for a class of non-linear time-delay systems in lower triangular form. The practical stability of the observer is guaranteed with an unknown variable but bounded delay under some conditions that have been presented. We have also shown the problem of the loss of observability when there is a lack of persistency and proposed a solution. Simulation results show the gain of the solution along with the practical stability of the observer for both state variables.

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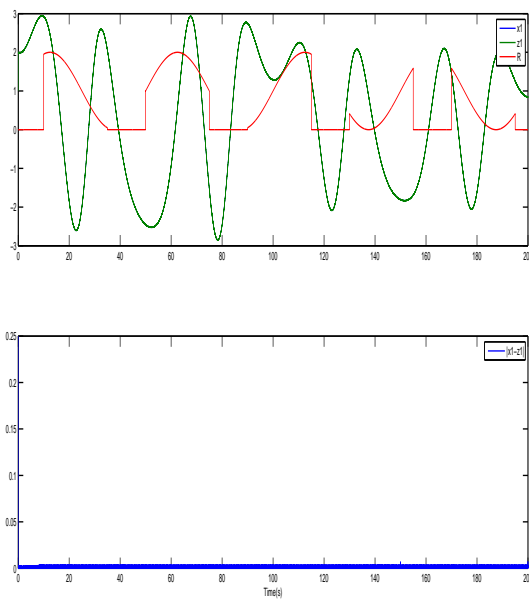
Figure 1: x_1 and its estimate z_1, time-varying delay $\tau(t)$ and the absolute value of the observation error $(z_2 - x_2)$.
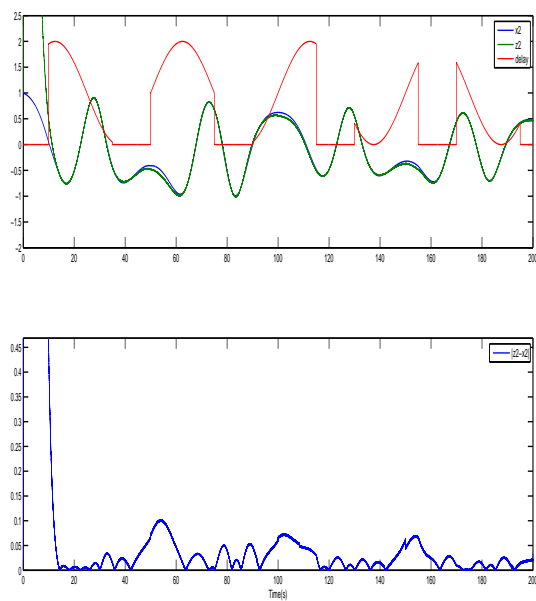
Figure 2: x_2 and its estimate z_2, time-varying delay $\tau(t)$ and the absolute value of the observation error $(z_1 - x_1)$.
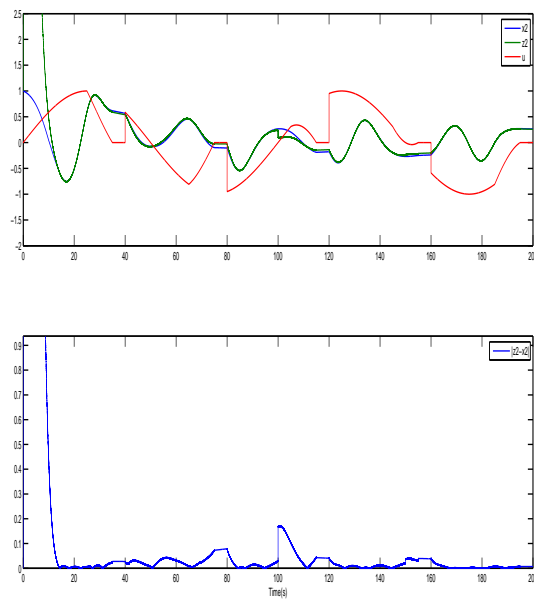
Figure 3: x_2 and its estimate z_2,input $u(t)$ and the absolute value of the observation error $(z_1 - x_1)$.

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