Self-Rating in a Community of Peers

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Abstract— Consider a community of agents, all performing a predefined task, but with different abilities. Each agent may be interested in knowing how well it performs in comparison with her peers. This general scenario is relevant, *e.g.*, in Wireless Sensor Networks (WSNs), or in the context of crowd sensing applications, where devices with embedded sensing capabilities collaboratively collect data to characterize the surrounding environment, but the performance is very sensitive to the accuracy of the gathered measurements.

This paper presents a distributed algorithm allowing each agent to self-rate her level of expertise/performance at the task, as a consequence of pairwise interactions with the peers. The dynamics of the proportions of agents with similar beliefs in their expertise are described using continuous-time state equations. The existence of an equilibrium is shown. Closed form expressions for the various proportions of agents with similar belief in their expertise is provided at equilibrium. Simulations match well theoretical results in the context of agents aiming at determining the performance of their sensors.

I. INTRODUCTION

We consider a situation where all the agents in a community are engaged in some activity (sensing, estimation, classification...), for which it is reasonable to expect heterogeneous levels of ability. The agents, who regularly meet their peers and interact with them in pairs, are incentivated to self-rate their own capabilities at the task in order to better address their future actions. This general scenario may well describe, *e.g.*, the needs of a group of amateur chess players trying to pair up with opponents of comparable strength; or the situation of a sensor network, where devices with embedded sensing capabilities collaboratively collect data to characterize the surrounding environment, and nodes with scarce sensing accuracy may prefer to withhold their contribution to avoid to pollute the data. This may happen, for instance, in the context of crowdsensing scenarios [1], where, in order to provide reliable sensing services such as those advocated by the S^2aaS (Sensing as a service) paradigm [2], there is a need for high quality measurements that are not always available at devices since this is tightly related to the accuracy of the sensors embedded.

This problem is counteracted in S^2aaS scenarios by introducing reputation-based mechanisms [3]–[6]. In such

schemes sensing data are collected from smart-phones, based on an auction mechanism. The sensing task is assigned to the nodes by the central authority on the basis of their reputation level, to maximize the utility for the crowd. Such reputation-based mechanisms are effective in rating the agents of the community, but are based on data centralization and need to establish a rating authority. Allowing agents to assess their ability has the benefit to help them providing measurements as well as their associated quality levels. This facilitates further measurement processing and may avoid compromising the reputation of an agent temporarily producing outliers. Moreover, in S^2aaS applications, agents knowing that their sensing ability is above the average may negotiate a better reward for their measurements.

This work considers instead distributed mechanism that nodes can employ to self-rate their capabilities. A distributed classification approach is taken in the context of WSN by the works in [7], [8], where different nodes are subject to different sensing noise models characterized by hidden parameters. Each node is willing to co-operate with the rest of the network via gossip or consensus algorithms to estimate the common observed physical quantity, and simultaneously learns its sensing model. For these algorithms to converge, however, the WSN architecture needs to guarantee large static connectivity for each node, and to disseminate node identification. In this work we propose an alternative approach to distributed self-rating where instead nodes can be mobile, connectivity can be dynamic, and identification is not used by the algorithm, allowing, *e.g.*, for node substitution or removal without disrupting the network functionality.

The proposed distributed self-rating (DSR) algorithm needs a Local Comparison Test (LCT) adequate to establish which agent of a pair of agents engaged in the test is better in performing the task. The self-rating algorithm consists then in performing a LCT when two agents meet, and in counting the number of positive test outcomes as well as the number of past meetings. The agents, who have knowledge of the expected proportion of nodes in each group, maintain a self-rating which evolves in time, and converges fast and reliably. The effectiveness of the proposed algorithm is measured by the correct grade rate (CGR), *i.e.*, the proportion of agents whose self-rating result corresponds to the real level, and by the false grade rate (FGR), *i.e.*, the proportion of agents who self-rate the wrong class. The behavior of the algorithm is described by state equations. The influence of the characteristics of the LCT of the CGR and the FGR at equilibrium provides insights to optimize design.

The paper is organized as follows. Section II introduces the system model and LCT. Section III introduces and analyzes

the proposed DSR algorithm. The CGR and the FGR at equilibrium is characterized in Section IV. Simulations results are reported in Section V and conclusions in Section VI.

II. SYSTEM MODEL AND LOCAL COMPARISON TEST

Consider a set $\mathcal{S}$ of N_{S} moving agents. Assume that the agents in $\mathcal{S}$ are partitioned into N_{G} groups $\mathcal{S}_1, \ldots \mathcal{S}_{N_{\mathrm{G}}}$, containing respectively a proportion $p_1, \ldots, p_{N_{\mathrm{G}}}$ of the agents. A group contains agents able to perform some task with a similar level of expertise or reliability. Groups are sorted in decreasing levels of expertise. Let $\theta_i(t) \in \mathcal{G} = \{1, \ldots, N_{\mathrm{G}}\}$ be the index of the group $\mathcal{S}_{\theta_i(t)}$ to which Agent i belongs at time t. Initially, Agent i is not aware of the actual value of θ_i but is willing to estimate it as fast as possible. We assume that the group to which Agent i belongs does not change over the time horizon of the experiment, so $\theta_i(t) = \theta_i$.

One will present a DSR algorithm allowing each Agent i to get an estimate $\widehat{\theta}_i$ of its group θ_i. For that purpose an LCT is introduced, performed by Agent i after meeting and interacting with any Agent j. Meetings are assumed to involve only two agents. The output $y_{i,j}$ of the LCT performed by Agent i upon interaction with Agent j is binary. The LCT determines whether Agent i is better at doing the considered task than Agent j ($y_{i,j} = 1$), or vice-versa ($y_{i,j} = 0$), without being able to determine the Agent i's absolute level of expertise. LCTs may provide erroneous conclusions and are characterized by their statistical properties

$$q(\theta_i, \theta_j) = \mathbb{P}\{Y_{i,j} = 1 | \theta_i, \theta_j\}. \quad (1)$$

One assumes that $\theta_i \leqslant \theta_j$ implies $q(\theta_i, 1) \geqslant q(\theta_j, 1)$, which appears reasonable, since the first group contains the agents with the highest level of expertise.

Interactions may take various forms depending on the application scenario, ranging from the exchange of noisy measurements m_i and m_j of the same physical quantity when the agents are nodes of a WSN, to a blitz-game between humans, willing, *e.g.*, to self-rate their level in chess. An example LCT is provided in Section V in the context of WSNs. A meeting between two nodes does not necessarily entail interaction: one assumes here that the probability of interaction $\alpha(\widehat{\theta}_i, \widehat{\theta}_j)$ is a function of the estimates $\widehat{\theta}_i(t)$ and $\widehat{\theta}_j(t)$ of the groups to which each agent belongs. When the agents aim is to self-rate their level in chess, for instance, $\alpha(\widehat{\theta}_i, \widehat{\theta}_j)$ will approach one when $\widehat{\theta}_i$ and $\widehat{\theta}_j$ are close and thus the outcome of the LCT is reputed unpredictable, and be small when $\widehat{\theta}_i$ and $\widehat{\theta}_j$ are very different, thus preventing the agents wasting time in a LCT whose outcome is reputed easily foreseeable. In practice, one assumes that when two agents i and j meet, they start exchanging the estimated index of their group, Agent i will request for a further interaction with probability $\alpha(\widehat{\theta}_i, \widehat{\theta}_j)$ and Agent j with a probability $\alpha(\widehat{\theta}_j, \widehat{\theta}_i)$.

III. DISTRIBUTED SELF-RATING ALGORITHM

In the proposed DSR algorithm, each agent manages two counters $c_{\mathrm{t},i}(t)$ and $c_{\mathrm{b},i}(t)$ initialized at 0 at $t = 0$. The number of LCTs performed by Agent i and following an

---

Algorithm 1 DSR algorithm for Agent i

1) Initialize $t_i^0 = 0$, $\widehat{\theta}_i(t_i^0) = 1$, $c_{\mathrm{t},i}(t_i^0) = c_{\mathrm{b},i}(t_i^0) = 0$, $\kappa = 1$, and $\mu = 0$.
2) Do $\widehat{\theta}_i(t) = \widehat{\theta}_i(t_i^{\kappa-1})$, $c_{\mathrm{t},i}(t) = c_{\mathrm{t},i}(t_i^{\kappa-1})$, $c_{\mathrm{b},i}(t) = c_{\mathrm{b},i}(t_i^{\kappa-1})$, and $t = t + \delta t$ until the κ-th meeting occurs at time t_i^κ with Agent $j^\kappa \in \mathcal{S}$.
3) Transmit $\widehat{\theta}_i(t_i^\kappa)$ to Agent j^κ and receive $\widehat{\theta}_{j^\kappa}(t_i^\kappa)$ from Agent j^κ.
4) With probability $\alpha\left(\widehat{\theta}_i(t_i^\kappa), \widehat{\theta}_{j^\kappa}(t_i^\kappa)\right)$, perform a LCT with outcome y_i^μ, then
   a) $\mu = \mu + 1$. Update $c_{\mathrm{t},i}$ and $c_{\mathrm{b},i}$ as
   $$\begin{cases} c_{\mathrm{t},i}(t_i^\kappa) = \min\{\mu, M\} \\ c_{\mathrm{b},i}(t_i^\kappa) = \sum_{m=\max\{1, \mu-M+1\}}^{\mu} y_i^m \end{cases} \quad (2)$$
   b) Update $\widehat{\theta}_i$ according to 3
5) $\kappa = \kappa + 1$.
6) Go to 2.

---

interaction it has requested is stored in $c_{\mathrm{t},i}(t)$. The number of LCTs concluding that Agent i is *better* than the agent met is stored in $c_{\mathrm{b},i}(t)$. An agent involved in an interaction it has not requested does not update $c_{\mathrm{t},i}(t)$ and $c_{\mathrm{b},i}(t)$. If the agents are randomly spread, the ratio $c_{\mathrm{b},i}(t)/c_{\mathrm{t},i}(t)$ mainly depends on the proportions of agents in each group, on the interaction probabilities, and on the properties of the LCT.

Intuitively, an agent belonging to Group 1 is likely to have a larger ratio $c_{\mathrm{b},i}(t)/c_{\mathrm{t},i}(t)$ than an agent belonging to Group N_{G}. Introducing the partition of $[0, 1]$ into N_{G} decision intervals $[\nu_k, \nu_{k-1})$ with $\nu_0 = 1$ and $\nu_{N_{\mathrm{G}}} = 0$, one may consider the decision rule

$$\widehat{\theta}_i(t) = k \text{ if } c_{\mathrm{b},i}(t)/c_{\mathrm{t},i}(t) \in [\nu_k, \nu_{k-1}), \ \forall k \in \mathcal{G}. \quad (3)$$

Agents with the largest value of $c_{\mathrm{b},i}(t)/c_{\mathrm{t},i}(t)$ choose the smallest group index. One will show that the decision rule (3) leads to a satisfying self-rating of the agents, for an appropriate choice of the value of ν_k, for all $k \in \mathcal{G}$. The values of $\alpha(\widehat{\theta}_i, \widehat{\theta}_j)$ can be adjusted to optimize the performance of the DSR algorithm.

A. Practical Self-Rating Algorithm

Let $\mathbf{x}_i(t) = (\theta_i, c_{\mathrm{t},i}(t), c_{\mathrm{b},i}(t))$ represent the *state* of Agent i. If all the LCT results obtained in the past are considered, one has $\lim_{t \to \infty} c_{\mathrm{t},i}(t) \to \infty$, which results an infinite number of possible values for $\mathbf{x}_i(t)$. The global (macroscopic) behavior of the algorithm is in this case difficult to analyze. To limit the number of possible states, one considers the evolution of $c_{\mathrm{t},i}(t)$ and $c_{\mathrm{b},i}(t)$ over a sliding variable-length time interval containing the time instants of the last M meetings during which Agent i has performed a LCT. Algorithm 1 summarizes the proposed DSR algorithm for an arbitrary Agent i.

B. Evolution of the state of an agent

At time t, among the agents in Group θ, let $X_\theta^{\ell,d}(t)$ be the proportion of agents in state $\mathbf{x}_i(t) = (\theta, \ell, d)$, *i.e.*, with
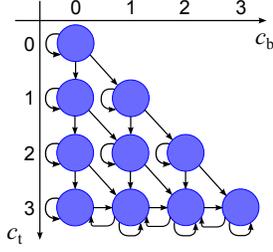
Fig. 1. Example of Markov model for the evolution of the state of an agent when $M = 3$.

$c_{t,i}(t) = \ell$ and $c_{b,i}(t) = d$. Since $\ell \in \{0, \dots, M\}$ and $d \in \{0, \dots, \ell\}$, the number of values that may be taken by the state of an agent is $(M+1)(M+2)/2$. The evolution of the state of Agent i, conditioned by the index θ of its actual group, follows a Markov model with state transition diagram similar to that shown in Figure 1 for $M = 3$.

There are N_G parallel chains conditioned by the value of $\theta \in \mathcal{G}$. Define $\pi_\theta^{\delta_t, \delta_b}$ as the transition probability from State (θ, ℓ, d) to State $(\theta, \ell + \delta_t, d + \delta_b)$. Note that $\pi_\theta^{\delta_t, \delta_b}$ depends on the current state (θ, ℓ, d) of the reference agent, and also on the current proportion of agents with estimation $\widehat{\theta}_i(t)$.

One has first to evaluate the probability that some Agent i with state (θ, ℓ, d) and estimated rating $\widehat{\theta}(\ell, d)$ performs a LCT upon meeting a random Agent J. This probability may be evaluated as $\beta(\ell, d) = \mathbb{E}(\alpha(\widehat{\theta}(\ell, d), \widehat{\theta}_J(t)))$ where the expectation has to be taken over $\widehat{\theta}_J(t)$. Then

$$\beta(\ell, d) = \sum_{k_1, k_2 \in \mathcal{G}} \alpha(\widehat{\theta}(\ell, d), k_2) \mathbb{P}\left\{\widehat{\theta}_J(t) = k_2, \theta_J = k_1\right\}$$
$$= \sum_{k_1, k_2 \in \mathcal{G}} \alpha(\widehat{\theta}(\ell, d), k_2) \mathbb{P}\left\{\widehat{\theta}_J(t) = k_2 | \theta_J = k_1\right\} \mathbb{P}\{\theta_J = k_1\}$$
$$= \sum_{k_1, k_2 \in \mathcal{G}} \alpha(\widehat{\theta}(\ell, d), k_2) p_{k_1} p^{k_1 k_2}(t). \quad (4)$$

In (4), one uses the fact that agents are randomly spread to get $\mathbb{P}\{\theta_J = k_1\} = p_{k_1}$. We introduce $p^{k_1 k_2}(t)$ as the proportion of agents in Group k_1 believing their group index is k_2. Finally, one has $\widehat{\theta}(\ell, d) = k$ when $d/\ell \in [\nu_k, \nu_{k-1})$. More specifically,

$$p^{k_1 k_2}(t) = \mathbb{P}\left\{\widehat{\theta}_J(t) = k_2 | \theta_J = k_1\right\}$$
$$= \begin{cases} X_{k_1}^{0,0}(t) + \sum_{\ell, d, d/\ell \in [\nu_{k_2}, \nu_{k_2-1})} X_{k_1}^{\ell, d}(t), & \text{if } k_2 = 1, \\ \sum_{\ell, d, d/\ell \in [\nu_{k_2}, \nu_{k_2-1})} X_{k_1}^{\ell, d}(t), & \text{else.} \end{cases}$$
$$(5)$$

Two phases have to be considered in Algorithm 1, depending on the value of $c_{t,i}(t)$. In the *transient phase,* for states with $c_{t,i}(t) = \ell < M$, one has $(\delta_t, \delta_b) \in \{(0,0), (1,0), (1,1)\}$, since ℓ may either increase or remain constant and $\delta_b \leqslant \delta_t$. The only possibility leading to $\delta_b = 0$ is that Agent i, once it has met a random Agent J, decides not to continue interaction. Then $\pi_\theta^{0,0}(t, \ell, d) = 1 - \beta(\ell, d)$.

A state transition occurs with $(\delta_t, \delta_b) = (1,1)$ when, once Agent i has met Agent J, they continue interaction *and* the LCT yields $y_i(t) = 1$. Since α only depends on the group

estimates, these two events can be assumed as independent. Considering all possible values taken by $\widehat{\theta}_J(t)$ one gets

$$\pi_\theta^{1,1}(t, \ell, d)$$
$$= \sum_{k_1, k_2 \in \mathcal{G}} \alpha(\widehat{\theta}(\ell, d), k_2) \mathbb{P}\{Y_i = 1, \theta_J = k_1, \widehat{\theta}_J(t) = k_2 | \theta_i = \theta\}$$
$$= \sum_{k_1, k_2 \in \mathcal{G}} \alpha(\widehat{\theta}(\ell, d), k_2) p_{k_1} p^{k_1 k_2}(t) q(\theta, k_1). \quad (6)$$

Finally, $\pi_\theta^{1,0}(t, \ell, d)$ is obtained similarly as (6),

$$\pi_\theta^{1,0}(t, \ell, d) = \sum_{k_1, k_2 \in \mathcal{G}} \alpha(\widehat{\theta}(\ell, d), k_2) p_{k_1} p^{k_1 k_2}(t) (1 - q(\theta, k_1)).$$
$$(7)$$

In the *permanent regime*, $c_{t,i}(t) = M$ and remains constant, thus $\delta_t = 0$. In Algorithm 1, μ is the number of LCTs performed by Agent i till time t. When $\mu \geqslant M$, only the last M LCT outcomes are considered. To determine the value taken by $\delta_b \in \{-1, 0, 1\}$ after the μ-th LODT, consider an arbitrary $y \in \{0, 1\}$ and the random event $\mathcal{E}_y(t) = \left\{Y_i^{\mu - M} = y | \sum_{m=\mu - M}^{\mu - 1} Y_i^m = d\right\}$, which corresponds to a situation where one knows that k LCTs yield 1 among the last M tests and $Y_i^{\mu - M} = y$ will be ignored once the new LCT outcome is available. $\mathbb{P}\{\mathcal{E}_1(t)\}$ is relatively complex to evaluate, since $\mathbb{P}\{Y_i^n = y\}$ depends on the actual group of the encountered agent and is time-varying. Assuming that LCT outcomes with $Y_i^m = y$, are independently distributed over the time horizon corresponding to $m = \mu - M, \dots, \mu - 1$, one gets $\mathbb{P}\{\mathcal{E}_1(t)\} = d/M$ and $\mathbb{P}\{\mathcal{E}_0(t)\} = 1 - d/M$.

Assume that the $(\mu - M)$-th LCT performed by Agent i occurred at time $\tilde{t}$, then $y_i^{\mu - M}$ can also be denoted as $y_i(\tilde{t})$ and the transition related to $c_{b,i}$ is such that $\delta_b = y_i(t) - y_i(\tilde{t}) \in \{-1, 0, 1\}$. To have $(\delta_t, \delta_b) = (0, 1)$, three independent events have to occur: 1) interaction has to continue once Agent J has been met; 2) $y_i(t) = 1$; 3) $y_i(\tilde{t}) = 0$, *i.e.*, $\mathcal{E}_0(t)$. Following derivations similar to (6),

$$\pi_\theta^{0,1}(t, M, d)$$
$$= \sum_{k_1, k_2 \in \mathcal{G}} \alpha(\widehat{\theta}(M, d), k_2) p_{k_1} p^{k_1 k_2}(t) q(\theta, k_1) \frac{M - d}{M}. \quad (8)$$

Similarly,

$$\pi_\theta^{0,-1}(t, M, d)$$
$$= \sum_{k_1, k_2 \in \mathcal{G}} \alpha(\widehat{\theta}(M, d), k_2) p_{k_1} p^{k_1 k_2}(t) (1 - q(\theta, k_1)) \frac{d}{M}. \quad (9)$$

Applying (8-9), $\pi_\theta^{0,0}(t, M, d)$ can be obtained from $\pi_\theta^{0,0}(t, M, d) = 1 - \pi_\theta^{0,1}(t, M, d) - \pi_\theta^{0,-1}(t, M, d)$.

C. Macroscopic evolution

All agent state transition probabilities evaluated in Section III-B are now used to determine the evolution of the various proportions $X_\theta^{\ell, d}(t)$ of agents in the corresponding states. In what follows, one assumes that agents are mobile and form a well-mixed population. Considering an inter-contact rate λ, during a short time interval $[t, t + \delta t]$, the number of agents with state (θ, ℓ, d) that will meet another agent can be estimated as $\lambda p_\theta N_S X_\theta^{\ell, d}(t) \delta t$. When $0 < d < \ell < M$, these agents will switch to the states $(\theta, \ell + \delta_t, d + \delta_b)$, with a probability $\pi_\theta^{\delta_t, \delta_b}(t, \ell, d)$, where $(\delta_t, \delta_b) \in \{(0,0), (1,0), (1,1)\}$. Moreover, agents in the

states $(\theta, \ell-1, d-1)$ and $(\theta, \ell-1, d)$ that have met another agent in the time interval $[t, t+\delta t]$ may reach state (θ, ℓ, d), respectively with a probability $\pi_\theta^{1,1}(t, \ell-1, d-1)$ and $\pi_\theta^{1,0}(t, \ell-1, d)$. The evolution of $X_\theta^{\ell,d}(t)$ is then described by the following differential equation (10-c), in which the time dependency is omitted to lighten notations. Similar derivations can be made for the remaining state components to obtain the following equations.

$$
\begin{cases}
\frac{dX_\theta^{0,0}}{dt} \overset{(a)}{=} -\lambda X_\theta^{0,0}\left(\pi_\theta^{1,0}(0,0) + \pi_\theta^{1,1}(0,0)\right), \\
\frac{dX_\theta^{\ell,0}}{dt} \overset{(b)}{=} \lambda\Big(-X_\theta^{\ell,0}\left(\pi_\theta^{1,0}(\ell,0) + \pi_\theta^{1,1}(\ell,0)\right) \\
\qquad\qquad + X_\theta^{\ell-1,0}\pi_\theta^{1,0}(\ell-1,0)\Big), \\
\frac{dX_\theta^{\ell,d}}{dt} \overset{(c)}{=} \lambda\Big(-X_\theta^{\ell,0}\left(\pi_\theta^{1,0}(\ell,0) + \pi_\theta^{1,1}(\ell,0)\right) \\
\qquad + X_\theta^{\ell-1,d}\pi_\theta^{1,0}(\ell-1,d) + X_\theta^{\ell-1,d-1}\pi_\theta^{1,1}(\ell-1,d-1)\Big), \\
\frac{dX_\theta^{\ell,\ell}}{dt} \overset{(d)}{=} \lambda\Big(-X_\theta^{\ell,\ell}\left(\pi_\theta^{1,0}(\ell,\ell) + \pi_\theta^{1,1}(\ell,\ell)\right) \\
\qquad\qquad + X_\theta^{\ell-1,\ell-1}\pi_\theta^{1,1}(\ell-1,\ell-1)\Big), \\
\frac{dX_\theta^{M,0}}{dt} \overset{(e)}{=} \lambda\Big(-X_\theta^{M,0}\pi_\theta^{0,1}(M,0) + X_\theta^{M-1,0}\pi_\theta^{1,0}(M-1,0) \\
\qquad\qquad + X_\theta^{M,1}\pi_\theta^{0,-1}(M,1)\Big), \\
\frac{dX_\theta^{M,d}}{dt} \overset{(f)}{=} \lambda\Big(-X_\theta^{M,d}\left(\pi_\theta^{0,1}(M,d) + \pi_\theta^{0,-1}(M,d)\right) \\
\quad + X_\theta^{M-1,d-1}\pi_\theta^{1,1}(M-1,d-1) + X_\theta^{M-1,d}\pi_\theta^{1,0}(M-1,d) \\
\quad + X_\theta^{M,d-1}\pi_\theta^{0,1}(M,d-1) + X_\theta^{M,d+1}\pi_\theta^{0,-1}(M,d+1)\Big), \\
\frac{dX_\theta^{M,M}}{dt} \overset{(g)}{=} \lambda\Big(-X_\theta^{M,M}\pi_\theta^{0,-1}(M,M) + X_\theta^{M,M-1}\pi_\theta^{0,1}(M,M-1) \\
\qquad\qquad + X_\theta^{M-1,M-1}\pi_\theta^{1,1}(M-1,M-1)\Big),
\end{cases}
$$
(10)

D. Equilibrium point of $X_\theta^{\ell,d}$

To analyze the asymptotic behavior of the state equations (10), one considers a special case where $\alpha(k_1, k_2) = 1$ if $k_2 = 1$ and $\alpha(k_1, k_2) = 0$ else. Interaction is only performed when an Agent i meets an Agent j believing it is in the best group. As will be seen in Section V, this strategy is efficient to self-rate agents equipped with sensors of different quality.

One investigates first the evolution of $X_\theta^{\ell,k}(t)$ when $\ell < M$. in the following proposition.

Proposition 1: For any $d \leqslant \ell < M$, $\lim_{t \to \infty} X_\theta^{\ell,d}(t) = 0$. Proposition 1 is proved by extending the proof of Proposition 2 in [9], to θ discrete. As a consequence, the only possible value at equilibrium of $X_\theta^{\ell,d}(t)$ with $\ell < M$ is 0. Let $\overline{X}_\theta^{M,d}$ be the value at equilibrium of $X_\theta^{M,d}$. The proportion of agents estimating their group as $\widehat{\theta}$ depends on the partition of the interval $[0,1]$ introduced in (3)

$$
\overline{p}^{\theta\widehat{\theta}} = \sum_{d: d/M \in [\nu_{\widehat{\theta}}, \nu_{\widehat{\theta}-1})} \overline{X}_\theta^{M,d}.
$$
(11)

Denote $\overline{\mathbf{p}}^1 = \left[\overline{p}^{11}, \ldots, \overline{p}^{N_G 1}\right]^{\mathrm{T}}$ and consider the functions

$$
h_\theta\left(\overline{\mathbf{p}}^1\right) = \left(\sum_{k \in \mathcal{G}} p_k \overline{p}^{k1} q(\theta, k)\right) \bigg/ \left(\sum_{k \in \mathcal{G}} p_k \overline{p}^{k1}\right),
$$
(12)

$$
F_\theta\left(\overline{\mathbf{p}}^1\right) = \sum_{d=\lceil \nu_1 M\rceil}^{M} \binom{M}{d}\left(h_\theta\left(\overline{\mathbf{p}}^1\right)\right)^d\left(1 - h_\theta\left(\overline{\mathbf{p}}^1\right)\right)^{M-d},
$$
(13)

and $\mathbf{F}\left(\overline{\mathbf{p}}^1\right) = \left[F_1\left(\overline{\mathbf{p}}^1\right), \ldots, F_{N_G}\left(\overline{\mathbf{p}}^1\right)\right]^{\mathrm{T}}$. The following proposition provides a non-linear equation that has to be satisfied by $\overline{\mathbf{p}}^1$. Once this equation is solved, one easily deduces the various $\overline{X}_\theta^{M,d}$ at equilibrium.

Proposition 2: Assume that (10) admits some equilibrium $\overline{X}_\theta^{\ell,d}$, then for any $\theta \in \mathcal{G}$ and $d \leqslant \ell$, $\overline{X}_\theta^{\ell,d} = 0$ when $\ell < M$ and

$$
\overline{X}_\theta^{\ell,d} = \binom{M}{d}\left(h_\theta\left(\overline{\mathbf{p}}^1\right)\right)^d\left(1 - h_\theta\left(\overline{\mathbf{p}}^1\right)\right)^{M-d}
$$
(14)

when $\ell = M$, where $\overline{\mathbf{p}}^1$ are obtained by solving

$$
\overline{\mathbf{p}}^1 = \mathbf{F}\left(\overline{\mathbf{p}}^1\right).
$$
(15)

See Section IV-A for the proof.

The existence of $\overline{X}_\theta^{M,d}$ depends on whether (15) has a solution $\overline{\mathbf{p}}^1$. One then shows the existence of an equilibrium in Proposition 3 in the special case where $N_G = 2$.

Proposition 3: In the case $N_G = 2$, for any $\nu_1 \in [0,1]$, (15) always admits a solution, and (10) admits an equilibrium.

See Section IV-B for the proof.

E. Approximations of the equilibrium

Explicit expressions for $\overline{p}^{\theta1}$ are difficult to obtain from (15). Since $\overline{p}^{\theta1}$ with $\theta \neq 1$ represent the proportions of agents that have wrongly estimated their group, the vector $\overline{\mathbf{p}}^1 = \left[\overline{p}^{11}, \ldots, \overline{p}^{N_G 1}\right]^{\mathrm{T}}$ should be close to $\widetilde{\mathbf{p}}^1 = [1, 0, \ldots, 0]^{\mathrm{T}}$. One has $\lim_{\mathbf{p}^1 \to \widetilde{\mathbf{p}}^1} h_\theta\left(\mathbf{p}^1\right) = q(\theta, 1)$. Assuming that at equilibrium, $h_\theta\left(\overline{\mathbf{p}}^1\right) \simeq q(\theta, 1)$, using (14), $\overline{X}_\theta^{M,d}$ can be approximated as

$$
\widetilde{X}_\theta^{M,d} = \binom{M}{d}\left(q(\theta, 1)\right)^d\left(1 - q(\theta, 1)\right)^{M-d},
$$
(16)

and follows thus a binomial distribution. Knowing $\widetilde{X}_\theta^{M,d}$ for all $\theta \in \mathcal{G}$ and all $d \leqslant M$ gives some insights to properly tune the decision thresholds introduced in (3). The values of the ν_θs may for example be adjusted to maximize $\sum_{\theta \in \mathcal{G}} \widetilde{p}^{\theta\theta}$, see the extended version of this paper [10] for details.

IV. PROOFS

A. Proof of Proposition 2

At equilibrium, one has $dX_\theta^{\ell,d}(t)/dt = 0$ for all $0 \leqslant d \leqslant \ell \leqslant M$ and the transition probabilities will not vary any more. From Proposition 1, one has $\overline{X}_\theta^{\ell,d} = 0$, for all $d \leqslant \ell < M$. Let $\overline{\boldsymbol{X}}_\theta^M = \left[\overline{X}_\theta^{M,1}, \ldots, \overline{X}_\theta^{M,M}\right]^{\mathrm{T}}$, $a_\theta(d) = \pi_\theta^{0,1}(M,d)$, and $b_\theta(d) = \pi_\theta^{0,-1}(M,d)$. Writing (10) at equilibrium, $\overline{\boldsymbol{X}}_\theta^M$ should satisfy $\boldsymbol{\Psi} \cdot \overline{\boldsymbol{X}}_\theta^M = \mathbf{0}$ where

$$
\boldsymbol{\Psi} = \begin{pmatrix}
-a_\theta(0) & b_\theta(1) & & \\
a_\theta(0) & -a_\theta(1) - b_\theta(1) & b_\theta(2) & \\
& \ddots & \ddots & \ddots \\
& & a_\theta(M-1) & -b_\theta(M)
\end{pmatrix}.
$$

Summing Lines 1 to $d+1$, for $d = 0, \ldots, M-1$, one obtains $b_\theta (d+1) \overline{X}_\theta^{M,d+1} = a_\theta (d) \overline{X}_\theta^{M,d}$, which leads to

$$\overline{X}_\theta^{M,d} = \overline{X}_\theta^{M,0} \prod_{j=0}^{d-1} \frac{a_0 (j)}{b_0 (j+1)} = \overline{X}_\theta^{M,0} \prod_{j=0}^{d-1} \frac{\pi_\theta^{0,1} (M, j)}{\pi_\theta^{0,-1} (M, j+1)}$$

$$= \overline{X}_\theta^{M,0} \prod_{j=0}^{d-1} \frac{(M-j) h_\theta}{(j+1)(1 - h_\theta)} = \overline{X}_\theta^{M,0} \binom{M}{d} \left(\frac{h_\theta}{1 - h_\theta} \right)^d \quad (17)$$

where h_θ is defined in (12). Since $\sum_{d=0}^{M} \overline{X}_\theta^{M,d} = 1$, one obtains for all $\theta \in \mathcal{G}$ and $d \in \{0, \ldots, M\}$

$$\overline{X}_\theta^{M,d} = \binom{M}{d} h_\theta^d (1 - h_\theta)^{M-d}. \quad (18)$$

Introducing (18) in (11), one obtains (15) with F_θ defined in (13). Thus one needs to solve (15) to determine $\overline{p}^{\theta 1}$ for all $\theta \in \mathcal{G}$, which are the used to deduce $\overline{X}_\theta^{M,d}$ using (18).

B. Proof sketch of Proposition 3

The existence of a solution of (15) when $N_\mathrm{G} = 2$ is shown using Brouwer's fixed-point [11]. One shows first that for any $(p_0^{11}, p_0^{21}) \in \mathcal{P}_0 = \{(x, y) \in [0, 1] \times [0, 1] \text{ and } (x, y) \neq (0, 0)\}$, the discrete-time system[1]

$$(p_{n+1}^{11}, p_{n+1}^{21}) = (F_1 (p_n^{11}, p_n^{21}), F_2 (p_n^{11}, p_n^{21})). \quad (19)$$

converges to a equilibrium point $(\overline{p}^{11}, \overline{p}^{21})$.

Both F_1 and F_2 are continuous functions, however $\mathcal{P}_0$ is not a compact set. Thus one needs to find a compact $\mathcal{P}_n$ such that $F_1 (p_n^{11}, p_n^{21})$ is a mapping $\mathcal{P}_n \to \mathcal{P}_n$. One starts showing some useful lemmas.

Lemma 4: If $(x, y) \in \mathcal{P}_0$, then $h_\theta(x, y)$ is bounded as $0 < h_{\theta,\min} \leqslant h_\theta (x, y) \leqslant h_{\theta,\max}$, where $h_{\theta,\max} = \max\{q(\theta, 1), q(\theta, 2)\}$ and $h_{\theta,\min} = \min\{q(\theta, 1), q(\theta, 2)\}$.

Lemma 5: Define $g(z) = \sum_{d=\lceil M\nu_1 \rceil}^{M} \binom{M}{d} z^d (1-z)^{M-d}$. If $0 < \nu_1 < 1$, then $g(z)$ is increasing over $[0, 1]$.

Lemma 4 can be easily proved by comparing h_θ with $h_{\theta,\max}$ and $h_{\theta,\min}$. The proof of Lemma 5 can be found in [9]. From Lemma 4 and 5, one obtains that for any $\theta \in \{1, 2\}$ and $(x, y) \in \mathcal{P}_0$

$$0 < g (h_{\theta,\min}) \leqslant F_\theta (x, y) \leqslant g (h_{\theta,\max}). \quad (20)$$

Define $p_{n,\max}^{\theta 1}$ and $p_{n,\min}^{\theta 1}$ as upper and lower bounds of $p_n^{\theta 1}$, i.e., $p_{n,\min}^{\theta 1} \leqslant p_n^{\theta 1} \leqslant p_{n,\max}^{\theta 1}$. When $n = 0$, $p_{0,\min}^{\theta 1} = 0$ and $p_{0,\max}^{\theta 0} = 1$. From (20), one gets

$$p_{1,\min}^{\theta 1} = g (h_{\theta,\min}) > 0, \quad p_{1,\max}^{\theta 1} = g (h_{\theta,\max}). \quad (21)$$

Define $\mathcal{P}_1 = [p_{1,\min}^{11}, p_{1,\max}^{11}] \times [p_{1,\min}^{21}, p_{1,\max}^{21}]$, then $(p_1^{11}, p_1^{21}) = \mathbf{F} ((p_0^{11}, p_0^{21})) \in \mathcal{P}_1$. Notice that $\mathbf{0} \notin \mathcal{P}_1$ and $\mathcal{P}_1$ is a compact set since $p_{1,\min}^{\theta 1} > 0$.

Consider then an arbitrary integer $n \in \mathbb{N}^*$. Assume that $(p_{n-1}^{11}, p_{n-1}^{21}) \in \mathcal{P}_1$, one needs to see whether $(p_n^{11}, p_n^{21}) \in \mathcal{P}_1$ is satisfied. Since $\mathcal{P}_1 \subseteq \mathcal{P}_0$, one still has

$$h_{\theta,\min} \leqslant h_\theta (p_{n-1}^{11}, p_{n-1}^{21}) \leqslant h_{\theta,\max},$$

[1] Notice that one never has $p_0^{11} = p_0^{21} = 0$, which can be obtained from Lemma 9 and Lemma 10 in [9].

which leads to

$$g (h_{\theta,\min}) \leqslant p_n^{\theta 1} = F_\theta (p_{n-1}^{11}, p_{n-1}^{21}) \leqslant g (h_{\theta,\max}). \quad (22)$$

Therefore $\mathbf{F}$ maps $\mathcal{P}_1$ to $\mathcal{P}_1$, which is compact. Applying Brouwer's fixed-point theorem one proves Proposition 3.

V. ILLUSTRATION

Consider a scenario where all agents are equipped with a sensor providing noisy observations of the scalar physical quantity $\phi (\boldsymbol{o}, t)$, at location $\boldsymbol{o}$ and at time t. For Agent i,

$$m_i (\boldsymbol{o}, t) = \phi (\boldsymbol{o}, t) + w_i, \qquad \forall i \in \mathcal{S}. \quad (23)$$

The components w_i of the measurement noise in (23) are assumed to be realizations of random Gaussian variables $W_i \sim \mathcal{N} (e_i, \sigma^2)$, where e_i is a constant agent-dependent bias. The agents can be classified according to the value of e_i. Assume that among different sensors, the absolute value $|e_I|$ of a random agent I follows an exponential distribution with parameter γ. For example, Group θ can be defined as $\mathcal{S}_\theta = \{i \in \mathcal{S} : \theta - 1 \leqslant |e_i| < \theta\}$ for any $\theta \in \mathcal{G} \setminus \{N_\mathrm{G}\}$ and $\mathcal{S}_{N_\mathrm{G}} = \{i \in \mathcal{S} : |e_i| \geqslant N_\mathrm{G} - 1\}$. Therefore, the proportion of agents in Group θ is $p_\theta = \exp (-\gamma (\theta - 1)) - \exp (-\gamma\theta)$.

Agent i does not know the characteristics of W_i and aim at using Algorithm 1 to self-rate its sensor. For that purpose, when meeting other agents, it will share measurements performed at close locations and run the LCT introduced in Section V-A.

A. LCT

Consider some tolerance ω and the interval $[m] = [m - \omega, m + \omega]$ of width 2ω centered around some measurement m. Assume that Agents i and j meet and exchange the measurements m_i and m_j. The *set* estimate [12] of ϕ obtained combining m_i and m_j is defined as $[\widehat{\phi} (m_i, m_j)] = [m_i] \cap [m_j]$, which may be used to define the low-complexity LCT $y_i = y_j = 1$, if $[\widehat{\phi} (m_i, m_j)] \neq \emptyset$ and $y_i = y_j = 0$ else. If $[\widehat{\phi} (m_i, m_j)] \neq \emptyset$ it is likely that the biases e_i and e_j are of the same order of magnitude and both agents can conclude that their sensor performs similarly. If $[\widehat{\phi} (m_i, m_j)] = \emptyset$, it is likely that e_i and e_j differ significantly more than ω. One is unable to indicate which agent has the best sensor in that case, so, both agents choose to conclude that its sensor behaves worse than that of the other agent.

For any pair of groups (θ_i, θ_j), one is able to evaluate the probability $q (\theta_i, \theta_j)$ as a function of ω, σ, and γ.

B. Numerical verification of theoretical results

This section presents first the evolution of the proportion of nodes in various states given by (10). Algorithm 1 is simulated considering a random displacement of agents without constraint on their speed. One takes $N_\mathrm{G} = 4$, $\sigma^2 = 0.25$, $\gamma = 0.7$, and $\omega = 1.8$, resulting in $p_1 = 0.503$, $p_2 = 0.25$, $p_3 = 0.124$, $p_4 = 0.123$, and

$$\mathbf{q} = \begin{pmatrix} 0.92 & 0.65 & 0.24 & 0.02 \\ 0.65 & 0.53 & 0.42 & 0.11 \\ 0.24 & 0.42 & 0.49 & 0.26 \\ 0.02 & 0.11 & 0.26 & 0.36 \end{pmatrix}.$$
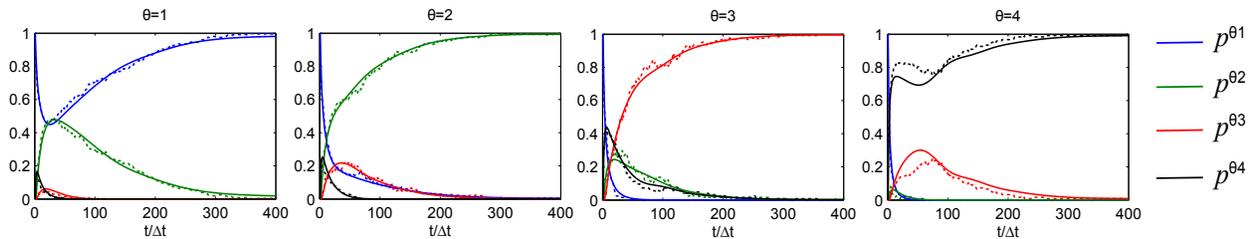
Fig. 2. Evolution of $p^{\theta\widehat{\theta}}(t)$, lines are for the theoretical values obtained solving (10), dots are simulation results using the random displacement model.
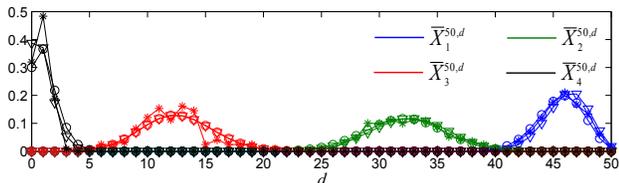

Fig. 3. Comparison of $\overline{X}_{\theta}^{50,d}$ at the equilibrium: circles are for the theoretical values obtained integrating (10), triangles are the approximations obtained from (16), and crosses are for the moving agents simulation.

Besides, one considers $M = 50$, and a sampling period Δt such that $\lambda\Delta t = 0.33$. The interaction probability α is as in Section III-D and the decision thresholds are chosen as $\nu_1 = 0.8$, $\nu_2 = 0.45$, $\nu_3 = 0.1$, and $\nu_4 = 0$. Figure 2 presents the evolution $p^{\theta\widehat{\theta}}(t)$, for $\theta, \widehat{\theta} \in \mathcal{G}$. One observes that the proportions of agents in each state converge. For any $\theta \in \mathcal{G}$, $p^{\theta\theta}$ is close to 1 for t sufficient large, while $p^{\theta\widehat{\theta}}$ tends to 0 for any $\widehat{\theta} \neq \theta$, showing the efficiency of Algorithm 1.

Consider now a set $\mathcal{S}$ of $N_S = 1000$ moving agents which initial position is uniformly distributed over a unit square. Agents randomly choose their location at time $(k+1)\Delta t$, independently from their previous location at time $k\Delta t$. Two agents communicate only at discrete time instant $k\Delta t$ when their distance is less than r_0. The value of r_0 can be adjusted so that the inter-contact probability during Δt is $\lambda\Delta t \approx 0.33$. The resulting evolutions of $p^{\theta\widehat{\theta}}(t)$ are in Figure 2 and match closely those predicted by the direct integration of the state equation.

Figure 3 further illustrates the good match between theory and simulation for the proportions of states $\overline{X}_{\theta}^{50,d}$ at equilibrium. The approximation of $\overline{X}_{\theta}^{50,d}$ using (16) is also provided in Figure 3, which is also close to its actual value.

Similar results have been obtained with agents following a Brownian motion model.

VI. CONCLUSIONS

This paper has investigated the problem of helping agents self-rating their expertise level at doing some task via exchange of information with peers. Using local comparison tests involving, *e.g.*, data exchanged during meetings with other agents, each agent is able to estimate the proportion of agents it is better at doing the considered task. With that information, each agent may then determine to which group of agents with similar expertise it belongs to.

The behavior of the proposed algorithm is described using dynamical equations. The existence of an equilibrium is established. The proportions of agents with similar beliefs in their expertise is characterized at equilibrium. This gives some insight in the tuning of the various parameters of the proposed algorithm.

The approach is illustrated with agents equipped with sensing devices of different sensing performance, which may be found in crowd sensing scenarios. Simulation results are in good match with theory.

Significant work remains to be done to analyze the behavior of the proposed algorithm with generic probabilities of interaction. The existence and uniqueness of the equilibrium has also to be shown in the general case. Nevertheless, the proposed approach may be useful to analyze other types of self-rating problems.

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