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Alejandro Montoya, Christelle Guéret, Jorge E. Mendoza, Juan G. Villegas. A hybrid metaheuristic for the electric vehicle routing problem with partial charging and nonlinear charging function. 2016. hal-01331293v1

HAL Id: hal-01331293

<https://hal.science/hal-01331293v1>

Preprint submitted on 13 Jun 2016 (v1), last revised 28 Nov 2016 (v3)

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A hybrid metaheuristic for the electric vehicle routing problem with partial charging and nonlinear charging function

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Abstract

This paper considers a variant of the electric vehicle routing problem including realistic assumptions about the charging process: the electric vehicle routing problem with partial charging and nonlinear charging function (eVRP-PNL). To tackle this problem, we propose an iterated local search (ILS) enhanced with heuristic concentration (HC). At each iteration of the ILS, a variable-neighborhood descent (VND) procedure with three different local search operators is applied, and the routes of the local optimum are added in a pool of routes. Once the ILS phase is completed, the HC component assembles the final solution from this pool of routes. One original feature of this approach is the use in the VND component of a local search operator which aims at improving the charging decisions on a route (detour to charging stations and amount of energy recharged). The use of this operator implies the solution of a new problem known as the fixed-route vehicle-charging problem (FRVCP) which consists in defining the charging decisions of a fixed-route. We solve the FRVCP by a heuristic and a mixed-integer linear programming formulation. To test our approach, we propose a new 120-instances testbed. The results show a high performance of the proposed hybrid metaheuristic and the importance of making optimal charging decisions on the routes of electric vehicle routing problems.

Keywords: Vehicle routing problem, Electric vehicles routing problem with nonlinear charging function, Iterated local search (ILS), Matheuristic

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1. Introduction

Electric vehicles are a promising technology for reducing greenhouse gas emissions and transportation costs in goods distribution (Electrification-Coalition 2013, TU Delft 2013). Therefore, several companies have significantly increased the use of electric vehicles in their operations in last years. The adoption of this technology by La Poste and UPS were popular success cases (Kleindorfer et al. 2012, UPS 2008). Since then, several companies of different sectors and sizes have started to use electric vehicles (Nesterova et al. 2015). For instances, the Portuguese postal company CTT operates with 10 small electric vans (Post & Parcel 2014). Heineken started to use the Europe’s largest electric truck to distribute the beer within Amsterdam (Heineken 2013). In Mexico, Bimbo the biggest food company signed orders to buy 100 electric vehicles which will use the energy from a wind farm (Ramirez 2015). In Colombia, a health care company uses electric vehicles for their home healthcare service (Loaiza 2014).

The use of electric vehicles in goods distribution and services introduces a new family of vehicle routing problems (VRPs), the so-called electric VRPs (eVRPs) (Pelletier et al. 2016). These problems extend classical VRPs to consider the limited driving range of electric vehicles and their long battery charging times. Because of the short driving range, eVRP solutions frequently include routes with planned detours to charging stations (CSs) where the vehicles recharge. In general, the work on eVRPs makes different simplifications concerning the capacity of the CSs, the consumption rate, and the modeling of the charging process. This study focuses on a more realistic modeling of the charging process.

To model the battery charging process, eVRP models make assumptions about the *charging policy* and the *charging function approximation*. The former defines how much of the battery capacity can be (or must be) restored when a vehicle visits a CS, and the latter models the relationship between battery charging time and charging level. With respect to the charging policies, the eVRP literature can be classified into two groups: studies assuming *full* and *partial* charging policies. As the name suggests, in full charging policies, the battery capacity is fully restored every time a vehicle reaches a CS. Some studies in this group assume that there is no charging function but rather a constant charging time (Conrad & Figliozzi 2011, Erdoğan & Miller-Hooks 2012, Montoya et al. 2015). This is a plausible assumption in applications where the CSs replace a (partially) depleted battery with a fully charged one. On the other hand, Schneider et al. (2014), Hiermann et al. (2016), Desaulniers et al. (2014), Goeke & Schneider (2015), and Schneider et al. (2015) consider full charging policies with a linear charging function approximation (i.e., the battery level is assumed to be a linear function of the charging time). In their models, the time spent at each CS depends on the battery level when the vehicle arrives and on the (constant) charging rate of the CS. In partial charging policies, the amount of charge (and thus the time spent at each charging point) is a decision variable. To the best of our knowledge, all existing eVRP models with partial charging consider linear function approximations (Felipe et al. 2014, Desaulniers et al. 2014, Sassi et al. 2014, Bruglieri et al. 2015, Schiffer & Walther 2015, Keskin & Çatay 2016).

In practice, using linear approximations to model the charging function may not be re-

alistic. Indeed, it is well documented that the charging level is a concave function of the charging time (Bruglieri et al. 2014, Hõimoja et al. 2012, Pelletier et al. 2015). Furthermore, (as shown in the Part I of the electronic companion) disallowing partial charging and neglecting the nonlinear nature of the charging function may lead to infeasible or overly expensive solutions. Therefore, in this paper we introduce a more realistic eVRP extension: the electric vehicle routing problem with partial charging and nonlinear charging functions (eVPR-PNL).

To solve the eVRP-PNL we propose a hybrid metaheuristic combining iterated local search (ILS) and heuristic concentration (HC). The ILS component uses a variable neighborhood descent (VND) procedure for the local search phase. The VND uses three local search operators. The first two operators are classical relocate and 2-Opt moves. On the other hand, the third one is a specialized operator for improving the charging decisions. This operator relies on solving the *fixed-route vehicle-charging problem* (FRVCP). A new problem defining the charging decisions (i.e., where and how much to charge) of an energy-infeasible fixed-route (i.e., a route infeasible with respect to the energy autonomy of the vehicle). To solve the FRVCP, we propose a heuristic and a mixed-integer linear programming (MILP) formulation. Finally, the HC component assembles the final solution from the set of all routes found in the local optima reached by the ILS.

The main contributions of this research are fourfold: i) we introduce a new eVRP problem that considers realistic assumptions about the charging process (i.e., partial charging and nonlinear charging functions); ii) we present a hybrid metaheuristic that solve the problem with competitive results; iii) we introduce the fixed-route vehicle-charging problem, which arises as a subproblem of the eVRP-PNL, when making charging decisions for a fixed-route, and propose two methods to solve it; iv) we show the importance of making optimal charging decisions when visiting multiple CSs in eVRP-PNL solutions.

The remainder of this paper is organized as follows. Section 2 describes the problem. Section 3 introduces our hybrid metaheuristic. Section 4 discusses the FRVCP and presents the approaches to solve it. Section 5 presents a computational evaluation of the proposed method. Finally, Section 6 concludes the paper and outlines future work. Additionally, the Part I of the electronic companion presents the MILP formulation of the eVRP-PNL and discuss the impact of different charging assumptions on the eVRP-PNL, and the Part II presents the detailed results of our hybrid metaheuristic.

2. Problem description

Formally, the eVRP-PNL is defined on a directed and complete graph $G = (V, A)$. The vertex set $V = \{0\} \cup I \cup F$ is made up of a depot (vertex 0), a set of customers I , and a set of CSs F . Each customer $i \in I$ has a service time p_i . Each CS $i \in F$ has a nonlinear charging function, which is modeled using a piecewise linear approximation (see more details about the charging function and the piecewise linear approximation in the Part I of the electronic companion). This approximation is defined by a set of breakpoints B , where each breakpoint $k \in B$ is associated to a charging time c_{ik} and a charge level a_{ik} (see Figure 1). The set $A = \{(i, j) : i, j \in V, i \neq j\}$ corresponds to arcs connecting vertices of V . Each arc (i, j)

has two associated nonnegative values: a travel time t_{ij} and a energy consumption e_{ij} . The customers are served using an unlimited homogeneous fleet of electric vehicles. All the vehicles have a battery of capacity Q (expressed in kWh) and a maximum tour duration T_{max} . It is assumed that the vehicles leave the depot with a fully charged battery, and that all the CSs can handle an unlimited number of vehicles simultaneously.

For the sake of brevity we present the MILP formulation of the eVRP-PNL in the Part I of the electronic companion. In sum, the routes in eVRP-PNL satisfy the following conditions: each customer is visited exactly once; the level of the battery when the vehicle arrives at any vertex is nonnegative; each route satisfies the maximum-duration limit; and each route starts and ends at the depot.

Since the distance is related directly to the energy consumption, most work on eVRPs with homogeneous fleet use the minimization of the distance as objective function (Schneider et al. 2014, Desaulniers et al. 2014, Hiermann et al. 2016, Keskin & Çatay 2016). However, this objective function ignores an important feature of eVRPs: the charging activity. This may lead to poor charging decisions such as: charging the batteries more than needed, or charging the batteries when their level is high (which degrades the batteries and implies longer charging times given the charging functions profile). All those decisions affect directly the battery degradation cost (which can represent about 3 times the energy cost Becker et al. (2009)) or affect the charging fees at CS applying a cost per time unit (Bansal 2015). Then, to make better decisions with reference to the travel and charging activities, we use the same objective function than Zündorf (2014) and Liao et al. (2016) for related eVRPs: the minimization of the travel time plus the charging time. This objective function can measure the costs associated to the charging activity and the energy cost without being too complex.

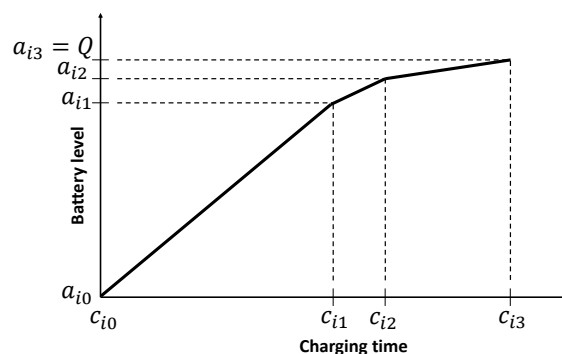


Figure 1: Piecewise linear approximation composed of 3 linear segments for a CS of 22 kW of power, charging a battery of 16 kWh.

With the aim that the reader can understand how the charging assumptions of this problem are integrated in the routing decisions, we present a numerical example of an eVRP-PNL solution. Figure 2 shows a solution with 7 customers and 3 CSs. The CSs have different technologies (slow and fast), and each technology has a particular piecewise-linear charging function. In the charging functions, variables q_i and o_i specify the battery levels

when a vehicle arrives at and departs from CS $i \in F$. The charging function maps these variables to charging times s_i and e_i , in order to estimate the time spent at CS (Δ_i). In this example, Route 1 does not visit any CS, because its total energy consumption is less than the battery capacity. On the other hand Route 2 visits CS 8. In this route, the vehicle arrives at the CS with a battery level $q_8 = 1.0$, and it charges the battery to a level $o_8 = 6.0$. To estimate the time spent at the CS, we use the piecewise-linear charging function: the charging time associated to q_8 and o_8 are $s_8 = 0.8$ and $e_8 = 6.0$, then the time spent at CS 8 is $\Delta_8 = 6.0 - 0.8 = 5.2$. The duration of Route 2 is then the sum of the travel time (13.0), the charging time (5.2), and the service time (1.0), that is 19.2 which is lower than T_{max} . And its cost is 18.2 (travel time + charging time). Finally, Route 3 visits CSs 10 and 9; and it spends $\Delta_{10} = 7.2$ and $\Delta_9 = 1.6$ time units charging in these CSs, respectively.

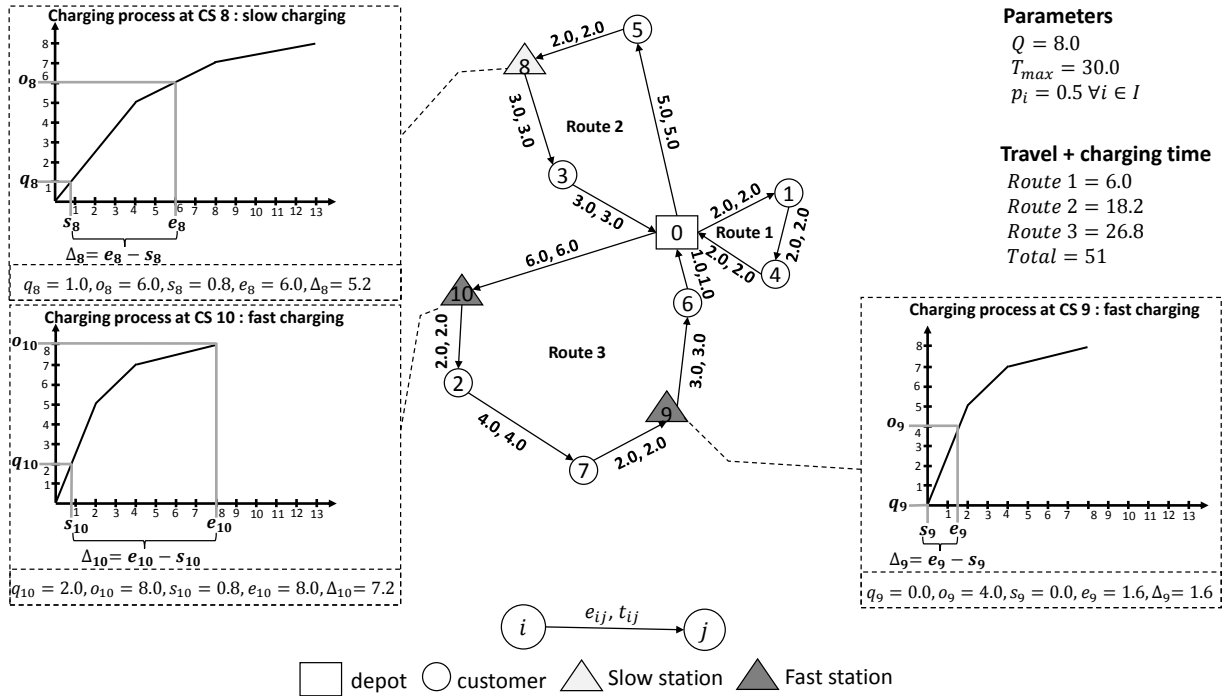


Figure 2: Example of a feasible eVRP-PNL solution

3. Hybrid metaheuristic

To solve the eVRP-PNL we developed a hybrid metaheuristic combining ILS (Lourenço et al. 2010) an HC (Rosing & ReVelle 1997). Figure 3 presents the general structure of ILS+HC for the eVRP-PNL. To build the initial solution we follow the route-first cluster-second approach which uses a constructive heuristic to build a TSP tour visiting all the customers, followed by a splitting procedure to retrieve an eVRP-PNL solution. Then each iteration of the ILS tries to improve the current solution with a VND based on three local search operators: relocate, 2-Opt, and global charging improvement (GCI). At the end of the iteration, we update the best solution and add the routes of the local optimum to a pool

of routes Ω . To diversify the search, we concatenate the routes of the local optimum to build a new TSP tour, and subsequently perturb the new TSP tour. We start a new iteration by splitting the perturbed TSP tour. After K iterations the ILS component stops, and we carry out the HC. In this phase, the hybrid metaheuristic solves a set partitioning problem over the set of routes Ω using the best solution found as upper bound. In the remainder of this section, we describe the main components of our method.

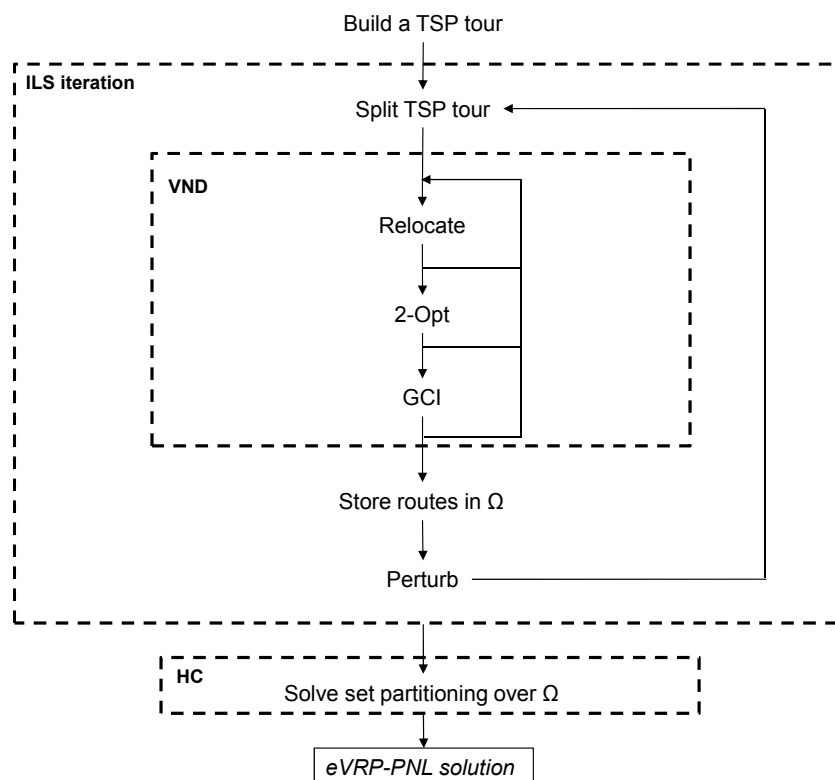


Figure 3: General structure of ILS+HC

3.1. Initial solution

We generate the initial TSP tour using the simple and well-known nearest neighbor heuristic (NN). For a description of NN see Rosenkrantz et al. (1974)

3.2. Split

To extract a feasible solution from a TSP tour, our approach uses an adaptation of the splitting procedure introduced by Prins (2004). The splitting procedure builds a directed acyclic graph $G^* = (V^*, A^*)$ composed of the ordered vertex set $V^* = (v_0, v_1, \dots, v_i, \dots, v_n)$

and the arc set A^* . Vertex $v_0 = 0$ is an auxiliary vertex, and each vertex v_i represents the customer in the i th position of the TSP tour. Arc $(v_i, v_{i+n_r}) \in A^*$ represents a feasible route $r_{v_i, v_{i+n_r}}$ with an energy consumption $e_{r_{v_i, v_{i+n_r}}}$ starting and ending at the depot and visiting customers in the sequence v_{i+1} to v_{i+n_r} .

Since TSP tour includes only customers, route $r_{v_i, v_{i+n_r}}$ may not satisfy the energy constraint (i.e., the route's energy consumption is greater than Q). If it does not, we try to repair it by inserting visits to CSs. If the visits to CSs increases the duration of the route beyond T_{max} then we do not include the arc associated with the route in G^* . To obtain an energy-feasible route, we solve a FRVCP, which is explained in Section 4. To obtain a feasible eVRP-PNL solution, the splitting procedure finds the set of arcs (i.e., routes) along the shortest path connecting 0 and v_n in G^* .

3.3. Variable neighborhood descent

To improve the solution generated by the splitting procedure we use a VND based on three local search operators: relocate, 2-Opt and GCI. For the first two operators we use intra-route and inter-route versions with best-improvement selection, without changing the current position of the CSs. Since the relocate and 2-Opt change the sequence of customers, the current charging decisions may not be valid, then to recalculate the energy charged at each CS we follow the approach proposed by Felipe et al. (2014): the vehicles charge at each CS only the necessary energy to reach the next CS or the depot. Note that this procedure may lead to a solution that is not feasible.

As its name suggest, the global charging improvement operator (GCI) optimizes the charging decisions of the routes of a given solution. For each route that visits CSs, GCI eliminates all the CSs of the sequence of the route. If the elimination of the CSs turns the route infeasible with respect to the energy constraint, then (as in the split procedure) we solve a FRVCP following the fixed sequence of the customers in the route to reinsert new CSs and to make optimal charging decisions.

3.4. Perturb

To diversify the search our approach concatenates the routes of the current best solution to build a TSP tour. Then, we perturb the resulting TSP tour with a randomized double bridge (DB) operator (Lourenço et al. 2010) and then apply the split procedure to obtain a new eVRP-PNL solution.

3.5. Heuristic concentration

The heuristic concentration component solves a set partitioning formulation over the pool of routes Ω . The objective is then to select the best subset of routes from Ω to build the final solution guaranteeing that each customer is visited by exactly one route.

4. The fixed-route vehicle-charging problem

As mentioned before, in the split and VND components, we solve a new fixed-route problem to make the charging decisions. This new problem is a variant of the well-known

fixed-route vehicle-refueling problem (FRVRP). The FRVRP seeks the minimum-cost refueling policy (which fuel stations to visit and the refueling quantity at each visited station) for a given origin-destination route (Suzuki 2014). Most of the research carried on the FRVRP applies to internal combustion vehicles, where the refueling time is negligible. Therefore, new variants of the FRVRP emerged for the case of electric vehicles. Most of these variants assume full charging policies (Montoya et al. 2015, Hiermann et al. 2016, Liao et al. 2016). Only Sweda et al. (2014) assume a partial charging policy, however they do not take into account the charging time, because their objective is to minimize the energy and degradation costs, and they do not consider maximum route duration constraints.

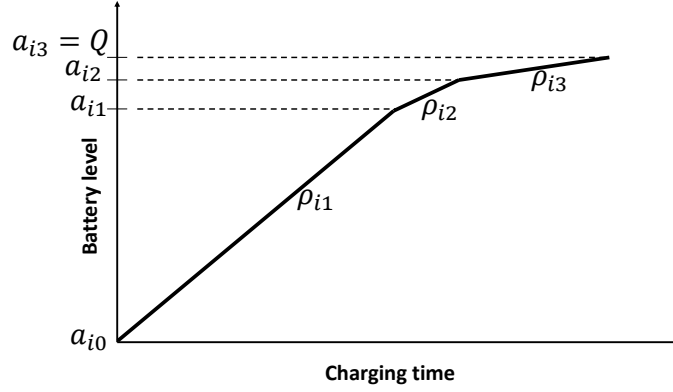
In this section, we introduce the fixed-route vehicle-charging problem (FRVCP) where: (i) vehicles have to detour of their fixed-route to visit the CSs; (ii) the amount of charge is a decision variable; and (iii) the charging function is piecewise linear. Since the FRVRP is NP-hard (Suzuki 2014) and the FRVCP generalizes the FRVRP, we can conclude that the FRVCP is also NP-hard. To solve this problem, we propose two approaches: a MILP formulation and a greedy heuristic.

Let $\Pi = \{\pi(0), \pi(1), \dots, \pi(i), \dots, \pi(j), \dots, \pi(n_r)\}$ be a route that violates the energy constraint, where $\pi(0)$ and $\pi(n_r)$ represent the depot. This route has a total time \bar{t} , which is the sum of the travel times plus the service times. The feasibility of Π may be restored by inserting visits to CSs. As mentioned in Section 2, each CS $j \in F$ has a piecewise-linear charging function defined by a set of breakpoints B . The piecewise linear function is composed by a set of segments. Each segment is defined between the breakpoints $k - 1$ and $k \in B$, it has a slope ρ_{jk} (representing a charging rate), and it is bounded between the battery levels a_{jk-1} and a_{jk} (see Figure 4a). In Figure 4b the values $e_{\pi(i-1)\pi(i)}$ and $t_{\pi(i-1)\pi(i)}$ represent respectively the energy consumption and the travel time between vertices $\pi(i - 1)$ and $\pi(i) \in \Pi$, $e_{\pi(i-1)j}$ and $t_{\pi(i-1)j}$ represent the energy consumption and the travel time between vertex $\pi(i - 1) \in \Pi$ and CS $j \in F$, and $e_{j\pi(i)}$ and $t_{j\pi(i)}$ represent the energy consumption and the travel time between the CS $j \in F$ and vertex $\pi(i) \in \Pi$.

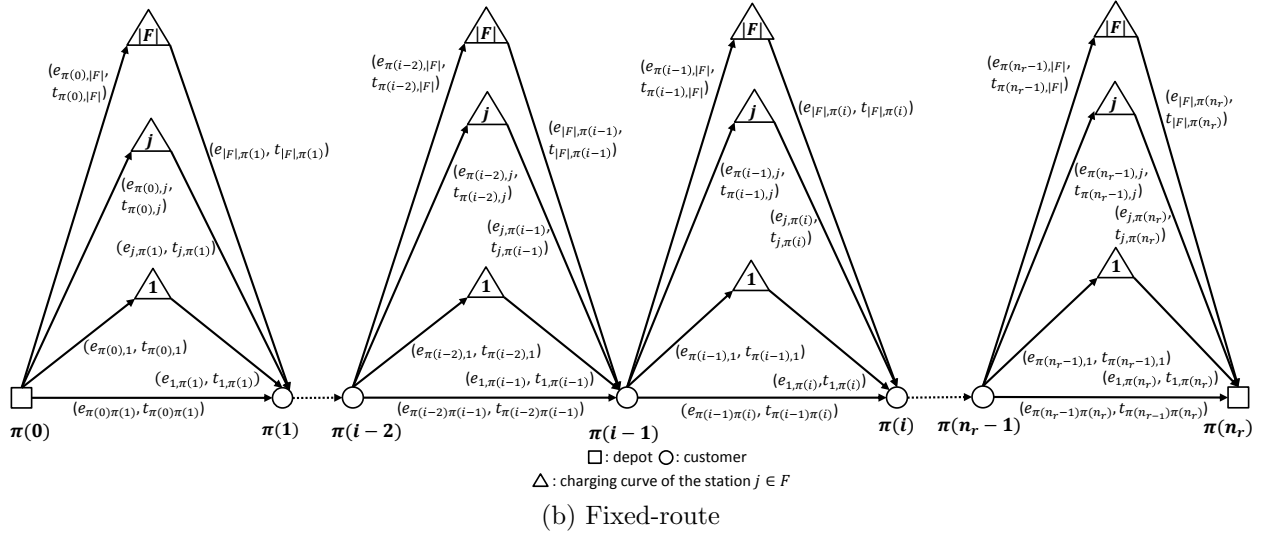
In the FRVCP the objective is to find the charging decisions (i.e., where and how much to charge) that minimize the sum of the charging times and detour times, satisfying the following conditions: the level of the battery when the vehicle arrives at any vertex is nonnegative; and the route satisfies the maximum-duration limit.

4.1. Mixed-integer linear programming formulation

We formulate the FRVCP using the following decision variables: variable $\varepsilon_{\pi(i)j}$ is equal to 1 if the vehicle charges at CS $j \in F$ before visiting vertex $\pi(i) \in \Pi$. Variable $\phi_{\pi(i)}$ tracks the battery level. If $\varepsilon_{\pi(i)j} = 0$, $\phi_{\pi(i)}$ is the battery level when the vehicle arrives at vertex $\pi(i)$. On the other hand, if $\varepsilon_{\pi(i)j} = 1$, $\phi_{\pi(i)}$ is the battery level when the vehicle arrives at CS $j \in F$ right before visiting vertex $\pi(i)$. Variable $\theta_{\pi(i)jk}$ is equal to 1 if the vehicle charges on the segment defined by breakpoints $k - 1$ and $k \in B$ at CS $j \in F$ before visiting vertex $\pi(i) \in \Pi$. Finally, variables $\delta_{\pi(i)jk}$ and $\mu_{\pi(i)jk}$ are respectively the amount of energy charged and the battery level when the charging finishes on the segment defined by breakpoints $k - 1$ and $k \in B$ at CS $j \in F$ before the vehicle visits vertex $\pi(i) \in \Pi$. The MILP formulation of the FRVCP follows:



(a) Segments of the piecewise-linear charging function



(b) Fixed-route

Figure 4: Piecewise-linear charging function and fixed-route for the FRVCP

$$\min \sum_{\pi(i) \in \Pi \setminus \{\pi(0)\}} \sum_{j \in F} \sum_{k \in B \setminus \{0\}} \frac{\delta_{\pi(i)jk}}{\rho_{jk}} + \sum_{\pi(i) \in \Pi \setminus \{\pi(0)\}} \sum_{j \in F} \varepsilon_{\pi(i)j} (t_{\pi(i-1)j} + t_{j\pi(i)} - t_{\pi(i-1)\pi(i)}) \quad (1)$$

Subject to

$$\phi_{\pi(1)} = Q - \sum_{j \in F} \varepsilon_{\pi(1)j} e_{\pi(0)j} - e_{\pi(0)\pi(1)} \left(1 - \sum_{j \in F} \varepsilon_{\pi(1)j}\right) \quad (2)$$

$$\phi_{\pi(i)} = \phi_{\pi(i-1)} + \sum_{j \in F} \sum_{k \in B \setminus \{0\}} \delta_{\pi(i-1)jk} - \sum_{j \in F} \varepsilon_{\pi(i-1)j} e_{j\pi(i-1)} -$$

$$\sum_{j \in F} \varepsilon_{\pi(i)j} e_{\pi(i-1)j} - e_{\pi(i-1)\pi(i)} \left(1 - \sum_{j \in F} \varepsilon_{\pi(i)j}\right) \quad \forall \pi(i) \in \Pi \setminus \{\pi(0), \pi(1), \pi(n_r)\} \quad (3)$$

$$\begin{aligned} \phi_{\pi(n_r)} &= \phi_{\pi(n_r-1)} + \sum_{j \in F} \sum_{k \in B \setminus \{0\}} \delta_{\pi(n_r-1)jk} + \\ &\sum_{j \in F} \sum_{k \in B \setminus \{0\}} \delta_{\pi(n_r)jk} - \sum_{j \in F} \varepsilon_{\pi(n_r-1)j} e_{j\pi(n_r-1)}^- \\ &\sum_{j \in F} \varepsilon_{\pi(n_r)j} (e_{\pi(n_r-1)j} + e_{j\pi(n_r)}) - e_{\pi(n_r-1)\pi(n_r)} (1 - \sum_{j \in F} \varepsilon_{\pi(n_r)j}) \end{aligned} \quad (4)$$

$$\begin{aligned} &\phi_{\pi(n_r-1)} + \sum_{j \in F} \sum_{k \in B \setminus \{0\}} \delta_{\pi(n_r-1)jk} - \sum_{j \in F} e_{j\pi(n_r-1)} \varepsilon_{\pi(n_r-1)j}^- \\ &\sum_{j \in F} e_{\pi(n_r-1)j} \varepsilon_{\pi(n_r)j} \geq 0 \end{aligned} \quad (5)$$

$$\mu_{\pi(i)j1} = \phi_{\pi(i)} + \delta_{\pi(i)j1} \quad \forall \pi(i) \in \Pi \setminus \{\pi(0)\}, \forall j \in F \quad (6)$$

$$\begin{aligned} \mu_{\pi(n_r)j1} &= \phi_{\pi(n_r-1)} + \sum_{l \in F} \sum_{k \in B \setminus \{0\}} \delta_{\pi(n_r-1)lk} - \\ &\sum_{l \in F} e_{l\pi(n_r-1)} \varepsilon_{\pi(n_r-1)l} - \varepsilon_{\pi(n_r)j} e_{\pi(n_r-1)j} + \delta_{\pi(n_r)j1} \end{aligned} \quad \forall \pi(i) \in \Pi \setminus \{\pi(0)\}, \forall j \in F \quad (7)$$

$$\mu_{\pi(i)jk} = \mu_{\pi(i)j,k-1} + \delta_{\pi(i)jk} \quad \forall \pi(i) \in \Pi \setminus \{\pi(0)\}, \forall j \in F, \forall k \in B \setminus \{0, 1\} \quad (8)$$

$$\mu_{\pi(i)jk} \geq a_{jk-1} \theta_{\pi(i)jk} \quad \forall \pi(i) \in \Pi \setminus \{\pi(0)\}, \forall j \in F, \forall k \in B \setminus \{0, 1\} \quad (9)$$

$$\mu_{\pi(i)jk} \leq a_{jk} \theta_{\pi(i)jk} + (1 - \theta_{\pi(i)jk}) Q \quad \forall \pi(i) \in \Pi \setminus \{\pi(0)\}, \forall j \in F, \forall k \in B \setminus \{0\} \quad (10)$$

$$\sum_{j \in F} \varepsilon_{\pi(i)j} \leq 1, \quad \forall \pi(i) \in \Pi \setminus \{\pi(0)\} \quad (11)$$

$$\theta_{\pi(i)jk} \leq \varepsilon_{\pi(i)j} \quad \forall \pi(i) \in \Pi \setminus \{\pi(0)\}, \forall j \in F, \forall k \in B \setminus \{0\} \quad (12)$$

$$\delta_{\pi(i)jk} \leq \theta_{\pi(i)jk} Q \quad \forall \pi(i) \in \Pi \setminus \{\pi(0)\}, \forall j \in F, \forall k \in B \setminus \{0\} \quad (13)$$

$$\begin{aligned} \bar{t} + \sum_{\pi(i) \in \Pi \setminus \{\pi(0)\}} \sum_{j \in F} \sum_{k \in B \setminus \{0\}} \frac{\delta_{\pi(i)jk}}{\rho_{jk}} + \\ \sum_{\pi(i) \in \Pi} \sum_{j \in F} \varepsilon_{\pi(i)j} (t_{\pi(i-1)j} + t_{j\pi(i)}) - t_{\pi(i-1)\pi(i)} \leq T_{max} \end{aligned} \quad (14)$$

$$\phi_{\pi(i)} \geq 0, \quad \forall \pi(i) \in \Pi \setminus \{\pi(0)\} \quad (15)$$

$$\varepsilon_{\pi(i)j} \in \{0, 1\}, \quad \forall \pi(i) \in \Pi \setminus \{\pi(0)\}, \forall j \in F \quad (16)$$

$$\theta_{\pi(i)jk} \in \{0, 1\} \quad \forall \pi(i) \in \Pi \setminus \{\pi(0)\}, \forall j \in F, \forall k \in B \setminus \{0\} \quad (17)$$

$$\delta_{\pi(i)jk} \geq 0 \quad \forall \pi(i) \in \Pi \setminus \{\pi(0)\}, \forall j \in F, \forall k \in B \setminus \{0\} \quad (18)$$

$$\mu_{\pi(i)jk} \geq 0 \quad \forall \pi(i) \in \Pi \setminus \{\pi(0)\}, \forall j \in F, \forall k \in B \setminus \{0\} \quad (19)$$

The objective function (1) seeks to minimize the total route time (including charging and detour times). Constraints (2-5) define the battery level when the vehicle arrives at vertex $\pi(i) \in \Pi$ if $\varepsilon_{\pi(i)j} = 0$, or to CS $j \in F$ before visiting the vertex $\pi(i) \in \Pi$, if $\varepsilon_{\pi(i)j} = 1$. Constraints (6-8) define the battery level when the vehicle finishes charging at CS $j \in F$ in the segment between breakpoints $k - 1$ and $k \in B$ before the vehicle visits vertex $\pi(i) \in \Pi$. Constraints (9-10) ensure that if the vehicle charges on a given segment, the battery level lays between the values of its corresponding break points ($a_{j,k-1}$ and a_{jk}). Constraints (11) state that only one CS is visited between any two vertices of the fixed route. Constraints (12) ensure that the vehicle only uses segments of visited CSs. Likewise, constraints (13) ensure

that the vehicle charges only at selected segments of visited CSs. Constraint (14) represents the duration constraint of the route. Finally, constraints (15-19) define the domain of the decision variables.

To reduce the size of the MILP formulation and consequently improve the solution time, we introduce three strategies to forbid visits to CSs that will not be included in any feasible route of the eVRP-PNL. All the strategies rely on the two following premises: (i) energy consumption and travel time between vertices meet the triangle inequality; (ii) since the piecewise-linear charging function is concave (i.e., $\rho_{jk-1} \geq \rho_{jk}$), the first segment has the fastest charging rate. We propose two types of strategies. The first two strategies identify infeasible visits to CSs between pairs of vertices i and $h \in I \cup \{0\}$, and before or after a vertex $i \in I \cup \{0\}$, without exploiting information on the fixed-route. These strategies are applied once before running ILS+HC. Whereas, the third strategy identifies infeasible visits to CSs between vertices i and $h \in \Pi$ considering the information of the fixed route.

Strategy 1 : This strategy estimates the minimum time τ needed to visit CS $j \in F$ between two vertices i and $h \in I \cup \{0\}$. This time is defined as the sum of a lower bound of the travel time (u) plus a lower bound of the charging time (v). To compute u , note that the shortest travel time to reach a vertex i is to come directly from the depot. Note also that the shortest travel time to finish a tour from vertex h is to go directly to the depot. Then the minimum travel time of a route visiting CS j between vertices i and h is $u = t_{0,i} + t_{i,j} + t_{j,h} + t_{h,0} + p_i + p_h$. To compute the minimum charging time v , we need to compute the minimum amount of energy (ec) that the vehicle coming from vertex i and traveling to vertex h must charge at CS j . This minimum amount is the charge needed to recover the energy consumed to make the detour to j , that is defined as $ec = e_{ij} + e_{jh} - e_{ih}$. Because the battery level when the vehicle arrives at i is impossible to estimate a priori, we consider that the battery is recharged with the charging rate ρ_{0j} (fastest charging rate). Then $v = \frac{ec}{\rho_{0j}}$. Finally, if $\tau = u + v$ is greater than T_{max} , then we forbid the visit to CS j between the two vertices i and h .

Strategy 2 : This strategy computes a lower bound q of the energy consumed by a vehicle which visits a CS j after a vertex i , and a lower bound o of the energy consumed by a vehicle which visits a CS j before a vertex h . The first lower bound is computed as follows: in terms of energy consumption, the best path to reach vertex i is to come directly from its closest charging station $c(i)$ (which may be the depot). Then $q = e_{c(i)i} + e_{ij}$. Similarly, the path consuming the least energy after visiting vertex h is the one going directly to its closest station $c(h)$. Thus, the minimum energy consumed by a vehicle which visits a CS j before a vertex h is $o = e_{jh} + e_{hc(h)}$. If $q > Q$ (resp. $o > Q$) then we forbid visiting CS j after i (resp. before h).

Strategy 3 : This strategy estimates the minimum time τ needed to visit CS $j \in F$ between two vertices $\pi(i)$ and $\pi(i+1) \in \Pi$, $i \neq n_r$, considering the current total time of the fixed-route (\bar{t}). This minimum time τ is defined as the sum of the travel time (u) plus a lower bound of the charging time (v). First, we know that $u = \bar{t} + t_{\pi(i),j} + t_{j,\pi(i+1)} - t_{\pi(i),\pi(i+1)}$. Second, we follow the same idea than in Strategy 1 to

obtain a lower bound on v . Therefore $v = \frac{ec}{\rho_{0j}}$, with $ec = e_{\pi(i)j} + e_{j\pi(i+1)} - e_{\pi(i)\pi(i+1)}$ the charge needed to recover the energy consumed in the detour to j . Finally, if $\tau = u + v$ is greater than T_{max} , then we forbid visiting CS j between vertices $\pi(i)$ and $\pi(i + 1)$.

4.2. Greedy heuristic

Existing metaheuristics for eVRPs use different approaches to make charging decisions. One of them is the recharge relocation operator proposed by Felipe et al. (2014) for the green vehicle routing problem with multiple technology and partial recharges (GVRP- MTPR). This approach considers the insertion of only one CS per route. It starts from an energy-feasible fixed-route composed of a set a customers and one CS. It deletes the current CS, and tries to improve the charging decisions by inserting each CS into each arc of the fixed-route. In addition, it uses a simple rule for the energy charged at the inserted CS: vehicles charge the energy needed to reach the depot (i.e., to complete the route). We propose here a heuristic to solve the FRVCP based on this approach.

Our heuristic is composed of two phases: *location of CSs* and *charge setting*. In the first phase, the heuristic iteratively inserts CSs into the arcs of the fixed-route Π in order to ensure the feasibility in terms of energy. In the second phase, the heuristic improves the charging decisions by adjusting the energy charged at each visited CS. Algorithm 1 describes the structure of our heuristic. It uses three important procedures `trackBattery`, `sumNegative`, and `totalTime`. Procedure `trackBattery` computes the battery level Y_i at each vertex $i \in \Pi$, supposing that the vehicle fully charges its battery in each visit to CSs. Procedure `sumNegative` computes the sum of the battery levels with negative values (i.e., $Y_i < 0$). Notice that `sumNegative` computes the aggregated infeasibility of the route Π in terms of energy. Finally, procedure `totalTime` computes the total time t of the route (supposing a full charging policy). This heuristic starts by the location phase (line 2-27). After computing the sum of battery levels with negative values for the current fixed-route Π , the heuristic enters the outer loop (line 7-27). Each pass through the outer loop evaluates all the possible insertions of each CS in F into each arc of Π supposing that the vehicle fully charges its battery, and keeps the best insertion in terms of the sum of battery levels s with negative value (line 8-25). If $s = 0$ (i.e., the route is energy-feasible), the heuristic keeps the best insertion in terms of the total time of the route t (line 19-23). Then, the heuristic inserts the best CS F_u into the best arc (π_v, π_{v+1}) (line 26). If the route Π is still energy-infeasible ($s < 0$) the heuristic inserts another CS starting again in line 8. In the charge setting phase (line 28), the heuristic invokes a procedure called `ruleMinEnergy(Π)` to set the energy charged at each inserted CS following Felipe et al. (2014) approach: the vehicle only charges at each CS the energy needed to arrive at the next CS or the depot. Finally, the heuristic evaluates if the repaired route satisfies the duration constraint (29-33). The heuristic returns a boolean variable indicating whether or not the fixed-route is feasible, and the repaired route Π , if found.

Algorithm 1 Greedy heuristic

```
1: function GREEDYHEURISTIC( $\Pi^0, F$ )
2:    $\Pi \leftarrow \Pi^0$ 
3:    $Y \leftarrow \text{trackBattery}(\Pi)$ 
4:    $s \leftarrow \text{sumNegative}(Y)$ 
5:    $t \leftarrow \infty$ 
6:    $f \leftarrow \text{false}$ 
7:   while  $s < 0$  do
8:     for  $i = 1$  to  $|F|$  do
9:       for  $j = 0$  to  $n_r - 1$  do
10:         $\Pi \leftarrow \{\pi_0, \dots, \pi_j, F_i, \pi_{j+1}, \dots, \pi_{n_r}\}$ 
11:         $Y' \leftarrow \text{trackBattery}(\Pi)$ 
12:         $s' \leftarrow \text{sumNegative}(Y')$ 
13:         $t' \leftarrow \text{totalTime}(\Pi)$ 
14:        if  $s' > s$  then
15:           $s \leftarrow s'$ 
16:           $u \leftarrow i$ 
17:           $v \leftarrow j$ 
18:        end if
19:        if  $s' = 0$  and  $t' < t$  then
20:           $t \leftarrow t'$ 
21:           $u \leftarrow i$ 
22:           $v \leftarrow j$ 
23:        end if
24:      end for
25:    end for
26:     $\Pi \leftarrow \{\pi_0, \dots, \pi_v, F_u, \pi_{v+1}, \dots, \pi_{n_r}\}$ 
27:  end while
28:   $\langle t, \Pi \rangle \leftarrow \text{RuleMinEnergy}(\Pi)$ 
29:  if  $t \leq T_{max}$  then
30:     $f \leftarrow \text{true}$ 
31:  else
32:     $\Pi \leftarrow \Pi^0$ 
33:  end if
34:  return  $\Pi, f$ 
35: end function
```

5. Computational experiments

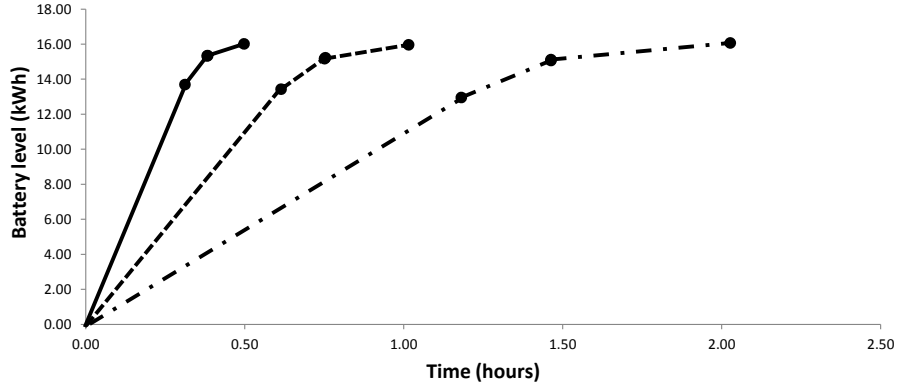
In this section, we present three computational studies. The first study compares the quality of the solutions obtained by two versions of our metaheuristic. The second study evaluates the CPU time of our metaheuristic, and assesses the impact of the preprocessing

strategies. Finally, the third study analyses the charging decisions of the best solutions found.

5.1. Test instances for the eVRP-PNL

To test our approach, we propose a new 120-instances testbed using real data of vehicle configuration and charging functions. In order to ensure the feasibility of the instances, we opt to generate our set of instances, instead of using classical instances from the literature. Our instances range from 10 to 320 customers. We locate the customers into a geographic space of 120 x 120 km. This geographic space represents a semi-urban operation, where the distance of the routes may exceed the autonomy of the vehicles (a setting where eVRPs are more interesting). To locate the customers, we use three options: random uniform distribution, random clustered distribution and a mixture of both. For each instance the customer location option is selected randomly with equal probability.

Considering that the number and location of the CSs impact the routing and charging decisions, for each number of customers we generate four types of configurations changing these factors. For the number of CSs, we evaluate two levels: low and high availability of CSs. The values of these levels are proportional to the number of customers in order to ensure the feasibility of the instances. For the location of the CSs, we use two approaches: random location and one using a simple p-median heuristic. This p-media heuristic starts with a random location and exchanges iteratively the location of the CSs, in order to minimize the total distance between the CSs and the customers. We use three types of CSs: slow, moderate, and fast. For each location of the CSs we select randomly with uniform probability the type of the station. Finally, for the vehicle and charging parameters of this set of instances, we use the real data of a Peugeot Ion. This vehicle has a consumption rate of 0.125 kWh/km, and a battery of 16 kWh. Figure 5 shows the piecewise linear charging function associate to each charging station. These piecewise linear functions were generated using real data provided by Uhrig et al. (2015). Finally, the maximum route duration was fixed to 10 hours.



Type of CSs	Slow	Moderate	Fast
Charging power (kWh/h)	11	22	44
Piecewise linear approximation	⋯	- - - -	—

Figure 5: Piecewise linear approximation for different types of CS charging a vehicle with a battery of 16 kWh.

5.2. Parameter settings & experimental environment

As mentioned before, we propose two approaches to solve the FRVCP in the split and VND components. Considering that the objective of the split procedure is only to find an initial solution for the VND, and to favor short running times we opt to use the greedy heuristic rather than the MILP formulation in the split procedure of ILS+HC. On the other hand, we use the two approaches to solve the FRVCP in the VND component. We thus present two ILS+HC versions: ILS(M)+HC and ILS(H)+HC. As their name suggest the former uses the MILP formulation to solve the FRVCP whereas the later uses the heuristic. This allow us to evaluate the impact of optimal versus heuristic charging decisions over the solution quality and the computing times of ILS+HC.

For the number of iterations of ILS (K), we conducted a parameter tuning, and found that $K = 80$ provides the best trade-off between solution quality and computing time. For the sake of brevity, we will not present these experiments.

We implemented our ILS in Java (jre V.1.8.0) and used Gurobi Optimizer (version 5.6.0) to solve the FRVCP and the set partitioning problem. We set a time limit of 800 seconds on Gurobi to control the running time of the heuristic concentration phase. All the experiments were run on a computing cluster with 2.33 GHz Inter Xeon E5410 processors with 16 GB of RAM running under Linux Rocks 6.1.1. The results for the two versions of the metaheuristic are computed over 10 runs. Each replication of the experiments was run on a single processor.

5.3. Solution quality evaluation

Initially, we compare the two versions of the metaheuristic to evaluate the impact of the optimal and heuristic charging decisions over the eVRP-PNL results. Table 1 summarizes the results of the two versions on the 20 small instances of 10 customers compared with the optimal solutions found using the MILP formulation presented in the Part I of the electronic

companion. The rows of Table 1 indicate the number of times that each metaheuristic version found the optimal solution, the average and maximum gap (in %) with respect to the optimal solution over the 10 runs, and the average best gap ¹ (in %). Table 2 presents the results obtained for the other instances (without proven optima). The first row contains the number of solutions found by the MILP formulation and the two versions of the metaheuristic. The following rows contain the same metrics than Table 1 but the comparison is done against the best solutions found (BKSs) by the MILP formulation or any of the two versions of the metaheuristic. The detailed results are reported in the Part II of the electronic companion.

As Table 1 shows, for the instances with proven optimal solution, ILS(M)+HC has a remarkable performance: it matches 20 out of 20 optimal solutions, and has an average gap of only 0.34%. On the other hand, ILS(H)+HC only matches 15 out of 20 optimal solutions, and has an average gap of 1.97%. The maximum gap reached by ILS(H)+HC (17.07%) also highlights that making heuristic charging decisions has an important impact on the solution quality even for these small instances. The results are similar on larger instances (Table 2). ILS(M)+HC matches 100 out of 100 BKSs and has an average and maximum gaps of 1.51% and 4.44% respectively. By contrast ILS(H)+HC only matches 7 out of 100 BKSs and has an average gap of 7.51% and a maximum gap of 28.68%. On the other hand, using the MILP formulation with a time limit of 10 hours, Gurobi found integer solutions (without guaranteeing optimality) for only 25 out of 100 instances, and obtains 7 out of 100 BKSs, with an average gap of 41.36%.

In conclusion, the proposed ILS(M)+HC has competitive results. It matches all the optimal solutions on small instances and all the BKSs of large instances. The clear out-performance of ILS(M)+HC over ILS(H)+HC shows the importance of making optimal charging decisions to obtain high-quality solutions of the eVRP-PNL.

Table 1: Comparison of the two versions of the metaheuristic on small instances (with proven optima)

Metric	ILS(H)+HC	ILS(M)+HC
Number of optimal solutions	15/20	20/20
Avg. Gap (%)	1.97	0.34
Max. Gap (%)	17.07	1.87
Avg. Best Gap (%)	1.20	0.00

5.4. Computing time performance

Table 3 summarizes the computing time (in seconds) of ILS(H)+HC, a version of ILS+HC without the preprocessing strategies (ILS(M')+HC), and the original ILS(M)+HC which includes these strategies. For each instance size (measured in number of customers), the first three columns indicate the average computing time of the three methods (in seconds), and the last column indicates the average speedup between ILS(M')+HC and ILS(M)+HC

¹The best gap is the gap between the best solution found over 10 runs and the optimal solution

Table 2: Comparison of the two versions of the metaheuristic on large instances

Metric	Gurobi	ILS(H)+HC	ILS(M)+HC
Number of solutions	25/100	100/100	100/100
Number of BKSs	7/100	7/100	100/100
Avg. Gap (%)	41.36	7.51	1.51
Max. Gap (%)	NA	28.68	4.44
Avg. Best Gap (%)	NA	5.28	0.00

(measured as the ratio of their computing times). The results show that ILS(H)+HC is the fastest approach. However, as mentioned before, it does not provide competitive results in terms of solution quality. Despite ILS(M)+HC has remarkable results, the use of an MILP formulation for the charging decision has an impact over the computing time (it is roughly five times slower than ILS(H)+HC). However, the use of the preprocessing strategies reduces significantly the computing time of the ILS(M)+HC. On average the speedup between ILS(M')+HC and ILS(M)+HC is 1.65.

Table 3: Average computing time (in seconds) of different variants of the metaheuristic

Instance size	ILS(H)+HC	ILS(M')+HC	ILS(M)+HC	Speedup
10	0.64	8.54	5.62	1.52
20	1.75	17.47	10.56	1.65
40	8.48	64.16	35.35	1.82
80	39.35	148.76	80.11	1.86
160	289.08	976.84	568.02	1.72
320	2,568.94	5,759.67	4,397.64	1.31
Average	484.71	1,162.57	849.55	1.65
Max	4,766.36	10,335.56	7,636.50	2.53
Min	0.49	3.71	2.36	1.16

ILS(M')+HC and ILS(M)+HC are the hybrid metaheuristic with and without preprocessing strategies

5.5. Analysis of the charging decisions in the solutions

As pointed out previously, there are several simplifications of the charging process in the eVRP literature: most of the studies assume that the battery is completely recharged at each visit to a CS, and/or that at most one CS can be visited in a route. In order to compare these assumptions against those introduced in the eVRP-PNL (i.e., the partial charging option and the possibility to visit several CSs in a route), we analyze the best solutions obtained for each instance.

Figure 6 presents the percentage of the routes with/without visits to CSs grouped by instance size (in terms of number of customers). The results show that on average 71.47%

of the routes visit at least one CS, and this percentage is roughly the same for each instance size. This shows that charging decisions impact most of the routes.

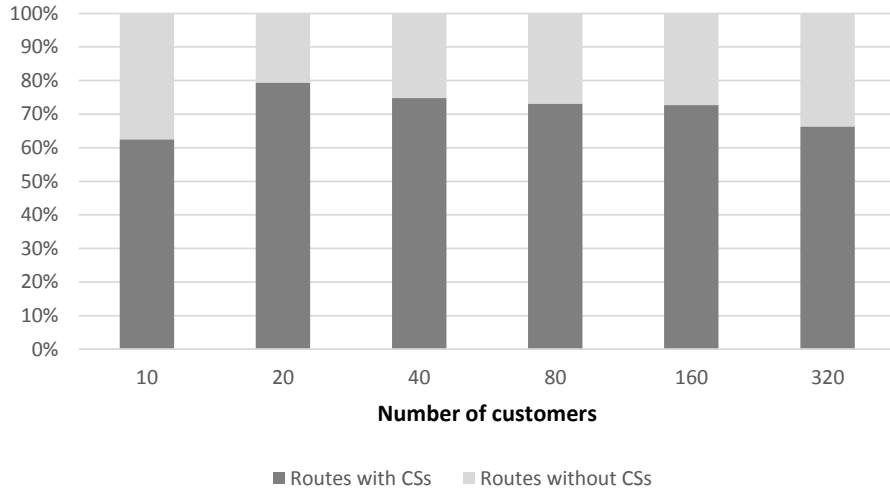
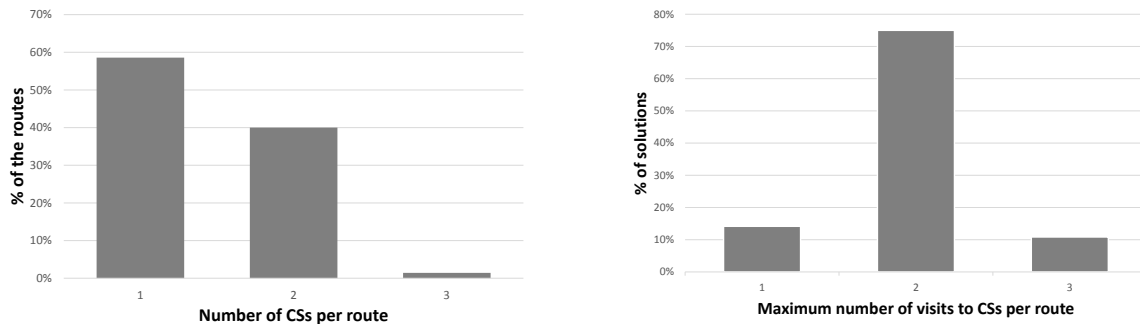


Figure 6: Percentage of the routes with/without visits to CSs by instance size.

Figure 7a and 7b present histograms of the number of visited CSs per route and the maximum number of visited CSs in the routes of each solution. Figure 7a shows that among the routes visiting CSs, 58.58% visit only one CS, 40.00% visit two CSs and 1.43% three CSs. Although a large portion of the routes visit only one CS, 85.83% of the solutions contain at least a route visiting more than one CS (Figure 7b). This confirms the importance of allowing the option of visiting several CSs in a route. Moreover, Figure 8 presents the histogram of the average amount of charge by route (in terms of percentage of the battery capacity). The cumulative curve shows that in roughly 99% of the routes the vehicles charge partially. This last result supports the fact that partial charging is necessary in the eVRPs context.



(a) Histogram of number of visits to CS per route. (b) Histogram of the maximum number of visited CSs in the routes of each solution.

Figure 7: Analysis of the number of visits to CSs

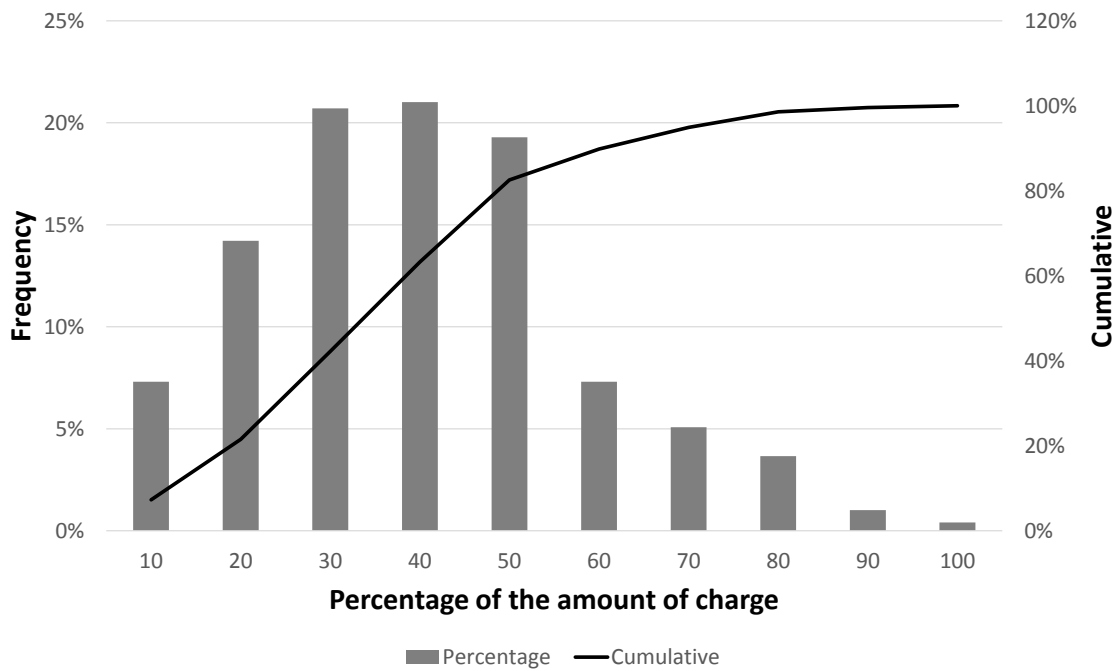


Figure 8: Histogram of the average charged amount per route

Finally, Figure 9 presents the average gap of ILS(H)+HC and ILS(M)+HC grouped by the maximum number of visited CSs in the routes of each solution. This figure shows that the average gap of the ILS(M)+HC remains roughly constant regardless of the number of visited CSs. On the other hand, the average gap of ILS(H)+HC increases significantly with the number of visited CSs. This confirms the importance of a procedure that makes optimal charging decisions when including the option to visit several CSs in a route.

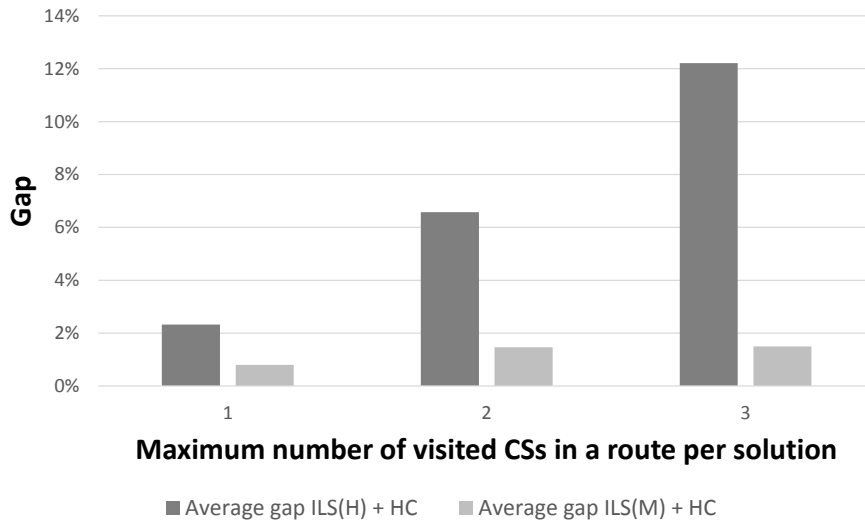


Figure 9: Average gap grouped by maximum number of visited CSs in the routes of each solution

6. Conclusion and future work

This paper considers an extension of the electric vehicle routing problem which includes realistic assumptions about the charging process: the electric vehicle routing problem with partial charging and nonlinear charging function (eVRP-PNL). We present an iterated local search (ILS) with a heuristic concentration (HC) to solve it. An important step in this metaheuristic is the resolution of a sub-problem known as the fixed-route vehicle-charging problem (FRVCP). We propose two solution methods to solve it: a heuristic and an MILP formulation. Computational results on a set of 120 instances of different sizes reveal that the version of the metaheuristic using the MILP formulation to solve the FRVCP obtains competitive results: it matches all the optimal solutions for small instances and all the best known solutions for the others. Finally, the analysis of charging decisions of the best solutions highlights the importance of making optimal charging decisions, including the option to visit multiple CSs, and allowing partial charging in the routes of eVRPs.

Interesting research directions include the resolution of the eVRP-PNL using other metaheuristics and the development of alternative methods to solve the FRVCP to optimality. In addition, in ongoing research we are extending the problem definition to consider a cost objective function, and time windows.

Acknowledgement

The authors would like to thank the Universidad EAFIT scientific computing center (APOLO) for their support in the computational experiments. This research was partly funded in Colombia by Universidad EAFIT, Programa de Movilidad Doctoral hacia Francia (Colfuturo - Emb. de Francia - ASCUN - Colciencias - Min. de Educación), and Programa Enlaza Mundos (Alcaldía de Medellín); and in France by ANR through project e-VRO.

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Electronic companion of:

A hybrid metaheuristic for the electric vehicle routing problem with partial charging and nonlinear charging function

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Part I

Comparison of different charging assumptions

The objective of this part of the electronic companion is to show the importance of considering a nonlinear function to model the battery charging process and the impact of partial recharges of the battery. We first present the approximations used for the charging function in transportation planning problems. Then, we propose a mixed-integer linear programming (MILP) formulation for the electric vehicle routing problem with partial charging and nonlinear charging function (eVRP-PNL) that, with a few modifications, can be adapted to model other charging policies and charging function approximations used in the extant eVRP literature. We use the MILP formulation and a commercial solver to evaluate the impact of including the nonlinear charging functions and partial charging in eVRPs. Our results suggest that disallowing partial charging and neglecting the nonlinear nature of the charging functions leads to solutions that may be infeasible or overly expensive.

1. Modeling of battery charging functions

The charging function of a battery models the relationship between battery charging time and charging level. In general, the charging functions are nonlinear, because the terminal

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voltage and current change during the charging process. This process is divided into two phases. In the first phase, the charging current is held constant, and thus the state of charge (SOC) increases linearly with time until the battery's terminal voltage increases to a specific maximum value (see Figure 1). In the second phase, the current decreases exponentially and the terminal voltage is held constant to avoid battery damages. The SOC increases then concavely with the time (Pelletier et al. 2015).

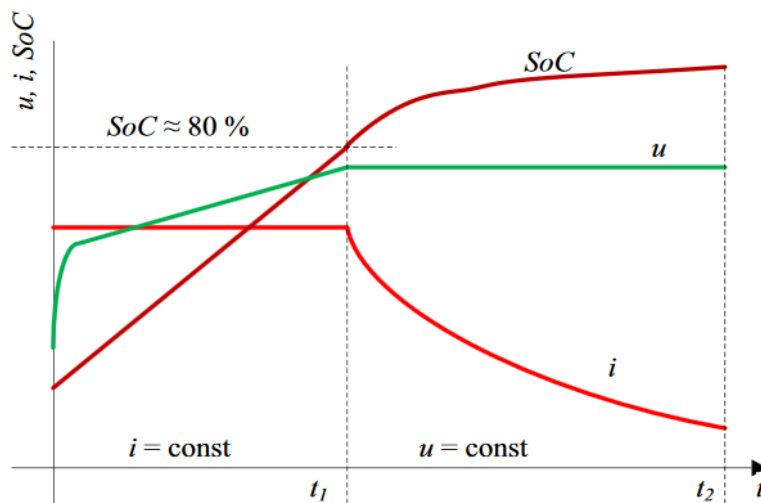


Figure 1: Typical charging curve, where i and u represent the current and terminal voltage respectively. (Source Hõimoja et al. (2012))

Although the shape of the charging functions are known, their exact modeling is very complex because it depends on many factors as: current, voltage, self-recovery and temperature, among others (Wang et al. 2013). The battery state of charge is then often described by differential equations. Since such equations are difficult to integrate in transportation problems, different approximations are used in those problems. These approximations are presented below compared with real data of a charging function provided by Uhrig et al. (2015). Each of them can be used in a full recharge policy (FR) or in a partial recharge policy (PR).

First segment (FS) : To avoid dealing with the nonlinear segment, Bruglieri et al. (2014) use a linear approximation that considers only the first segment (Figure 2).

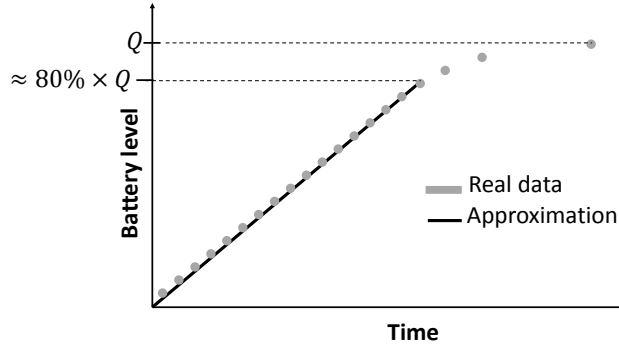
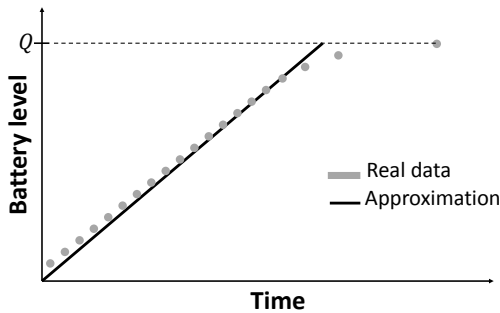
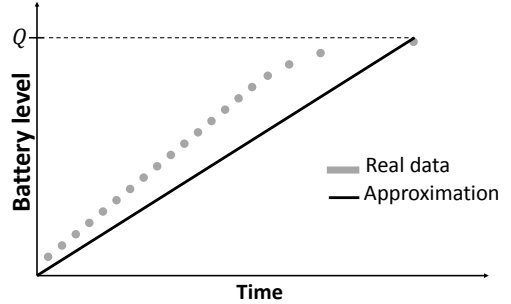


Figure 2: First segment approximation (FS)

Linear approximations (L1 and L2) : Although several authors assume a linear approximation (Felipe et al. 2014, Sassi et al. 2014, Bruglieri et al. 2015, Desaulniers et al. 2014, Schiffer & Walther 2015, Keskin & Çatay 2016), they do not explain how the approximation is estimated. Two options can be considered. In the first one (L1) the charging rate of the function corresponds to the slope of the first segment of the piecewise linear approximation (see Figure 3a). This approximation is optimistic, because it assumes that batteries charge up to Q faster than they do in reality. In the second approximation (L2) the charging rate is the slope of the line connecting the first and last observations (see Figure 3b) of the charging curve. This approximation tends to be pessimistic, because over a large portion of the curve, the charging rate is slower than in reality.



(a) Linear approximation 1 (L1)



(b) Linear approximation 2 (L2)

Figure 3: Linear approximations of charging functions.

Piecewise linear approximations (PL) : This approximation, proposed by Zündorf (2014) for a shortest path problem with electric vehicles, consists in approximating the charging function by a series of linear segments (see Figure 4a).

In this study, we use the same approximation proposed by Zündorf (2014). To assess the validity of this approximation, we use the data provided by Uhrig et al. (2015). These

authors conducted experiments to estimate the charging time for different charge levels with two types of electric vehicles and three types of CSs. We fit piecewise linear functions to the data and obtain approximations with an average relative absolute error of 0.90%, 1.24%, and 1.90% for CSs of 11, 22, and 44 kW, respectively. Figure 4b shows the piecewise linear approximation for a CS i of 22 kW charging a vehicle equipped with a battery of 16 kWh. In the plot, c_{ik} and a_{ik} represent the charging time and the charge level for the breakpoint $k \in B$ of the CS $i \in F'$, where $B = \{0, \dots, b\}$ is the set of breakpoints of the piecewise linear approximation.

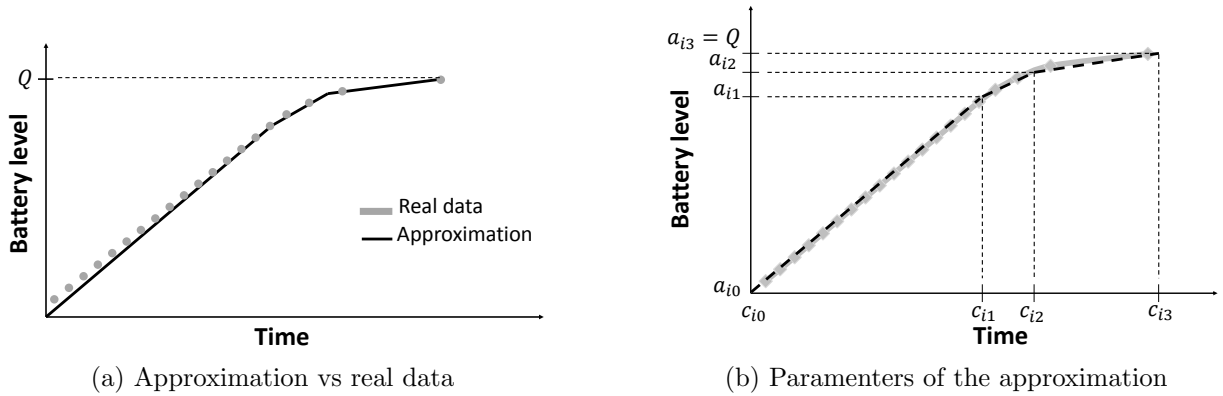


Figure 4: Piecewise linear approximation (PL).

2. Mixed-integer linear programming formulation

We present in this section the MILP formulation of the eVRP-PNL. We also show how it can be adapted to model other approximations and full charging policies

To formulate the eVRP-PNL, we introduce the set F' that contains the set F and β copies of each CS (i.e., $|F'| = |F| \times (1 + \beta)$). The value of $1 + \beta$ corresponds to the number of times that each CS can be visited. For this MILP formulation, we use the following decision variables: variable x_{ij} is equal to 1 if a vehicle travels from vertex i to j , and 0 otherwise. Variables τ_j and y_j track the time and charge level when the vehicle departs from vertex $j \in V$. Variables q_i and o_i specify the charge levels when a vehicle arrives at and departs from CS $i \in F'$, and s_i and d_i are the associated charging times (see Figure 5). Variable $\Delta_i = d_i - s_i$ represents the time spent at CS $i \in F'$. Variables z_{ik} and w_{ik} are equal to 1 if the charge level is between $a_{i,k-1}$ and a_{ik} , with $k \in B \setminus \{0\}$, when the vehicle arrives at and departs from CS $i \in F'$ respectively. Finally, variables α_{ik} and λ_{ik} are the coefficients of the breakpoint $k \in B$ in the piecewise linear approximation, when the vehicle arrives at and departs from CS $i \in F'$ respectively. The MILP formulation follows:

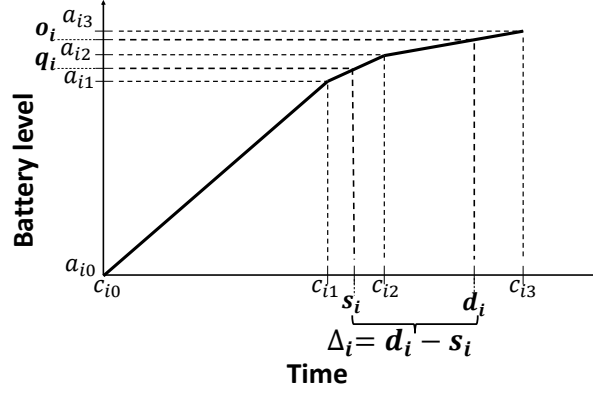


Figure 5: Battery charge levels and charging times $i \in F'$

$$\min \sum_{i,j \in V} t_{ij} x_{ij} + \sum_{i \in F'} \Delta_i \quad (1)$$

subject to

$$\sum_{j \in V, i \neq j} x_{ij} = 1, \quad \forall i \in I \quad (2)$$

$$\sum_{j \in V, i \neq j} x_{ij} \leq 1, \quad \forall i \in F' \quad (3)$$

$$\sum_{j \in V, i \neq j} x_{ji} - \sum_{j \in V, i \neq j} x_{ij} = 0, \quad \forall i \in V \quad (4)$$

$$e_{ij} x_{ij} - (1 - x_{ij})Q \leq y_i - y_j \leq e_{ij} x_{ij} + (1 - x_{ij})Q, \quad \forall i \in V, \forall j \in I \quad (5)$$

$$e_{ij} x_{ij} - (1 - x_{ij})Q \leq y_i - q_j \leq e_{ij} x_{ij} + (1 - x_{ij})Q, \quad \forall i \in V, \forall j \in F' \quad (6)$$

$$y_i \geq e_{i0} x_{i0}, \quad \forall i \in V \quad (7)$$

$$y_i = o_i, \quad \forall i \in F' \quad (8)$$

$$y_0 = Q \quad (9)$$

$$q_i \leq o_i, \quad \forall i \in F' \quad (10)$$

$$q_i = \sum_{k \in B} \alpha_{ik} a_{ik}, \quad \forall i \in F' \quad (11)$$

$$s_i = \sum_{k \in B} \alpha_{ik} c_{ik}, \quad \forall i \in F' \quad (12)$$

$$\sum_{k \in B} \alpha_{ik} = \sum_{k \in B} z_{ik}, \quad \forall i \in F' \quad (13)$$

$$\sum_{k \in B} z_{ik} = \sum_{j \in V} x_{ij}, \quad \forall i \in F' \quad (14)$$

$$\alpha_{ik} \leq z_{ik} + z_{i,k+1}, \quad \forall i \in F', \forall k \in B \setminus \{b\} \quad (15)$$

$$\alpha_{ib} \leq z_{ib}, \quad \forall i \in F' \quad (16)$$

$$o_i = \sum_{k \in B} \lambda_{ik} a_{ik}, \quad \forall i \in F' \quad (17)$$

$$d_i = \sum_{k \in B} \lambda_{ik} c_{ik}, \quad \forall i \in F' \quad (18)$$

$$\sum_{k \in B} \lambda_{ik} = \sum_{k \in B} w_{ik}, \quad \forall i \in F' \quad (19)$$

$$\sum_{k \in B} w_{ik} = \sum_{j \in V} x_{ij}, \quad \forall i \in F' \quad (20)$$

$$\lambda_{ik} \leq w_{ik} + w_{i,k+1}, \quad \forall i \in F', \forall k \in B \setminus \{b\} \quad (21)$$

$$\lambda_{ib} \leq w_{ib}, \quad \forall i \in F' \quad (22)$$

$$\Delta_i = d_i - s_i, \quad \forall i \in F' \quad (23)$$

$$\tau_i + (t_{ij} + p_j)x_{ij} - T_{max}(1 - x_{ij}) \leq \tau_j, \quad \forall i \in V, \forall j \in I \quad (24)$$

$$\tau_i + \Delta_j + t_{ij}x_{ij} - (S_{max} + T_{max})(1 - x_{ij}) \leq \tau_j, \quad \forall i \in V, \forall j \in F' \quad (25)$$

$$\tau_j + t_{j0} \leq T_{max}, \quad \forall j \in V \quad (26)$$

$$\tau_0 \leq T_{max} \quad (27)$$

$$x_{ij} = 0, \quad \forall i, j \in F' : m_{ij} = 1 \quad (28)$$

$$\tau_i \geq \tau_j, \quad \forall i, j \in F' : m_{ij} = 1, j \leq i \quad (29)$$

$$\tau_j \leq T_{max} \sum_{i \in V} x_{ij}, \quad \forall j \in F' \quad (30)$$

$$\sum_{i \in V} x_{ih} \geq \sum_{j \in V} x_{jf}, \quad \forall h, f \in F' : m_{hf} = 1, h \leq f \quad (31)$$

$$x_{ij} \in \{0, 1\}, \quad \forall i, j \in V \quad (32)$$

$$\tau_i \geq 0, y_i \geq 0 \quad \forall i \in V \quad (33)$$

$$z_{ik} \in \{0, 1\}, w_{ik} \in \{0, 1\}, \alpha_{ik} \geq 0, \lambda_{ik} \geq 0, \quad \forall i \in F', \forall k \in B \quad (34)$$

$$q_i \geq 0, o_i \geq 0, s_i \geq 0, d_i \geq 0, \Delta_i \geq 0, \quad \forall i \in F' \quad (35)$$

The objective function (1) seeks to minimize the total time (travel times plus charging times). Constraints (2) ensure that each customer is visited once. Constraints (3) ensure that each CS is visited at most once. Constraints (4) impose the flow conservation. Constraints (5) and (6) track the battery charge level at each vertex. Constraints (7) ensure that, if the vehicle travels between a vertex and the depot, it has sufficient energy to reach its destination. Constraints (8) reset the battery tracking to o_i upon departure from CS $i \in F'$. Constraint (9) ensures that the battery charge level is Q at the depot. Constraints (10) couple the charge levels when a vehicle arrives at and departs from any CS. Constraints (11–16) define the charge level (and its corresponding charging time) when a vehicle arrives at CS $i \in F'$ (based on the piecewise linear approximation of the charging function). Similarly, constraints (17–22) define the charge level (and its corresponding charging time) when a vehicle departs from CS $i \in F'$. Constraints (23) define the time spent at any CS. Constraints (24) and (25) track the departure time at each vertex, where $S_{max} = \max_{i \in F'} \{c_{ib}\}$. Constraints (26) and (27) ensure that the vehicles return to the depot no later than T_{max} . Constraints (28) and (31) help to avoid the symmetry generated by the copies of the CSs. The parameter m_{ij} is equal to 1 if i and $j \in F'$ represent the same CS. Finally, constraints (32–35) define the domain of the decision variables.

This MILP formulation can be easily adapted to model the other approximations (FS,

L1, and L2) and the different charging policies (FR and PR):

- Full charge and piecewise linear approximations (FR-PL): we replace constraints (17) and (18) by

$$o_i = \sum_{k \in B} \lambda_{ik} a_{ib}, \forall i \in F' \quad (36)$$

$$e_i = \sum_{k \in B} \lambda_{ik} c_{ib}, \forall i \in F' \quad (37)$$

- Partial charge using the first segment (PR-FS): To run our MILP with this assumption, we modify the input data to include only the first segment.
- Partial charge and linear approximations (PR-L): To run our MILP with PR-L1 and PR-L2, we modify the input data so that in the piecewise linear approximation there is a single segment with the corresponding charging rate.

3. Results of the comparison

3.1. Experimental settings

As mentioned in the last section, the MILP formulation uses β copies of the CSs to model multiple visits to the same CS. Although several authors followed this strategy (Conrad & Figliozzi 2011, Erdoğan & Miller-Hooks 2012, Schneider et al. 2014, Sassi et al. 2014, Goeke & Schneider 2015, Hiermann et al. 2016), they do not explain how the value of β is set. It is worth noting that β plays an important role in the definition of the solution space, and therefore it restricts the optimal solution of the model. For instance, an optimal solution found with $\beta = 3$ may not be optimal for $\beta = 4$. In practice, there is no restriction on the number of times that a CS can be visited, but large values of β result in models that are computationally intractable. To overcome this difficulty, we designed an iterative procedure to solve the MILP for increasing values of β . Starting with $\beta = 0$, at each iteration, our procedure (i) tries to solve the MILP to optimality with a time limit of 100 h, and (ii) sets $\beta = \beta + 1$. The procedure stops when the time limit is reached or an iteration ends with a solution s_β satisfying $f(s_\beta) = f(s_{\beta-1}^*)$, where $f(\cdot)$ denotes the objective function and $*$ an optimal solution.

3.2. Results

To assess the value of considering partial charging and nonlinear charging function approximations in eVRPs, we conducted an experiment to compare our battery charging assumptions with the assumptions commonly used in the literature. We compare only optimal solutions of the different approaches so that the comparison is independent of the solution method.

Table 1 presents the results. For each charging assumption, we give the objective function value (of), the percentage gap between of and the PR-PL solution (G), the number of routes in the solution (r), and the value of β . Since in practice the charging time is controlled by the nonlinear charging function, the charging decisions of the PR-L solutions are evaluated a posteriori using the piecewise linear approximation. The last rows of Table 1 summarize the results. We present, for each assumption, the average and maximum percentage gap, the number of solutions employing more vehicles than in the PR-PL solution, and the number of infeasible solutions.

The results show that solutions based on the full charging policy perform poorly in terms of both objective function (+20.11% on average) and number of routes (8 solutions use a larger fleet) with respect to those based on the partial charging policy. This is because the vehicles spend more time than necessary at the CSs. The main motivation for a full charging policy is to avoid complex charging-quantity decisions. However, according to our results, the gain in simplicity does not offset the loss of solution quality.

In the PR-FS assumption vehicles can charge their batteries up to only around 80% of the actual capacity. Artificially constraining the capacity may force vehicles to detour to CSs more often than necessary when traveling to distant customers. Because the maximum route duration is limited, the time spent detouring and recharging the battery reduces the number of customers that can be visited. Consequently, more routes may be needed to service the same number of customers. Our results confirm this intuition: in 3 of the 20 instances the PR-FS assumption increases the number of routes. Furthermore, in practice some distant customers may not be included in routes unless the vehicles can fully use their battery capacity. In our experiments, 9 instances become infeasible under PR-FS. In conclusion, although PR-FS simplifies the problem (avoiding the nonlinear segment of the charging function) it may lead to solutions that are infeasible, or with larger fleets and (on average) 2.70% more expensive.

As mentioned before, PR-L1 assumes that batteries charge faster than they do in reality (Figure 3a). As a consequence, routes based on PR-L1 may in practice need more time to reach the planned charge levels. The extra time may make a route infeasible if there is little slack in the duration constraint. Indeed, a post-hoc evaluation shows that for 14 instances, the PR-L1 solutions are infeasible in practice. On the other hand, PR-L2 assumes that batteries charge slower than in reality (Figure 3b). Overestimating the charging times does not lead to feasibility issues, but the resulting routes may be overly conservative. For instance, in our experiments PR-L2 leads to solutions that are (on average) 1.45% more expensive, and it increases the number of routes in 2 instances.

Table 1: Comparison of our charging assumptions with charging assumptions from the literature

Instance	PR-PL			FR-PL			PR-FS				PR-L1				PR-L2										
											Solution		Evaluation		Solution		Evaluation								
	<i>of</i>	<i>r</i>	β	<i>of</i>	$G(\%)$	<i>r</i>	β	<i>of</i>	$G(\%)$	<i>r</i>	β	<i>of</i>	<i>r</i>	β	<i>of</i>	$G(\%)$	<i>r</i>	β	<i>of</i>	$G(\%)$	<i>r</i>	β			
tc0c10s2cf1	19.75	3	2	20.82	5.42	3	2	NFS	NFS	NFS	NFS	19.61	3	2	NFE	NFE	NFE	NFE	20.50	3	2	20.22	2.38	3	2
tc0c10s2ct1	12.30	2	0	12.53	1.87	2	0	12.61	2.52	3	0	12.22	2	0	12.42	0.98	2	0	12.46	2	0	12.30	0.00	2	0
tc0c10s3cf1	19.75	3	2	20.82	5.42	3	2	NFS	NFS	NFS	NFS	19.61	3	2	NFE	NFE	NFE	NFE	20.50	3	2	20.22	2.38	3	2
tc0c10s3ct1	10.80	2	0	11.10	2.78	2	0	10.80	0.00	2	0	10.79	2	0	11.03	2.13	2	0	10.97	2	0	10.80	0.00	2	0
tc1c10s2cf2	9.03	3	0	9.19	1.77	3	0	9.03	0.00	3	0	9.03	3	0	9.12	1.00	3	0	9.14	3	0	9.03	0.00	3	0
tc1c10s2cf3	16.37	3	2	21.33	30.30	3	2	NFS	NFS	NFS	NFS	15.99	3	1	NFE	NFE	NFE	NFE	16.89	3	2	16.37	0.00	3	2
tc1c10s2cf4	16.10	3	2	25.31	57.20	4	3	NFS	NFS	NFS	NFS	15.66	3	2	NFE	NFE	NFE	NFE	16.43	3	2	16.23	0.81	3	2
tc1c10s2ct2	10.75	3	1	11.14	3.63	3	0	10.75	0.00	3	1	10.75	3	0	10.76	0.09	3	0	10.94	3	0	10.78	0.28	3	0
tc1c10s2ct3	13.17	2	2	22.76	72.82	3	3	15.98	21.34	3	2	13.06	2	2	NFE	NFE	NFE	NFE	13.60	2	2	13.17	0.00	2	2
tc1c10s2ct4	13.83	2	1	17.61	27.33	3	1	NFS	NFS	NFS	NFS	13.34	2	1	NFE	NFE	NFE	NFE	14.17	2	1	14.17	2.46	2	1
tc1c10s3cf2	9.03	3	0	9.19	1.77	3	0	9.03	0.00	3	0	9.03	3	0	9.12	1.00	3	0	9.14	3	0	9.03	0.00	3	0
tc1c10s3cf3	16.37	3	1	21.33	30.30	3	2	NFS	NFS	NFS	NFS	15.99	3	1	NFE	NFE	NFE	NFE	16.89	3	2	16.37	0.00	3	2
tc1c10s3cf4	14.90	3	1	18.43	23.69	4	0	NFS	NFS	NFS	NFS	14.56	2	1	NFE	NFE	NFE	NFE	15.18	3	0	15.18	1.88	3	0
tc1c10s3ct2	9.20	3	0	11.14	21.09	3	0	9.20	0.00	3	0	9.19	3	0	NFE	NFE	NFE	NFE	10.80	3	0	10.57	14.89	3	0
tc1c10s3ct3	13.02	2	0	17.06	31.03	3	0	13.07	0.38	2	1	12.98	2	0	13.16	1.08	2	0	13.60	2	0	13.02	0.00	2	0
tc1c10s3ct4	13.21	2	0	15.54	17.64	3	1	13.58	2.80	3	1	12.92	2	1	NFE	NFE	NFE	NFE	13.71	2	0	13.21	0.00	2	0
tc2c10s2cf0	21.77	3	3	25.24	15.94	4	2	NFS	NFS	NFS	NFS	14.53	2	2	NFE	NFE	NFE	NFE	22.78	4	4	22.15	1.75	4	4
tc2c10s2ct0	12.45	3	2	15.05	20.88	3	3	12.45	0.00	3	2	12.44	3	3	NFE	NFE	NFE	NFE	12.93	3	2	12.45	0.00	3	2
tc2c10s3cf0	21.77	3	2	25.24	15.94	4	2	NFS	NFS	NFS	NFS	14.53	2	2	NFE	NFE	NFE	NFE	23.02	4	3	22.20	1.98	4	3
tc2c10s3ct0	11.51	3	0	13.27	15.29	2	0	11.51	0.00	3	0	11.50	3	0	NFE	NFE	NFE	NFE	11.92	3	0	11.54	0.26	3	0
Avg. Difference (%)					20.11				2.70							1.04									1.45
Max. Difference (%)					72.82				21.34							2.13									14.89
Solutions with larger fleet						8				3							0								2
Infeasible solutions				0				9						14							0				

NFS: Non-feasible solution, NFE: Non-feasible evaluation
 $G(\%) = (of - of_{PR-PL}) / of_{PR-PL} \times 100$

Part II

Detailed results of the hybrid metaheuristic

Tables 2 and 3 show the results of our two ILS versions (i.e., ILS(M)+HC and MILP(H)+HC) for the small and large eVRP-PNL instances. In Table 2, we compare our results with the optimal solutions found by Gurobi using the MILP formulation. In Table 3, we compare our results with the best results obtained with Gurobi. For each instance, we report the problem name¹, and the best solution (BKS) taken from the results of Gurobi, ILS(H)+HC or ILS(M)+HC.

For the results obtained with Gurobi, we report the best solution (Best), and the gap with respect to the BKS (G)². For the results obtained with the ILS(H) + HC and ILS(M) + HC, we report the best solution, the average solution (Avg.), and the average computing time (t in seconds) over ten runs. For the ILS(M) + HC, we also report the average computing time using the preprocessing procedure (t^*). For the two ILS + HC versions, we provide the gap of the average solution and best solution with reference to the BKS. The last rows of the table summarize the average and maximum BKS gap, the number of times each method found the BKS, and the average and maximum running time. Values in bold indicate that

¹ $tc\alpha c\beta s\mu\epsilon\#$, where α is the type of the location of the customers (i.e., 0:randomize, 1:cluster, 2: mixture of both), β is the number of customers, μ is the number of the CSs, ϵ is 't' if we use a p-media heuristic to locate the CSs and 'f' otherwise, and $\#$ is the number of the instance for each combination of parameters (i.e., $\# = 0, 1, 2, 3, 4$)

² $G(\%) = (of - of_{BKS}) / of_{BKS} \times 100$

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