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## Competition in access to Content

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Abstract. We study competition between users over access to content in a telecom market that includes several access providers and several content providers. We focus situations where exclusive agreements exist between content and access providers, which allows access providers to offer content services for free for their subscribers. We call access providers having such agreements "super" providers or "enhanced" service providers. We show that the competition between the users results in a paradoxical phenomenon in which subscribers of enhanced providers prefer to defer part of their demand to other content providers whose content is costly. We show how this phenomena can be exploited by the content providers so as to maximize their benefits.

#### 1 Introduction

The last years have seen much public debate and legislation initiatives concerning access to the global Internet. Some of the central issues concerned the possibility of discrimination of packets by service providers according to their source or destination, the protocol used. A discrimination of a packet can occur when preferential treatment is offered to it either in terms of the quality of service it receives or in terms of the cost to transfer it. Much of this debate took part in anticipation of legislation over the "Net Neutrality", and several public consultations were lounged in 2010 (e.g. in the USA, in France and in the E.U.). Network neutrality asserts that packets should not be discriminated. Two of the important issues concerning discrimination of traffic are whether (i) an ISP may or may not request payment from a content provider in order to allow it to offer services to the subscribers of that service provider, and (ii) whether or not a service provider can have an exclusive agreement with a given content provider resulting in a vertical monopoly. Indeed, for Hahn and Wallsten [2], net neutrality "usually means that broadband service providers charge consumers only once for Internet access, do not favor one content provider over another, and do not charge content providers for sending information over broadband lines to end users".

The Network Neutrality legislation will determine much of the socio-economic role of the Internet in the future. The Internet has already had a huge impact on economy and communication, but also on the exercise of socio-cultural and fundamental rights. Directive 2002/22/EC of the European Union, as amended

by the Directive  $2009/136$ /EC, established Internet access as a universal service<sup>3</sup>. The Ministry of Transport and Communication of Finland has passed a Decree in October 2009 that goes beyond the recognition of the right for Internet access: it guarantees the right for broadband Internet connection as a universal service.

The first objective of this paper is to model exclusive agreements between service and content providers and study their economic impact. We propose an unusual way of modeling this consisting in a transformation of the problem into a routing game.

Our second objective is to get insight on the behavior of the equilibrium as a function of the parameters. To that end, we choose to study simple models that allow one to obtain explicit expressions for the equilibrium behavior. In particular, we choose to restrict to symmetric conditions. We then make use of a recent result that shows that under some convexity conditions, when we have symmetry in routing games, any equilibrium has to inherit the symmetry properties of the network [10]. This allows us to reduce the number of unknown variables considerably. Note that for similar games, it has been shown that the the worst possible values ofthe price of anarchy (i.e. the ratio between the global costs at eqilibrium and that at the social optimum) are obtained obtained for symmeetric games [1, 13].

Related Works Although there have been many papers discussing network neutrality issues, there have been very few proposing economic analysis of neutral or non-neutral features.

Several papers study the impact of the ISP charging the content providers on the welfare. Economides and Tag [3] show that there is a decrease in welfare when in addition to the consumers, the content provider is also charged. In [4] it is shown that allowing ISPs to determine the amount they charge the content providers can result in a dramatic decrease in the demand and in losses not only to the content providers but also to the ISPs. These losses can be avoided using some regulation to determine the side payments as is shown in [5].

Another important question is whether network neutrality gives incentives for ISPs to invest or not. Economic analysis of the question is provided in [6–8].

Exclusive agreement between an ISP and a content provider are called a vertical monopoly in the economic literature. References [7, 5] study the impact of such behavior, considered as non neutral, on the welfare.

#### 2 Model

We consider the network depicted in Figure 1 that contains Internet Service Providers (ISPs) and Content Providers (CPs). Some ISPs and some CPs have exclusive agreements between them. More precisely, we assume that there are  $n$ pairs of ISP - CP, where each such pair is are tied together by some exclusive

<sup>&</sup>lt;sup>3</sup> A universal service has been defined by the EU, as a service guaranteed by the government to all end users, regardless of their geographical location, at reasonable quality and reliability, and at affordable prices that does not depend on the location.

agreement. In addition there may be  $m$  CPs and  $k$  ISPs that do not have any such agreement. We call these independent CPs and ISPs. Those are denoted respectively iCP and iISP. The other are called super CPs and super ISPs, respectively.



Fig. 1. Routing Game Representation of the Networking Game between n ISPs each having an exclusive agreement with some CP, m Independent Content Providers (ICP) and k Independent Internet Service Providers (iISP).

Each ISP (iISP and super ISP)  $i$  creates a demand for content at a rate of  $\phi^i$ . All the subscribers for each ISP and iISP are identical. The total demand comes from their subscribers. We assume that the same content is available at all CPs ( $i$ CP and super CP). Users connected to service provider i split their demand between the content providers, they download an amount  $x_j^i$  from CP j for  $j \in \{1, \ldots, n+m\}$  (n CP with agreement and m independent CP). Let  $x_j$  be the total demand presented to CP j, i.e.  $x_j = \sum_i x_j^i$ . The total demand coming from ISP i has the flow constraint:

$$
\sum_j x_j^i = \phi^i, \quad x_j^i \ge 0, \ \forall j \in \{1, \dots, n+m\}.
$$

We assume that there is a congestion cost at content provider  $j$  that is paid by each packet that is downloaded from it. This cost is assumed to be a convex increasing function of the total demand offered to the content provider. In particular, this function may represent the expected download delay per packet for traffic from the content provider. We denote the function that corresponds to the per-packet cost of content provider j by  $D_{cp}^{j}(x_j)$ .

We assume further that there is a fixed per packet cost of  $d_{ij}$  that user connected to service provider  $i$  is charged per content unit it requests from CP j. This can represent a monetary cost or an additional constant delay due to propagation. The disutility or cost function of subscriber connected to service provider  $i$  is given by

$$
C^{i}(\underline{\mathbf{x}}) = \sum_{j=1}^{n+m} x_{j}^{i} (D_{cp}^{j}(x_{j}) + d_{ij}).
$$

We assume that an ISP that has an exclusive agreement with a given CP is not charged for receiving contents from that CP, but pays an amount of d per unit of traffic that it fetches from a CP that has an exclusive agreement with a competing ISP. It pays an amount of  $\delta$  per unit of traffic that it fetches from independent CPs.

The aim of the paper is to study interactions between service providers (meanwhile the end-users) through their demand sharing between the several type of content providers. We compute in the next section the equilibrium of this networking game.

#### 3 Computing the equilibrium

In order to compute the equilibrium of this networking game, we associate a Lagrange multiplier  $\lambda_i$  with each subscriber connected to ISP i. We use it so as to relax the constraint corresponding to the total flow conservation of player  $i$ . Write the Lagrangian as

$$
L^{i}(\underline{\mathbf{x}}) = C^{i}(\underline{\mathbf{x}}) - \lambda^{i}(\sum_{j} x_{j}^{i} - \phi^{i}),
$$

where **x** is the vector of demand for all the end users of the system. Now, according to Karush-Kuhn-Tucker theorem, (under our convexity conditions) for each  $i, x^i = \{x^i_j\}$  is a best response for player i if and only if there exist  $\lambda^i$  such that  $\lambda^{i}(\sum_{j} x_{j}^{i} - \phi^{i}) = 0$  and such that  $x^{i}$  minimize  $L^{i}$ . The best response  $x_{j}^{i}$  for player's  $i$  demand to content provider  $j$  should thus satisfy:

$$
0 \le \frac{\partial L^i(\mathbf{x})}{\partial x_j^i} = D^j_{cp}(x_j) + d_{ij} + x_j^i \frac{\partial D_{cp}}{\partial x_j^i}(x_j) - \lambda^i.
$$

Moreover, the above equals zero if  $x_j^i > 0$ .

We make the following assumption concerning symmetry. The costs  $d := d_{ij}$ are the same for each subscriber  $i$  and iCP  $j$  not under contract with  $i$ . Moreover the demand for each subscriber connected to ISP *i*, which we denote by  $\Phi = \phi^i$ , is the same all i's. We assume that due to exclusive agreement,  $d_{ij} = 0$  if j is the CP under contract with subscriber connected to ISP  $i$ .

There exists two kinds of subscribers: the set  $S_c$  for those whose ISP has an agreement with an CP, and the set  $S_{nc}$  for those that do not have any agreement.

Each subscriber  $i \in S_c$  can split his demand between: his super-CP, all the independent iCPs and all the concurrent CPs. Whereas, each subscriber  $i \in S_{nc}$ has to split his demand only among iCPs and CPs.

In the rest of the paper we consider the linear cost function  $D_{cp}^{j}(x_j) = ax_j =$  $ax_j^i + a \sum_{i' \neq i} x_j^{i'}$  with  $a > 0$  for all CP j. Then we have  $\frac{\partial D_{cp}^j}{\partial x_j^i}(x_j) = a$ ,  $\forall i, j$ . and the best response  $x_j^i$  for subscriber's i to CP j satisfies at equilibrium:

$$
ax_j + d_{ij} + ax_j^i - \lambda^i = 0
$$
, which is equivalent to 
$$
x_j^i = \frac{\lambda^i - d_{ij} - a \sum_{i' \neq i} x_j^{i'}}{2a}.
$$
 (1)

#### 3.1 Equilibrium

The game is seen to be equivalent to a standard splitable routing game as studied in [9], in which each user is a source, in which there is one common destination node, and in which each ISP and CP are represented as links. The access costs d and  $\delta$  are also associated to links.

The system possesses several symmetries: (I) all players among  $S_c$  are interchangeable, (II) all players among  $S_{nc}$  are interchangeable, (III) if the flows sent to each iCP by all users other than  $i$  are the same, then for player  $i$ , the iCPs are interchangeable. Similarly, if the flows sent to each CP by all users other than  $i$  are the same, then for player  $i$ , the CPs are interchangeable. These symmetric properties implies that any equilibrium in this routing game inherits also these symmetric properties, as was recently shown in [10]. We thus restrict below, without loss of generality, to a symmetric equilibria.

Let  $w$  be the equilibrium rate of traffic requested by a subscriber of a super ISP from the CP associated to that ISP. Let  $y$  be the amount it requests from each super CP that is not associated with that ISP, and let z be the amount it requests from each independent ISP.

Let  $\xi$  be the equilibrium rate of traffic requested by a subscriber of each independent ISP from each super CP and let  $\zeta$  be the amount it requests from each independent ISP.

We have  $w + (n-1)y + mz = n\xi + m\zeta = \Phi$ . Let  $\rho = w + (n-1)y + k\varepsilon$  be the amount of traffic at a super CP and let  $\eta = nz + k\zeta$  be the amount of traffic at the iCPs.

Assume first that the equilibrium  $w, y, z, \zeta, \varepsilon$  is an interior equilibrium. We rewrite below eq. (1) while substituting for  $x_i^j$  the five different values they can take  $(w, y, z, \epsilon \text{ and } \zeta)$ . We thus obtain the following 5 equations. We write below in parenthesis the variable with respect to which (1) is given, i.e. with respect to which the Lagrangian was differentiated.

$$
0 = a\rho + aw - \lambda^i \quad \text{(with respect to } w\text{)},\tag{2}
$$

$$
0 = a\rho + d + ay - \lambda^i \quad \text{(with respect to } y),\tag{3}
$$

$$
0 = a\eta + \delta + az - \lambda^i \quad \text{(with respect to } z\text{)}.
$$
 (4)

For a subscriber  $i'$  of an independent ISP we have:

$$
0 = a\rho + d + a\varepsilon - \lambda^{i'} \quad \text{(with respect to } \varepsilon\text{)},\tag{5}
$$

$$
0 = a\eta + \delta + a\zeta - \lambda^{i'} \quad \text{(with respect to } \zeta\text{)}.
$$
 (6)

These 5 linear equaations with 5 unknowns allow us to compute the equilibrium.

#### 3.2 Computing the equilibrium

We obtain the following equilibrium.

Proposition 1. Whenever the equilibrium is in the interior, it is given by

$$
y^* = \frac{\Phi}{n+m} - d\frac{n+k+1+2m}{a(n+k+1)(n+m)} + \delta \frac{m}{a(n+k+1)(n+m)},
$$
  

$$
z^* = \frac{\Phi}{n+m} + d\frac{n-1-k}{a(n+k+1)(n+m)} - \delta \frac{n}{a(n+k+1)(n+m)},
$$
  

$$
w^* = \frac{\Phi}{n+m} + d\frac{m(n-1+k)+(n-1)(n+k+1)}{a(n+k+1)(n+m)} + \delta \frac{m}{a(n+k+1)(n+m)},
$$
  

$$
\epsilon^* = \frac{\Phi}{n+m} - d\frac{2m}{a(n+k+1)(n+m)} + \delta \frac{m}{a(n+k+1)(n+m)},
$$
  

$$
\zeta^* = \frac{\Phi}{n+m} + d\frac{2n}{a(n+k+1)(n+m)} - \delta \frac{n}{a(n+k+1)(n+m)}.
$$

 $Proof$ : We first substract at equilibrium equations (3) and (2), which gives:

$$
(3) - (2) = ay + d - aw = 0, \text{ implying } w = y + \frac{d}{a}.
$$

From equations (3) and (4), we get:

$$
(4) - (3) = 0 \Leftrightarrow z - y = \rho - \eta + \frac{d - \delta}{a}.
$$

Similarly, we have:

$$
(6) - (5) = 0 \Leftrightarrow \zeta - \epsilon = \eta - \rho + \frac{\delta - d}{a}.
$$

Thus we conclude that:

$$
z - y = \zeta - \epsilon. \tag{7}
$$

d

Recall that we have:  $\rho = w + (n-1)y + k\epsilon$  and  $\eta = nz + k\zeta$ . then,

$$
\rho - \eta = w + (n-1)y + k(\epsilon - \zeta) - nz = w + (n-1)y + k(y-z) - nz = w + y(n+k-1) - z(n+k).
$$

Moreover, we have proved previously that  $w = y + \frac{d}{a}$ . So,

$$
z - y = \rho - \eta + \frac{d - \delta}{a} = w + y(n + k - 1) - z(n + k) + \frac{d - \delta}{a},
$$
  
\n
$$
z = w + y(n + k) - z(n + k) + \frac{d - \delta}{a},
$$
  
\n
$$
z(n + k + 1) = y + \frac{d}{a} + y(n + k) + \frac{d - \delta}{a},
$$
  
\n
$$
z = y + \frac{2d - \delta}{a(n + k + 1)}.
$$

We thus obtain the following linear equation for computing  $y$  at equilibrium, which leads to the expression in the Theorem. Substituting in the

$$
\Phi = w + (n - 1)y + mz = y + \frac{d}{a} + (n - 1)y + my + m\frac{2d - \delta}{a(n + k + 1)},
$$
  
=  $y(n + m) + \frac{d(n + k + 1) + m(2d - \delta)}{a(n + k + 1)},$ 

Substituting in the previous equations yields the expressions for  $w$  and  $z$ .

For the other type of subscribers, we have the following relation:  $\Phi = n\epsilon +$  $m\zeta \Leftrightarrow \zeta = \frac{\Phi - n\epsilon}{m}$ . Moreover, we have that:  $\epsilon - \zeta = y - z = -\frac{2d - \delta}{a(n+k+1)}$ , This yields the expressions for  $\epsilon$  and  $\zeta$  of the Proposition.

We observe that at equilibrium that  $y^*$  and  $\epsilon^*$ , the rates of the demands sent to CPs with agreement are decreasing linearly with the cost d associated to that kind of demand and linearly increasing with  $\delta$ , the cost for a demand to an independent CP. There is the same kind of equilibrium behavior with  $\zeta^*$  the rate of an iISP to an iCP. Indeed, this demand is linearly increasing with d and linearly decreasing with  $\delta$ . Finally, the demand  $w^*$  of an ISP with agreement to his own CP is linearly increasing with d and  $\delta$ . We have also several properties on the different demand rates at the equilibrium.

**Proposition 2.** At equilibrium the demand of a subscriber to his own super CP is never zero, i.e.  $w^* > 0$ .

*Proof* We prove this result by studying the expression of  $w^*$  obtained in proposition 1. We get

$$
w^* = \frac{\Phi}{n+m} + \frac{d(n+k+1)(n+m-1) - m(2d-\delta)}{a(n+k+1)(n+m)},
$$
  
= 
$$
\frac{a\Phi(n+k+1) + d(n+k+1)(n+m-1) - m(2d-\delta)}{a(n+k+1)(n+m)},
$$
  
= 
$$
\frac{a\Phi(n+k+1) + d((n+k+1)(n+m-1) - 2m) + m\delta)}{a(n+k+1)(n+m)},
$$
  
= 
$$
\frac{a\Phi(n+k+1) + d(n^2 - 1 + m(n-1) + k(n-1+m)) + m\delta)}{a(n+k+1)(n+m)}.
$$

But as we have  $n > 1$ , we have  $w^* > 0$ .

Then, we have proved in this proposition that for all costs, an ISP with agreement always sends part of his demand to the CP with which he has an agreement. This induces that the cost for a user connected to an ISP with agreement is always strictly lower than the cost for a user with an independent ISP.

**Proposition 3.** At equilibrium, we have the following equivalence:  $\frac{\delta}{d} > 2 \Leftrightarrow$  $y^* > z^*$ .

Proof : We consider the difference:

$$
y^* - z^* = \frac{-d(n+k+1) - m(2d - \delta) + d(n+k+1) - n(2d - \delta)}{a(n+k+1)(n+m)},
$$
  
= 
$$
-\frac{m(2d - \delta) + n(2d - \delta)}{a(n+k+1)(n+m)} = \frac{\delta - 2d}{a(n+k+1)}.
$$

This result shows an interesting ratio of 2 between the costs for sending demand to concurrent or independent CP. This ratio determines for a subscriber if it is better to send more demand to a concurrent CP or to an iCP. Given the expressions of the equilibrium, we determine the condition on the costs d and  $\delta$ such that all the rates are strictly positive.

**Proposition 4.** If the costs d and  $\delta$  satisfy:

$$
\delta > d\frac{2m+n+k+1}{m} - a\Phi\frac{n+k+1}{m}
$$

and

$$
\delta < d\frac{n-k-1}{n} + a\varPhi\frac{n+k+1}{n},
$$

then  $y^*$ ,  $z^*$ ,  $w^*$ ,  $\epsilon^*$  and  $\zeta^*$  are strictly positive.

*Proof* : First of all, we have already proved that  $w^* > 0$ . Moreover, considering the expressions of the rates at the equilibrium we have

$$
y^* > 0 \Leftrightarrow \delta > d \frac{2m+n+k+1}{m} - a\Phi \frac{n+k+1}{m},
$$
  
and  $z^* > 0 \Leftrightarrow \delta < d \frac{n-k-1}{n} + a\Phi \frac{n+k+1}{n}.$ 

Now we prove the two following results: if  $y^* > 0$  (resp.  $z^* > 0$ ) then  $\epsilon^* > 0$ (resp.  $\zeta^* > 0$ ). First assume  $y^* > 0$ . We have that  $\epsilon^* > 0$  if and only if:

$$
\delta > 2d - a\Phi \frac{n+k+1}{m}.
$$

But  $2m + n + k + 1 > 2m$  which implies that  $d\left(\frac{2m+n+k+1}{m}\right) > 2d$ . Thus, we have:

$$
\delta > d \frac{2m + n + k + 1}{m} - a\Phi \frac{n + k + 1}{m} > 2d - a\Phi \frac{n + k + 1}{m},
$$

which leads to  $\delta > 2d - a\Phi \frac{n+k+1}{m}$ . Then,  $y^* > 0$  implies  $\epsilon^* > 0$ . Now we assume that  $z^* > 0$ . We have that  $\zeta^* > 0$  if and only if:

$$
\delta<2d+a\varPhi\frac{n+k+1}{n}.
$$

But  $z^* > 0$  is equivalent to  $\delta < d \frac{n-k-1}{n} + a \Phi \frac{n+k+1}{n}$  and moreover  $n-k-1 < 2n$ . Then we have  $\delta < 2d + a\Phi \frac{n+k+1}{n}$  which leads to  $\zeta^* > 0$ . Thus we have proved also that  $z^* > 0$  implies that  $\zeta^* > 0$  and that prove the proposition.

#### 4 Paradox and Price of Anarchy

We are interested in showing that there exist some conditions under which the behavior of the system is not as desired. For example, if the ISPs increase their cost it can result in a lower total cost for the users at equilibrium. This is a Braess type Paradox, named after Dieter Braess who first observed and computed such paradoxes in a traffic network ([11]).

We focus on the special network game with only contractual subscribers and super-CP. Numerical results for the general case are differed to the following section. With  $m = k = 0$  we have  $\rho = w + (n - 1)y = \Phi$  and  $w - y = \Phi - ny$ . The expected delay at each content provider does not depend on the price d.

We thus get

$$
y = \frac{1}{n} \left( \Phi - \frac{d}{D'_{cp}(\Phi)} \right), \qquad w = \frac{1}{n} \left( \Phi + (n-1) \frac{d}{D'_{cp}(\Phi)} \right)
$$

This is compatible with the assumption of the theorem if

$$
d \leq \Phi D_{cp}'(I\Phi)
$$

If this is not satisfied then at the equilibrium,  $y = 0$  and  $x = \Phi$  which coincides with the globally optimal solution. The cost at equilibrium for  $d \leq \Phi D'_{cp}(I\Phi)$  is

$$
C^{i}(\underline{x}) = wD(\rho) + (n-1)y(d+D(\rho)) = \Phi D(\rho) + (n-1)yd,
$$

$$
= \Phi D(\rho) + d\frac{n-1}{n} \left(\Phi - \frac{d}{D'_{cp}(\Phi)}\right).
$$

and is otherwise  $\Phi D(\rho)$ .

We observe two types of paradoxes. The first is similar to the original Braess paradox in which eliminating a link improves the cost to all users. In our case, forcing users to download only from the CP that has a contract with their ISP can be viewed as eliminating a link. This is equivalent to taking  $d = \infty$  which results in a globally optimal behavior at equilibrium.

Thus if  $d < \Phi D'_{cp}(I\Phi)$  then the equilibrium cost strictly decreases by eliminating each ISP i the links to all CPs that except the one with which it has an exclusive agreement.

Another variant of Braess paradox studied in the literature consists of the impact of adding capacity to links. A paradoxical behavior is one in which the equilibrium cost increases when the capacity is increased. Translated to our model, we shall say that we have a paradox if by increasing the cost  $d$  the equilibrium cost would decrease. From the above calculations, an increase of the cost d from any value such that  $d < \Phi D'_{cp}(I\Phi)$  to a value satisfying  $d \ge \Phi D'_{cp}(I\Phi)$  creates a paradox of this kind. However, we can identify yet another such paradox. Indeed, the subscriber cost is decreasing for  $d \in [\frac{\Phi D_{cp}^{'}(\Phi)}{2}, \Phi D_{cp}'(\Phi)].$ 

To see that, note that the cost  $C^i$  at equilibrium is expressed by:

$$
C^{i}(d) = \Phi D(\rho) + d \frac{n-1}{n} \left( \Phi - \frac{d}{D'_{cp}(\Phi)} \right).
$$

Then the cost of a subscriber is an hyperbolic function with a maximum when  $d = d^* := \frac{\Phi D_{cp}'(I\Phi)}{2}$ . Then the subscriber cost is first increasing and decreasing depending on  $\overline{d}$ , which proves the existence of a Braess type paradox.

We now look at the performance of the distributed system compared to the centralized solution. The centralized solution is obtained when a central entity determine the actions to take for all users. In order to do that, we use the concept of Price of Anarchy (PoA) [12].

This metric is defined as the ratio between the maximum user cost at equilibrium and the cost for the optimal centralized problem. Our important result is that the PoA is unbounded which is not generally the case in economic problems.

**Proposition 5.** In the particular case where  $m = k = 0$ , the PoA is unbounded.

*Proof* : The optimal subscriber cost at equilibrium, depending on  $d$ , is:

$$
C^i(d^*)=\varPhi D(\rho)+\frac{n-1}{n}\frac{1}{4}\varPhi^2 D'_{cp}(\rho)
$$

The globally optimal solution is obtained at  $y = 0$  for which the subscriber cost is  $\Phi D(\rho)$ . Thus, the price of anarchy is given by

$$
PoA = \frac{C^i(d^*)}{\Phi D(\rho)} = 1 + \frac{(n-1)\Phi D_{cp}'(I\Phi)}{4n D_{cp}(I\Phi)}
$$

In particular, let  $D_{cp}(\rho) = \exp(4nsF(\rho)/(n-1))$  for some F. Then

$$
D'_{cp}(\rho) = 4sn D_{cp}(\rho)F'(\rho)/(n-1)
$$
 so that  $PoA = 1 + \Phi sF'(\rho)$ 

Thus the PoA is unbounded.

We see that in spite of the fact that exclusive agreements offer subscribers with incentives to download from one specific CP (the one that has an exclusive agreement with the subscriber's ISP), the competition between subscribers results in an equilibrium behavior in which subscribers also download from other CPs provided that they are not much more expensive than the one suggested by their ISP.



Fig. 2. Braess type paradox. Fig. 3. User costs for each type of subscribers.

## 5 Numerical Illustrations

First we present numerically the Braess type paradox in the general case when  $n = 4, m = 2, k = 3, \Phi = 1, a = 2$  and  $\delta = 1$ . For these values, we can observe that the individual user cost increases when increasing d from 0 to 0.7, but then it decrases. In particular, when  $d = 2$  the individual user cost at equilibrium is lower than for  $d = \delta = 1$ . Next, we evaluate (Fig. 3) the individual cost at the equilibrium depending on the number of ISPs with agreement  $n$  and the number of independent CPs  $m$ . We observe that the user cost is decreasing in  $m$  for both type of users (independent and with agreement). It is also decreasing in  $n$  for small values of  $n$  and then slowly increasing. The figure shows the same behavior for both types of users, but the individual cost for the independent users is always greater.

### 6 Conclusions

In this paper we model exclusive agreements between service (ISP) and content providers (CP) and study their economic impact. We propose an unusual way of modeling transforming the problem into a routing game.

We compute the Nash equilibrium for this routing game and show the conditions that describe the domain where the equilibrium rates are positive.

We focus on the situation where exclusive agreements exist between content and access providers, which allows access providers to offer content services for free for their subscribers.

We show that the competition between the users results in a paradoxical phenomenon in which subscribers of super-providers prefer to defer part of their demand to other content providers whose content is costly. We show how this phenomena can be exploited by the content providers so as to maximize their benefits. This shows that in spite of the fact that exclusive agreements offer subscribers with incentives to download from one specific CP (the one that has an exclusive agreement with the subscriber's ISP), the competition between subscribers results in an equilibrium in which subscribers also download from other CPs provided that they are not much more expensive than the one suggested by their ISP.

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