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Methods for the characterization of sound packages containing visco-elastic and anisotropic porous materials.

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ABSTRACT

A wide range of commercial applications, in both building acoustics as well as automotive industry, can be found for sound packages containing poro-elastic materials. Accurate material characterization is vital for the use of these multilayered systems due to the increasing demands in acoustic comfort and legislation. There is however a distressing lack in raw, accurate material data and measurement methods concerning the characterization of these materials. Especially the effect of anisotropy and visco-elasticity on the acoustic properties of multilayered systems remains barely understood.

Measuring and modeling methods for the characterization of these materials at an acoustically interesting frequency range will be presented. The experimental technique for the determination of the elastic properties is based upon the excitation of waveguides in anisotropic and visco-elastic porous materials. The other parameters of the Biot-Allard model are determined using the classical methods.

Results concerning surface wave phase velocities show that anisotropy and visco-elasticity influence the acoustical properties of porous media which are calculated using a transfer matrix approach.

1 INTRODUCTION

Demands involving legislation and acoustical comfort have lead to an optimization of modeling techniques regarding porous materials as being part of an overall solution of a noise problem. Advances in numerical techniques are however tampered with a lack of accurate measurement techniques and material data needed as input parameters for the numerical models. In many applications the materials are used in multilayered systems and the porous material itself is visco-elastic and/or anisotropic. The estimation of the material properties of a porous material, with respect to the multilayer, is therefore often the hardest task at hand. The acoustical behavior and the damping of the porous material play, on the other hand, often a critical role in the multilayered system.

In the past years considerable research effort has gone to the determination of the parameters of the Biot-Allard [1] model which determine the airborne wave propagation throughout porous media. These parameters solely determine the acoustical behavior in many applications where a sound wave is impinging from the air directly onto the surface of an open-celled porous material; elastic coupling can in these cases often be neglected. Classical ultrasound methods [2, 3] provide an elegant tool for estimating the Biot-Allard parameters. They are fast, non-destructive and user-friendly. Furthermore ultrasonic characterization has the potential of being used as an inline technique for quality control and for the validation of the acoustical properties.

In many cases the porous material can not be treated as an equivalent fluid and accurate knowledge of the elastic constants of the porous material is needed for modeling purposes. For many applications it is also not clear in which geometrical configuration the porous material is combined with the other components of the multilayered system since the manufacturer differs from the end-user of the porous material. Furthermore, the elastic response of these materials is influenced by visco-elasticity, anisotropy and high attenuation. Elastic constants are therefore very difficult to measure accurately. Classical techniques [4] are limited to the lower frequency range and accuracy is often dependent on the size and shape of the sample under investigation. Complicated numerical inversion schemes (e.g. 3D finite element modeling) are often needed to obtain the elastic constants.

Recently surface waves [5] have been proposed as a good method for investigating the elastic response of porous materials. The use of surface waves as a tool for material research has some convenient advantages in that their application does not depend on the size and shape of the sample, their use is not restricted to the lower frequency range and the measurement techniques are non-destructive. Indeed surface wave propagation depends only on the bulk wave propagation throughout the porous material and on the boundary conditions at the free surface.

An adapted formulation of the Christoffel wave equation for surface wave propagation in anisotropic porous media will be described in the next sections. Measurement results concerning surface wave phase velocities at the interface of a transverse isotropic material are presented. They indicate that the anisotropy of these materials influences their acoustical performance.

2 MEASURING AND MODELING SURFACE WAVES ON POROUS MEDIA

2.1 Christoffel matrix for anisotropic porous materials

Surface wave and guided wave propagation on elastic layers has been extensively investigated by many authors. Elaborate reviews can be found in [6, 7]. Only recently however these formalisms have been adapted to describe surface wave propagation in isotropic poro-elastic materials. Allard et al. [5] investigated surface waves at the interface between a fluid and a semi-infinite isotropic poro-elastic layer. The phase velocity of this Raleigh wave was estimated by searching the pole k_p of the plane wave reflection coefficient

 $R(k_r)$ of a semi-infinite isotropic poro-elastic layer. Khurana and Vashishth [8, 9] presented a similar technique to derive surface wave propagation in the basal plane of transverse anisotropic porous media.

An alternative approach to describe guided waves in slabs of isotropic porous materials was presented by [10]. The guided waves are described in terms of potential functions which describe bulk wave propagation throughout the isotropic porous material. A characteristic determinant was derived, which is an expression of the boundary conditions at free surface. The vanishing of this determinant corresponds to the existence of guided waves propagating through the slab of porous material. The frequency dependency of the guided wave phase velocities in the material was used to investigate the visco-elasticity of porous media.

For anisotropic porous media however an alternative approach in terms of partial waves should be followed. The formalism is more or less similar to the one presented by [6, 7] for surface waves in anisotropic elastic materials. The coordinate system of interest is that in which a semi-infinite anisotropic poro-elastic material occupies a space $x_3 \le 0$. The x_3 -axis is taken as the outward normal to the free-surface and x_1 and x_2 are chosen in some convenient direction. Similar to [8] we are interested in transverse isotropic porous media in which the symmetry axis is taken to be the x_3 -axis. We are interested in a surface wave which decays exponentially with depth below $x_3 = 0$ and which propagates according to a random direction n in the $x_3 = 0$ plane. In this case, surface waves are a linear combination in terms of:

$$u_i = \alpha_i \exp(ikn_3x_3) \exp\left[ik\left(n_1x_1 + n_2x_2 - vt\right)\right]$$
(1)

 kn_1 and kn_2 are the projections of the wave vector on the x_1 and x_2 axis and α_i is the amplitude. Equation (1) has to satisfy simultaneously the wave equation for an anisotropic porous material and the boundary conditions imposed to the free surface. The quantity n_3 in each of the terms of the solution must be such that the amplitudes of the displacement components vanish as $x_3 \rightarrow -\infty$.

The wave equation for anisotropic elastic solids is usually given in terms of a 6×6 Christoffel matrix $[\Gamma_{ij}]$, which describes the stress strain relations in an anisotropic solid. The Christoffel equation has the form:

$$\left(\Gamma_{ij} - \delta_{ij}\rho v^2\right)\alpha_i = 0 \tag{2}$$

 ρ is the density and v is the phase velocity. Setting the determinant of the coefficients of the Christoffel equation equal to zero for a given direction (n_1, n_2, n_3) yields the bulk wave propagation. Adapting equation (2) for the description of wave propagation in porous materials with a random anisotropy in the fluid and solid phase of the porous material:

$$\begin{bmatrix} SS_{ij} - \delta_{ij} \tilde{\rho}_{ij}^{SS} v^2 \end{bmatrix} \begin{bmatrix} SF_{ij} - \delta_{ij} \tilde{\rho}_{ij}^{SF} v^2 \end{bmatrix} \begin{bmatrix} \alpha^S \\ \alpha^F \end{bmatrix} = 0$$
(3)

 α^{s} , α^{F} are the amplitudes of respectively solid and fluid displacement. The 3×3 sub matrices $\left[SS_{ij} - \delta_{ij}\tilde{\rho}_{ij}^{SS}v^{2}\right]$, $\left[FF_{ij} - \delta_{ij}\tilde{\rho}_{ij}^{FF}v^{2}\right]$, $\left[SF_{ij} - \delta_{ij}\tilde{\rho}_{ij}^{SF}v^{2}\right]$, $\left[FS_{ij} - \delta_{ij}\tilde{\rho}_{ij}^{FS}v^{2}\right]$ for a transverse anisotropic porous material are given in appendix A. For any given value of v, eq. (3) gives a secular equation of degree 8 in terms of n_{3} which corresponds to a solution of the form (1) travelling through both fluid and solid phases. In general there are only 4 lower-halfplane roots of equation (3) which vanish at infinite depth. A surface wave is therefore given by a linear combination of partial waves of the form of equation (1):

$$u_{i} = \sum_{l=1}^{4} C_{l} \alpha_{i}^{l} \exp(ik \left[n_{1} x_{1} + n_{2} x_{2} + n_{3}^{l} x_{3} - vt \right])$$
(4)

The problem now is to determine the four weighting factors C_l in such a manner that the boundary conditions are satisfied everywhere on the free surface. A surface wave can be found for only a specific value of v and the corresponding values of n_3^l that make the boundary-value determinant zero.

2.2 Measured phase velocities at a transverse isotropic porous medium

Surface waves were excited on a thick (20 cm) slab of porous material. The material properties were measured and can be found in Table 1 of [5]. The measurement setup is depicted on the left hand side of *Figure 1*. A detailed description of a similar measurement setup to induce surface waves or guided waves at the interface of a porous material can be found in reference [10]. Due to the fabrication process, sound absorbing foams are transverse isotropic in the direction dependent physical properties of the solid and fluid phase. For simplicity only transverse isotropy of the elastic properties is considered here. However anisotropy in the fluid parameters (e.g. flow resistivity) can be taken into account by adapting the appropriate densities ($\tilde{\rho}_{33}^{SF}$, $\tilde{\rho}_{33}^{SF}$ and $\tilde{\rho}_{33}^{FF}$) with the introduction of a flow resistivity, a tortuosity and a viscous characteristic length measured parallel and perpendicular to the symmetry axis of the material. The surface wave phase velocities were measured with a Laser Doppler in the $x_3 = 0$ plane and in the $x_1 = 0$ plane in a concentric circle around the location of a point source (an LDS shaker, right hand side of *Figure 1*). A sine burst of 20 periods centred at 2 kHz, was used to excite the surface waves.



Fig. 1: Measurement setup for measuring surface waves in porous materials.

As expected no variation was noticed in $x_3 = 0$ plane (isotropic plane, perpendicular to the x_3 -axis). This is indicated by the dashed line of *Figure 2*. However, a decrease of the surface wave velocity was measured with an increasing angle from the x_3 -axis in the $x_1 = 0$ plane (indicated by the full circles in *Figure 2*). Measured phase velocities were best fitted using values for f=1.2 and l=0.8, f and l were defined in [8]. The magnitude of f and l corresponds to the case of "strong" anisotropy as described in [8]. The small variations in surface wave phase velocity tend to suggest however that the material is only slightly anisotropic in comparison to materials like e.g. glass wool [11].



Fig. 2: Surface wave phase velocity as a function of the angle from the x_1 -axis ($x_3=0$ plane) (dashed line) and from x_3 -axis($x_1=0$ plane) (full circles). Crosses (×) indicate calculated values

3 CONCLUSIONS

An adapted version of the Christoffel equation (2) was presented for the investigation of surface waves at the interface of anisotropic porous media. The model was used to describe measured surface wave phase velocities at the basal and sagittal plane of a transverse anisotropic sound absorbing material. The anisotropic constants that were obtained correspond to the case of a "strong" anisotropic material as described by [8]. The estimated magnitude of anisotropic constants f and l indicate that the anisotropy of the material does have a measurable influence on the absorption and transmission coefficients of this sound absorbing foam, as was predicted by [8].

APPENDIX A

$$\begin{bmatrix} SS_{ij} - \delta_{ij}\tilde{\rho}_{ij}^{SS}v^{2} \end{bmatrix} = \begin{bmatrix} Pn_{1}^{2} + Nn_{2}^{2} + Ln_{3}^{2} - \tilde{\rho}_{11}^{SS}v^{2} & (P - N)n_{1}n_{2} & (F + L)n_{1}n_{3} \\ (P - N)n_{1}n_{2} & Pn_{2}^{2} + Nn_{1}^{2} + Ln_{3}^{2} - \tilde{\rho}_{22}^{SS}v^{2} & (F + L)n_{2}n_{3} \\ (F + L)n_{1}n_{3} & (F + L)n_{2}n_{3} & L(n_{1}^{2} + n_{2}^{2}) + Cn_{3}^{2} - \tilde{\rho}_{33}^{SS}v^{2} \end{bmatrix},$$

$$\begin{bmatrix} SF_{ij} - \delta_{ij}\tilde{\rho}_{ij}^{SF}v^{2} \end{bmatrix} = \begin{bmatrix} Mn_{1}^{2} - \tilde{\rho}_{11}^{SF}v^{2} & Mn_{1}n_{2} & Mn_{1}n_{3} \\ Mn_{1}n_{2} & Mn_{2}^{2} - \tilde{\rho}_{22}^{SF}v^{2} & Mn_{2}n_{3} \\ Qn_{1}n_{3} & Qn_{2}n_{3} & Qn_{3}^{2} - \tilde{\rho}_{33}^{SF}v^{2} \end{bmatrix} = \begin{bmatrix} FS_{ij} - \delta_{ij}\tilde{\rho}_{ij}^{FS}v^{2} \end{bmatrix}^{T},$$

$$\begin{bmatrix} FF_{ij} - \delta_{ij}\tilde{\rho}_{ij}^{FF}v^{2} \end{bmatrix} = \begin{bmatrix} Rn_{1}^{2} - \tilde{\rho}_{11}^{FF}v^{2} & Rn_{1}n_{2} & Rn_{1}n_{3} \\ Rn_{1}n_{3} & Rn_{2}n_{3} & Rn_{3}^{2} - \tilde{\rho}_{33}^{FF}v^{2} \end{bmatrix}.$$

The constants P, N, Q, F, M, L, C and R can be found in [8], where A=P-2N with P given by equation (A1) in [5]. Using $\tilde{\rho}_{11}$, $\tilde{\rho}_{12}$ and $\tilde{\rho}_{22}$ (equations (A5-A7) of [5]) the densities can be identified as $\tilde{\rho}_{ii}^{SS} = \tilde{\rho}_{11}$, $\tilde{\rho}_{ii}^{FS} = \tilde{\rho}_{ii}^{SF} = \tilde{\rho}_{12}$ and $\tilde{\rho}_{ii}^{FF} = \tilde{\rho}_{22}$.

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