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Self-Rating in a Community of Peers

Wenjie Li, Francesca Bassi, Laura Galluccio, and Michel Kieffer

Abstract—Consider a community of agents, all performing a predefined task, but with different abilities. Each agent may be interested in knowing how well it performs in comparison with her peers. This general scenario is relevant, *e.g.*, in Wireless Sensor Networks (WSNs), or in the context of crowd sensing applications, where devices with embedded sensing capabilities collaboratively collect data to characterize the surrounding environment, but the performance is very sensitive to the accuracy of the gathered measurements.

In this paper we present a distributed algorithm allowing each agent to self-rate her level of expertise/performance at the task, as a consequence of pairwise interactions with the peers. The dynamics of the proportions of agents with similar beliefs in their expertise are described using continuous-time state equations. The existence of an equilibrium is shown. Closed form expressions for the various proportions of agents with similar belief in their expertise is provided at equilibrium. Simulation results match well theoretical results in the context of agents equipped with sensors aiming at determining the performance of their sensors.

I. INTRODUCTION

We consider a situation where all the agents in a community are engaged in the same activity, for which it is reasonable to expect heterogeneous levels of ability. The agents, who regularly meet their peers and interact with them in pairs, are incentivated to self-rate their own capabilities at the task in order to better address their future actions. This general scenario may well describe, *e.g.*, the needs of a group of amateur chess players trying to pair up with opponents of comparable strength; or the situation of a sensor network, where devices with embedded sensing capabilities collaboratively collect data to characterize the surrounding environment, and nodes with scarce sensing accuracy may prefer to withhold their contribution to avoid to pollute the data. This may happen, for instance, in the context of crowdsensing scenarios [1], where, in order to provide reliable sensing services such as those advocated by the S²aaS (Sensing as a service) paradigm [2], there is a need for high quality measurements that are not always available at devices since this is tightly related to the accuracy of the sensors embedded.

This problem is counteracted in S²aaS scenarios by introducing reputation-based mechanisms, see, *e.g.*, the Trust-

worthy Sensing for Crowd Management (TSCM) discussed in [3]–[5]. In this scheme sensing data are collected from smart-phones, based on an auction mechanism. The sensing task is assigned to the nodes by the central authority on the basis of their reputation level, to maximize the utility for the crowd.

Reputation-based mechanisms as TSCM are effective in rating the agents of the community, but are based on data centralization and need to establish a rating authority. In this work we are instead interested in distributed mechanism that nodes can employ to self-rate their capabilities. A distributed classification approach is taken in the context of WSN by the works in [6], [7], where different nodes are subject to different sensing noise models characterized by hidden parameters. Each node is willing to co-operate with the rest of the network via gossip or consensus algorithms to estimate the common observed physical quantity, and simultaneously learns its sensing model. For these algorithms to converge, however, the WSN architecture needs to guarantee large static connectivity for each node, and to disseminate node identification. In this work we propose an alternative approach to distributed self-rating where instead nodes can be mobile, connectivity can be dynamic, and identification is not used by the algorithm, allowing, *e.g.*, for node substitution or removal without disrupting the network functionality.

The proposed self-rating algorithm needs a Local Compatibility Test (LCT) adequate to establish which element of the pair of agents engaging in the test is better in performing the task. The self-rating algorithm consists then in performing a LCT when two nodes meet, and in counting the number of positive tests outcomes as well as the number of past meetings. The agents, who have knowledge of the expected proportion of nodes in each group, maintain a self-rating which evolves in time, and converges fast and reliably. The effectiveness of the proposed algorithm is measured by the correct grade rate (CGR), *i.e.*, the proportion of individuals whose self-rating result corresponds to the real level, and by the false grade rate (FGR), *i.e.*, the proportion of individuals who self-rate the wrong class. The behavior of the algorithm is analyzed by describing its state equations. The analysis of the equilibrium allows to uncover the relationship between CGR and FGR and the characteristics of the local comparison test, providing a powerful tool to optimize design.

The paper is organized as follows. Section II introduces the system model and LCT. Section III introduces and analyzes the proposed distributed self-rating algorithms. Proofs for the analysis of the equilibrium are presented in Section IV. Simulations results are reported in Section V before drawing some conclusions in Section VI.

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II. SYSTEM MODEL AND LOCAL COMPATIBILITY TEST

Consider a set \mathcal{S} of N_S moving agents. Assume that the agents in \mathcal{S} can be partitioned into N_G groups denoted $\mathcal{S}_1, \dots, \mathcal{S}_{N_G}$, containing respectively a proportion p_1, \dots, p_{N_G} of the agents. A group contains agents able to perform some task with a similar level of expertise or reliability. Groups are sorted in decreasing levels of expertise. Let $\theta_i(t) \in \mathcal{G} = \{1, \dots, N_G\}$ be the index of the group $\mathcal{S}_{\theta_i(t)}$ to which Agent i belongs at time t . Initially, Agent i is not aware of the actual value of θ_i but is willing to estimate it as fast as possible. We assume that the group to which Agent i belongs does not change over the time horizon of the experiment, so $\theta_i(t) = \theta_i$.

In what follows, one will present a distributed algorithm allowing each Agent i to get an estimate $\hat{\theta}_i$ of its group θ_i . For that purpose a *local comparison test* (LCT) is introduced, performed by Agent i after meeting and interacting with any Agent j . Meetings are assumed to involve only two agents. The output $y_{i,j}$ of the LCT performed by Agent i upon interaction with Agent j is binary. The LCT determines whether Agent i is better at doing the considered task than Agent j ($y_{i,j} = 1$), or vice-versa ($y_{i,j} = 0$), without being able to determine the Agent i 's absolute level of expertise. LCTs may provide erroneous conclusions and are characterized by their statistical properties

$$q(\theta_i, \theta_j) = \mathbb{P}\{Y_{i,j} = 1 | \theta_i, \theta_j\}. \quad (1)$$

One has not necessarily $q(\theta_i, \theta_j) = 1 - q(\theta_j, \theta_i)$. One assumes that $\theta_i \leq \theta_j$ implies $q(\theta_i, 1) \geq q(\theta_j, 1)$ and $q(1, \theta_i) \geq q(1, \theta_j)$, which appears reasonable, since the first group contains the agents with the highest level of expertise.

Interactions may take various forms depending on the application scenario, ranging from the exchange of noisy measurements m_i and m_j of the same physical quantity when the agents are nodes of a WSN, to a blitz-game between humans, willing, *e.g.*, to self-rate their level in chess. An example LCT is provided in Section V in the context of WSNs. A meeting between two nodes does not necessarily entail interaction: one assumes here that the probability of interaction $\alpha(\hat{\theta}_i, \hat{\theta}_j)$ is a function of the estimates $\hat{\theta}_i(t)$ and $\hat{\theta}_j(t)$ of the groups to which each agent belongs. When the agents aim is to self-rate their level in chess, for instance, $\alpha(\hat{\theta}_i, \hat{\theta}_j)$ will approach one when $\hat{\theta}_i$ and $\hat{\theta}_j$ are close and thus the outcome of the LCT is reputed unpredictable, and be small when $\hat{\theta}_i$ and $\hat{\theta}_j$ are very different, thus preventing the agents wasting time in a LCT whose outcome is reputed easily foreseeable. In practice, one assumes that when two agents i and j meet, they start exchanging the estimated index of their group, Agent i will request for a further interaction with probability $\alpha(\hat{\theta}_i, \hat{\theta}_j)$ and Agent j with a probability $\alpha(\hat{\theta}_j, \hat{\theta}_i)$.

III. DISTRIBUTED SELF-RATING ALGORITHM

In the proposed Distributed Self-Rating (DSR) algorithm, each agent manages two counters $c_{t,i}(t)$ and $c_{b,i}(t)$ initialized

at 0 at $t = 0$. The number of LCT *tests* performed by Agent i and following an interaction it has requested is stored in $c_{t,i}(t)$. The number of tests concluding that Agent i is *better* than the agent met is stored in $c_{b,i}(t)$. An agent involved in an interaction it has not requested does not update $c_{t,i}(t)$ and $c_{b,i}(t)$. If the agents are randomly spread, the ratio $c_{b,i}(t)/c_{t,i}(t)$ will mainly depend on the proportions of agents in each group, on the interaction probabilities, and on the statistical properties of the LCT.

Intuitively, an Agent belonging to Group 1 is likely to have a larger ratio $c_{b,i}(t)/c_{t,i}(t)$ than an Agent belonging to Group N_G . One may thus introduce a partition of the interval $[0, 1]$ into N_G decision intervals $[\nu_k, \nu_{k-1})$ with $\nu_0 = 1$ and $\nu_{N_G} = 0$ and consider the decision rule

$$\hat{\theta}_i(t) = k \text{ if } c_{b,i}(t)/c_{t,i}(t) \in [\nu_k, \nu_{k-1}), \quad k = 1, \dots, N_G. \quad (2)$$

Agents with the largest value of $c_{b,i}(t)/c_{t,i}(t)$ choose the smallest group index. The aim of this work is to show that the decision rule (2) leads to a satisfying self-rating of the agents, for appropriate choice of the value of ν_k , for all $k = 1, \dots, N_G$. The values of $\alpha(\hat{\theta}_i, \hat{\theta}_j)$ are design parameters that can be adjusted to optimize the performance of the self-rating algorithm.

A. Practical Self-Rating Algorithm

Let $\mathbf{x}_i(t) = (\theta_i, c_{t,i}(t), c_{b,i}(t))$ represent the *state* of each Agent i . If all the LCT results obtained in the past are considered, one has $\lim_{t \rightarrow \infty} c_{t,i}(t) \rightarrow \infty$, which results an infinite number of possible values for $\mathbf{x}_i(t)$. The global (macroscopic) behavior of the algorithm is in this case difficult to analyze. To limit the number of possible states, one considers the evolution of $c_{t,i}(t)$ and $c_{b,i}(t)$ over a sliding variable-length time interval containing the time instants of the last M meetings during which Agent i has performed a LCT. Algorithm 1 summarizes the proposed DSR algorithm for an arbitrary reference Agent i .

B. Evolution of the state of an agent

At time t , among the agents in Group θ , let $X_\theta^{\ell,d}(t)$ be the proportion of agents in state $\mathbf{x}_i(t) = (\theta, \ell, d)$, *i.e.*, with $c_{t,i}(t) = \ell$ and $c_{b,i}(t) = d$. Since $\ell \in \{0, 1, \dots, M\}$ and $d \in \{0, \dots, \ell\}$, the number of values that may be taken by the state of an agent is $(M+1)(M+2)/2$. The evolution of the state of Agent i , conditioned by the index θ of its actual group, follows a Markov model with state transition diagram similar to that shown in Figure 1 for $M = 4$.

There are N_G parallel chains conditioned by the value of $\theta \in \mathcal{G}$. Define $\pi_\theta^{\delta_t, \delta_b}$ as the transition probability from State (θ, ℓ, d) to State $(\theta, \ell + \delta_t, d + \delta_b)$. Note that $\pi_\theta^{\delta_t, \delta_b}$ depends on the current state (θ, ℓ, d) of the reference Agent i , but also on the current proportion of agents with estimated self-rating $\hat{\theta}_i(t)$.

One has first to evaluate the probability that some Agent i with state (θ, ℓ, d) and estimated rating $\hat{\theta}_i(\ell, d)$ performs a LCT upon meeting a random Agent J . This probability may

Algorithm 1 DSR algorithm for Agent i

- 1) Initialize $t_i^0 = 0$, $\hat{\theta}_i(t_i^0) = 1$, $c_{t,i}(t_i^0) = c_{b,i}(t_i^0) = 0$, $\kappa = 1$, and $\mu = 0$.
- 2) Do

$$\begin{cases} \hat{\theta}_i(t) = \hat{\theta}_i(t_i^{\kappa-1}), \\ c_{t,i}(t) = c_{t,i}(t_i^{\kappa-1}), \\ c_{b,i}(t) = c_{b,i}(t_i^{\kappa-1}), \end{cases} \quad (3)$$

$$t = t + \delta t \quad (4)$$

until the κ -th meeting occurs at time t_i^κ with Agent $j^\kappa \in \mathcal{S}$.

- 3) Transmit $\hat{\theta}_i(t_i^\kappa)$ to Agent j^κ and receive $\hat{\theta}_{j^\kappa}(t_i^\kappa)$ from Agent j^κ .
- 4) With probability $\alpha(\hat{\theta}_i(t_i^\kappa), \hat{\theta}_{j^\kappa}(t_i^\kappa))$, perform a LCT with outcome y_i^μ , then
 - a) $\mu = \mu + 1$. Update $c_{t,i}$ and $c_{b,i}$ as

$$\begin{cases} c_{t,i}(t_i^\kappa) = \min\{\mu, M\} \\ c_{b,i}(t_i^\kappa) = \sum_{m=\max\{1, \mu-M+1\}}^\mu y_i^m \end{cases} \quad (5)$$

- b) Update $\hat{\theta}_i$ according to 2

- 5) $\kappa = \kappa + 1$.
 - 6) Go to 2.
-

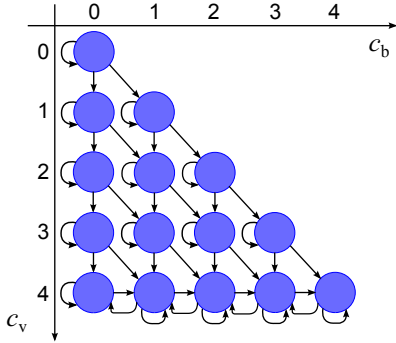


Fig. 1. Example of Markov model for the evolution of the state of an agent when $M = 4$.

be evaluated as

$$\beta(\ell, d) = E\left(\alpha(\hat{\theta}(\ell, d), \hat{\theta}_J(t))\right) \quad (6)$$

where the expectation has to be taken over $\hat{\theta}_J(t)$. Then

$$\begin{aligned} \beta(\ell, d) &= \sum_{k_1, k_2 \in \mathcal{G}} \alpha(\hat{\theta}(\ell, d), k_2) \mathbb{P}\{\hat{\theta}_J(t) = k_2, \theta_J = k_1\} \\ &= \sum_{k_1, k_2 \in \mathcal{G}} \alpha(\hat{\theta}(\ell, d), k_2) \mathbb{P}\{\hat{\theta}_J(t) = k_2 | \theta_J = k_1\} \mathbb{P}\{\theta_J = k_1\} \\ &= \sum_{k_1, k_2 \in \mathcal{G}} \alpha(\hat{\theta}(\ell, d), k_2) p_{k_1} p^{k_1 k_2}(t). \end{aligned} \quad (7)$$

In (7), one uses the fact that Agents are randomly spread to get $\mathbb{P}\{\theta_J = k_1\} = p_{k_1}$. We introduce $p^{k_1 k_2}(t) = \mathbb{P}\{\hat{\theta}_J(t) = k_2 | \theta_J = k_1\}$ as the proportion of agents in Group k_1 believing their group index is k_2 . Finally, one has

$\hat{\theta}(\ell, d) = k$ such that $d/\ell \in [\nu_k, \nu_{k-1})$. More specifically, it is

$$\begin{aligned} p^{k_1 k_2}(t) &= \mathbb{P}\{\hat{\theta}_J(t) = k_2 | \theta_J = k_1\} \\ &= \begin{cases} X_{k_1}^{0,0}(t) + \sum_{\ell, d, d/\ell \in [\nu_{k_2}, \nu_{k_2-1})} X_{k_1}^{\ell, d}(t), & \text{if } k_2 = 1, \\ \sum_{\ell, d, d/\ell \in [\nu_{k_2}, \nu_{k_2-1})} X_{k_1}^{\ell, d}(t), & \text{else.} \end{cases} \end{aligned} \quad (8)$$

Two phases have to be considered in Algorithm 1, depending on the value of $c_{t,i}(t)$. In the *transient phase*, for states with $c_{t,i}(t) = \ell < M$, one has $(\delta_t, \delta_b) \in \{(0, 0), (1, 0), (1, 1)\}$, since ℓ may either increase or remain constant and $\delta_b \leq \delta_t$. The only possibility leading to $\delta_t = 0$ is that Agent i , once it has met a random Agent J , decides not to continue interaction. Then

$$\pi_\theta^{0,0}(t, \ell, d) = 1 - \beta(\ell, d). \quad (9)$$

A state transition occurs with $(\delta_t, \delta_b) = (1, 1)$ when, once Agent i has met Agent J , they continue interaction and the LCT yields $y_i(t) = 1$. Since α only depends on the group estimates, these two events can be assumed as independent and one has to consider all possible values taken by $\hat{\theta}_J(t)$ to get

$$\begin{aligned} \pi_\theta^{1,1}(t, \ell, d) &= \sum_{k_2 \in \mathcal{G}} \alpha(\hat{\theta}(\ell, d), k_2) \mathbb{P}\{Y_i = 1, \hat{\theta}_J(t) = k_2 | \theta_i = \theta\} \\ &= \sum_{k_1, k_2 \in \mathcal{G}} \alpha(\hat{\theta}(\ell, d), k_2) \mathbb{P}\{Y_i = 1, \theta_J = k_1, \hat{\theta}_J(t) = k_2 | \theta_i = \theta\} \\ &= \sum_{k_1, k_2 \in \mathcal{G}} \alpha(\hat{\theta}(\ell, d), k_2) \mathbb{P}\{Y_i = 1 | \theta_i = \theta, \theta_J = k_1\} \\ &\quad \cdot \mathbb{P}\{\hat{\theta}_J(t) = k_2 | \theta_J = k_1\} \mathbb{P}\{\theta_J = k_1\} \\ &= \sum_{k_1, k_2 \in \mathcal{G}} \alpha(\hat{\theta}(\ell, d), k_2) p_{k_1} p^{k_1 k_2}(t) q(\theta, k_1). \end{aligned} \quad (10)$$

Finally, $\pi_\theta^{1,0}(t, \ell, d)$ is obtained similarly as (10),

$$\pi_\theta^{1,0}(t, \ell, d) = \sum_{k_1, k_2 \in \mathcal{G}} \alpha(\hat{\theta}(\ell, d), k_2) p_{k_1} p^{k_1 k_2}(t) (1 - q(\theta, k_1)). \quad (11)$$

In the *permanent regime*, $c_{t,i}(t) = M$ and remains constant, thus $\delta_t = 0$. In Algorithm 1, μ is the number of LCTs performed by Agent i till time t . When $\mu \geq M$, only the last M LCT outcomes are considered. To determine the value taken by $\delta_b \in \{-1, 0, 1\}$ after the μ -th LODT, consider an arbitrary $y \in \{0, 1\}$ and the random event

$$\mathcal{E}_y(t) = \left\{ Y_i^{\mu-M} = y \mid \sum_{m=\mu-M}^{\mu-1} Y_i^m = d \right\}. \quad (12)$$

which corresponds to a situation where one knows that k LCTs yield 1 among the last M tests and $Y_i^{\mu-M} = y$ will be ignored once the new LCT outcome is available. $\mathbb{P}\{\mathcal{E}_1(t)\}$ is relatively complex to evaluate, since $\mathbb{P}\{Y_i^m = y\}$ depends on the actual group of the encountered agent and is time-varying. In what follows, we assume that LCT outcomes with $Y_i^m = y$, are independently distributed over the time

horizon corresponding to $m = \mu - M, \dots, \mu - 1$. One obtains then $\mathbb{P}\{\mathcal{E}_1(t)\} = d/M$ and $\mathbb{P}\{\mathcal{E}_0(t)\} = 1 - d/M$.

Assume that the $(\mu - M)$ -th LCT performed by Agent i occurred at time \hat{t} , then $y_i^{\mu - M}$ can also be denoted as $y_i(\hat{t})$ and the transition related to $c_{b,i}$ is such that

$$\delta_b = y_i(t) - y_i(\hat{t}) \in \{-1, 0, 1\}. \quad (13)$$

To have $(\delta_t, \delta_b) = (0, 1)$, three independent events have to occur: 1) interaction has to continue once Agent J has been met; 2) $y_i(t) = 1$; 3) $y_i(\hat{t}) = 0$, *i.e.*, $\mathcal{E}_0(t)$. The transition probability is obtained using derivations similar to (10)

$$\begin{aligned} & \pi_{\theta}^{0,1}(t, M, d) \\ &= \sum_{k_1, k_2 \in \mathcal{G}} \alpha(\hat{\theta}(M, d), k_2) p_{k_1} p^{k_1 k_2}(t) q(\theta, k_1) \frac{M - d}{M}. \end{aligned} \quad (14)$$

Consider now $(\delta_t, \delta_b) = (0, -1)$, one has

$$\begin{aligned} & \pi_{\theta}^{0,-1}(t, M, d) \\ &= \sum_{k_1, k_2 \in \mathcal{G}} \alpha(\hat{\theta}(M, d), k_2) p_{k_1} p^{k_1 k_2}(t) (1 - q(\theta, k_1)) \frac{d}{M}, \end{aligned} \quad (15)$$

in the same way. Applying (14-15), $\pi_{\theta}^{0,0}(t, M, d)$ can be obtained from $\pi_{\theta}^{0,0}(t, M, d) = 1 - \pi_{\theta}^{0,1}(t, M, d) - \pi_{\theta}^{0,-1}(t, M, d)$.

C. Macroscopic evolution

All agent state transition probabilities evaluated in Section III-B are now used to determine the evolution of the various proportions $X_{\theta}^{\ell,d}(t)$ of agents in the corresponding states, with $\theta = 1, \dots, N_G$, $\ell = 0, \dots, M$ and $d \leq \ell$. In what follows, one assumes that agents are mobile and form a well-mixed population. Considering an inter-contact rate λ , during a short time interval $[t, t + \delta t]$, the number of agents with state (θ, ℓ, d) that will meet another agent can be estimated as $\lambda p_{\theta} N_S X_{\theta}^{\ell,d}(t) \delta t$. When $0 < \ell < M$ and $0 < d < \ell$, these agents will switch to the states $(\theta, \ell + \delta_t, d + \delta_b)$, with $(\delta_t, \delta_b) \in \{(0, 0), (1, 0), (1, 1)\}$ with a probability $\pi_{\theta}^{\delta_t, \delta_b}(t, \ell, d)$. Moreover, agents in the states $(\theta, \ell - 1, d - 1)$ and $(\theta, \ell - 1, d)$ that have met another agent in the time interval $[t, t + \delta t]$ may reach state (θ, ℓ, d) , respectively with a probability $\pi_{\theta}^{1,1}(t, \ell - 1, d - 1)$ and $\pi_{\theta}^{1,0}(t, \ell - 1, d)$. The evolution of $X_{\theta}^{\ell,d}(t)$ is then described by the following differential equation, where the time dependency is omitted to lighten notations

$$\begin{aligned} \frac{dX_{\theta}^{\ell,d}}{dt} &= -\lambda X_{\theta}^{\ell,d} (\pi_{\theta}^{1,0}(\ell, d) + \pi_{\theta}^{1,1}(\ell, d)) \\ &+ \lambda X_{\theta}^{\ell-1, d-1} \pi_{\theta}^{1,1}(\ell-1, d-1) + \lambda X_{\theta}^{\ell-1, d} \pi_{\theta}^{1,0}(\ell-1, d). \end{aligned} \quad (16)$$

When $\ell = M$ and $0 < d < M$, agents in state (θ, M, d) will switch to the states $(\theta, M, d + \delta_b)$, $\delta_b \in \{-1, 0, 1\}$ with a probability $\pi_{\theta}^{0, \delta_b}(t, M, d)$. Agents in the states $(\theta, M - 1, d - 1)$ and $(\theta, M - 1, d)$ that have met another agent in the time interval $[t, t + \delta t]$ may reach state (θ, M, d) , with probability $\pi_{\theta}^{1,1}(t, M - 1, d - 1)$ and $\pi_{\theta}^{1,0}(t, M - 1, d)$, respectively. As a consequence, the evolution of $X_{\theta}^{M,d}(t)$ can

be described by

$$\begin{aligned} \frac{dX_{\theta}^{M,d}}{dt} &= -\lambda X_{\theta}^{M,d} (\pi_{\theta}^{0,1}(M, d) + \pi_{\theta}^{0,-1}(M, d)) \\ &+ \lambda X_{\theta}^{M-1, d-1} \pi_{\theta}^{1,1}(M-1, d-1) + \lambda X_{\theta}^{M-1, d} \pi_{\theta}^{1,0}(M-1, d) \\ &+ \lambda X_{\theta}^{M, d-1} \pi_{\theta}^{0,1}(M, d-1) + \lambda X_{\theta}^{M, d+1} \pi_{\theta}^{0,-1}(M, d+1). \end{aligned} \quad (17)$$

Similar derivations can be made for the remaining state components to obtain

$$\left\{ \begin{array}{l} \frac{dX_{\theta}^{0,0}}{dt} \stackrel{(a)}{=} -\lambda X_{\theta}^{0,0} (\pi_{\theta}^{1,0}(0, 0) + \pi_{\theta}^{1,1}(0, 0)), \\ \frac{dX_{\theta}^{\ell,0}}{dt} \stackrel{(b)}{=} \lambda \left(-X_{\theta}^{\ell,0} (\pi_{\theta}^{1,0}(\ell, 0) + \pi_{\theta}^{1,1}(\ell, 0)) \right. \\ \quad \left. + X_{\theta}^{\ell-1,0} \pi_{\theta}^{1,0}(\ell-1, 0) \right), \\ \frac{dX_{\theta}^{\ell,\ell}}{dt} \stackrel{(c)}{=} \lambda \left(-X_{\theta}^{\ell,\ell} (\pi_{\theta}^{1,0}(\ell, \ell) + \pi_{\theta}^{1,1}(\ell, \ell)) \right. \\ \quad \left. + X_{\theta}^{\ell-1, \ell-1} \pi_{\theta}^{1,1}(\ell-1, \ell-1) \right), \\ \frac{dX_{\theta}^{M,0}}{dt} \stackrel{(d)}{=} \lambda \left(-X_{\theta}^{M,0} \pi_{\theta}^{0,1}(M, 0) + X_{\theta}^{M-1,0} \pi_{\theta}^{1,0}(M-1, 0) \right. \\ \quad \left. + X_{\theta}^{M,1} \pi_{\theta}^{0,-1}(M, 1) \right), \\ \frac{dX_{\theta}^{M,M}}{dt} \stackrel{(e)}{=} \lambda \left(-X_{\theta}^{M,M} \pi_{\theta}^{0,-1}(M, M) + X_{\theta}^{M, M-1} \pi_{\theta}^{0,1}(M, M-1) \right. \\ \quad \left. + X_{\theta}^{M-1, M-1} \pi_{\theta}^{1,1}(M-1, M-1) \right), \end{array} \right. \quad (18)$$

for any $\ell = 1 \dots M-1$, with the initial conditions $X_{\theta}^{0,0}(0) = 1$ and $X_{\theta}^{\ell,d}(0) = 0$, $\forall \ell, d \neq 0$. Note that the state equations (16-18) are nonlinear, since each $\pi_{\theta}^{\delta_t, \delta_b}$ depends on $X_{\theta}^{\ell,d}$.

D. Equilibrium point of $X_{\theta}^{\ell,d}$

In this section, the asymptotic behavior of the state equations (16-18) is characterized. This analysis will be performed considering the following special case for the α s. One considers

$$\alpha(k_1, k_2) = \begin{cases} 1 & \text{if } k_2 = 1 \\ 0 & \text{else.} \end{cases} \quad (19)$$

This hypothesis corresponds to a situation where interaction is only performed when an Agent i meets an Agent j believing it is in the best group. As will be seen in Section V, this strategy is efficient to self-rate agents equipped with sensors of different quality.

To perform the analysis of the equilibrium, one investigates first the evolution of $X_{\theta}^{\ell,k}(t)$ when $\ell < M$ in the following proposition.

Proposition 1: For any $\ell < M$ and $d \leq \ell$, $\lim_{t \rightarrow \infty} X_{\theta}^{\ell,d}(t) = 0$.

The proof of Proposition 1 is in Section IV-A. As a consequence, the only possible value at equilibrium of $X_{\theta}^{\ell,d}(t)$ with $\ell < M$ is 0. Let $\bar{X}_{\theta}^{M,d}$ be the value at equilibrium of $X_{\theta}^{M,d}$. The proportion of agents estimating their group is $\hat{\theta}$ depends on the partition of the interval $[0, 1]$ introduced in (2)

$$\bar{p}^{\theta \hat{\theta}} = \sum_{d: d/M \in [\nu_{\hat{\theta}}, \nu_{\hat{\theta}+1})} \bar{X}_{\theta}^{M,d}. \quad (20)$$

In what follows, the case $\hat{\theta} = 1$ is analyzed in details. The analysis of the other cases is similar.

Denote $\bar{\mathbf{p}}^1 = [\bar{p}^{11}, \dots, \bar{p}^{N_G 1}]^T$ and consider the functions

$$h_\theta(\bar{\mathbf{p}}^1) = \frac{\sum_{k \in \mathcal{G}} p_k \bar{p}^{k1} q(\theta, k)}{\sum_{k \in \mathcal{G}} p_k \bar{p}^{k1}}, \quad (21)$$

$$F_\theta(\bar{\mathbf{p}}^1) = \sum_{d=\lceil \nu_1 M \rceil}^M \binom{M}{d} (h_\theta(\bar{\mathbf{p}}^1))^d (1 - h_\theta(\bar{\mathbf{p}}^1))^{M-d}, \quad (22)$$

and $\mathbf{F}(\bar{\mathbf{p}}^1) = [F_1(\bar{\mathbf{p}}^1), \dots, F_{N_G}(\bar{\mathbf{p}}^1)]^T$. The following proposition provides a non-linear equation that has to be satisfied by $\bar{\mathbf{p}}^1$. Once this equation is solved, one easily deduces the various $\bar{X}_\theta^{M,d}$ at equilibrium.

Proposition 2: Assume that the dynamic system described by (16-18) admits some equilibrium $\bar{X}_\theta^{\ell,d}$, then for any $\theta \in \mathcal{G}$ and $d \leq \ell$,

$$\bar{X}_\theta^{\ell,d} = \begin{cases} 0, & \forall \ell < M, \\ \binom{M}{d} (h_\theta(\bar{\mathbf{p}}^1))^d (1 - h_\theta(\bar{\mathbf{p}}^1))^{M-d}, & \ell = M, \end{cases} \quad (23)$$

where $\bar{\mathbf{p}}^1$ are obtained by solving

$$\bar{\mathbf{p}}^1 = \mathbf{F}(\bar{\mathbf{p}}^1). \quad (24)$$

See Section IV-B for the proof.

The existence of $\bar{X}_\theta^{M,d}$ mainly depends on whether (24) has a solution $\bar{\mathbf{p}}^1$. For that purpose, in the special case where $N_G = 2$, one shows the existence of an equilibrium in Proposition 3 using Brouwer's fixed-point theorem.

Proposition 3: In the case $N_G = 2$, for any $\nu_1 \in [0, 1]$, (24) always admits a solution, and the dynamical system (16-18) has an equilibrium.

See Section IV-C for the proof.

E. Approximations of the equilibrium

Explicit expressions for $\bar{p}^{\theta 1}$ are difficult to obtain from (24). Since $\bar{p}^{\theta 1}$ with $\theta \neq 1$ represent the proportions of agents that have wrongly estimated their group, the vector $\bar{\mathbf{p}}^1 = [\bar{p}^{11}, \dots, \bar{p}^{N_G 1}]^T$ should be close to $\tilde{\mathbf{p}}^1 = [1, 0, \dots, 0]^T$. One has $\lim_{\mathbf{p}^1 \rightarrow \tilde{\mathbf{p}}^1} h_\theta(\mathbf{p}^1) = q(\theta, 1)$. Assuming that at equilibrium, $h_\theta(\bar{\mathbf{p}}^1) \simeq q(\theta, 1)$, using (23), $\bar{X}_\theta^{M,d}$ can be approximated as

$$\tilde{X}_\theta^{M,d} = \binom{M}{d} (q(\theta, 1))^d (1 - q(\theta, 1))^{M-d}, \quad (25)$$

and follows thus a binomial distribution. Knowing $\tilde{X}_\theta^{M,d}$ for all $\theta \in \mathcal{G}$ and all $d \leq M$ gives some insights to properly tune the decision thresholds introduced in (2). The value of the ν_θ s may for example be adjusted to maximize the CGR under some cumulated FGR constraint evaluated using (25), but alternative performance requirements may be considered.

IV. PROOFS

A. Proof of Proposition 1

For the proof, one considers first the following lemmas.

Lemma 4: If

$$\lim_{t \rightarrow \infty} \int_0^t \left(\sum_{k \in \mathcal{G}} p_k p^{k1}(\tau) \right) d\tau = \infty \quad (26)$$

then $\sum_{k \in \mathcal{G}} p_k p^{k1}(\tau) > 0$ for all $t \in \mathbb{R}^+$.

Proof: Since $p_k > 0$ at least for some $k \in \mathcal{G}$, it suffices to prove that

$$\sum_{k \in \mathcal{G}} p^{k1}(\tau) \neq 0 \quad \forall t > 0. \quad (27)$$

Assume that there exists a time instant $t^* > 0$, such that $\sum_{k \in \mathcal{G}} p^{k1}(t^*) = 0$. As a consequence, at time t^* , all agents have an estimate $\hat{\theta} \neq 1$. Considering the choice of α s done in (19), no LCT will be performed after time t^* and the state of agents will remain constant. Hence, if $\sum_{k \in \mathcal{G}} p^{k1}(t^*) = 0$ for some t^* , then $\sum_{k \in \mathcal{G}} p^{k1}(t) = 0$ for all $t > t^*$. Consequently, $\lim_{t \rightarrow \infty} \int_0^t \left(\sum_{k \in \mathcal{G}} p_k p^{k1}(\tau) \right) d\tau = \int_0^{t^*} \left(\sum_{k \in \mathcal{G}} p_k p^{k1}(\tau) \right) d\tau$, which contradicts (26). ■

Lemma 5: Property (26) is always satisfied.

Proof: From (18-a), one has

$$X_\theta^{0,0}(t) = \exp \left(-\lambda \int_0^t \left(\sum_{k \in \mathcal{G}} p_k p^{k1}(\tau) \right) d\tau \right). \quad (28)$$

Assume that there exists C^* such that $\lim_{t \rightarrow \infty} \int_0^t \left(\sum_{k \in \mathcal{G}} p_k p^{k1}(\tau) \right) d\tau \leq C^*$. Then

$$X_\theta^{0,0}(t) \geq \exp(-\lambda C^*) > 0. \quad (29)$$

However, from (8), one has $p^{\theta 1}(\tau) \geq X_\theta^{0,0}(\tau)$, leading to

$$\int_0^t \left(\sum_{k \in \mathcal{G}} p_k p^{k1}(\tau) \right) d\tau \geq \int_0^t \left(\sum_{k \in \mathcal{G}} p_k X_\theta^{0,0}(\tau) \right) d\tau > \exp(-\lambda C^*) t$$

and $\exp(-\lambda C^*) t \rightarrow \infty$ as $t \rightarrow \infty$, which violates (29). Hence, one always has (26). ■

The proof of Proposition 1 is then by induction. Starting with (18.a), one has (28). Since (26) is satisfied according to Lemma 5, for any $\xi > 0$, there exists $t_{00} > 0$ such that $t > t_{00}$ implies $X_\theta^{0,0}(t) < \xi$ and $\lim_{t \rightarrow \infty} X_\theta^{0,0}(t) = 0$.

Then, assume that for any $\ell \leq M - 1$, and $\xi > 0$, there exists $t_{(\ell-1)0} > \dots > t_{00}$ such that $t > t_{(\ell-1)0}$ implies $X_\theta^{j,0}(t) < \xi$ for $j = 0, \dots, \ell - 1$. One has to show now that there exists $t_{\ell 0} > t_{(\ell-1)0}$ such that $X_\theta^{\ell,0}(t) < \xi$ for all $t > t_{\ell 0}$.

Define $Z_\theta^{\ell,0}(t) = \sum_{j=0}^{\ell} X_\theta^{j,0}(t)$. From (18a) and (18b), one has

$$\frac{dZ_\theta^{\ell,0}}{dt} = -\lambda \left(v(t) Z_\theta^{\ell-1,0}(t) + \sum_{k \in \mathcal{G}} p_k p^{k1}(\tau) X_\theta^{\ell,0}(t) \right),$$

where $v(t) = \pi_\theta^{1,1}(t, \ell, d)$, since $\pi_\theta^{1,0}$ and $\pi_\theta^{1,1}$ do not depend on ℓ and k when $\ell < M$. Using (27) one has $dZ_\theta^{\ell,0}/dt < 0$ for any $X_\theta^{\ell,0} > 0$. As a consequence, $Z_\theta^{\ell,0}(t)$ decreases until $X_\theta^{\ell,0}(t)$ reaches 0. Hence, for any $\xi > 0$, there exists $t_{\ell 0} > t_{(\ell-1)0}$, such that $X_\theta^{\ell,0} < \xi$ and $\lim_{t \rightarrow \infty} X_\theta^{\ell,0}(t) = 0$.

In the same way, using (18c) and the previous results that $X_\theta^{\ell,d}(t) \rightarrow 0$ with $d = 1, \dots, M - 2$ and $\ell = d, \dots, M - 2$, one can prove that for any $d = 1, \dots, M - 1$, $X_\theta^{\ell',(k+1)}(t)$ tends to zero as $t \rightarrow \infty$, with any $\ell' = d + 1, \dots, M - 1$.

B. Proof of Proposition 2

According to Proposition 1, one has $\bar{X}_\theta^{\ell,d} = 0$, for all $\ell < M$ and $d \leq \ell$. To evaluate $\bar{X}_\theta^{M,d}$, one considers the following simplified dynamics derived from (17-18),

$$\begin{cases} \frac{dX_\theta^{M,0}}{dt} = \lambda \left(-X_\theta^{M,0} \pi_\theta^{0,1}(M,0) + X_\theta^{M,1} \pi_\theta^{0,-1}(M,1) \right), \\ \frac{dX_\theta^{M,M}}{dt} = \lambda \left(-X_\theta^{M,M} \pi_\theta^{0,-1}(M,M) + X_\theta^{M,M-1} \pi_\theta^{0,1}(M,M-1) \right), \\ \frac{dX_\theta^{M,d}}{dt} = \lambda \left(-X_\theta^{M,d} (\pi_\theta^{0,-1}(M,d) + \pi_\theta^{0,1}(M,d)) \right. \\ \quad \left. + X_\theta^{M,d+1} \pi_\theta^{0,-1}(M,d+1) + X_\theta^{M,d-1} \pi_\theta^{0,1}(M,d-1) \right). \end{cases} \quad (30)$$

At equilibrium, one has $dX_\theta^{M,d}(t)/dt = 0$ for all $k \leq M$. Moreover, the transition probabilities will not vary any more. Let $\bar{X}_\theta^M = [\bar{X}_\theta^{M,1}, \dots, \bar{X}_\theta^{M,M}]^T$, $a_\theta(d) = \pi_\theta^{0,1}(M,d)$, and $b_\theta(d) = \pi_\theta^{0,-1}(M,d)$. The vector \bar{X}_θ^M should satisfy $\Psi \cdot \bar{X}_\theta^M = \mathbf{0}$ where

$$\Psi = \begin{pmatrix} -a_\theta(0) & b_\theta(1) & & & \\ a_\theta(0) & -a_\theta(1) - b_\theta(1) & b_\theta(2) & & \\ & \ddots & \ddots & \ddots & \\ & & & a_\theta(M-1) & -b_\theta(M) \end{pmatrix}.$$

Summing Lines 1 to $d+1$, for $d = 0, \dots, M-1$, one obtains $b_\theta(d+1) \bar{X}_\theta^{M,d+1} = a_\theta(d) \bar{X}_\theta^{M,d}$, which leads to

$$\begin{aligned} \bar{X}_\theta^{M,d} &= \bar{X}_\theta^{M,0} \prod_{j=0}^{d-1} \frac{a_\theta(j)}{b_\theta(j+1)} = \bar{X}_\theta^{M,0} \prod_{j=0}^{d-1} \frac{\pi_\theta^{0,1}(M,j)}{\pi_\theta^{0,-1}(M,j+1)} \\ &= \bar{X}_\theta^{M,0} \prod_{j=0}^{d-1} \frac{(M-j) \sum_{k \in \mathcal{G}} p_k \bar{p}^{k-1} q(\theta, k)}{(j+1) \sum_{k \in \mathcal{G}} p_k \bar{p}^{k-1} (1-q(\theta, k))} \\ &= \bar{X}_\theta^{M,0} \binom{M}{d} \left(\frac{h_\theta}{1-h_\theta} \right)^d \end{aligned} \quad (31)$$

where h_θ is defined in (21). Since $\sum_{d=0}^M \bar{X}_\theta^{M,d} = 1$, one obtains for all $\theta \in \mathcal{G}$ and $d \in \{0, \dots, M\}$

$$\bar{X}_\theta^{M,d} = \binom{M}{d} h_\theta^d (1-h_\theta)^{M-d}. \quad (32)$$

Introducing (32) in (20), one obtains (24) with F_θ defined in (22). Thus one needs to solve (24) to determine $\bar{p}^{\theta 1}$ for all $\theta \in \mathcal{G}$, which are the used to deduce $\bar{X}_\theta^{M,d}$ using (32).

C. Proof of Proposition 3

To show the existence of a solution of (24) when $N_G = 2$, one uses Brouwer's fixed-point [8]. For that purpose, one has to show that for any $(p_0^{11}, p_0^{21}) \in \mathcal{P}_0 = \{(x, y) \in [0, 1] \times [0, 1] \text{ and } (x, y) \neq (0, 0)\}$, the discrete-time system

$$\begin{cases} p_{n+1}^{11} = F_1(p_n^{11}, p_n^{21}), \\ p_{n+1}^{21} = F_2(p_n^{11}, p_n^{21}). \end{cases} \quad (33)$$

converges to an equilibrium point $(\bar{p}^{11}, \bar{p}^{21})$. Recall that one never has $p_0^{11} = p_0^{21} = 0$ from Lemma 4.

Both F_1 and F_2 are continuous functions, however \mathcal{P}_0 is not a compact set. Thus one needs to find a compact \mathcal{P}_n such that $F_1(p_n^{11}, p_n^{21})$ is a mapping $\mathcal{P}_n \rightarrow \mathcal{P}_n$. One starts

showing some monotonicity properties. The monotonicity of h_θ (defined in (21)) depends on the values taken by q .

Lemma 6: For all $(x, y) \in \mathcal{P}_0$, $h_1(x, y)$ is increasing with x and decreasing with y . If $q(2, 1) > q(2, 2)$, then $h_2(x, y)$ has the same monotonicity as h_1 ; If $q(2, 1) < q(2, 2)$, then $h_2(x, y)$ is decreasing with x and increasing with y ; If $q(2, 1) = q(2, 2)$, then h_2 is a constant.

Lemma 6 is proved by evaluating the partial derivative of h_1 and h_2 and using the fact that $q(1, 1) \geq q(1, 2)$, see Section II.

Lemma 7: If $0 < \nu_1 < 1$, the function

$$g(z) = \sum_{d=\lceil M\nu_1 \rceil}^M \binom{M}{d} z^d (1-z)^{M-d}, \quad (34)$$

is increasing for all $z \in [0, 1]$.

Proof: For $z \in [0, 1]$ and $i = 0, \dots, M$, define the elementary function $f_d(z) = z^d (1-z)^{M-d}$. Evaluating its derivative with respect to z , f_d is easily shown to be increasing over $[0, \frac{d}{M}]$ and decreasing over $[\frac{d}{M}, 1]$. Now, to evaluate the monotonicity of (34), first consider $z \in [0, \nu_1]$. Each i in the sum is such that $z < \nu_1 \leq \frac{d}{M}$. Since $f_d(z)$ is an increasing function for any $z \leq \frac{d}{M}$, the sum $g(z)$ in (34) is also increasing with z . Now, consider $z \in [\nu_1, 1]$. One rewrites (34) as $g(z) = 1 - \sum_{d=0}^{\lceil M\nu_1 \rceil - 1} \binom{M}{d} f_d(z)$, in which each d in the sum is such that $\frac{d}{M} < \nu_1 \leq z$. As $f_d(z)$ is decreasing for any $\frac{d}{M} < z$, $g(z)$ is increasing with z . Thus $g(z)$ is a increasing function of z over $[0, 1]$. ■

One considers the first case where $q(2, 1) > q(2, 2)$. From Lemma 6 and Lemma 7, one obtains that $F_\theta(x, y) = g(h_\theta(x, y))$ are increasing functions of x and decreasing functions of y , for any $\theta \in \{1, 2\}$ and $(x, y) \in \mathcal{P}_0$. Define $p_{\max}^{\theta 1}(n)$ and $p_{\min}^{\theta 1}(n)$ as the upper and lower bounds of $p^{\theta 1}(n)$, i.e., $p_{\min}^{\theta 1}(n) \leq p^{\theta 1}(n) \leq p_{\max}^{\theta 1}(n)$. When $n = 0$, one has $p_{\min}^{\theta 1}(0) = 0$ and $p_{\max}^{\theta 1}(0) = 1$. Based on the monotonicity of F_θ , one gets

$$\begin{cases} F_\theta(p^{\theta 1}(n), p^{\theta 1}(n)) \geq F_\theta(p_{\min}^{\theta 1}(n), p_{\max}^{\theta 1}(n)), \\ F_\theta(p^{\theta 1}(n), p^{\theta 1}(n)) \leq F_\theta(p_{\max}^{\theta 1}(n), p_{\min}^{\theta 1}(n)), \end{cases}$$

thus

$$\begin{cases} p_{\min}^{\theta 1}(n+1) = F_\theta(p_{\min}^{\theta 1}(n), p_{\max}^{\theta 1}(n)), \\ p_{\max}^{\theta 1}(n+1) = F_\theta(p_{\max}^{\theta 1}(n), p_{\min}^{\theta 1}(n)). \end{cases} \quad (35)$$

Lemma 8: For any $\theta \in \{1, 2\}$ and $n \in \mathbb{N}^*$, if $q(1, 1) > q(1, 2)$ and $q(2, 1) > q(2, 2)$, then

$$\begin{cases} p_{\min}^{\theta 1}(n) > p_{\min}^{\theta 1}(n-1), \\ p_{\max}^{\theta 1}(n) < p_{\max}^{\theta 1}(n-1). \end{cases} \quad (36)$$

Proof: The proof is by induction. At the beginning, one has $p_{\min}^{\theta 1}(0) = 0$ and $p_{\max}^{\theta 1}(0) = 1$, then

$$\begin{cases} p_{\min}^{\theta 1}(1) = F_\theta(0, 1) = g(q(\theta, 2)) > 0, \\ p_{\max}^{\theta 1}(1) = F_\theta(1, 0) = g(q(\theta, 1)) < 1, \end{cases} \quad (37)$$

thus (36) is true for $n = 1$.

Consider then an arbitrary $n \in \mathbb{N}^*$ and $n > 1$. Assume that (36) is satisfied for $n-1$, one needs to see whether (36)

is still satisfied for n . From (35) and the monotonicity of F_θ , one has

$$\begin{aligned} p_{\min}^{\theta 1}(n) &= F_\theta(p_{\min}^{\theta 1}(n-1), p_{\max}^{\theta 1}(n-1)) \\ &> F_\theta(p_{\min}^{\theta 1}(n-2), p_{\max}^{\theta 1}(n-2)) = p_{\min}^{\theta 1}(n-1), \end{aligned}$$

and

$$\begin{aligned} p_{\max}^{\theta 1}(n) &= F_\theta(p_{\max}^{\theta 1}(n-1), p_{\min}^{\theta 1}(n-1)) \\ &< F_\theta(p_{\max}^{\theta 1}(n-2), p_{\min}^{\theta 1}(n-2)) = p_{\max}^{\theta 1}(n-1), \end{aligned}$$

which completes the proof. \blacksquare

From Lemma 8, one finds that as n increases, the upper bound $p_{\max}^{\theta 1}(n)$ keeps decreasing while the lower bound $p_{\min}^{\theta 1}(n)$ keeps increasing. In the other situations where $q(2,1) \leq q(2,2)$, one can have the same result using a similar approach.

Since $(F_0(p_n^{00}, p_n^{10}), F_1(p_n^{00}, p_n^{10}))$ maps $[p_{\min}^{00}, p_{\max}^{00}] \times [p_{\min}^{10}, p_{\max}^{10}]$ to $[p_{\min}^{00}, p_{\max}^{00}] \times [p_{\min}^{10}, p_{\max}^{10}]$, $\forall n \in \mathbb{N}^+$, one can apply Brouwer's fixed-point theorem to prove Proposition (3).

V. ILLUSTRATION

Consider a scenario where each agent i is equipped with a sensor providing noisy observations of the scalar physical quantity $\phi(\mathbf{o}, t)$, at location \mathbf{o} and at time t ,

$$m_i(\mathbf{o}, t) = \phi(\mathbf{o}, t) + w_i, \quad \forall i \in \mathcal{S}. \quad (38)$$

The components w_i of the measurement noise in (38) are assumed to be realizations of random Gaussian variables $W_i \sim \mathcal{N}(e_i, \sigma^2)$, where e_i is a constant agent-dependent bias. The agents can be classified according to the value of e_i , *i.e.*, the sensors with small values of e_i provide better measurements. Assume that among different sensors, the absolute value $|e_I|$ of a random agent I follows an exponential distribution with parameter γ , *i.e.*, $\mathbb{P}\{|e_I| \leq x\} = 1 - \exp(-\gamma x)$. For example, Group θ can be defined as

$$\mathcal{S}_\theta = \{i \in \mathcal{S} : \Lambda_{\theta-1} \leq |e_i| < \Lambda_\theta\},$$

where

$$\Lambda_\theta = \begin{cases} \theta, & \forall \theta \in \mathcal{G} \setminus \{N_G\}, \\ \infty, & \theta = N_G. \end{cases}$$

Therefore, the proportion of agents in Group θ is $p_\theta = \exp(-\gamma(\theta-1)) - \exp(-\gamma\theta)$.

Agent i does not know the characteristics of W_i and aim at using Algorithm 1 to self-rate its sensor. For that purpose, when meeting other agents, it will share measurements performed at close locations and run the LCT introduced in Section V-A.

A. LCT

Consider some tolerance ω and the interval $[m] = [m - \omega, m + \omega]$ of width 2ω centered around some measurement m . Consider now two sensors i and j meeting and exchanging the measurements m_i and m_j . The *set* estimate [9] of ϕ obtained combining m_i and m_j is defined as

$$[\widehat{\phi}(m_i, m_j)] = [m_i] \cap [m_j]. \quad (39)$$

It may be used to define the following low-complexity LCT

$$y_i = y_j = \begin{cases} 1, & \text{if } [\widehat{\phi}(m_i, m_j)] \neq \emptyset, \\ 0, & \text{else.} \end{cases} \quad (40)$$

If $[\widehat{\phi}(m_i, m_j)] \neq \emptyset$ it is likely that the biases e_i and e_j are of the same order of magnitude and both agents can conclude that their sensor performs similarly. If $[\widehat{\phi}(m_i, m_j)] = \emptyset$, it is likely that e_i and e_j differ significantly more than ω . One is unable to indicate which agent has the best sensor in that case, so, both agents choose to conclude that its sensor behaves worse than that of the other agent.

For any arbitrary pair of groups (θ_i, θ_j) , one is able to evaluate the probability $q(\theta_i, \theta_j) = \mathbb{P}\{Y_{i,j} = 1 | i \in \mathcal{S}_{\theta_i}, j \in \mathcal{S}_{\theta_j}\}$ as a function of ω, σ , and γ .

B. Numerical verification of theoretical results

This section presents first the solution of the state equation (16)-(18) describing the evolution of the proportion of nodes in various states. Algorithm 1 is analyzed first considering a random displacement of agents without constraint on their speed.

For the numerical example, one takes $N_G = 4$, $\sigma^2 = 0.25$, $\gamma = 0.7$, and $\omega = 1.8$, resulting in

$$p_1 = 0.503, p_2 = 0.25, p_3 = 0.124, p_4 = 0.123, \quad (41)$$

and

$$\mathbf{q} = \begin{pmatrix} 0.92 & 0.65 & 0.24 & 0.02 \\ 0.65 & 0.53 & 0.42 & 0.11 \\ 0.24 & 0.42 & 0.49 & 0.26 \\ 0.02 & 0.11 & 0.26 & 0.36 \end{pmatrix}. \quad (42)$$

Besides, one considers $M = 50$, and a sampling period Δt such that $\lambda \Delta t = 0.33$. The decision thresholds are chosen as $\nu_1 = 0.8$, $\nu_2 = 0.45$, $\nu_3 = 0.1$, and $\nu_4 = 0$. Figure 2 presents the evolution $p^{\theta \hat{\theta}}(t)$, for $\theta, \hat{\theta} \in \mathcal{G} = \{1, 2, 3, 4\}$. One observes that the proportion agents of each state converges. Moreover, for any $\theta \in \mathcal{G}$, $p^{\theta \hat{\theta}}$ is close to 1 for t sufficient large, while $p^{\theta \hat{\theta}}$ tends to 0 for any $\hat{\theta} \neq \theta$. Algorithm 1 behaves thus in a satisfying way.

Consider now a set $\mathcal{S} = N_S = 1000$ moving agents which initial position is uniformly distributed over a unit square. At first, one considers Agent i randomly chooses its location at time instant $(k+1)\Delta t$, independently from its previous location at time $k\Delta t$. Two agents communicate only at discrete time instant $k\Delta t$ when their distance is less than r_0 . Agent i has its neighbors in the set $\mathcal{N}_i = \{j \in \mathcal{S} : 0 < R_{i,j} \leq r_0\}$, where $R_{i,j}$ is the distance between i and j . Then, we assume that Agent i can contact its neighbor if and only if $|\mathcal{N}_i| = 1$. Denote $\rho = \pi r_0^2 N_S$ as the average value of $|\mathcal{N}_i|$. The cardinality of \mathcal{N}_i approximately follows a Poisson distribution provided that N_S is large enough, the inter-contact probability during Δt is thus $\lambda \Delta t = \mathbb{P}\{|\mathcal{N}_i| = 1\} = \rho \exp(-\rho)$. In the simulations, we used $r_0 = 0.014$, so that $\rho \approx 0.6$ and $\lambda \Delta t \approx 0.33$.

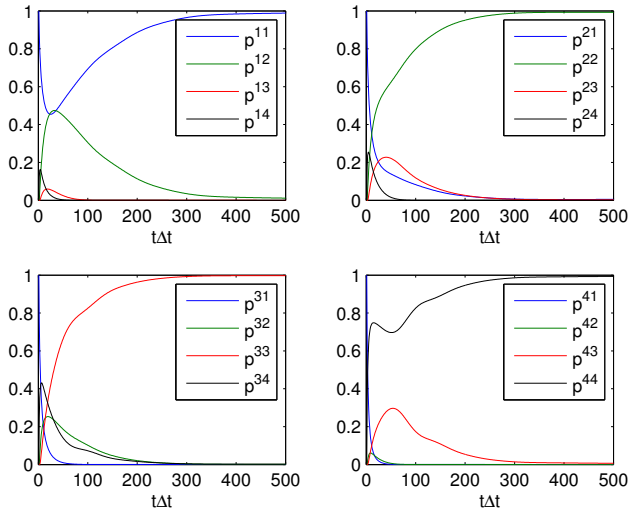


Fig. 2. Evolution of $p^{\theta\hat{\theta}}(t)$ obtained solving (16-18).

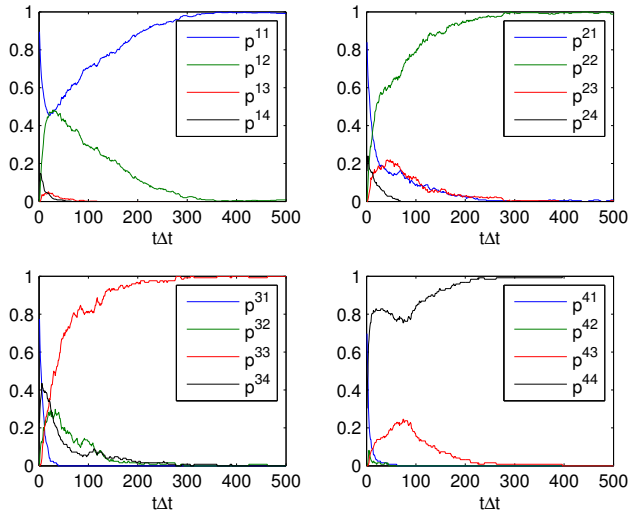


Fig. 3. Evolution of $p^{\theta\hat{\theta}}(t)$ by simulations with the random displacement model.

The resulting evolutions of $p^{\theta\hat{\theta}}(t)$ are shown in Figure 3 and are very close to those predicted by the direct integration of the state equation as shown in Figure 2.

Figure 4 further illustrates the good match between theory and simulation for the proportions of states $\bar{X}_{\theta}^{50,d}$ at equilibrium. The approximation of $\bar{X}_{\theta}^{50,d}$ using (25) is also provided in Figure 4, which is also close to its actual value.

Similar results have been obtained with agents following a Brownian motion model.

VI. CONCLUSIONS

This paper has investigated the problem of helping agents self-rating their expertise level at doing some task via exchange of information with peers. Using local compatibility tests involving, *e.g.*, data exchanged during meetings with other agents, each agent is able to estimate the proportion of agents it is better at doing the considered task. With that information, each agent may then determine to which group

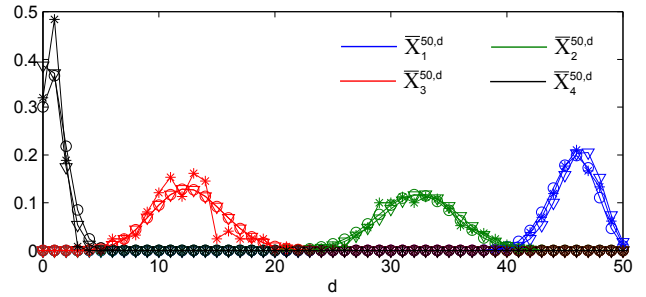


Fig. 4. Comparison of $\bar{X}_{\theta}^{50,d}$ at the equilibrium: circles are for the theoretical values obtained integrating (16)-(18), triangles are the approximations obtained from (25), and crosses are for the moving agents simulation.

of agents with similar expertise it belongs to.

The behavior of the proposed algorithm is described using dynamical equations. The existence of an equilibrium is established. The proportions of agents with similar beliefs in their expertise is characterized at equilibrium. This gives some insight in the tuning of the various parameters of the proposed algorithm.

The approach is illustrated with agents equipped with sensing devices of different sensing performance, which may be found in crowd sensing scenarios. Simulation results are in good match with theory.

Significant work remain to be done to analyze the behavior of the proposed algorithm with generic probabilities of interaction. The existence and uniqueness of the equilibrium has also to be shown in the general case. Nevertheless, the proposed approach may be useful to analyze other types of self-rating problems.

REFERENCES

- [1] B. Guo, Z. Wang, Z. Yu, Y. Wang, N. Yen, R. Huang, and X. Zhou, "Mobile crowd sensing and computing: the review of an emerging human-powered sensing paradigm," *ACM Computing Surveys*, vol. 48, no. 1, September 2015.
- [2] X. Sheng, J. Tang, X. Xiao, and G. Xue, "Sensing as a Service: challenges, solutions and future directions," *IEEE Sensors Journal*, vol. 13, October 2013.
- [3] B. Kantarci and H. T. Mouftah, "Trustworthy sensing for public safety in cloud-centric internet of things," *IEEE Internet Of Things Journal*, vol. 1, no. 4, August 2014.
- [4] —, "Reputation-based sensing-as-a-service for crowd management over the cloud," in *Proc. of IEEE Int. Conf. on Communications, ICC*, June 2014.
- [5] Z. Shen and A. Neumaier, "The Krawczyk operator and Kantorovich's theorem," *J. Math. Anal. Appl.*, vol. 149, pp. 437–443, 1990.
- [6] A. Chiuso, F. Fagnani, L. Schenato, and S. Zampieri, "Gossip algorithms for simultaneous distributed estimation and classification in sensor networks," August 2011.
- [7] F. Fagnani, S. M. Fosson, and C. Ravazzi, "Input driven consensus algorithm for distributed estimation and classification in sensor networks," in *Proc. IEEE Conf. on Decision and Control*, December 2011.
- [8] A. Granas and J. Dugundji, *Fixed point theory*. Springer Science & Business Media, 2013.
- [9] G. Belforte, B. Bona, and V. Cerone, "Parameter estimation algorithms for a set-membership description of uncertainty," *Automatica*, vol. 26, no. 5, pp. 887–898, 1990.