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Parametric compact modelling of dynamical systems using meshfree method with multi-port technique

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Abstract: Several dynamical systems require parametric compact modelling of physical behaviour taking into account the tight coupling with the control system. In this paper, we present an approach combining meshfree method with multi-port technique for the analysis of physical systems modelled with partial differential equations. The meshfree technique used is based on radial basis functions and differential quadrature method. Thermal modelling and simulation of a power electronic converter is considered as an application to validate the proposed approach. Obtained results of temperatures and heat fluxes in different layers of the power converter are compared with finite element analysis. It is shown that with small stencil sizes, it is possible to produce solutions for complex systems with an accuracy comparable to finite element method, but with less computation time.

Keywords: Partial Differential Equations; Meshfree; Radial Basis Functions; Differential Quadrature Method; Multi-port Technique.

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1 Introduction

Several dynamical systems such as mechatronic systems require integrated design of mechanical-based systems and their embedded control systems (Amerongen, 2003). The mechanical-based system could also be considered as a multi-physics system coupling the mechanical structure with other multi-physical sub-systems: fluid, thermal, electrical, electromagnetic, etc. This means that the optimal dynamical system performance can be obtained when the overall system (physical system + control system) is designed and optimized in an integrated way.

In most of cases, the dynamical system behaviour is modelled with partial differential equations (PDEs) and the control system is modelled using ordinary differential equations (ODEs) or differential-algebraic equations (DAEs). For instance, control of flexible robots (Lee et al., 2001), electro-thermal simulation of Micro-electromechanical systems (MEMS) (Bechtold et al., 2007), simulation of integrated power electronic converters (Emadi et al., 2009) and dynamics of smart structures (Vepa, 2010) are examples of dynamical systems concerned with this subject.

Multi-port technique is a graphical representation for modelling of dynamical systems addressing problems associated with physical behaviour modelling and design of control systems. Multi-port modelling tools, such as Bond-Graph (Thoma, 1975) and Modelica (Elmqvist et al., 1998), support modelling with DAEs. However, PDE modelling is currently not quite supported by these tools, which is considered as a limitation for multi-port technique.

PDEs can be solved numerically using mainly mesh-based methods, such as finite element method (FEM), finite difference method (FDM) and finite volume method (FVM). However, modelling tools based on meshing methods do not currently support design of control systems.

In recent years, meshfree (or meshless) methods have been used by many researchers for treating a wide range of linear and non-linear phenomena and engineering problems (Tiago and V.M.A.Leito, 2006). Compared to mesh-based techniques, meshfree methods require only nodal data without explicit connectivity between nodes (Alfaro et al., 2006). Among a variety of meshfree methods, radial basis function collocation method is one of the most frequently used. Radial basis functions (RBF) were first used to solve partial differential equations by Kansa (Kansa, 1990a; Kansa, 1990b), to model fluid dynamics using the method of collocation with Multiquadric RBF.

However, accuracy in most of meshfree methods depends on a shape parameter "c" that needs to be correctly selected. Choosing optimal values of shape parameter has been addressed in several previous works (Carlson and Foley, 1991; Shmuel,

1999; Larsson and Fornberg, 2003; Fornberg and Wright, 2004; Fornberg and Zuev, 2007). For more information about meshfree techniques, the reader may refer to (Nguyen et al., 2008).

To overcome some of the drawbacks of the classic RBF collocation method, (Shu et al., 2003) proposed the radial basis function-differential quadrature method RBF-DQM. In RBF-DQM, the differential operators in a given node are approximated as a linear weighted sum of values of the unknown function at some surrounding nodes. In (Bayona et al., 2010), authors showed that there is a range of values of the shape parameter for which the convergence of RBF-DQM is significantly more accurate than the classic DQM technique.

Few previous works for implementing mesh-based PDE modelling with multi-port technique (Saldamli et al., 2005; Li et al., 2008) did not show practical use, due to the difficulties of implementing mesh and long computation time, compared to standard mesh-based tools. Other approaches based on multi-tool integration cannot be easily applied to all dynamical systems. For instance, a technique like co-simulation requires developing exchange interfaces between the tools to be coupled, and also to define efficient strategies for data exchange between solvers. Otherwise, the co-simulation could fail or would be with long computing time (Hammadi et al., 2011).

In this paper we propose a meshfree method based on RBF-DQM to solve problems modelled with PDEs in multi-port environments. We show through an application to thermal modelling of a power converter, that the method can be applied to model multi-component systems with an accuracy compared to finite element method, but with a shorter computation time. The parametric compact model proposed is suited for sensitivity analysis, control system design and overall design optimization.

The rest of the paper is organized as follows: in section 2, the mathematical formulation of RBF-DQM is presented. In section 3, RBF-DQM is applied to elaborate a one-dimension thermal component in the multi-port modelling language Modelica. In section 4, a thermal model of a power converter is considered using the Modelica component developed with RBF-DQM. In section 5, results of temperatures and heat fluxes obtained in different layers of the power converter are exposed and compared to finite element analysis. Finally, a conclusion is given in section 6.

2 RBF-DQM formulation

Considering a PDE-based problem modelled in a bounded domain $\Omega \subset \mathbb{R}^d$

$$\begin{cases} \mathcal{L}_t[u(x, t)] + \mathcal{L}_x[u(x, t)] &= f(x, t), \text{ in } \Omega, \\ \mathcal{L}_x[u(x, t)] &= g(x, t), \text{ on } \partial\Omega, \\ \mathcal{L}_t[u(x, 0)] &= h(x), \text{ at } t = 0. \end{cases} \quad (1)$$

where $\mathcal{L}_t[\cdot]$ and $\mathcal{L}_x[\cdot]$ are time differential operator and space differential operator, respectively.

In RBF-DQM, the unknown function u is interpolated using radial basis func-

tions(Bayona et al., 2010),

$$u(\mathbf{x}, t) = \sum_{j=1}^N \lambda_j(t) \cdot \Phi(\mathbf{r}_j(\mathbf{x}), \mathbf{c}_j), \quad (2)$$

where $\lambda_j(t)$ are the unknown variables of the interpolation, $\mathbf{x} \in \mathbb{R}^d$, $\mathbf{r}_j = \|\mathbf{x} - \mu_j\|$ is the distance to the RBF centre μ_j , $\mu_j \in \Sigma$ and Σ is a set of nodes ($\Sigma \subset \mathbb{R}^d$). $\Phi(\mathbf{r}_j(\mathbf{x}), \mathbf{c}_j)$ is a radial basis function which depends on a shape parameter c_j . In this study, we consider a constant value shape parameter, thus $c_j = c$. Commonly used RBFs are:

$$\text{Gaussian} : \Phi(r) = e^{-\frac{r^2}{c^2}}, \quad (3)$$

$$\text{Multiquadric} : \Phi(r) = \sqrt{r^2 + c^2}, \quad (4)$$

$$\text{Inverse Quadratic} : \Phi(r) = \frac{1}{r^2 + c^2}, \quad (5)$$

$$\text{Inverse Multiquadric} : \Phi(r) = \frac{1}{\sqrt{r^2 + c^2}}. \quad (6)$$

In the RBF-DQM, the $\mathcal{L}_x[\cdot]$ operator is approximated at nodes x_i using the values of the unknown function u at the N scattered nodes surrounding x_i , thus

$$\mathcal{L}_x[u(x_i)] \approx \sum_{k=1}^N a_{ik} \cdot u(x_k). \quad (7)$$

Substituting Eq. 2 in Eq. 7

$$\mathcal{L}_x[u(x_i)] \approx \sum_{k=1}^N a_{ik} \cdot \sum_{j=1}^N \lambda_j(t) \cdot \Phi(r_j(x_k), c). \quad (8)$$

Applying the operator $\mathcal{L}_x[\cdot]$ to Eq. 2 for $x = x_i$

$$\mathcal{L}_x[u(x_i)] = \sum_{j=1}^N \lambda_j(t) \cdot \mathcal{L}_x[\Phi(r_j(x_i), c)]. \quad (9)$$

Using Eq. 8 and Eq. 9 a linear algebraic system is obtained and can be solved to determine the coefficients a_{ik} ; a_{ik} are only dependent on radial function and the shape parameter c .

The $\mathcal{L}_t[\cdot]$ will not be approximated in this study, since multi-port tools are able to solve differential equations with time derivatives.

3 Application of RBF-DQM to one-dimensional heat conduction

Considering the one-dimensional heat conduction problem

$$\rho \cdot C_p \cdot \frac{\partial T(z, t)}{\partial t} = k \cdot \frac{\partial^2 T(z, t)}{\partial z^2}, \quad (10)$$

where $\rho[kg/m^3]$ is the mass density of the material, $C_p[J.kg^{-1}.K^{-1}]$ is the specific heat capacity and $k[W.m^{-1}.K^{-1}]$ the thermal conductivity, which is supposed constant in this study.

With two boundary conditions:

$$k \cdot \frac{\partial T(z, t)}{\partial z} = -Q_z, \text{ for } z = H_z, \quad (11)$$

and

$$T(z = 0, t) = T_{z_0}, \quad (12)$$

with an initial condition

$$T(z, t = 0) = T_i, \quad (13)$$

where $Q_z[W/m^2]$ is the heat flux imposed to one boundary ($z = H_z$), $T_{z_0}[^\circ C]$ is a temperature imposed to the other boundary ($z = z_0$) and $T_i[^\circ C]$ is the initial temperature.

Considering the Multiquadric RBF expressed as

$$\Phi(z - \mu_j) = \sqrt{(z - \mu_j)^2 + c^2}. \quad (14)$$

The first derivative of Φ with respect to z can easily be determined

$$\frac{\partial \Phi}{\partial z} = \frac{z - \mu_j}{\sqrt{(z - \mu_j)^2 + c^2}}, \quad (15)$$

and the second derivative is

$$\frac{\partial^2 \Phi}{\partial z^2} = \frac{1}{\sqrt{(z - \mu_j)^2 + c^2}} - \frac{(z - \mu_j)^2}{(\sqrt{(z - \mu_j)^2 + c^2})^3}. \quad (16)$$

By substituting Eq. 16 in Eq. 10 and Eq. 15 in Eq. 11, a DAE system is obtained.

Both regular and irregular stencils can be used in RBF-DQM. The so-called Gauss-Lobatto-Chbyshev sampling technique has been used to generate an irregular stencil

$$z_i = \frac{H_z}{2} \left[1 - \cos\left(\frac{i-1}{N-1}\pi\right) \right] \text{ (for } i = 1, 2, \dots, N). \quad (17)$$

The multi-port modelling language Modelica has been chosen to encapsulate the DAE system to define a thermal element as shown in Figure 1. Compared to other modeling tools, such as Bond-Graph, Modelica is an object-oriented and acausal language, which offers a high flexibility and simplicity to build complex physical systems.

Connectors are used in multi-port tools to define interfaces between components. Two type of variables can be defined in this case, flow variable which corresponds to heat flux and potential variable which is temperature in our case. Thus, boundary conditions defined by Eq. 11 and Eq. 12 can be associated to connector variables. The thermal element developed can then be used to model more complex multi-component systems such as thermal modelling of power converters.

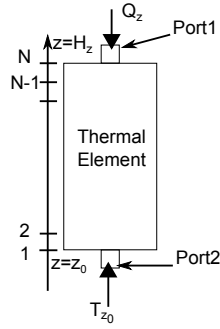


Figure 1 Thermal RBF-DQM model in Modelica.

4 Case study: Thermal modelling of a power converter

Several dynamic technical systems include power electronic converters to transduce electric energy from one form to another (DC/DC, DC/AC, AC/DC and AC/AC). Changes in temperature have a significant influence on electrical and mechanical behaviour of power converters. These devices may fail catastrophically if the junction temperature becomes high enough to cause melting and thermal runaway. Heat generation, caused primarily by semiconductors, must be removed as efficiently as possible by thermal exchange with ambient. Thus, designing an optimal layout with an optimal control system using integrated configurations is a real challenge for industry looking to improve packaging and power density. Parametric compact thermal models are therefore required for optimizing both the layout and control systems of power converters.

In the case of the thermal modelling of a power converter, the thickness of components is small relative to other dimensions, so the system may be governed by 1-D heat transfer equations. Assuming the temperature is uniform over cross-sections, heat transfer is therefore performed by conduction along z-axis.

To apply the proposed approach based on the RBF-DQM, the power converter presented in Figure 2 is considered.

For simplification reasons we consider only one chip semiconductor and its associated components. Figure 2-a shows the superposition of the components making the power converter. Figure 2-b shows a 3D-FEM model of the power converter used to validate the RBF-DQM. The physical and geometrical characteristics of the power converter are given in Table 1.

Table 1 Dimensions and material characteristics of power converter components.

	Ref.	$L_x(mm)$	$L_y(mm)$	$H_z(mm)$	$\rho(kg/m^3)$	$k(W/(m.K))$	$Cp(J/kg.K)$
Chip	1	10	10	0.35	2330	124	771
Solder1	2	10	10	0.1	7300	60	180
Copper plate	3	10	10	0.3	8900	390	385
Solder2	4	10	10	0.1	7300	60	180
Substrate	5	45	25	0.5	3960	23	880
Solder3	6	45	25	0.1	7300	60	180
Heat Sink	7	45	25	0.5	2700	237	897

The chip device (Ref. 1), which is a transistor, is supposed to receive a variable heat

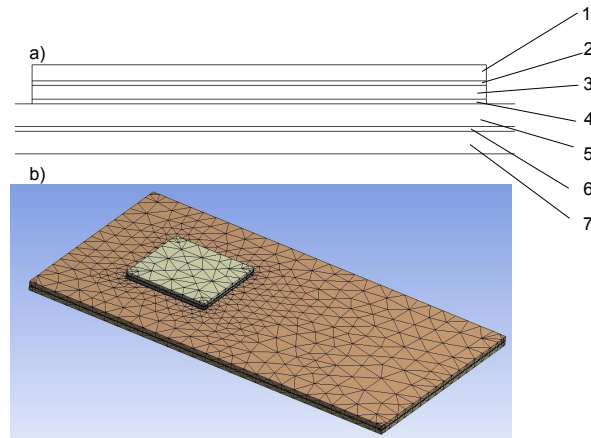


Figure 2 Power converter: a) Layout of the components, b) FEM model.

flux Q_{in} , due to electric dissipation during device switching. Electric modelling is not considered in this study. The bottom face of the heat sink (Ref. 7) is supposed to be at a constant temperature T_{in} . The initial temperature T_0 is supposed the same for all the components.

Figure 3 shows the thermal model of the power converter performed in Modelica by connecting instances of the thermal element developed using RBF-DQM.

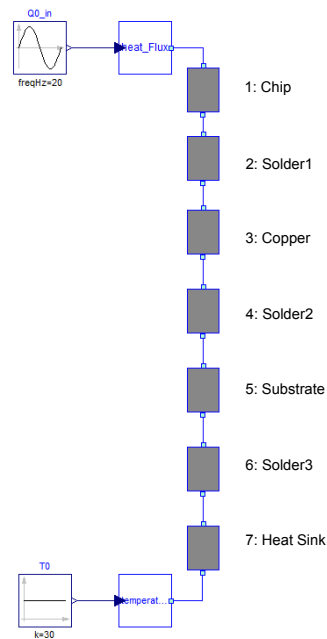


Figure 3 Thermal model of the power converter using RBF-DQM and Modelica.

The results obtained with the RBF-DQM-Modelica approach are compared to those obtained with FEM analysis using Ansys Workbench software. Thus, temperatures and heat fluxes are measured in different layers of FEM study and compared to

temperatures and heat fluxes measured in the interfaces of the thermal model in Modelica.

5 Results and discussion

For FEM and RBF-DQM simulations, a transient thermal analysis were performed considering a sinusoidal input heat flux with a frequency of 20Hz and an amplitude of $4W/cm^2$. The temperature of the bottom face of the heat sink (Ref. 7) is $T_{in} = 30^\circ C$. The initial temperature for all components is $T_0 = 30^\circ C$. The time of simulation is chosen to be 0.25 second with a step equal to $125 \cdot 10^{-5}$ second, for both FEM and RBF-DQM analysis.

To study the stability and convergence of the proposed approach and its application to the thermal modelling of a power converter, the number of nodes per thermal component and the shape parameter were firstly considered.

Figure 4 shows the relative error variation of the junction temperature depending on time for different number of nodes per thermal element N . The junction temperature is measured in the face receiving the input heat flux of the chip device (Ref. 1).

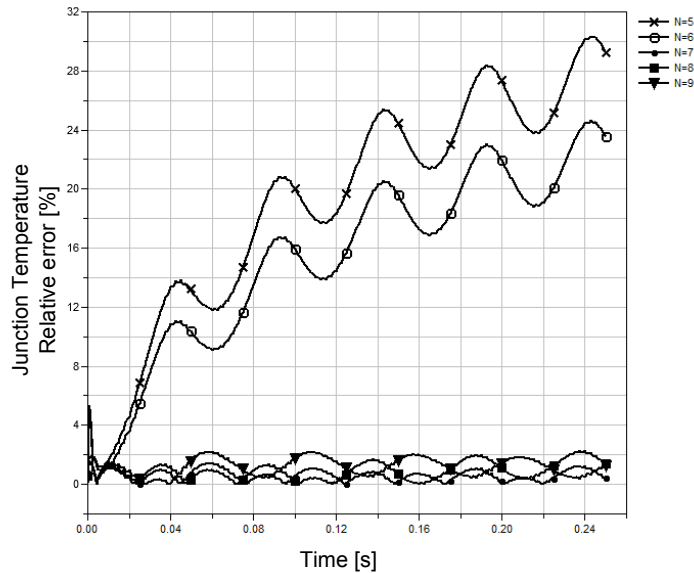


Figure 4 Relative error of junction temperature depending on number of nodes per thermal element.

Results show that for a number of nodes per thermal element less than or equal to 6, the relative error can be more than 24%. However for a number of nodes equal to or more than 7 the relative error is less than 2%. This shows that with small stencil sizes, it is possible to produce accurate results with RBF-DQM comparable to finite element method.

Figure 5 shows the mean value of the relative error for the junction temperature depending on shape parameter c_1 of plates (Refs: 1, 3, 5 and 7).

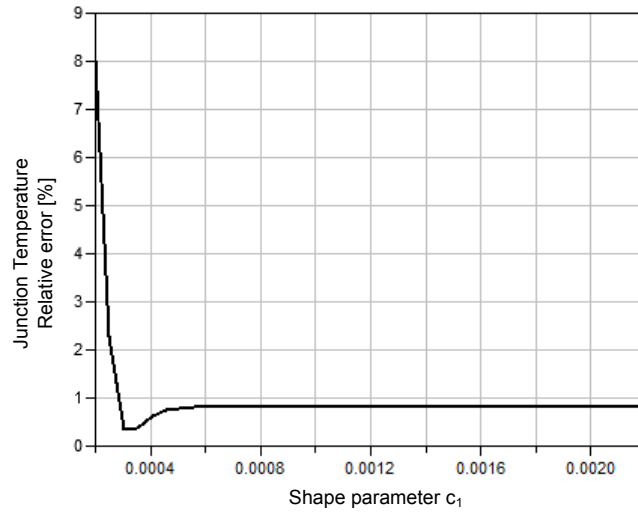


Figure 5 Relative error depending on shape parameter c_1 .

Results show that for c_1 between 0.0003 and 0.002 the mean relative error remains less than 1%, which confirms the existence of a domain for which convergence of the solution is achieved.

For the next RBF-DQM analysis, we chose a configuration defined by a number of nodes per thermal element $N = 7$ and shape parameter for components (1, 3, 5 and 7) $c_1 = 0.0003$ and $c_2 = 0.0001$ for other components.

Figure 6 shows results for heat fluxes obtained with FEM and RBF-DQM analysis.

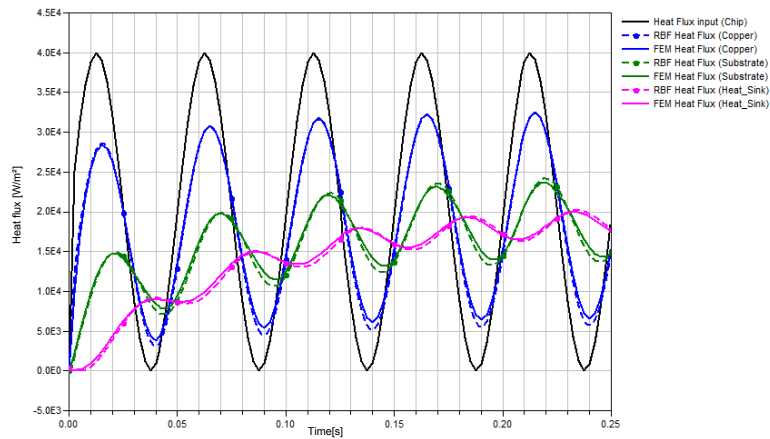


Figure 6 RBF-DQM and FEM analysis: Heat fluxes in three interfaces and input heat flux.

Heat fluxes were measured in three layers: top face of the copper plate (Ref. 3), top face of substrate plate (Ref. 5) and top face of heat sink plate (Ref. 7).

It can be deduced that results show a good agreement between RBF-DQM and

FEM method. The relative error between the two results varies between 0.2% and 1%, which shows a good approximation of the first derivatives used to calculate heat fluxes with RBF-DQM. Heat fluxes, calculated at thermal element boundaries, have a direct influence on temperatures calculated in thermal elements and in their interfaces. Therefore, they have to be calculated as accurately as possible.

Results of temperature simulation with FEM and RBF-DQM are superposed in Figure 7. These results are arranged as following: junction temperature in 'Port1'

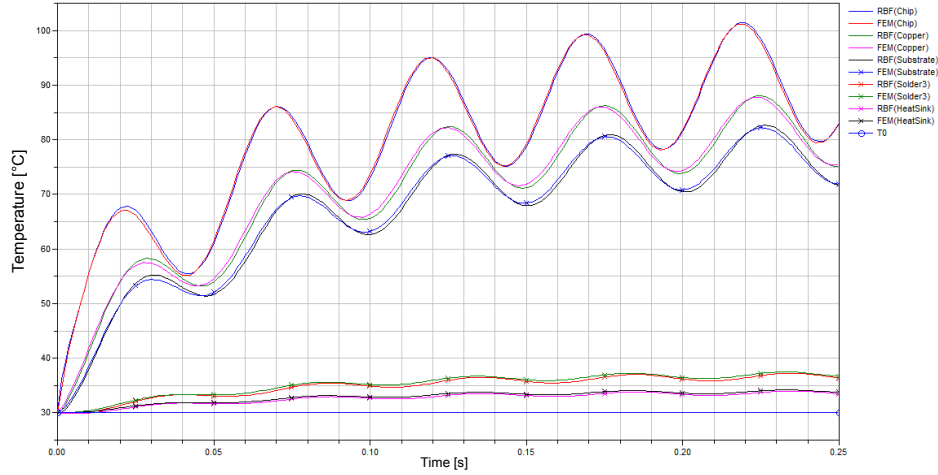


Figure 7 Comparison of temperature results for FEM and RBF-DQM simulation.

of Chip component, temperature in interface between Solder1 (Ref. 2) and Copper plate (Ref. 3), temperature in interface between Solder2 (Ref. 4) and Substrate (Ref. 5), temperature in interface between Substrate (Ref. 5) and Solder3 (Ref. 6) and temperature in interface between Solder3 (Ref. 6) and Heat Sink (Ref. 7).

As for heat fluxes, RBF-DQM and FEM analysis show a good correlation for temperatures measured in different interfaces. These results help to understand the effect of every component in the thermal exchange process with ambient, allowing better thermal management of the power converter. Results show that a significant drop in temperature is found in the substrate layer (Ref. 5) due to its low conductivity. We can also remark delays introduced in different temperatures due to specific heat capacities related to component materials used.

Figure 8 shows the relative error of the junction temperature. The value of the relative error in the starting phase is between 5% and 6%. This relatively high value can be explained with the complexity of numerical solving of the transient heating phase. Therefore, the numerical solutions given by FEM and RBF-DQM are not exactly the same. However, the relative error in the steady-state phase is less than 2%, which is coherent with the results previously presented.

To compare computation time, both FEM and RBF-DQM analysis were performed on the same computer (4 Go RAM, Intel Processor 2.8 Ghz). Computing time for FEM analysis was 94 seconds, whereas it was only 11 seconds for the RBF-DQM analysis. This gain of time is significantly important especially when dealing with

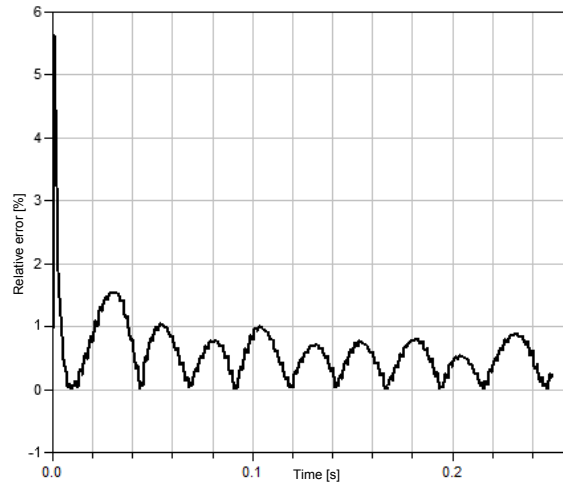


Figure 8 Relative error for junction temperature.

sensitivity analysis and optimization of complex power converters with a bigger number of components.

The parametric thermal model developed in this study allows sensitivity analysis to measure the effect of design parameters on the desired results. For this we chose to study the effect of component thickness on the junction temperature. In this analysis four thicknesses (noted H_z) are analyzed: Substrate (Sb), Solder (So), Heat Sink (Hs) and Copper (Co). Figure 9 shows the results of the sensitivity analysis.

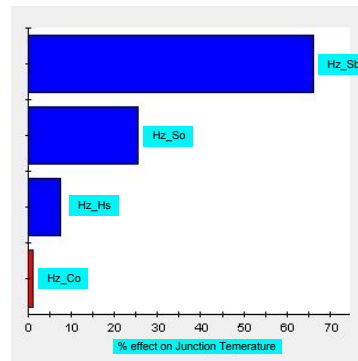


Figure 9 Component thickness effect on junction temperature

Results show that the substrate plate thickness has the most important effect (66%), and the copper plate thickness has the least effect (1%). Other thickness effects are: Solder (25 %) and Heat sink (6%).

The Figure 10 (left) shows the relationship between the junction temperature and the substrate thickness which varies between 0.1 and 1 mm. The junction temperature is varying quasi-linearly between 70 and 130° C.

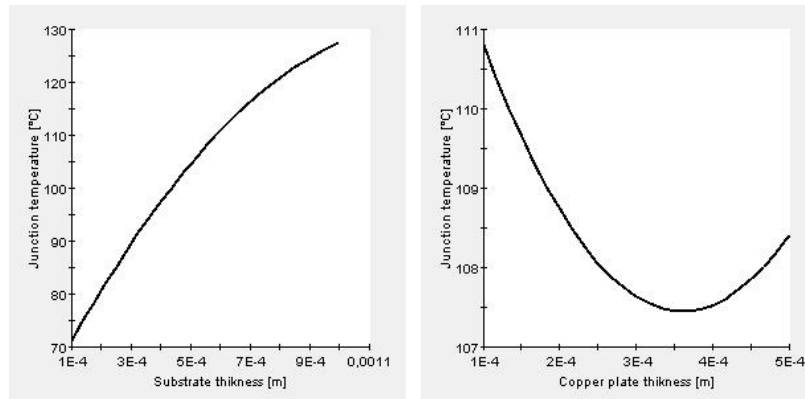


Figure 10 Substrate (left) and copper plate (right) thickness effects on junction temperature

The Figure 10 (right) shows the variation of the junction temperature depending on the copper plate thickness, which is varying between 0.1 and 0.5 *mm*. The figure shows that the difference between the maximum and minimum temperatures is 4° *C*. It is also shown that a minimum value for the junction temperature is found for a copper plate thickness around 0.35 *mm*.

Therefore, The parametric compact thermal model developed allows us to analyse the sensitivity and then can help to perform design optimization. Moreover, the model can be used as a support for the design of the control system.

6 Conclusion

In this paper a meshfree method is presented for parametric compact modelling of PDE-based systems in multi-port modelling environments. One-dimensional thermal model is developed using RBF-DQM and applied to thermal modelling of a power converter. Modelica language with Dymola software have been used for the implementation of the method. Any other multi-port modeling tool could also be used. Then, a comparison is made between RBF-DQM and FEM. Results considered are temperatures and heat fluxes in different layers of the power converter. Results show that with small stencil sizes, it is possible to produce solutions with accuracy comparable to FEM, but with less computation time. The parametric model elaborated allows sensitivity analysis, design optimization and helps for elaborating integrated control models.

Due to its dependence on the distance between centres and not the location, the method could be easily applied to 2D and 3D problems. RBF-DQM has been applied to thermal modelling, but it is also extendible to other dynamical problems.

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