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# Prediction and Measurements of the Influence of Boundary Conditions in a Standing Wave Tube

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## Summary

The paper concerns a comparison between measurement results of the acoustic absorption coefficient and impedance using two different methods, the standing wave tube (Kundt's tube) and a free field method. The mounting of the sample in the Kundt's tube is known to be very important. In this paper, the effect of the constraints at the tube wall on the absorption coefficient of an elastic porous material is analysed. It will be shown that the effect of the friction offered by the tube wall is to stiffen the material, compared to its behaviour in the free field. Measurement results together with model calculations are presented. The material parameters used in the models have been determined by separate measurements.

## 1. Introduction

The Kundt's tube remains a very important measuring device for determining the acoustic performance of sound absorbing materials. The method is simple, easy to use and requires only a small amount of sample material. The popularity of the method is reflected in a recent international standardisation, both of the classical method using a probe microphone (ISO 10534-1) and the more novel two-microphone method (ISO 10534-2, being at the stage of a draft).

However, the method has some drawbacks: in normal use, only measurements at normal sound incidence are possible. This implies the use of small diameter tubes for measurements up to high frequencies in order to avoid cross modes. For instance, in order to obtain a maximum frequency of 5000 Hz, a tube diameter of approximately 4 cm is necessary. This may give problems obtaining representative and homogeneous samples at these high frequencies.

Apart from these drawbacks it has been known for a long time that the mounting conditions can have a profound effect on the results. A sealing of unintentional small gaps or slits between the sample and the tube wall or between the sample and the rigid backing is important. In addition, using elastic porous samples where the vibration of the frame contributes to the absorption makes the results very sensitive to the actual boundary conditions obtained. Several papers have addressed these problems [1, 2, 3], the latter using the finite element method to calculate the effect of a deliberately introduced gap between the wall in the tube and the sample. The finite element representation of the porous material was based on the Biot theory [4]. Another recent paper [5] also uses the finite element method to investigate the effect of the edge

constraints on the surface normal impedance of a porous sample of foam showing that the edge constraints gives a stiffening of the foam acoustically.

This paper addresses the same problem by comparing measurements on a porous material with calculations using the finite element method. Measurements using a two-microphone free-field method [1] are first performed on an extended sample of the material and compared with finite element calculations simulating a tube measurement where the sample is allowed to slide along the tube wall. These calculations are also compared with an analytical method using a matrix representation of the Biot model [6]. Secondly, small samples were cut and measured within a tube with a relatively small diameter, 4 cm. These measurements are again compared with finite element calculation introducing boundary conditions in the tube representing a gluing of the sample to the wall.

## 2. Calculation models

### 2.1. Finite element model

The finite element program used is FEMAK, a general program especially developed for handling acoustic problems [7, 3]. The program incorporates a variety of elements: porous material, equivalent fluid models as well as a model described by the Biot-theory, elastic solids and plates as well as elements for handling the coupling of these elements together. Similar programs have been developed by Coyette [8] and Easwaran *et al.* [9]. Of special interest here is the coupling between a region containing a fluid (air) and a porous medium described by the Biot theory. The finite element formulation for handling this case has been described by Johansen *et al.* [3] and shall not be repeated here. Suffice it to say that the finite element code contained in FEMAK is based on the second formulation of the Biot model [4]. The displacement

of the solid frame of the porous material is described by the vector  $\mathbf{u}^s$ , and a vector  $\mathbf{w}$  is used to describe the relative motion between the pore fluid and the frame  $\mathbf{w} = h(\mathbf{u}^f - \mathbf{u}^s)$ , where  $\mathbf{u}^f$  is the fluid displacement and  $h$  is the porosity. Apart from the porosity, the other material properties used to characterise the porous material are the airflow resistivity  $r$ , the tortuosity  $k_s$  and two dimensions  $\Lambda$  and  $\Lambda'$  related to the viscous and thermal losses. The model of Johnson *et al.* [10] is used for the frequency dependence of the viscous interaction. The thermal losses are accounted for by a complex bulk modulus of the fluid, Champoux and Allard [11]. Finally, for the frame one needs the density  $\rho$  and two elastic moduli, i.e. the Young's modulus  $E$  and the shear modulus  $G$ . The methods to determine all these parameters are briefly described below.

## 2.2. Analytical model

The analytical model used in this paper has previously been described in detail (see e.g. [6]). It is based on a transfer matrix approach of a plane parallel layer of material. Fluid layers can be modelled by  $2 \times 2$  matrices, elastic solid layers by  $4 \times 4$  matrices and elastic porous layers by  $6 \times 6$  matrices. In the model, these matrices can be combined and the resulting reflection and transmission coefficient of the system can be calculated. For the application in this paper, the free field measurement was modelled as a porous layer, stuck on to a very thick elastic solid layer simulating a rigid termination.

## 3. Measuring methods

### 3.1. Material parameters

The porous material used was a polyurethane foam FIRE-FLEX, fabricated by Recticel, Wetteren, Belgium. It is the same type of material as used by Allard [12], but with a different pore size. The thickness  $l$  and the material parameters used in the analysis are given in Table I.

Most material parameters have been determined with standard equipment. The porosity  $h$  and the airflow resistivity have been measured with methods described by Stinson and Daigle [13] and Champoux *et al.* [14]. The tortuosity  $k_s$  has been measured with a new method based on the phase velocity of the air borne wave at high frequencies [15, 16].

The Young's modulus  $E$  and the shear modulus  $G$  have been measured with a device similar to the one used by Pritz [17]. A rod shaped specimen with a circular cross section, diameter 1.5 cm and length 20 cm, has been cut from the sample. As the sample are slabs of 5 cm thickness, the specimen had to be cut in a direction perpendicular to the direction of sound propagation. Due to a slight anisotropy in the foam it was expected that the measured values had to be adjusted to obtain a good fit between the free field measurement data and the predictions of the analytical model. The measured values were  $2.2 \cdot 10^5 \text{ N/m}^2$  and  $7.8 \cdot 10^4 \text{ N/m}^2$  for  $E$  and  $G$ , respectively, with the imaginary part being of the order of 15–21%. The values used in the calculations are given in Table I.

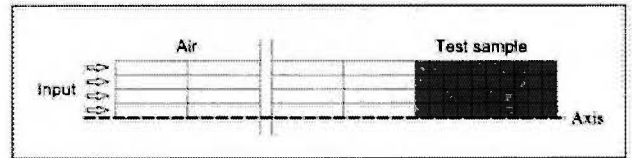


Figure 1. Mesh of the tube used in the finite element calculations.

The characteristic lengths  $\Lambda$  and  $\Lambda'$  have been determined from the high frequency sound attenuation per cycle using different types of gases saturating the pores [18, 19]. The measured values were  $\Lambda \approx 14 \mu\text{m}$  and  $\Lambda' \approx 100 \mu\text{m}$  and the large difference between these values can be explained by the following. For a polyurethane foam, the cells have approximately the shape of a pentagonal dodecahedron. The cells are interconnected with windows showing small perforations [20, see p.71]. The thermal characteristic lengths  $\Lambda$  is therefore mainly determined by the size of the cells whereas the viscous characteristic lengths  $\Lambda'$  is determined by the diameter of the perforations of the cell windows.

The measurements of  $\Lambda$  and  $\Lambda'$  are not very accurate, however, due to the very high sound attenuation in the materials in the frequency range used, 50 kHz–700 kHz. As seen from Table I, the value of  $\Lambda$  used in the calculations is smaller than the measured one.

### 3.2. Free-field measurements

The free field measurements have been carried out with a two microphone method described by Allard and Delage [1]. The sample, glued on a rigid backing, had dimension of about 2 m by 3 m and the microphones were placed at the centre of the sample as close as possible to the sample surface. The microphone separation was varied between 2 cm and 5 cm, depending on the frequency interval used. Pure tone signals were used for the excitation and measurements were performed using two different loudspeaker sources. These were placed at different heights above the sample.

### 3.3. Tube measurements

Several samples have been cut from the centre of the material used in the free field measurement. The absorption coefficient has been measured in a Kundt's tube, with a diameter of 4 cm, using a two-microphone configuration. The sample was glued on to a rigid backing and attention was paid that no air gaps were present at the side wall of the tube.

## 4. Finite element model for the Kundt's tube

The air and the porous material in the tube used in the experiments described above are modelled as shown in Figure 1. The model is axisymmetric using 8-noded elements for both the air and the porous material. The thickness of the sample is 5 cm and with a diameter of 4 cm, the 40 elements in the material are all  $5 \times 5 \text{ mm}$ . Due to the short wavelength of the slow wave in the material this fine meshing was necessary when performing calculations up to a frequency of 3000 Hz. The elements representing the air in the tube were 25 mm

Table I. Material parameters of the porous layer used in the calculations.

$l$ [m]	$\rho$ [kg/m <sup>3</sup> ]	$h$	$r$ [Ns/m <sup>4</sup> ]	$k_s$	$E$ [N/m <sup>2</sup> ]	$G$ [N/m <sup>2</sup> ]	$\Lambda$ [ $\mu$ m]	$\Lambda'$ [ $\mu$ m]
0.05	30	0.93	80 000	2.5	$4.3 \cdot 10^5 + j1.0 \cdot 10^5$	$1.6 \cdot 10^5 + j3.0 \cdot 10^4$	10	100

long. When simulating similar measurements performed in larger diameter tubes, the model was only "stretched" in the radial direction thereby increasing the size of the elements in this direction. This was accounted for by decreasing the high frequency limit of the calculations.

The program allows for a modelling of an input source in two different ways. A defined normal velocity may be given at the boundary of a calculation domain, here in the input plane of the tube representing a piston or loudspeaker input to the tube. Specifying the piston to have a velocity of 1 m/s, a calculation of the pressure at the input nodes immediately gives the input impedance, a result which easily may be transformed to find the input impedance in the plane of the sample. Alternatively, one may mimic the working of the two-microphone method, picking out the pressure at two nodes along the tube calculating the transfer function and using the formulas developed for this measuring method.

The other way of modelling the source offered by the program is to specify an incoming plane wave to the tube [21]. The tube is modelled as being of infinite length in one direction. Using this approach it is possible to separate the incoming and the reflected waves. Furthermore, the displacement of the material caused by a given acoustic pressure may be calculated. The input impedance of the material may again be calculated from the pressure at the input end of the tube, although with a different expression than used for the piston source. A comparison of these different method for evaluating the input impedance of the material was performed to test the calculation models.

## 5. Results and discussion

Figure 2 shows the result of the free field measurements of the absorption coefficient together with the calculated data. The open and the filled circles represent the measurements using the two different sound sources. As stated in paragraph 3.3, the Young's modulus  $E$ , the shear modulus  $G$  and the viscous length  $\Lambda$  parameter have been adjusted by using the analytical model to make a best fit to the measured data. The result is given by the hatched line in the figure. As pointed out,  $E$  and  $G$  have both been measured using a rod of 20 cm length, which has been cut in a direction perpendicular to the surface (and hence perpendicular to the direction of propagation of the waves). Since plastic foams are often slightly anisotropic, measurement of the elastic moduli will only give a first approximation of the value to be used in the calculations.

Figure 2 also shows the results of the finite element calculation, simulating the free-field conditions by making the sample free to slide along the wall of the tube. As seen from

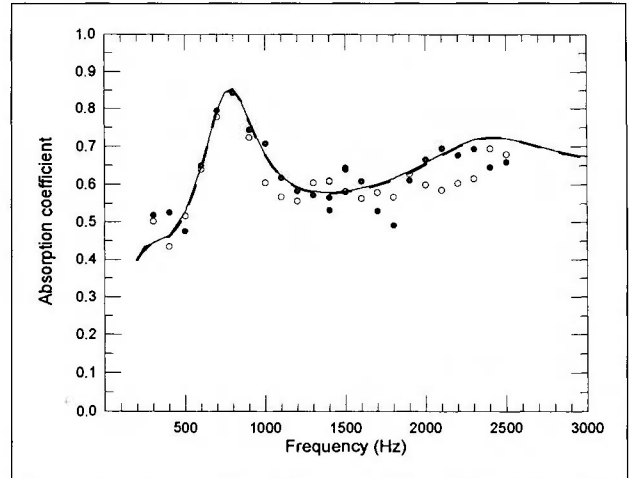


Figure 2. Absorption coefficient of the porous material measured in the free field. ●: Measurements, source 1; ○: Measurements, source 2; — —: Analytical model; —: FEM model.

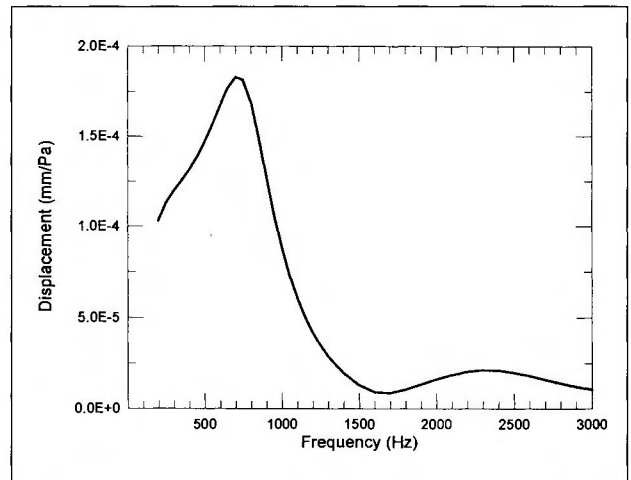


Figure 3. Calculated axial frame displacement at the sample surface using FEM model. Incident sound pressure of 1 Pa.

the figure, the difference between the results from the two calculation models are negligible. Generally, there is also a good agreement between calculated and measured data. The relative large scatter in the free field data is probably due to inhomogenities in the material since the actual area that determines the measuring results depends both on the frequency and the material properties. There is a large peak in the absorption coefficient near 800 Hz, which can be identified as a frame resonance. This can clearly be seen in Figure 3, which shows the predicted frame axial displacement calcu-

lated using the FEM-model. The incidence sound pressure at the top surface of the material is 1 Pa. The position of this peak is strongly (and almost exclusively) determined by the elastic constants of the frame. A second weak maximum near 2500 Hz is not strongly influenced by the frame elastic parameters and is more determined by the flow resistivity.

These results suggest that it is possible to determine the elastic moduli of the frame (or at least the longitudinal velocity of the frame wave) from the acoustic data if the peak can be attributed to a frame resonance. This can be verified for instance by measuring the frame displacement with a laser vibrometer.

Figure 4 and Figure 5 show the normalised impedance and the corresponding absorption coefficient, measured in the Kundt's tube of 4 cm diameter. A large shift in frequency of the peak near 800 Hz can be observed. Samples have both been glued to the tube wall or simply placed tightly fit in the tube. The friction force between the tube and the material seems also in the latter case to be large enough to retain the frame from moving. Only a slight difference in the results using the two configurations was measured. As explained earlier [5] the constraint of the sample at the tube wall results in a shear wave to be excited in the sample. This shear wave is neither present when the sample sides are allowed to slide in the tube nor in our free field configuration since only normal incidence measurements were performed. The result of the constraint is that the sample has a stiffer behaviour as compared to its free field behaviour.

The experimental data show a large variability due to a variation in the sample properties but the general behaviour is well predicted by the FEM-model. A simulation of the absorption data using a larger tube diameter (6 cm), as indicated by the hatched line in Figure 5, shows that the shift is less important for this larger tube. For a tube of 10 cm diameter, calculations indicate that the peak is almost down to the frequency found from the free field measurement.

The Figures 6 and 7 show the axial displacement of the frame and the fluid inside the material. In Figure 6 the data are calculated for a frequency approximately at the highest peak (1500 Hz) showing a relatively large frame displacement as compared with the results shown in Figure 7 for a frequency near to the lower frequency maximum (300 Hz). This shows that the maximum in the absorption coefficient near 1500 Hz is strongly influenced by the frame displacement, whereas the maximum near 300 Hz is mainly due to the fluid displacement while the frame stays motionless. This latter effect has also been noticed by Bardot *et al.* [22]. The displacement data on the two figures should not be compared as velocity sources have been used. The actual driving pressure at the two frequencies is therefore not the same.

## 6. Conclusions

In this paper, measurements of the absorption coefficient of an elastic porous material in the free field have been compared with measurements obtained in a Kundt's tube. The results can be simulated using a finite element model and in the free-field case also by using an analytical model. The

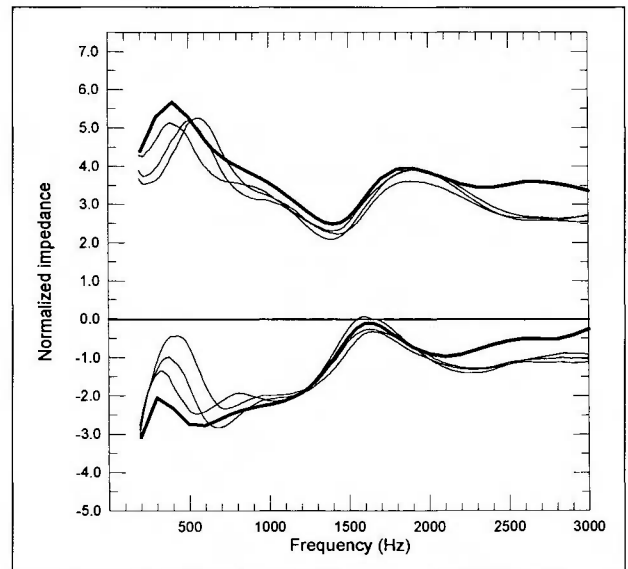


Figure 4. Real and imaginary component of the normalised impedance, real component in upper part and imaginary in the lower part of the figure. Samples cut from the material of Figure 2, measured in an impedance tube with a diameter of 4 cm. Thin solid lines: measurements on three different samples; thick solid line: results from FEM calculation.

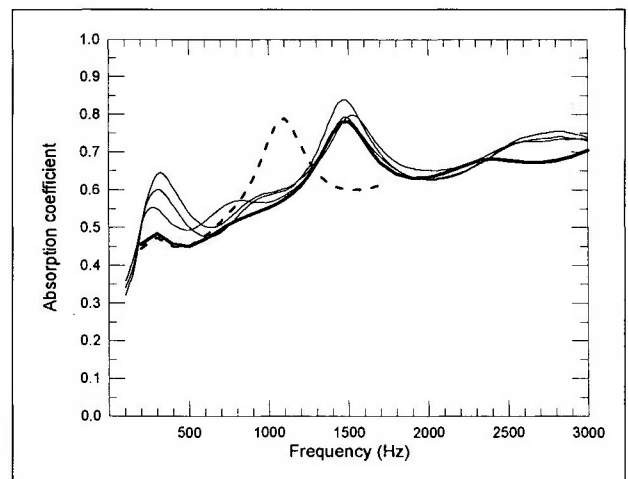


Figure 5. Absorption coefficient of samples cut from the material of Figure 2, measured in an impedance tube with a diameter of 4 cm. Thin solid lines: measurements on three different samples; thick solid line: results from FEM calculation; hatched line: calculations using a tube with a diameter of 6 cm.

input to the models are independently measured material parameters, except for the parameter  $\Lambda$  which has been adapted slightly (from  $14 \mu\text{m}$  to  $10 \mu\text{m}$ ) with the help of the analytical model and the free field data. As shown by the differences in the results the choice of measuring techniques can be very important. Due to the constraint imposed by the tube wall, a shear wave can be generated which was not present in the free field configuration used. This clearly proves that impedance tube measurements should be used carefully in those cases where the elasticity of the frame contributes to the surface impedance of the material. This applies to elastic porous materials with a high coupling (high tortuosity and high flow

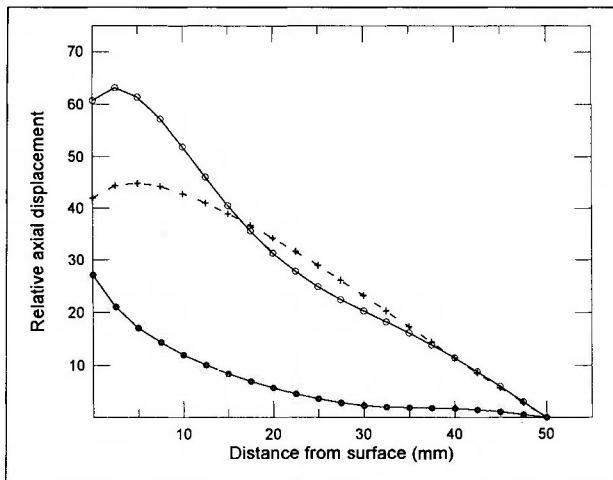


Figure 6. Axial displacement of the sample in the impedance tube, as a function of depth in the material. The frequency is 1500 Hz, coinciding with the peak in Figure 5. —○—: Fluid on the axis; —●—: Fluid at the wall; —+—: Frame on the axis.

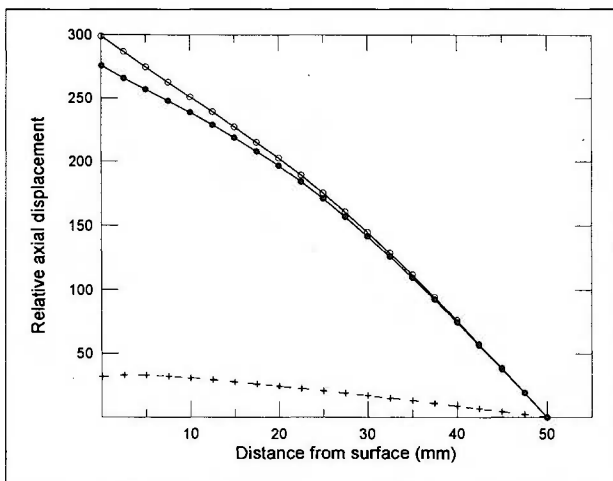


Figure 7. Axial displacement of the sample in the impedance tube, as a function of depth in the material. The frequency is 300 Hz. Figure symbols as in Figure 6.

resistivity) such as the material presented in this paper but may also be the case for materials with impervious or resistive screens. In the latter case, as shown earlier [23], a frame wave can easily be generated by the membrane movements forcing the frame.

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