Dynamic approach of the feedrate interpolation for trajectory planning process in multi-axis machining

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Abstract Feedrate interpolation, which consists in generating the set points sent to the axis controllers from the initial path, constitutes a major line of the trajectory planning process. In multi-axis high speed machining the feedrate is usually evaluated by a kinematic method as the maximum feedrate respecting the joint velocity, acceleration an jerk limits. However this approach requires a difficult experimental tuning of the kinematic limits based on the observation of the axis behaviour that leads to a loss of productivity.

The multiplication of the kinds of multi-axis machines in manufacturing leads to the need to take the dynamic behaviour into account, particularly for highly dynamic systems such as serial robots. A novel and efficient dynamic approach of the feedrate interpolation process is proposed in this paper. The system performance characteristics are checked through torque and torque rate limits at the actuator level. The integration of the dynamic model of the system enables to include different chosen effects such as inertia, centrifugal, Coriolis effects, gravity, and friction forces. The dynamic model can be also easily adapted to different systems and processes such as serial manufacturing robots or multi-axis machining centers. Then, these new dynamic constraints lead to limit values based on the dynamic behaviour of the whole axis mechanical chain from the actuator to the end-effector, and allow to be close from the system components technology. The efficiency of the proposed approach is demonstrated by several simulations on different systems and test paths and is compared to the usual kinematic method results.

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1 Introduction

The current issue of trajectory planning in multi-axis machining represents a crucial challenge. In manufacturing, both productivity and part quality are essential objectives constrained by the drive, control, and process capabilities. In this context, four main lines can be explored to improve the trajectory planning process. The first step of the process consists in creating a sufficiently continuous tool path from the part geometry, respecting the permitted milling tolerance. Then the feedrate interpolation is performed on the path, by considering the machine performance characteristics, to generate the axis set points to send to the drive controllers. The two other steps deal with the drive control improvement and the process and machine geometry understanding to forecast the resulting error. This paper focuses on the temporal interpolation to determine the feedrate along the path in a time-optimal trajectory planning process.

In multi-axis milling, kinematic constraints are usually evaluated to perform the temporal interpolation. In order to optimize the productivity the highest feedrate under axis kinematic limits is calculated at each point of the discretized path. Different techniques can be found in the scientific literature to solve the feedrate planning problem.

Seven phases jerk-constrained feedrate profiles are associated end-to-end by Erkorkmaz and Altintas [11], resulting in a trapezoidal evolution of the acceleration with specified slopes matching the maximum tangential jerk value. Liu et al. [18] suggest a similar method in an interpolation scheme for NURBS curves adjusting the feedrate according to the chord error, admissible deceleration/acceleration

at sharp corners and high jerk reduction. In [17] and [26] the authors introduce an additional module based on a cascade control structure with feedforward, with the aim of bounding the contour error. Such a jerk-limited approach is also implemented on an industrial 6-axis machining robot by Olabi et al. [19] with an additional extra check of the joint's kinematic constraints. Indeed, although the jerk-constrained profile has been frequently used in the literature, only the tangential jerk and acceleration are limited with this solution. Then, as the inverse kinematic transformation is non linear, it is impossible to verify in this way each axis own limitations.

Some papers have elaborated feedrate interpolation algorithms in 5-axis machining respecting the axis kinematic constraints. Lavernhe et al. [16] present a predictive model of the feedrate using an iterative time inverse method considering both the axis synchronisation and the kinematic performances, but only suited for G1 tool path. In [23] a smooth feedrate scheduling algorithm based on an iterative SQP optimization considering kinematic limits of the drives and minimizing the machining time is developed for cubic or NURBS splines tool path. Recently, iterative and discretized interpolation methods ensuring the respect of all the kinematic constraints (velocity, acceleration and jerk limits) at each time step, efficiently on both linear and polynomial tool paths, have been built-up. Erkorkmaz et al. ([12],[10]) propose to bypass the split points meeting a constraint activation by combining two basic feasible velocity profiles and integrate an additional maximum cutting force constraint. Beudaert et al. [4] develop the VPOp (Velocity Profile Optimization) algorithm which iteratively optimizes the feedrate profile, using dichotomy and the principle of constraints intersection, with a constant time step discretization.

Thus, the kinematic constraints are used most of the time to perform the feedrate interpolation in a multi-axis machining context. These constraints are joint limits of velocity, acceleration, and jerk tuned by the manufacturer to reduce the errors caused by the penalizing dynamic effects and avoid an excessive use of the drives according to the dynamic response of the machine. The tuning of theses parameters has to consider as well the mechanical characteristics of the actuators, mechanical transmission and structure as the dynamics of the control [7][22]. Yet, the feedrate interpolation by kinematic methods requires a complex off-line preprocessing, computationally expensive and not suitable for real time applications. In light of the difficult tuning of the axis constraints, one can ask if a direct dynamic approach could not lead to more relevant and consistent results.

Trajectory planning is a regular and large issue in other scientific fields. In robotics, the feedrate interpolation approaches slightly differ from the previously detailed methods because the robot end-effector path is often constituted of only few points and not a continuous geometry. As this paper deals with the trajectory planning in machining, the tool path is necessarily entirely known. But it seems interesting to compare the different planning processes and the associated constraints from system performance characteristics.

The planning process in robotics has been fully solved as PCTOM (Path-Constrained Time-Optimal Motions) trajectory optimization under maximum articular torque constraint. Bobrow et al. [5] and Shin and McKay [24] propose a two-pass iterative algorithm based on the constraints intersection principle knowing dynamic equations of motion and joint torque limitations. Unfortunately, the implementation of PCTOM trajectories on robots leads to oscillatory phenomenons of the joints that are susceptible to damage the whole robot. To control the vibratory behaviour and avoid these oscillations, Kyriakopoulos and Saridis [15] are the first to introduce a minimum-jerk optimization solved with the Pontryagin Maximum Principle. Later, Dong et al. [9] integrate a jerk constraint to the two-pass algorithm but with difficulties to connect the forward and the backward pass. In [13], the planning process is implemented by means of a SQP optimization algorithm minimizing both the execution time and the squared-jerk integral. But these methods built to minimize a jerk factor give the smoothest trajectory and not the fastest one without vibration as expected for high productivity machining applications.

The jerk notion is transformed in a dynamic constraint by Constantinescu and Croft [8] with the torque rate, that is the time derivative of torque, calculated at the actuator level after the differentiation of the robot's dynamic model. The actuator torque rate limits are chosen to eliminate important changes in torque which can cause highly jerky motions and severe vibrations in the joints. The authors carry out a time-optimal SPCTOM (Smooth Path-Constrained Time-Optimal Motions) trajectory optimization under both joint torque and torque rate limits using an iterative flexible tolerance method. This dynamic approach appears to be efficient and has been employed on several applications in the literature, for instance on flexible-link robotic manipulators by Boscariol and Gasparetto [6].

Thus, in robotics the trajectory planning is equivalent to the resolution of an optimization problem with different constraints and cost functions built on the system performance characteristics. A dynamic approach is frequently chosen integrating constraints such as joint torque and sometimes torque rate limits. This approach has the benefit of respecting system dynamics and actuation capabilities. But the resolution of these optimization problems requires complex algorithms and high time-consuming calculation that could be penalizing for machining path interpolation.

As underlined by the preceding state of the art, the system dynamics is a fundamental aspect of the feedrate interpolation in trajectory planning. Moreover the current multi-axis machining development results in the increase of the number and types of machines, such as multi-axis milling centers, serial or parallel industrial robots, which have very different dynamic behaviours. Consequently, in this paper a new dynamic approach is presented to reach several objectives. On one hand this article aims at finding alternative interpolation constraints built on the machine dynamic performance characteristics and transposing the problem from the axis kinematics level to the actuator torque level. On the other hand the influence of the dynamic behaviour on the feedrate interpolation has to be analysed according to the chosen drive limits. Thus, one of this study main goals aims at identifying the different dynamic effects on the machine performance characteristics along the tool path.

The paper highlights the influence of the system dynamics on the feedrate planning process in multi-axis machining. The integration of the dynamic model of the system enables to include different chosen effects such as inertia, centrifugal, Coriolis effects, gravity, and friction forces and can be easily adapted to different systems and processes. The new dynamic constraints lead to limit values based on the dynamic behaviour of the whole axis mechanical chain from the actuator to the end-effector, and allow to be close from the system components technology.

This paper is organized as follows. In Section 2 new interpolation constraints of actuator torques and torque rates, representing the dynamic performance of the system, are introduced. Section 3 details the simulation results of the dynamic approach on two test paths compared to the feedrate profiles obtained by the usual kinematic method in order to validate the proposed approach. Then, new limit values based on the actuators' capabilities are proposed and simulated on industrial machines and robots in Section 4 to observe the improvement leads by this method on different multi-axis machines.

2 Interpolation constraints from the dynamic performance characteristics

2.1 Evaluation of the maximum velocity profile

Usually, feedrate interpolation in multi-axis milling is performed respecting the tuned axis velocity, acceleration and jerk limits ($\dot{q}_{max,j}$, $\ddot{q}_{max,j}$ and $\dddot{q}_{max,j}$) recorded in the numerical controller. Thus, the kinematic constraints can be described for each axis j as follows:

$$| \dot{q}_j | \leq \dot{q}_{max,j} \; ; \; | \ddot{q}_j | \leq \ddot{q}_{max,j} \; ; \; | \dddot{q}_j | \leq \dddot{q}_{max,j} \qquad (1)$$

Beudaert et al. introduced in [3] the approximation $\ddot{s} = \dddot{s} = 0$ around tough zones where the curvature varies strongly. Thus articular velocity $\dot{q}_j$, acceleration $\ddot{q}_j$ and jerk $\dddot{q}_j$ can be expressed in terms of the tangential velocity $\dot{s}$.

$$\dot{q}_j = \frac{dq_j}{ds} \times \frac{ds}{dt} = q_{s,j} \times \dot{s} \; ; \; \ddot{q}_j \simeq q_{ss,j} \times \dot{s}^2 \qquad (2)$$
$$\dddot{q}_j \simeq q_{sss,j} \times \dot{s}^3$$

These simplified joint derivatives allow to formulate the constraints as functions of the tangential velocity. An approximative maximal feedrate profile $\dot{s}_{lim}$ is evaluated along the discretized path as the minimum of the constraints on all the axis of the machine. The maximum velocity profile constitutes a good way to predict the results of the feedrate interpolation and will be used to analyse the proposed dynamic approach further in the article.

New alternative interpolation constraints based on the system dynamic performance characteristics are introduced in next subsection. By integrating the dynamic model the problem can be transposed from the axis kinematic limits to the actuation torque and torque rate limits. Then the simplified constraints are calculated to plot the maximum velocity profile $\dot{s}_{lim}$ along the tool path.

2.2 Torque and torque rate constraints

The maximum actuator torques $\tau_{max,j}$ and torque rates $\dot{\tau}_{max,j}$ (time derivative of torque) have been chosen to embody the performance characteristics of the machine in the proposed approach. Indeed the dynamic constraints can be expressed for each axis j in terms of the actuation permissible limits.

$$| \tau_j | \leq \tau_{max,j} \; ; \; | \dot{\tau}_j | \leq \dot{\tau}_{max,j} \qquad (3)$$

The system dynamics leads to the equations of motion which can be described in a compact matrix way (Eq.(4)). The Newton-Euler equations and the new formalism of calculation developed by Bertrand and Bruneau [2] are used to evaluate the dynamic model. More details on the way to express and compute the dynamic terms are given in Appendix A.

$$\tau = H(q)\ddot{q} + B(q)\dot{q}^2 + C(q)\dot{q}_i\dot{q}_j + G(q) + \Gamma_f(\dot{q}) \qquad (4)$$

The differentiated dynamic model is written as follows:

$$\dot{\tau} = H(q)\dddot{q} + \dot{H}(q,\dot{q})\ddot{q} + 2B(q)\dot{q}\ddot{q} + \dot{B}(q,\dot{q})\dot{q}^2 + \\ \dot{C}(q,\dot{q})\dot{q}_i\dot{q}_j + C(q)(\ddot{q}_i\dot{q}_j + \dot{q}_i\ddot{q}_j) + \dot{G}(q,\dot{q}) + \dot{\Gamma}_f(\dot{q},\ddot{q}) \quad (5)$$

This formalism enables to identify the different dynamic terms : H is the inertia tensor, B and C represent successively the centrifugal and Coriolis effects, G the gravity forces, and

Γ_f the friction torques. The friction law of each joint i is chosen to be a linear function of the joint velocity:

$$\Gamma_{f,i} = K_{ti} (a_i \dot{q}_i + b_i) \tag{6}$$

where K_{ti} is the motor torque constant, the linear part $a_i \dot{q}_i$ is the current equivalent to the viscous friction and the constant part b_i represents the dry friction. The coefficients (a_i, b_i) are identified from experimental results by recording the actuator's current for various displacements with constant velocities, such as in [21] for the Mikron machine. The constraints are developed by integrating the dynamic model and its derivative form. Thus the influence of each dynamic effect on the feedrate interpolation can be dissociated and analysed.

To evaluate the proposed dynamic method, the constraints are expressed by using the approximation in Eq.(2). Thus, the equations Eq.(3), Eq.(4), and Eq.(5) lead to:

$$|H[q_{ss}]\dot{s}^2 + B[q_s^2]\dot{s}^2 + C[q_{s,i}q_{s,j}]\dot{s}^2 + G + \Gamma_f| \le [\tau_{max,j}] \tag{7}$$

$$|H[q_{sss}]\dot{s}^3 + \dot{H}[q_{ss}]\dot{s}^2 + 2B[q_s q_{ss}]\dot{s}^3 + \dot{B}[q_s^2]\dot{s}^2 +$$
$$C[q_{s,i}q_{s,j} + q_{s,i}q_{ss,j}]\dot{s}^3 + \dot{C}[q_{s,i}q_{s,j}]\dot{s}^2 + \dot{G} + \dot{\Gamma}_f|$$
$$\le [\dot{\tau}_{max,j}] \tag{8}$$

The maximum velocity profile $\dot{s}_{lim}$ is calculated at each point of the path as the maximum value of velocity permitted by the torque and torque rate constraints on all the axes:

$$\dot{s}_{lim} = \min_{j=1:n} (\dot{s}(s)_{\tau_j}, \dot{s}(s)_{\dot{\tau}_j}) \tag{9}$$

2.3 Choice of limit values

The actuator permissible torque and torque rate limits ($\tau_{max,j}$ and $\dot{\tau}_{max,j}$) have to be defined to carry out the feedrate interpolation.

The first study, in Section 3, aims at validating the new dynamic approach with respect to the usual kinematic method. In order to be able to properly compare the results of both approaches it is necessary to first choose the actuators' dynamic limits according to the axis maximal acceleration and jerk values. Indeed, the kinematic limits are tuned at the installation of the machine to obtain acceptable axis dynamic responses, and depend on various non-identifiable physic parameters. By using equivalent dynamic values the kinematic and dynamic approaches can be compared.

To obtain the joint's dynamic limits, the diagonal terms of the inertia tensor are considered as the axis equivalent inertia values J_j^{eq}. To be able to compare simulation results to the kinematic approach it is necessary to determine equivalent dynamic limits. This approximation is valid under the assumption that the coupled terms of the inertia tensor are negligible. The actuator dynamic limits are then calculated as functions of the axis equivalent inertias and the tuned

kinematic limits, where r_j is the transmission ratio of the drive and motor:

$$\tau_{max,j}^{eq} = \frac{J_j^{eq}}{r_j} \times \ddot{q}_{max,j} \; ; \; \dot{\tau}_{max,j}^{eq} = \frac{J_j^{eq}}{r_j} \times \dddot{q}_{max,j} \tag{10}$$

The Section 4 deals with the choice of torque and torque rate limits according to the machine performance characteristics. These new limits are those to use forward in the proposed dynamic method. They should directly depend on the axis components' technology and should be easy to calculate.

3 Simulation of the dynamic approach

Some simulation results are analysed in this section to validate and explore the proposed dynamic approach. The maximum feedrate profile respecting the torque and torque rate constraints is first evaluated on two tests paths. The influence of the different dynamic effects on the feedrate profile is observed by modifying the complexity of the dynamic model integrated into the constraints. Then these results are compared to the maximum feedrate profiles obtained by the usual kinematic method. In that way one can expect the dynamic approach will be approved and its benefits highlighted.
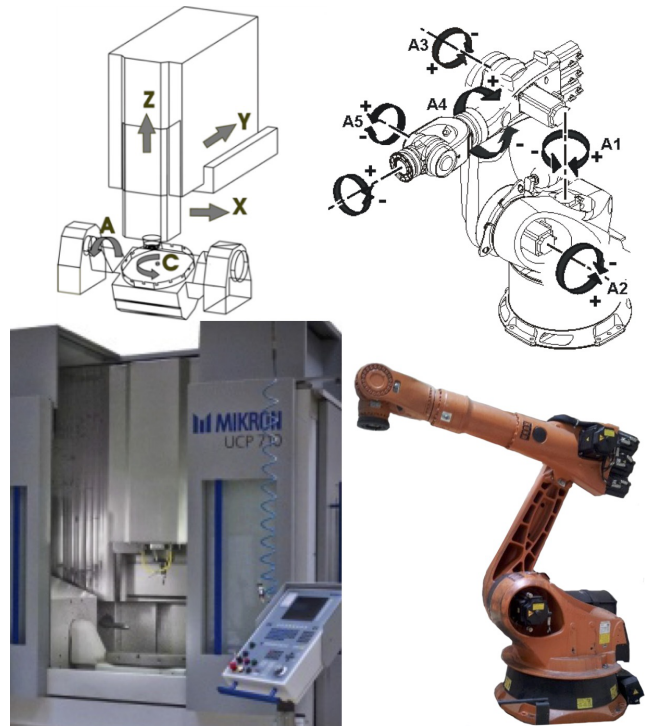

Fig. 1 Mikron UCP710 and KUKA KR180-L130

3.1 Test definition and machine characteristics

The development of multi-axis machining has been accompanied by increased amounts and kinds of machine structures with very different dynamics. In order to show that the new dynamic approach is sufficiently generic, two kinds of systems are studied: serial manufacturing robots and multi-axis machining centers. Simulations are carried out on a Mikron machine UCP710 and a Kuka robot KR180-L130 represented on the Fig.1.

The two studied paths are represented on the Fig.2 by the cutter location, the cutter contact positions, and the tool orientations. The first one is a simple test path in the yz plane, used to compare both the kinematic and the dynamic approach results. The second one is an hydro-turbine blade's 5-axis milling path, chosen to observe the effects of important kinematic needs on the axes caused by passing close to the Inverse Geometric Transformation (IGT) singularity. The joint positions q_j are obtained from these paths with the IGT of each system.
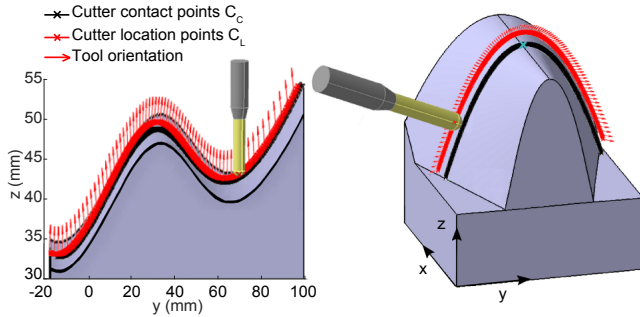

Fig. 2 Test paths 1 & 2

The next paragraph deals with the simulation results of both kinematic and dynamic approaches on the two example paths. The maximum velocity profiles under kinematic constraints (Eq.(1)) and dynamic constraints (Eq.(7) and Eq.(8)) are plotted. The dynamic parameters of these systems have been identified in [21] for the Mikron machine. These ones are approximated according to the CAD model for the Kuka robot, but a complete identification process as described in [20] should be applied to obtain more accurate results. The Table 1 details the axis tuned kinematic limits and the actuator dynamic equivalent values evaluated with the Eq.(10) for the studied Mikron machine UCP710.

Although one can expect the dynamic limits to be close between the axis, because the 5 axis' actuators are identical, the calculated values are far from each other. For example, the torque rate limit is much more higher for the axis Z_m and C, hence less restrictive for the feedrate interpolation. This observation underlines that the kinematic limits are experimentally tuned to avoid vibratory phenomenons by observing the frequency responses of each axis.

Table 1 Machine drive tuned kinematic limits and actuator dynamic equivalent values - Mikron UCP710

Axis	Unit	X_m	Y_m	Z_m	A (tr)	C (tr)
$\dot{\mathbf{q}}_{\mathbf{max,j}}$	m/min	30	30	30	15	20
$\ddot{\mathbf{q}}_{\mathbf{max,j}}$	m.s^{-2}	2.5	3	2.1	0.83	0.83
$\dddot{\mathbf{q}}_{\mathbf{max,j}}$	m.s^{-3}	5	5	50	5	50
$\tau^{\mathbf{eq}}_{\mathbf{max,j}}$	Nm	44	45.2	29.7	10.7	9.29
$\dot{\tau}^{\mathbf{eq}}_{\mathbf{max,j}}$	Nm/s	88	75.4	707	64.2	555

3.2 Feedrate profile under torque and torque rate constraints

The maximum velocity profile $\dot{s}_{lim}$ allows to predict the results of a complete feedrate interpolation. The feedrate profile is evaluated on the Fig.3 under the constraints of actuator torques Eq.(7) and torque rates Eq.(8) for the Mikron machine on the first test path presented above. The resulting maximum feedrate profile of the dynamic method is the minimum envelope of these two limits.
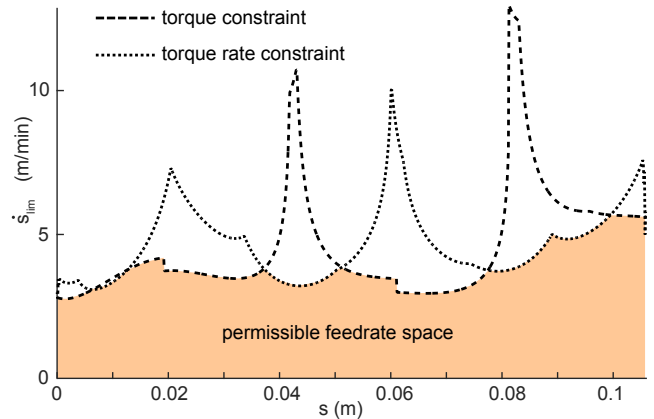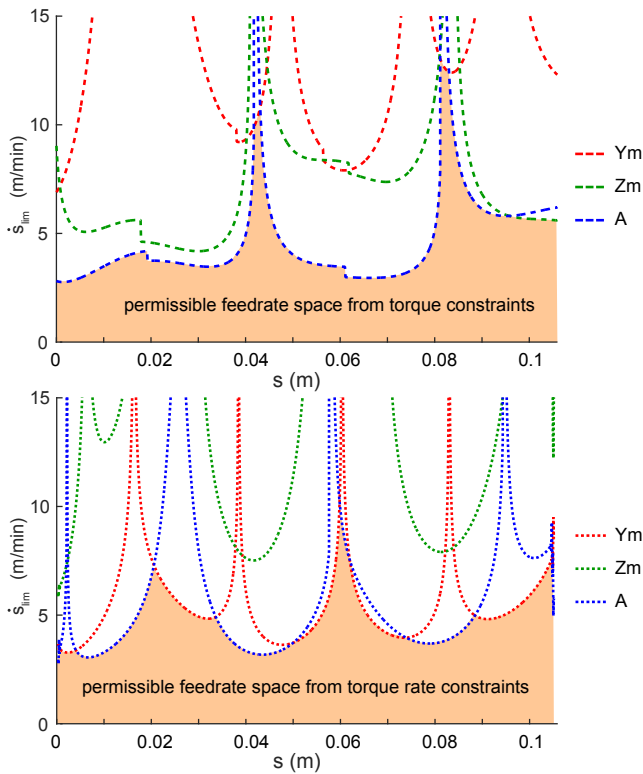

Fig. 3 Feedrate profiles under tuned dynamic constraints, path 1

The previous profile is determined as the minimum of each dynamic constraints on the five axis of the machine along the path. The details of the feedrate evolutions obtained for each axis at the actuator torque and torque rate tuned limits appear on the Fig.4. The simulation results allow to observe the impact of each axis' dynamic performance characteristics, in terms of actuator torque and torque rate limits, along a simple planar path. One can notice that the limiting axis is not the same in the torque constrained feedrate or in the torque rate constrained profile. The two dynamic constraints have influence on the feedrate interpolation. Rationally, the calculated feedrate depends greatly on the geometry of the path, because high curvature changes lead to important acceleration and jerk phases, and so important actuator torques and torque rates.

Fig. 4 Feedrate profiles under joint actuator torque and torque rate constraints on path 1

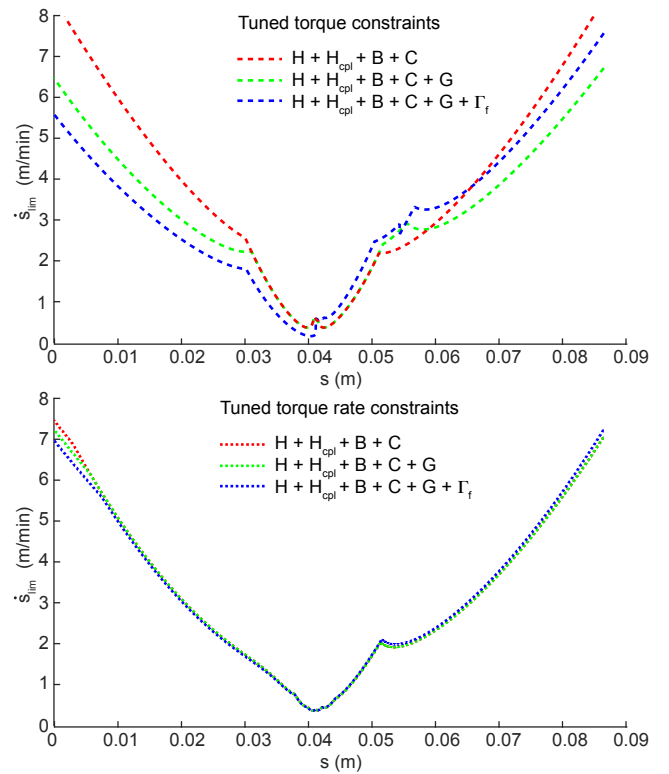**Fig. 5** Influence of the dynamic effects on the torque and torque rate constraints, path 2

The dynamic approach, simulated on the first path with limit values equivalent to the kinematic ones tuned in the Numerical Command, brings some interesting results and seems adapted to carry out the feedrate interpolation according to the machine performance characteristics. The next subsection tries to identify the influence of the different dynamic effects on the feedrate profile simulated for the second test path that requests high dynamics.

3.3 Influence of the different dynamic effects

The dynamic model of the machine tool is complex and includes non-dissociable factors sometimes coupled between the axis. These dynamic effects are mainly: H the uncoupled inertia, H_{cpl} the coupled inertia, B and C the centrifugal and Coriolis effects, G the gravity forces, and Γ_f the friction forces.

Here the influence of the integration of each dynamic effect is observed on the maximum feedrate profile to attempt to identify its contribution according to the constraints along the path.

The feedrate profiles respecting the actuator torque and torque rate constraints are plotted on the Fig.5, for the second path, considering the different dynamic effects. As the terms of the dynamic model are all added up together, the dynamic effects can not be considered as disjointed con-

straints. Consequently, the velocity limit is first evaluated under the complete dynamic constraints, then by removing the effects one by one to observe their influence.

Inertia H, gravity G, and friction Γ_f effects have a dominating influence on the dynamic performance characteristics for the Mikron machine studied in this part. The maximum velocity profiles calculated considering successively the effects of H, $H + H_{cpl}$ and $H + H_{cpl} + B$ are not presented here because they are identical to the one obtained with the model $H + H_{cpl} + B + C$. Thus the inertial, centrifugal and Coriolis couplings (H_{cpl}, B, C) do not modify the velocity limits considering only the uncoupled inertia of the axis, even around the singular point at $s = 0.04m$, in need of high axis dynamics. Hence the contribution of the couplings can be neglected on this machine-tool structure compared to the whole dynamic behaviour.

Several aspects have to be pointed on the profiles represented in Fig.5. First, the gravity and friction forces have a significant influence on the velocity limit calculation. G is directly linked to the joint positions of the axis subject to gravity. Thus gravity effect can be globally positive or negative for the machine dynamics along the path.

Similarly, the friction forces can be dynamically advantageous or restrictive along the path. This variable contribution is coherent because the friction leads to an additional resistive torque when the joint velocity is constant or in-

creases, while it helps the axis actuation to slow-down. Otherwise, some discontinuities appear on the torque-constrained profiles which take into account the friction effect. This phenomenon can be explained when the articular velocity crosses zero, where the apparition of dry friction to overcome causes a torque discontinuity and so a feedrate discontinuity.

Finally, this analysis aimed at observing the contribution of each dynamic effect on the actuator constraints along the path, in a feedrate interpolation process. By comparing different dynamic models more or less complex, the feedrate profiles were detailed and the dominating terms were identified. These first results of the dynamic approach are promising and merit to be compared to those from a classical kinematic method.

3.4 Comparison to the kinematic method results

In the literature the usual method considers kinematic constraints. This paragraph details and compares the simulation results of the proposed dynamic approach and the kinematic method.

The maximum velocity profiles under the constraints of actuator torque Eq.(7) and torque rate Eq.(8) on one hand and of axis velocity, acceleration and jerk Eq.(1) on the other hand are compared in the Fig.6. It is important to remind that the values of actuator dynamic limits have been calculated equivalent to the tuned kinematic limits read in the Mikron machine NC. Thus the comparison between the results of both approaches is usable.

The first aspect to underline on Fig.6 is that the velocity limit evaluated along the path using the dynamic method is close to the one from the usual kinematic method. Also, as one can assume by observing the expressions of the dynamic model, the actuator torque is linked to the axis acceleration limit and the torque rate to the jerk limit.

However, the results of the two approaches are not entirely similar, in particular on the torque/acceleration constrained profiles. The observed differences come from the gravity and friction forces integrated in the dynamic model and influencing the feedrate estimation. Indeed, the kinematic method takes the acceleration and jerk limits into account, which match the uncoupled inertial effects, but not the gravity and friction forces. This explanation is verified outside by checking that the velocity limits are exactly the same calculated with the torques and torque rates without gravity and friction effects or the acceleration and jerk respectively. This likeness is possible because the inertial, centrifugal and Coriolis couplings are negligible for the studied Mikron machine. Otherwise the couplings would affect the dynamic method results while they are not included in the kinematic limits tuned axis by axis.
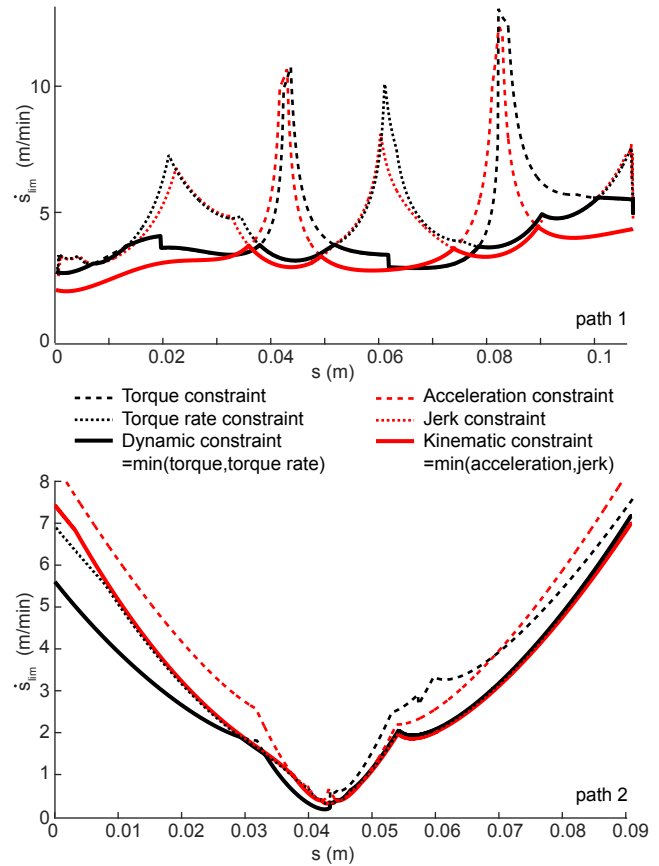
Fig. 6 Comparison of the feedrate profiles under dynamic and kinematic constraints, path 1 & 2

The simulation results detailed in this section, validate the use of the proposed dynamic approach to incorporate the machine performance characteristics in a feedrate interpolation process for machining trajectory planning. By comparison to the usual kinematic method, the torque and torque rate constraints allow to evaluate additional important effects according to the machine dynamics: the inertial, centrifugal and Coriolis couplings, the gravity and friction effects.

For now, the limit values of the dynamic constraints have been calculated to be equivalent to the axis acceleration and jerk limits, in order to apply a comparative analysis. The current kinematic values are tuned by limiting the vibratory behaviour of the axis for particular input control laws, and can be more restrictive than necessary. The goal of the following work is to find new dynamic limits built according to the system performance characteristics and observe the improvements lead on different machines.

4 Towards new limits based on the system performance characteristics

New dynamic limit values are introduced and discussed in this section based on the system performance characteris-

tics. In that way the feedrate interpolation issue is transposed at the actuator level, that should lead to more accurate values based directly on the components' technology and not on a global axis dynamic behaviour. The influence of these new limits is analysed by plotting the maximum feedrate profile on the path 1 for the Mikron machine and the Kuka robot.

4.1 Maximum actuator torques and torque rates

In this subsection new torque and torque rate values are discussed according to the actuators' capabilities.

4.1.1 Actuator torque limit

The actuators' technical datasheet from Siemens [25] provides the characteristics of the actuators for each Mikron axis. In particular this technical documentation gives immediately the maximum torque that the actuators can generate. This value of $65Nm$ is retained as the torque limit for the 5 axis which are actuated by the same motor type. This choice of constant limits can be discussed because the maximum torque actually depends on the actuator speed. An additional torque constraint, equivalent to the actuator power limit, can be proposed to include this dependence as a linear function of speed based on the motor speed-torque diagram. A such additional limitation on the torque constraint will have a significant influence on the feedrate calculation results but will not be detailed here for the sake of simplicity.

Thus, the dynamic approach owns the interest of ensuring torque constraints directly based on the actuators' technical characteristics.

4.1.2 Actuator torque rate limit

The torque rate limit is not a known motor characteristic and should be evaluated according to the mechanical chain dynamic behaviour depending for instance on the actuator, the transmission and the structure. For that an identification procedure has to be proposed to calculate the permissible actuator torque rates. A clear understanding of the dynamic effects of the torque rate on the system is necessary. As seen in the previous part, the torque rate limits impose the same kinds of constraints as the jerk limits. Also these two limits aim at reducing the vibratory phenomena of the axis and the structure that are harmful during important acceleration phases.

Different papers in the literature study the jerk dynamic effect and its link with the structure and axis oscillations. Barre et al. [1] introduce a relationship between the maximum amplitude of the vibratory error and the jerk limit. The maximum admissible joint displacement is empirically expressed as a function of the first natural frequency, the jerk limit and the structure damping in [14]. Finally a thorough

study of the machine stiffness and flexibilities should lead to accurate values of the axis jerk limit. However these flexibilities' analysis are complex and high time-consuming methods often based on finite elements modeling, that explains why the jerk limit remains experimentally tuned at the machine installation according to the axis dynamic response.

The torque rate limits should be identified from the technological parameters of the system components. Some experiments have been performed on the Mikron machine to attempt to evaluate the torque rate limits according to the vibratory behaviour of the mechanical chain observed at the actuation level. The vibratory mode, produced by the whole axis structure from the motor to the effector, should be removed by integrating the joint torque rate limit. Several paths are tested on the X_m Mikron axis with no and high jerk limits. The motor torque is calculated from the current measurements knowing the torque constant K_t. A first limit is considered as the maximum torque that the actuator is able to generate in one second. However, even if the actuator is able to provide an important torque rate, harmful vibratory phenomenon appears on the torque curves. New admissible values of torque rate limits have to be chosen according to the oscillation mode. An empirical relation can be introduced from these experimental results to approximate the torque rate limit as a function of the first natural frequency and the permissible maximal amplitude of the torque oscillations. Based on this relation, to reduce the oscillations to a maximal amplitude of $2Nm$, the torque rate limit of the X_m Mikron axis has to be chosen as $\dot{\tau}_{max,Xm} = 240Nm/s$. However the experimental identification procedure has to be carried out anew on each axis with more trials to obtain complete results and build a more accurate relation. Moreover the components' characteristics have to be linked to the vibratory behavior to propose a simple and versatile relationship between the axis components' technology and the torque rate limit. This point remains the prospect of current research and is not the major concern of this paper.

To keep a relative simple relation in order to fulfill this study, the actuator torque rate limits are calculated by considering that the ratio $\tau_{max,j}/\dot{\tau}_{max,j}$ stays identical between the tuned equivalent values evaluated from Eq.(10) and Table 1 and the new limits from the actuators' capabilities. This assumption is valid for the Mikron machine because the inertial, centrifugal, and Coriolis couplings are negligible in a feedrate interpolation process (see paragraph 3.3). The conservation of this ratio leads immediately to approximative actuator torque rate limits at the maximum capabilities:

$$\dot{\tau}_{max,j} = \tau_{max,j} \times \frac{\dot{\tau}^{eq}_{max,j}}{\tau^{eq}_{max,j}} \tag{11}$$

The values calculated for the Mikron actuators at the equivalent ratio are written below in the Table 2.

Table 2 Dynamic values from the actuator's capabilities - Mikron UCP710

Axis	Unit	X_m	Y_m	Z_m	A	C
$\tau_{\mathbf{max,j}}$	Nm	65	65	65	65	65
$\dot{\tau}_{\mathbf{max,j}}$	Nm/s	130	108.3	1547.5	390.1	3885

Table 3 Dynamic values from the actuator's capabilities - Kuka KR180-L130

Axis	Unit	q_1	q_2	q_3	q_4	q_5	q_6
$\tau_{\mathbf{max,j}}$	Nm	6.3	10.5	6.3	5	5	5
$\dot{\tau}_{\mathbf{max,j}}$	Nm/s	63	105	63	50	50	50

Here again the actuator torque rate limits vary strongly between the axes even if they are all equipped with the same motors and so should have similar torque rate values. Indeed the chosen calculation method necessarily transfers the axes differences from the equivalent tuned limits. The X_m torque rate limit is smaller than the one obtained experimentally, that confirms that kinematic limits could be more restrictive than necessary.

Approximative values are evaluated for the torque and torque rate limits of the Kuka robot KR180-L130, knowing the motors' maximum power and velocity. All these approximations used to evaluate the robot dynamics are appropriate here to observe the dynamic approach results on industrial machining robots. These actuator dynamic values are detailed in the following Table 3.

## 4.2 Results on Mikron and KUKA applications

The maximum feedrate profiles under dynamic constraints (Eq.(7) and Eq.(8)) are evaluated on the first path for the Mikron machine and the Kuka robot using the limits from the actuators' capabilities (Table 2 and Table 3).

4.2.1 Simulation results - Mikron machine

The feedrate profiles evaluated according to the torque and torque rate limits from the actuators' capabilities and from the tuned values are compared on Fig.7 for the Mikron machine.

The maximum feedrate profiles from the dynamic approach are higher with the new limits at the actuators' capabilities than those from the tuned values. Both torque and torque rate profiles are raised up by the use of these new limits. In particular, the torque-constrained feedrate, with the value directly defined by the actuators' technical characteristics that better illustrates the actuation capabilities, has been more than doubled.

This study shows that the dynamic approach allows to consider new constraint limits from the actuators' capabil-
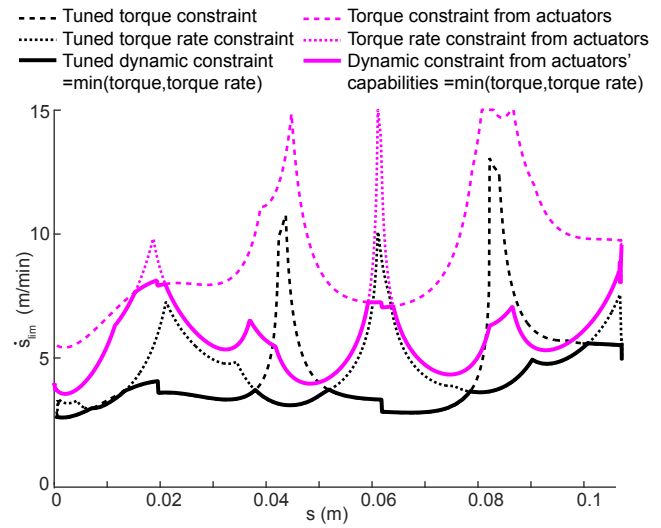
Fig. 7 Comparison of the feedrate profiles under dynamic constraints from the actuators' capabilities or tuned, Mikron machine, path 1

ities that seem less restrictive for the feedrate interpolation process. Moreover these new limits come immediately from the actuators' technology and technical capabilities. Thus the values can be easy to tune and more accurate than the kinematic ones tuned experimentally according to a global axis dynamic behaviour.

4.2.2 Simulation results - Kuka robot

The feedrate profiles are evaluated according to the torque and torque rate limits from the actuators' capabilities for the Kuka robot. This paragraph aims at observing the dynamic approach results on an industrial machining robot application which has different dynamics than a machine tool and should react otherwise. The influence of the integration of each dynamic effect on the constraints is observed on the Fig.8. Inertial, inertial couplings, centrifugal, Coriolis, gravity and friction effects are added up successively to attempt to identify the associated contribution.

The results point out that inertial, centrifugal and Coriolis couplings are not negligible in a feedrate planning process for a machining robot application contrary to the observations done on the milling machine. That can be explained because the robot has a serial structure with numerous components that leads to high coupling effects. Also, here again, the gravity and friction forces are dominant in the feedrate evaluation under the actuator torque constraint and can be as beneficial as limiting. These points underline one of the advantages of the dynamic approach that takes these effects into account in the velocity limit calculation unlike the usual kinematic method.

Compared to the machine tool structures designed to be very rigid, serial machining robots can contain considerable joint flexibilities. These flexibilities have an important im-
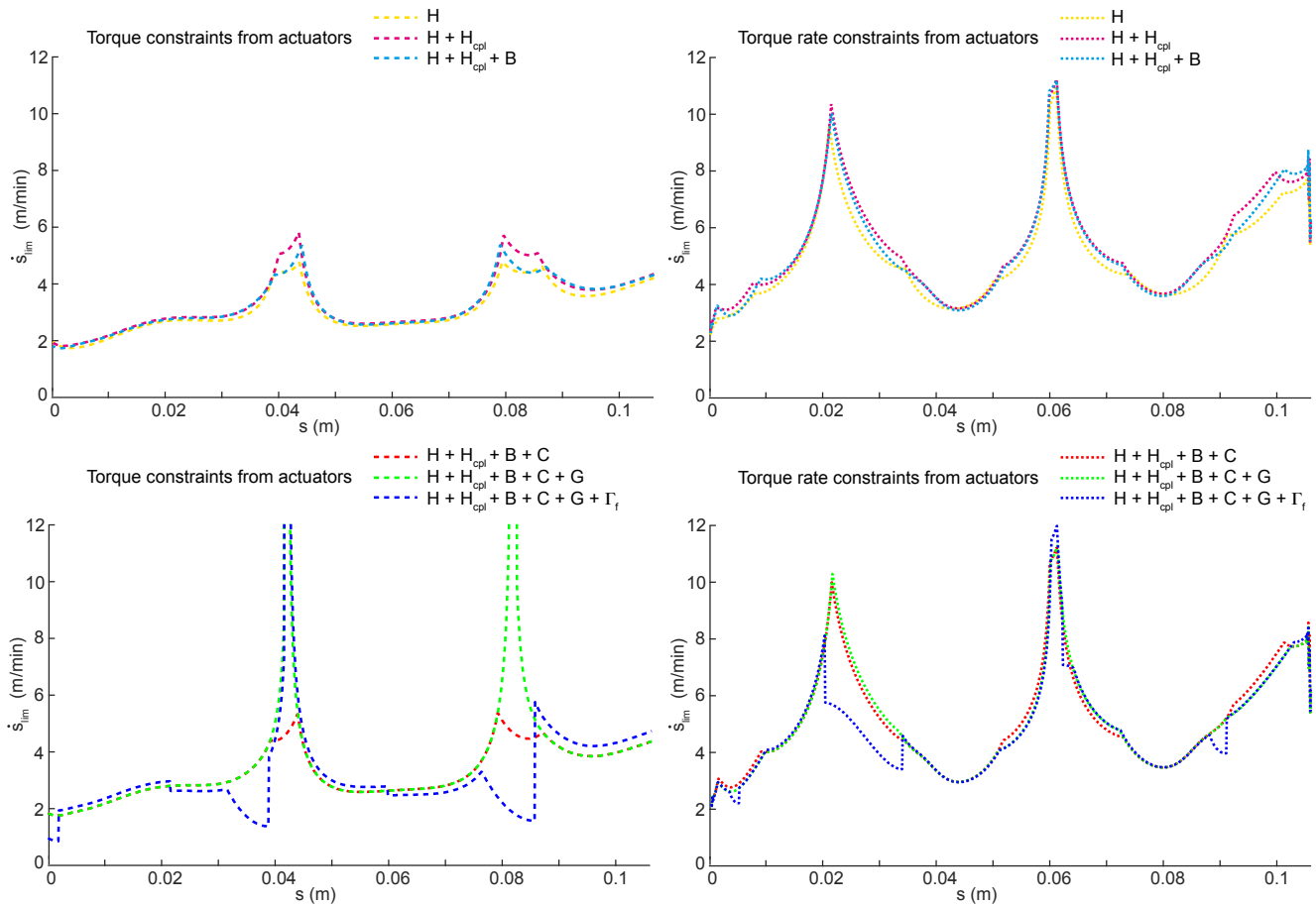
Fig. 8 Influence of the dynamic effects on the dynamic constraints from the actuators' capabilities, Kuka robot

pact on the vibratory behaviour of the structure and should be limited by the torque rate constraint. With this goal, the proposed method has the asset to include these joint flexibilities in the dynamic model to be more effective to reduce the oscillations.

5 Conclusion

This paper introduces a new dynamic approach to evaluate the maximum reachable feedrate respecting the performance characteristics of the system in a machining trajectory planning process.

This novel dynamic method brings about several improvements. First, the built model integrates the inertial, centrifugal an Coriolis couplings, the gravity effect, and the friction forces, which are dynamic effects not included in the usual kinematic constraints. Yet, they have an important influence on the feedrate calculation. Moreover, the dynamic model can be adapted to different systems that accentuates this interest for dynamically high sensitive structures such as serial robots. Then, the new dynamic constraints of torque and torque rate transpose the feedrate planning issue from the

global axis behaviour to the actuator level. In this way new limits based on the actuators' technical characteristics and the whole chain dynamic behaviour can be considered and appear to be clearer and more consistent than the kinematic values. Also, the dynamic model can be adapted to different systems and processes with different desired levels of complexity and precision.

Simulation results on two paths for a 5 axis milling machine and a 6 axis serial robot are analysed. The proposed approach is validated by comparison to the kinematic method usually employed in the literature. However, for now the maximum actuator torque rates have been approximatively evaluated considering the conservation of the ratio between torques and torque rates from the initial tuned values. Closely connected with the present study, more dedicated investigations on vibrations have to establish a relationship between the torque rate limit and the components' characteristics. Such a relation would lead to accurate values of joint torque rate limits respecting the system performance characteristics and reducing the harmful vibratory phenomenons.

A Dynamic model of the Kuka Robot

The compact dynamic model (Eq.(12)) and the compact differentiated dynamic model (Eq.(13)) of the Kuka robot are given in this Appendix. For further details about each component, one can refer to [2].

$$\tau = H(q)\ddot{q} + B(q)\dot{q}^2 + C(q)\dot{q}_i\dot{q}_j + G(q) + \Gamma_f(\dot{q}) \qquad (12)$$

$$\dot{\tau} = H(q)\dddot{q} + \dot{H}(q,\dot{q})\ddot{q} + 2B(q)\dot{q}\ddot{q} + \dot{B}(q,\dot{q})\dot{q}^2 + \qquad (13)$$
$$\dot{C}(q,\dot{q})\dot{q}_i\dot{q}_j + C(q)(\ddot{q}_i\dot{q}_j + \dot{q}_i\ddot{q}_j) + \dot{G}(q,\dot{q}) + \dot{\Gamma}_f(\dot{q},\ddot{q})$$

In these models the different dynamic terms are successively: H the inertia tensor, B and C the centrifugal and Coriolis effects, G the gravity forces, and Γ_f the friction torques. They are calculated for each joint i as follows, where all components are expressed in the local coordinate system of the joint:

$$\tau_i = \sum_{k=i}^{6}\sum_{j=1}^{k}\left(H_{ij}^k\ddot{q}_j + B_{ij}^k\dot{q}_j{}^2 + 2\dot{q}_j\sum_{n=1}^{j-1}C_{ijn}^k\dot{q}_n\right) + G_i^k + \Gamma_{f,i} \qquad (14)$$

with

$$H_{ij}^k = ((I_k - m_k\hat{c_{ik}}\hat{c_{jk}})\cdot u_j)\cdot u_i$$

$$B_{ij}^k = ((\hat{u}_j I_k - m_k\hat{c_{ik}}\hat{u}_j\hat{c_{jk}})\cdot u_j)\cdot u_i$$

$$C_{ijn}^k = ((dual(J_k u_n) - m_k\hat{c_{ik}}\hat{u}_n\hat{c_{jk}})\cdot u_j)\cdot u_i$$

$$G_i^k = -m_k g(\hat{c_{ik}}z_0)\cdot u_i \text{ and } \Gamma_{f,i} = K_{ti}(a_i\dot{q}_i + b_i)$$

The Kuka robot parameterization is described in the Fig.9. In the dynamic model u_i is the ith joint axis, $c_{ik} = O_iC_k$ where C_k is the center of mass of the body k, m_k, I_k and $J_k = -I_k + \frac{1}{2}tr(I_k)Id$ are respectively the mass, the inertia and pseudo-inertia tensors of the solid k, the operator $\hat{u}_j = dual(u_j)$ appoints the dual tensor.

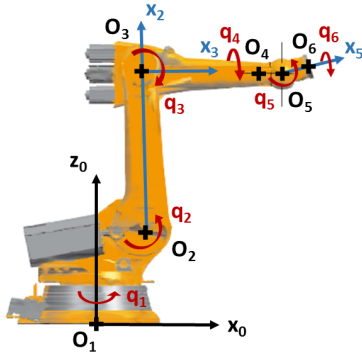**Fig. 9** Kuka KR180L130 robot parameterization

Then the torque rate of ith joint is:

$$\dot{\tau}_i = \sum_{k=i}^{6}\sum_{j=1}^{k}(H_{ij}^k\dddot{q}_j + \dot{H}_{ij}^k\ddot{q}_j + 2B_{ij}^k\ddot{q}_j + \dot{B}_{ij}^k\dot{q}_j{}^2 \qquad (15)$$

$$+ 2\dot{q}_j\sum_{n=1}^{j-1}\dot{C}_{ijn}^k\dot{q}_n + 2\ddot{q}_j\sum_{n=1}^{j-1}C_{ijn}^k\dot{q}_n$$

$$+ 2\dot{q}_j\sum_{n=1}^{j-1}C_{ijn}^k\ddot{q}_n) + \dot{G}_i^k + \dot{\Gamma}_{f,i}$$

with

$$\dot{\Gamma}_{f,i} = K_{ti}a_i\ddot{q}_i \qquad (16)$$

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