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# Simultaneous slow and fast light involving the Faraday effect

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We theoretically study the linear transmission of linearly polarized light pulses in an ensemble of cold atoms submitted to a static magnetic field parallel to the direction of propagation. The carrier frequency of the incident pulses coincides with a resonance frequency of the atoms. The transmitted light, the electric field of which is transversal, is examined in the polarizations parallel and perpendicular to that of the incident pulses. We give explicit analytic expressions for the transfer functions of the system for both polarizations and for the corresponding group delays. We demonstrate that slow light can be observed in a polarization, whereas fast light is simultaneously observed in the perpendicular polarization. Moreover, we point out that, due to the polarization post selection, the system is not necessarily minimum phase shift. Slow light can then be obtained in situations where an irrelevant application of the Kramers-Kronig relations could lead one to expect fast light. When the incident light is step modulated, we finally show that, in suitable conditions, the system enables one to separate optical precursor and main field.

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Dilute atomic or molecular media are precious tools for the study of the propagation of light in material [1, 2] and, more specifically, of the phenomena of slow light, fast light and optical precursors [3–6]. With their refractive index  $n$  being very close to unity, the parasitic reflections at the input and the output of the medium (“etalon effects”) that may complicate the analysis of the transmitted signals [7] are practically eliminated. On the other hand, the narrowness of their absorption or gain lines originates the singular group velocities  $v_g$  required to observe significant slow light ( $0 < v_g \ll c$ , where  $c$  is the light velocity in vacuum) or fast light ( $v_g > c$  or  $v_g < 0$ ) [3]. These group velocities are often obtained when the carrier frequency of the probe pulses coincides with a well-marked peak in the medium transmission in the slow-light case or a well-marked dip in the fast-light case. In most of the experiments, these conditions are created by applying extra fields (pump and/or coupling fields) whose interaction with the medium is nonlinear [3–5, 8]. It is, however, worth recalling that the pioneering experimental demonstrations of slow and fast light have been performed without any pump or coupling fields [7, 9, 10]. More recent experiments in “natural” atomic media are reported in [11–15]. The challenge in all these experiments is to obtain significant effects with moderate pulse distortion.

Fast-light and slow-light experiments have also been performed in nondispersive media by exploiting the effects of field polarization [16–19]. In these experiments, a medium [16, 17, 19] or fiber [18] with linear birefringence is placed between two linear polarizers. Note that fast light and slow light are then observed in different experimental conditions, the transition from fast to slow

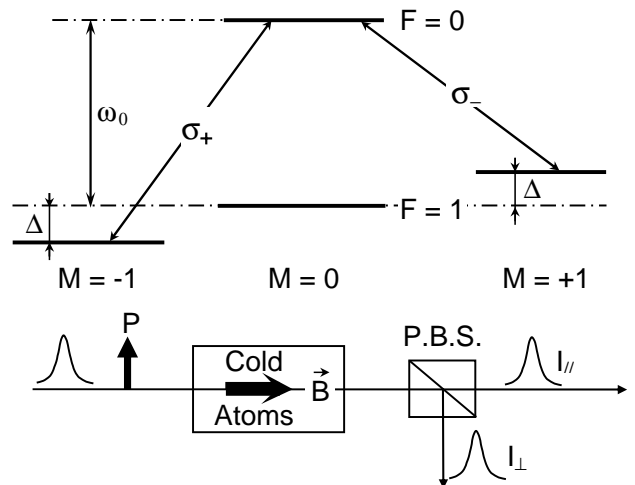


Figure 1: Proposed level arrangement (top) and experimental setup (bottom).  $\vec{B}$  is the magnetic field parallel to the direction of the light propagation, P and PBS respectively designate a linear polarizer and a polarization beam splitter separating the polarizations parallel ( $\parallel$ ) and perpendicular ( $\perp$ ) to the polarization of the incident pulse.

light being achieved by changing the orientation of the output polarizer. A similar behavior is obtained in the related system considered in [20].

In the present article, we propose a system combining effects of light polarization and of medium dispersion. It involves two detection channels, enabling one to observe fast light and slow light in the same experiment. The medium consists of a cloud of cold atoms submitted to a uniform [21], static magnetic field parallel to the direction of propagation of the (unique) probe field (Fig. 1). The incident field is transversal and linearly polarized by a polarizer (P) and the field transmitted by the medium is received on a polarization beam splitter (PBS) that

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separates the polarizations parallel and perpendicular to that of the incident field, with both directions of polarization being perpendicular to the direction of propagation. Cold atoms avoid the complications of Doppler broadening and ensure significant Faraday effects (circular birefringence and dichroism) for moderate values of the magnetic field [22]. For simplicity, the carrier-frequency  $\omega_c$  of the incident pulse is assumed to coincide with the frequency  $\omega_0$  of the transition from a ground level of total angular momentum  $F = 1$  to an excited state  $F = 0$ . In the presence of magnetic field the transition is split in two components of frequency  $\omega_0 \pm \Delta$  associated with the circular polarizations  $\sigma_{\pm}$  [23]. In a frame rotating at the angular frequency  $\omega_0$ , the transfer functions  $H_{\pm}(\Omega)$  relating the Fourier transforms of the envelopes of the incident and transmitted pulses for the polarizations  $\sigma_{\pm}$  read [24, 25]

$$H_{\pm}(\Omega) = \exp\left(-\frac{\alpha\ell\gamma}{2(\gamma + i\Omega \mp i\Delta)}\right), \quad (1)$$

where  $\alpha$  is the resonance absorption coefficient for the intensity of the medium,  $\ell$  is the medium thickness and  $\gamma$  is the half width at half maximum (HWHM) of the resonances. Notice that Eq. (1) is obtained by using a time retarded by the transit time at the velocity  $c$ . Decomposing the unit vector of the linearly polarized incident field in the complex unit vectors associated with the circular polarizations  $\sigma_{\pm}$ , we apply the transfer functions  $H_{\pm}(\Omega)$  to these polarizations and project each of the resulting fields on the directions parallel and perpendicular to that of the incident field. In this way we get the transfer functions  $H_{\parallel}(\Omega)$  and  $H_{\perp}(\Omega)$  corresponding to the fields transmitted in each polarization. They read

$$H_{\parallel}(\Omega) = \frac{1}{2} [H_{+}(\Omega) + H_{-}(\Omega)], \quad (2)$$

$$H_{\perp}(\Omega) = \frac{i}{2} [H_{+}(\Omega) - H_{-}(\Omega)]. \quad (3)$$

When the probe wave is a continuous wave (cw) of optical frequency  $\omega_0$  ( $\Omega = 0$ ), the amplitude transmissions are reduced to  $H_{\parallel}(0) = e^{-\gamma\theta/\Delta} \cos\theta$  and  $H_{\perp}(0) = e^{-\gamma\theta/\Delta} \sin\theta$  where

$$\theta = \frac{\alpha\ell\Delta}{2\gamma(1 + \Delta^2/\gamma^2)} \quad (4)$$

is nothing more than the Faraday rotation angle of the field polarization in the medium.

The study of the time-dependent regime is greatly simplified by remarking that the transfer functions given in Eqs. (2) and (3) are such that  $H(\Omega) = H^*(-\Omega)$ , where the asterisk stands for the complex conjugate. Assuming that the envelope  $x(t)$  of the incident pulse is real positive (amplitude modulation), the envelope  $y(t)$  of the transmitted pulse will be also real, and a simple application of

the moment theorem [25] shows that its center of gravity is delayed by the group delay

$$\tau_g = \frac{i}{H(0)} \frac{dH(\Omega)}{d\Omega} \Big|_{\Omega=0} = -\frac{d\Phi(\Omega)}{d\Omega} \Big|_{\Omega=0}, \quad (5)$$

where  $\Phi(\Omega)$  is the argument of  $H(\Omega)$  [26]. It also results from the relation  $H(\Omega) = H^*(-\Omega)$  that  $|H(\Omega)|$  and  $d\Phi(\Omega)/d\Omega$  are stationary around  $\Omega = 0$ . If the spectrum of the incident pulse is narrow enough, the envelope  $y(t)$  of the transmitted pulse can thus be determined by approximating  $H(\Omega)$  by  $H(0) \exp(-i\Omega\tau_g)$  and reads as

$$y(t) \approx H(0) x(t - \tau_g). \quad (6)$$

The incident pulse will then be transmitted without significant distortion. Keeping in mind that we use retarded times, the transmission will be superluminal (or even have absolute time advancement) when  $\tau_g < 0$  and subluminal when  $\tau_g > 0$ .

From Eqs.(1)-(3) and (5), we easily derive the group delays for the parallel and perpendicular polarizations. They respectively read

$$\tau_{g\parallel} = -\frac{\theta}{\gamma(1 + \Delta^2/\gamma^2)} \left[ (1 - \Delta^2/\gamma^2) + \frac{2\Delta \tan\theta}{\gamma} \right], \quad (7)$$

$$\tau_{g\perp} = -\frac{\theta}{\gamma(1 + \Delta^2/\gamma^2)} \left[ (1 - \Delta^2/\gamma^2) - \frac{2\Delta}{\gamma \tan\theta} \right]. \quad (8)$$

The previous results are valid for arbitrary values of  $\Delta$ . We restrict ourselves in the following the analysis to the particular case  $\Delta = \gamma$ , a condition easily met with cold atoms for reasonable magnetic fields [22]. This case is of special interest because the group velocities for the circular polarizations  $\sigma_{+}$  and  $\sigma_{-}$  are both equal to  $c$  (luminal propagation) at the carrier-frequency of the incident pulse [27]. We then get  $\theta = \alpha\ell/4$ ,  $H_{\parallel}(0) = e^{-\theta} \cos\theta$ ,  $H_{\perp}(0) = e^{-\theta} \sin\theta$ ,  $\gamma\tau_{g\parallel} = -\theta \tan\theta$ , and  $\gamma\tau_{g\perp} = \theta/\tan\theta$ . A remarkable result is obtained when  $\theta = \pi/4$  ( $\alpha\ell = \pi$ ), for which  $|H_{\parallel}(0)| = |H_{\perp}(0)| = e^{-\pi/4}/\sqrt{2}$ ,  $\tau_{g\parallel} = -\pi/(4\gamma)$  and  $\tau_{g\perp} = \pi/(4\gamma)$ . Figure 2 shows the normalized intensity profiles of the transmitted pulses for an incident Gaussian pulse of envelope  $x(t) = \exp(-t^2/(2\sigma^2))$  with  $\sigma = 2.6/\gamma$ , a value conciliating moderate pulse distortion with significant fractional time advancement or delay. As expected, there is advancement for the polarization parallel and delay for the polarization perpendicular, these quantities both being nearly equal to  $\pi/(4\gamma)$ . Figure 2 also shows the corresponding amplitude transmission  $|H(\Omega)|$  and phase  $\Phi(\Omega)$  as functions of  $\Omega$ . We see that fast light and slow light obtained in the parallel and perpendicular polarizations are, respectively, associated with a minimum and a maximum of transmission at  $\Omega = 0$ . This result is often considered as a consequence of the Kramers-Kronig relations from which one gets

$$\Phi(\Omega) = \Phi_{KK}(\Omega) = -\mathcal{H} \{ \ln |H(\Omega)| \}, \quad (9)$$

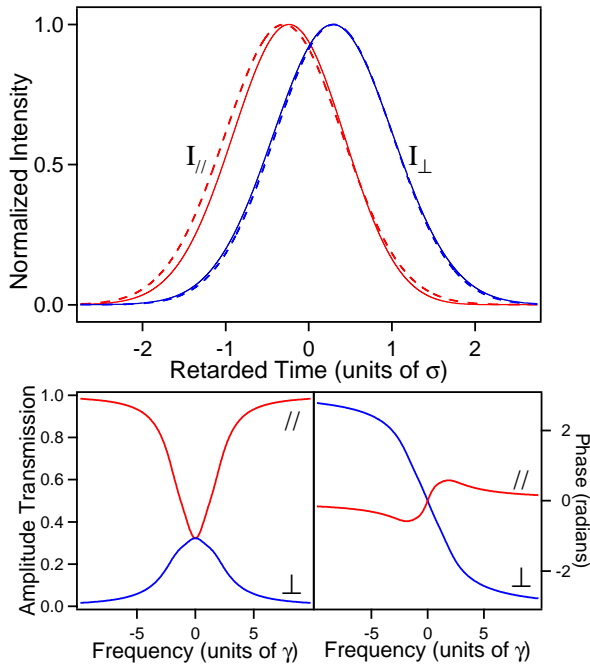


Figure 2: Top: normalized intensity profiles of the transmitted pulses in the polarizations  $\parallel$  and  $\perp$  for an incident pulse of Gaussian envelope  $x(t) = \exp[-t^2/(2\sigma^2)]$ . The solid (dashed) lines are the exact profiles obtained by Fourier transform (the approximate analytical profiles given by Eq.(6)). Parameters:  $\Delta = \gamma$ ,  $\theta = \pi/4$  ( $\alpha\ell = \pi$ ), and  $\sigma = 2.6/\gamma$ . Bottom: corresponding amplitude transmission and phase as a function of  $\Omega$ .

where  $\mathcal{H}$  designates the Hilbert transform [25]. The group delay then reads

$$\tau_g = \tau_{KK} = -\left. \frac{d\Phi_{KK}(\Omega)}{d\Omega} \right|_{\Omega=0} \quad (10)$$

In reality, Eq. (9) only holds if  $H(\Omega)$  is a minimum-phase-shift (MPS) function [25, 28]. This condition is met for purely propagative systems but may fail for systems involving polarizers [16, 18–20]. With  $\Omega$  being continued in the complex plane, the condition to ensure that  $H(\Omega)$  is MPS is that all its zeros lie in the upper half plane [ $\text{Im}(\Omega) > 0$ ]. Assuming again that  $\Delta = \gamma$ , it is easily shown that the zeros that may eventually have a negative imaginary part will occur at

$$\Omega = i\gamma \left( 1 - \sqrt{\frac{4\theta}{(2p-1)\pi} - 1} \right) \quad (11)$$

for  $H_{\parallel}(\Omega)$  and at

$$\Omega = i\gamma \left( 1 - \sqrt{\frac{2\theta}{p\pi} - 1} \right) \quad (12)$$

for  $H_{\perp}(\Omega)$ , where  $p$  is a positive integer. No such zeros exist for  $0 < \theta < \pi/2$  and  $H(\Omega)$  is MPS for both polarizations. In the conditions of Fig. 2 ( $\theta = \pi/4$ ), we have

numerically verified that the group delays derived from Eqs. (9) and (10) are actually equal to the exact values  $\pm\pi/(4\gamma)$ . When  $\theta > \pi/2$ ,  $H(\Omega)$  is not MPS, at least for one polarization. It can then be written as the product of a MPS transfer function by the transfer function  $H_{AP}(\Omega)$  of a causal all-pass filter of the form

$$H_{AP}(\Omega) = \prod_p \left( \frac{\Omega - \Omega_p}{\Omega - \Omega_p^*} \right), \quad (13)$$

where  $\Omega_p$  designates the zeros of  $H(\Omega)$  actually lying in the lower complex half plane [25, 28, 29]. As shown in Eqs.(11) and (12), these zeros are purely imaginary and the contributions of  $H_{AP}(\Omega)$  to add to the phase shift  $\Phi_{KK}(\Omega)$  and to the group delay  $\tau_{KK}$  respectively given by Eqs.(9) and (10) take the simple forms

$$\Phi_{AP}(\Omega) = -2 \sum_p \tan^{-1} \left( \frac{\Omega}{i\Omega_p} \right), \quad (14)$$

$$\tau_{AP} = -\left. \frac{d\Phi_{AP}(\Omega)}{d\Omega} \right|_{\Omega=0} = 2 \sum_p \frac{1}{i\Omega_p}. \quad (15)$$

Note that all the terms intervening in Eq. (15) are positive and thus contribute to an increase of the group delay. When  $\tau_{KK} < 0$ , this contribution can even change the advancement predicted by the Kramers-Kronig relations in a delay of comparable magnitude. Figure 3, obtained for  $\theta = 3\pi/4$  ( $\alpha\ell = 3\pi$ ), illustrates such a case. We have then  $|H_{\parallel}(0)| = |H_{\perp}(0)| = e^{-3\pi/4}/\sqrt{2}$ ,  $\tau_{g\parallel} = 3\pi/(4\gamma)$  and  $\tau_{g\perp} = -3\pi/(4\gamma)$ . For both polarizations, the amplitude transmission  $|H(\Omega)|$  has a well-marked dip at  $\Omega = 0$ . Equation (12) shows that  $H_{\perp}(\Omega)$  has no zeros in the lower half-plane and thus is MPS. As usual, the corresponding transmitted pulse is advanced, and we have again verified that  $\tau_g = \tau_{KK} = -3\pi/(4\gamma)$ . On the other hand,  $H_{\parallel}(\Omega)$  is not MPS. Equation (11) indeed shows that it has one active zero  $\Omega_1 = -i\gamma(\sqrt{2}-1)$  in the lower half plane. The associated time delay reads as  $\tau_{AP} = 2/(i\Omega_1) = 2/[\gamma(\sqrt{2}-1)] \approx 4.828/\gamma$ . From Eqs. (9) and (10), we get in this case  $\tau_{KK} = -2.472/\gamma$  and, finally,  $\tau_{g\parallel} = \tau_{KK} + \tau_{AP} \approx 2.356/\gamma$  in agreement with the expression  $\tau_{g\parallel} = 3\pi/(4\gamma)$  given above. We are actually in a case where the time delay has a value nearly opposite to that derived by an irrelevant application of the Kramers-Kronig relations. The transmitted pulse is then delayed in spite of a dip in the system transmission at the pulse carrier frequency.

Singular behaviors are obtained when  $\theta = (2k-1)\pi/2$  for  $H_{\parallel}(\Omega)$  and when  $\theta = k\pi$  for  $H_{\perp}(\Omega)$  where  $k$  is a positive integer. The cw transmission  $|H(0)|$  is then null, and the phase  $\Phi(\Omega)$  displays a  $\pi$  discontinuity at  $\Omega = 0$ . Note that such phase singularities have been clearly recognized in the analysis of the pioneering experiments on non dispersive birefringent media [17, 18]. As  $|H(0) = 0|$ , the area  $\int_{-\infty}^{+\infty} y(t) dt$  of the envelope  $y(t)$  of the transmitted pulse is also null [30], its center of gravity is not defined

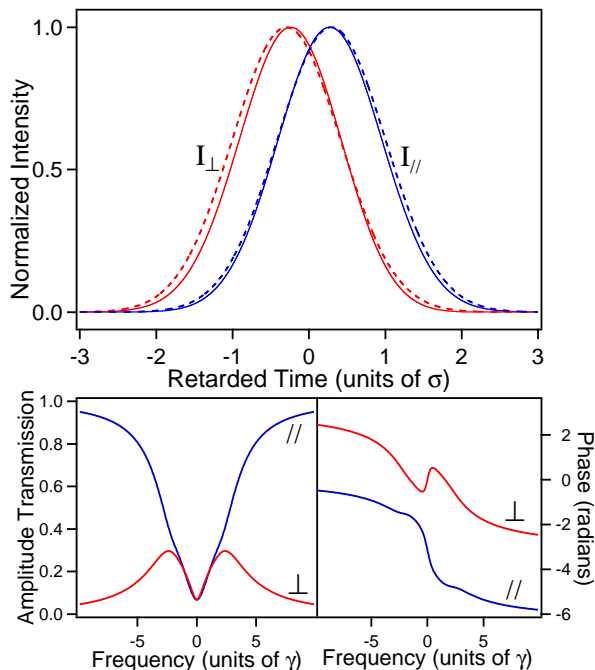


Figure 3: Same as Fig. 2 for  $\Delta = \gamma$ ,  $\theta = 3\pi/4$  ( $\alpha l = 3\pi$ ) and  $\sigma = 8.0/\gamma$ . The duration  $\sigma$  of the incident pulse has been chosen in order that the distortion of the transmitted pulse does not exceed that of Fig. 2.

and the pulse distortion is considerable, as narrow as the spectrum of the incident pulse may be. Figure 4 shows the results obtained for  $\theta = \pi/2$ . There is no problem for  $H_{\perp}(\Omega)$  with  $H_{\perp}(0) = e^{-\pi/2}$ ,  $\tau_{g\perp} = 0$  and, if its spectrum is narrow enough, the incident pulse will propagate without significant distortion at the velocity  $c$ . On the other hand  $H_{\parallel}(0) = 0$ . The phase  $\Phi_{\parallel}(\Omega)$  has the predicted discontinuity and the area of  $y_{\parallel}(t)$  is actually null. Again if the spectrum of  $x(t)$  is narrow enough,  $y_{\parallel}(t)$  can be calculated by using the power-series expansion of  $H_{\parallel}(\Omega)$  at the first order in  $\Omega$ , which reads  $H_{\parallel}(\Omega) \approx \theta e^{-\theta} (i\Omega/\gamma)$ . Passing in the time domain ( $i\Omega \rightarrow d/dt$ ), we get

$$y_{\parallel}(t) = \frac{\pi e^{-\pi/2}}{2\gamma} \frac{dx}{dt} = -\frac{\pi e^{-\pi/2}}{2\gamma\sigma} \left(\frac{t}{\sigma}\right) \exp\left(-\frac{t^2}{2\sigma^2}\right). \quad (16)$$

For the value  $\sigma = 2/\gamma$  used in Fig. 4,  $y_{\perp}(t) = e^{-\pi/2}x(t)$  and  $y_{\parallel}(t)$  given by Eq. (16) appear to be good approximations of the exact envelope of the transmitted pulses.

When  $\theta$  is close to the pathologic values considered in the previous paragraph, the phase discontinuity is replaced by a rapid variation around  $\Omega = 0$ , and in agreement with the relations  $\gamma\tau_{g\parallel} = -\theta \tan\theta$  and  $\gamma\tau_{g\perp} = \theta/\tan\theta$ , one of the group delays takes very large values. However, the fractional delays (delays in units of  $\sigma$ ) with moderate distortion that can be obtained are not significantly larger than those evidenced in the reference case  $\theta = \pi/4$  (Fig.2). Indeed, the spectral domain where  $d\Phi(\Omega)/d\Omega$  and  $|H(\Omega)|$  do not vary too considerably is very narrow and large pulse durations  $\sigma$  are necessary

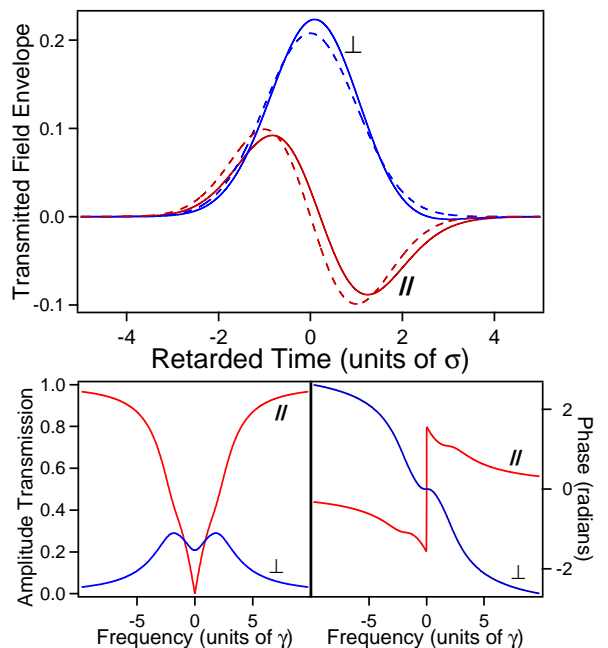


Figure 4: Top: envelopes  $y_{\parallel}(t)$  and  $y_{\perp}(t)$  of the transmitted pulses for  $x(t) = \exp[-t^2/(2\sigma^2)]$ . The solid (dashed) lines are the exact envelopes obtained by fast Fourier transforms (the approximate analytical envelopes  $y_{\parallel}(t)$  given by Eq. (16) and  $y_{\perp}(t) = e^{-\pi/2}x(t)$ ). Parameters:  $\Delta = \gamma$ ,  $\theta = \pi/2$  ( $\alpha l = 2\pi$ ), and  $\sigma = 2.0/\gamma$ . Bottom: corresponding amplitude transmission and phase as a function of  $\Omega$ .

to avoid significant distortion of the transmitted pulse. In addition, the corresponding transmission is very low. When, e.g.,  $\theta = (\pi/2) - (\pi/20)$ , we have  $\gamma\tau_{g\parallel} = -19$  but a numerical simulation shows that  $\sigma$  as large as  $47/\gamma$  is required to obtain a distortion comparable to that of the reference case. The fractional time-advancement of the transmitted pulse is then about 40% instead of 30% in the reference case but this is at the expense of a reduction of the pulse intensity by a factor exceeding 300!

The observation of slow light and, particularly, of fast light requires the use of incident pulses with ideally smooth envelope. We now consider briefly the opposite case where the incident field is switched with a rise time infinitely short with respect to all the characteristic times of the system but long compared to  $1/\omega_0$ , so that the slowly varying envelope approximation remains valid [1]. If the envelope  $x(t)$  of the incident pulse is a Heaviside unit step function  $u_H(t)$ , as currently considered in the study of optical precursors [6], that of the transmitted pulse reads

$$y(t) = \int_{-\infty}^t h(t') dt', \quad (17)$$

where  $h(t)$  is the impulse response of the system, the inverse Fourier transform of its transfer function  $H(\Omega)$ .

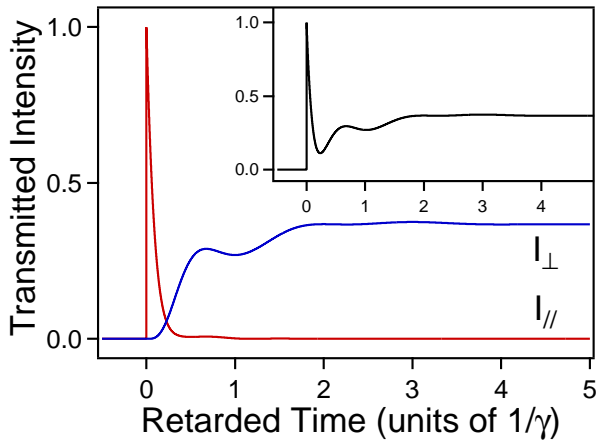


Figure 5: Numerically determined intensity profiles of the transmitted pulses when the incident pulse is step modulated with  $x(t) = u_H(t)$ . Parameters:  $\theta = \pi/2$  and  $\Delta = \pi\gamma$  ( $\alpha\ell = 1 + \pi^2$ ). The optical precursor and the main field are, respectively, detected in the parallel and perpendicular polarizations. The inset shows the intensity profile obtained in the absence of the polarization beam splitter.

Using standard results of Laplace transforms [31], we get

$$h_{\parallel}(t) = \delta(t) - \gamma\alpha\ell \frac{J_1(\sqrt{2\gamma\alpha\ell t})}{\sqrt{2\gamma\alpha\ell t}} \cos(\Delta t) e^{-\gamma t} u_H(t), \quad (18)$$

$$h_{\perp}(t) = \gamma\alpha\ell \frac{J_1(\sqrt{2\gamma\alpha\ell t})}{\sqrt{2\gamma\alpha\ell t}} \sin(\Delta t) e^{-\gamma t} u_H(t). \quad (19)$$

where  $\delta(z)$  and  $J_1(z)$  respectively designate the Dirac delta function and the Bessel function of the first kind

and index 1. Interesting features are obtained when  $\theta = \pi/2$  whatever  $\Delta$  is. Figure 5 shows the result obtained in this case for  $\Delta = \pi\gamma$ , a value chosen so that the precursor and steady state or main field have comparable intensities and are not clearly distinguishable in the absence of the PBS (see inset of Fig. 5). A similar situation occurs in experiments involving a single absorption line. See Ref. [32] and the related discussion in [33]. When the PBS is operating, our system separates the transmitted field into two parts. The part detected in the parallel polarization can be attributed to the precursor. Indeed, insofar as the field polarization does not rotate instantaneously, its initial intensity equals 1 but tends to zero as soon as the polarization rotates by  $\pi/2$ . On the contrary, the part detected in the perpendicular polarization starts from zero before asymptotically increasing to a steady value and is nothing but the so-called main-field in the precursor theory.

To summarize, we have proposed a hybrid system associating the effects of medium dispersion with the effects of polarization selection. At least conceptually, this system is relatively simple. It enables one to obtain simultaneous fast and slow light. Due to the non minimum phase shift of its transfer function, it presents a great variety of behaviors only a few of which have been explored. Additionally it can also be used to separate optical precursor and main field when the incident wave is step modulated.

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