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# Utilization of Spreading Codes as dedicated waveforms for Active Multi-Static Primary Surveillance Radar

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**Abstract:** *This paper deals with the selection of some waveforms family sets for use in Active Multi-Static Primary Surveillance Radar (MSPSR). The selection takes into account the correlation properties, the ambiguity function properties, the Peak to Average Power Ratio (PAPR) reduction, the cardinality and the diversity of each set. According to these criteria, the application of spreading codes in the MSPSR context is investigated.*

**Key words:** *active MSPSR, correlation, ambiguity function, PAPR, spreading codes.*

## 1. Introduction

Nowadays, MSPSR systems are sustainably settled in air surveillance program [1]. Compared to mono-static radar currently in use, an MSPSR system is based on a sparse network of transmitters (Tx) and receivers (Rx) interconnected to a Central Unit and offers advantages in terms of reliability, cost and performance.

Two kinds of MSPSR systems exist: the Passive form and the Active one. While the Passive MSPSR uses transmitters of opportunity such as radio Frequency Modulation (FM) transmitters and/or Digital Video Broadcasting-Terrestrial (DVB-T) transmitters [2], the Active MSPSR uses dedicated transmitters, which emit a waveform that is controlled and designed for a radar application. Each receiver processes the signal coming from all transmitters and reflected on the targets; and the Central Unit restores the target location by intersecting “ellipsoids” from all (transmitter, receiver) pairs.

Compared to passive MSPSR, the main advantages of the active MSPSR are the use of dedicated waveforms that allow reaching better performances (like a better association of the transmitters' contributions at the receiver level); more flexibility in the deployment of transmitters and receivers station (in order to meet the requirements in localisation accuracy and in horizontal and altitude coverages); and the guarantee of having a service continuity.

Building on the works carried out by the Telecommunication field for solving multi-user issues, this paper investigates the application of Spreading Codes in MSPSR concept.

Thus, the purpose of this paper is to compare different spreading codes taking into account some criteria such as the family size, length diversity, auto-correlation and cross-correlation properties, and PAPR. Then according to these criteria, some families are selected.

## 2. Codes criteria

### 2.1 Cardinality and diversity

The family size and the length of the sequences are very important parameters for designing a set of waveforms for use in MSPSR systems. The first reason is, assuming that each transmitter has its own dedicated waveform; it can be shown that the family must be composed of at least twenty sequences. The second reason is that each transmitter emits a Continuous Wave (CW) composed of the repetition of the same pattern: for example the spreading sequence. Therefore, when the receivers process the signal it appears range ambiguities that corresponds to the sequence length. Thus, to eliminate the range ambiguities the length of the sequences must be greater than the radar instrumented range.

### 2.2 Correlation properties

One of the critical stages in the process of selecting the waveform family sets is the correlation functions study. In MSPSR systems, each receiver correlates the received signal and replicas of each transmit signals. Therefore it is necessary to minimize the interference between the different transmitters, which means to minimize the cross-correlation level; and to maximize the Peak to Side Lobe Ratio (PSLR) of the auto-correlation function.

### 2.3 Ambiguity function properties

Another critical stage is the analysis of delay and Doppler effects on the waveform. Using the ambiguity functions (auto- and cross-ambiguities), robustness against delay and Doppler effects must be investigated for the various waveform family sets. Such as the correlation properties, the cross-ambiguity function must have the lowest possible level for all Doppler shifts and all delays; and the auto-ambiguity must look like a thumbtack function.

### 2.4 Peak to Average Power Ratio (PAPR)

PAPR describes the ratio between peak power and average power of the continuous signal. A high PAPR reduces the efficiency of the Power Amplifier in the sense that a back-off is necessary for working in the linear range. Thus the lower the PAPR is, the more efficient is the waveform for a given peak power.

## 3. Spreading Sequences

### 3.1 Gold codes / Pseudo-Gold

Gold codes have been developed to solve the cardinality issue of the *maximal connected set* of the *maximal-length sequences (m-sequences)* [3][4]. They are generated by taking a *preferred pair* of *m-sequence* of length  $N$  and doing the modulo-2 addition of one sequence of the *preferred pair* with a shifted version of the other one. The result is a family of  $N+2$  binary sequences of length  $N$  with cross-correlation and out-of-phase auto-correlation taking values in the set  $\{-1; -t(n); t(n) - 2\}$  with:

$$N = 2^n - 1 \quad \text{with } n \in \mathbb{N}^* \neq 4k \quad (1)$$

$$t(n) = \begin{cases} 2^{\frac{n+1}{2}} + 1 & \text{if } n \text{ odd} \\ 2^{\frac{n+2}{2}} + 1 & \text{if } n \text{ even} \end{cases} \quad (2)$$

$n$  represents the number of registers required for generating the *m-sequence*. And in case where  $n$  is a multiple of four there is no *preferred pair*, therefore no Gold sequence can be generated. But based on the work of Niho [3][5], it is possible to find a pseudo preferred pair

of length  $N$ . Therefore it is possible to generate a family that we call “pseudo-Gold” by using this pseudo preferred pair in the Gold generation procedure. The result is a family of  $N+2$  binary sequences of length  $N$  with cross-correlation and out-of-phase auto-correlation taking values in the set  $\{-1; t(n) - 2; -s(n); s(n) - 2\}$ .

$$N = 2^n - 1 \quad \text{with } n \in \mathbb{N}^* = 4k \quad (3)$$

$$s(n) = 2^{\frac{n}{2}} + 1 \quad (4)$$

### 3.2 Kasami codes

Kasami codes are composed of two sets, the Small Set and the Large Set [6][7][8]:

- The Small Set is generated by taking a  $m$ -sequence  $u(t)$  of length  $N$  and the  $q$ -decimation  $v(t)$  of  $u(t)$  with  $q = 2^{n/2} + 1$  and by doing the modulo-2 addition between the first sequence  $u(t)$  and a shift version of the second one  $v(t)$ . The result is a family of  $2^{n/2}$  binary sequences of length  $N$  with cross-correlation and out-of-phase auto-correlation taking values in the set  $\{-1; -s(n); s(n) - 2\}$ .

$$N = 2^n - 1 \quad \text{with } n \in \mathbb{N}^* = 2k \quad (5)$$

- The Large Set is generated by taking a  $m$ -sequence  $u(t)$  of length  $N$ , the  $q$ -decimation  $v(t)$  of  $u(t)$  with  $q = 2^{n/2} + 1$  and the  $q'$ -decimation  $w^{(k)}(t)$  of the  $k$ -shifted version of  $u(t)$  with  $q' = 2^{(n+2)/2} + 1$  and  $k \in [0; \text{gcd}(q', N) - 1]$ , and by doing the modulo-2 addition between the first sequence  $u(t)$  and a shift version of the second  $v(t)$  and third one  $w^{(k)}(t)$ . The result is a family of  $2^{n/2}(2^n + 1)$  or  $2^{n/2}(2^n + 1) - 1$  binary sequences of length  $N$  depending on whether  $n=2[4]$  or  $n=0[4]$  with cross-correlation and out-of-phase auto-correlation taking values in the set  $\{-1; -t(n); t(n) - 2; -s(n); s(n) - 2\}$ .

$$N = 2^n - 1 \quad \text{with } n \in \mathbb{N}^* = 2k \quad (6)$$

### 3.3 Legendre and Weil

Legendre and Weil codes are binary sequences built from a Legendre sequence  $a(t)$  of length  $N$  prime and generated by doing the modulo-2 addition between  $a(t)$  and a shift version of itself [9][10]. The result is a family of  $(N+1)/2$  binary sequences of length  $N$  with cross-correlation and out-of-phase auto-correlation bounded by  $2\sqrt{N} + 5$ .

### 3.4 No codes

No codes are binary sequences used in the cryptographic field. Introduced by Kumar and No [11] and generalized by No [12] they are generated by using the property of the primitive element on the Galois Field  $\text{GF}(2^n)$  and the trace function [13]. The characteristics of these sequences are a family of  $\sqrt{N+1} = 2^{n/2}$  binary sequences of length  $N$  with cross-correlation and out-of-phase auto-correlation bounded by  $\sqrt{N+1} + 1$ .

$$N = 2^n - 1 \quad \text{with } n \in \mathbb{N}^* = 2k \quad (7)$$

### 3.5 Trace Norm (TN) codes

TN codes are also binary sequences used in the cryptographic field. Their generation is almost similar to the No codes generation except the additional use of the Norm function [14]. The characteristics of these sequences are a family of  $\sqrt{N+1} = 2^{n/2}$  binary sequences of length  $N$  with cross-correlation and out-of-phase auto-correlation bounded by  $\sqrt{N+1} + 1$ .

$$N = 2^n - 1 \quad \text{with } n \in \mathbb{N}^* = 2km \quad (8)$$

### 3.6 Zadoff-Chu codes

Zadoff-Chu codes are complex sequences with ideal periodic auto-correlation functions and defined by [15][16]:

$$ZC_{q,i}(u) = \begin{cases} \exp\left(-j2\pi \frac{r_i}{N} \left(\frac{u^2}{2} + qu\right)\right) & \text{if } n \text{ even} \\ \exp\left(-j2\pi \frac{r_i}{N} \left(\frac{u(u+1)}{2} + qu\right)\right) & \text{if } n \text{ odd} \end{cases} \quad (9)$$

Where  $u \in [0 : N - 1]$ ,  $n \in \mathbb{N}$  and  $r_i$  the index of  $i$ -th sequence such as  $\text{gcd}(r_i, N) = 1$ . If  $N$  is a prime, the characteristics of these sequences are a family of  $N - 1$  polyphase sequences of length  $N$  with cross-correlation and out-of-phase auto-correlation bounded by  $\sqrt{N}$  [16].

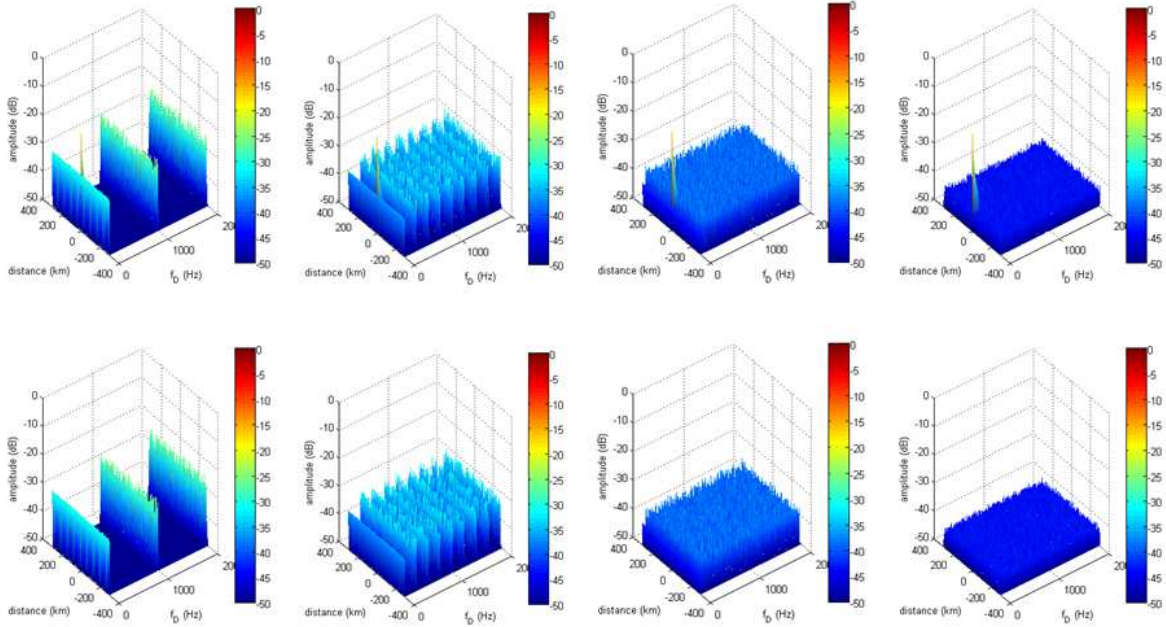
## 4. Simulation results

In order to compare all families presented above, all ambiguity functions have been calculated with different sequence's lengths in the configuration describes in **Tab. 1**.

**Table 1.** Simulation parameters

Simulation Parameters	Integration Time	Chip duration	Target Doppler Max	Target Delay Max
Value	130 ms	1 $\mu$ s	2000 Hz (= 300 m/s in L Band)	1 ms

Let  $f_D$  denotes the Doppler variable, **Fig. 1** shows the behaviour of the auto-ambiguity and cross-ambiguity function of a Small Set of Kasami sequence.

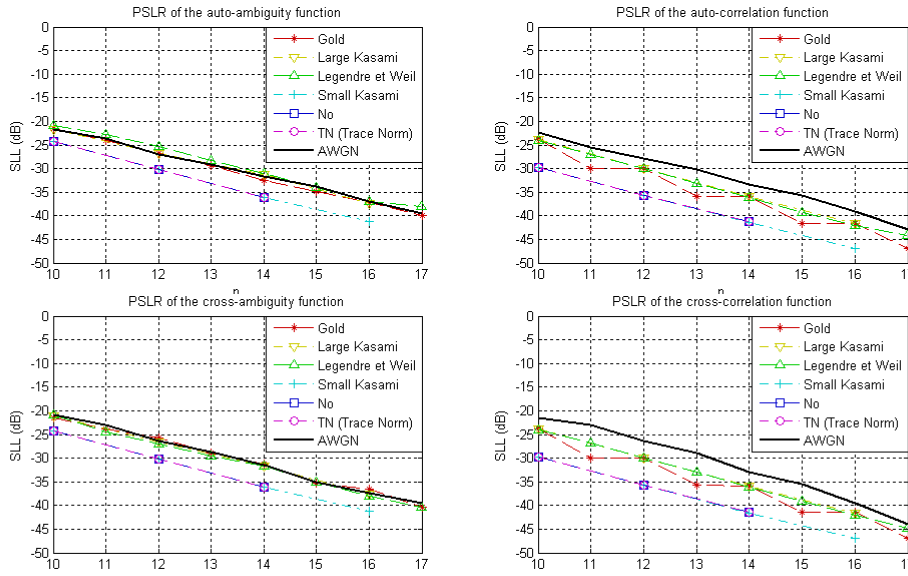


**Figure 1.** Auto-ambiguity (top) and cross-ambiguity (bottom) functions of a sequence of the Small Set of Kasami for  $N = 2^n - 1$  with  $n = \{10; 12; 14; 16\}$  (from left to right), using a chip duration of  $1\mu\text{s}$  and an integration time of 130 ms, which corresponds to a number of periods  $N_{\text{period}}$  of  $\{128; 32; 8; 2\}$  respectively.

Different observations can be done. First the recurrent lobes at Doppler intervals of the inverse of the pulse repetition interval in **Fig. 1** are a penalty related to the use of a coherent train of  $N_{\text{period}}$  periods [17][18]. Second the longer the sequences are, the better the ambiguity functions

tend to a thumbtack for the auto-ambiguity function and a very low and flat for the cross-ambiguity function. Other simulations, here due to lack of space not presented, have shown a similar behaviour for all families presented above; except for the Zadoff-Chu family, where the Doppler effect causes a shift in time delay of the compressed pulse like in Linear Frequency Modulation (LFM).

To complete the comparison between the different families, **Fig. 2** shows the PSLR of the correlation functions and the ambiguity functions compared to those of an Additive White Gaussian Noise (AWGN) of the same length and periodized for reaching the same integration time.



**Figure 2.** Evolution of the PSLR of the auto-ambiguity (top left), cross-ambiguity (bottom left), auto-correlation (top right), and cross-correlation (bottom right) functions for all families depending of the sequence length  $N$  with  $n = \log 2(N)$ , using a chip duration of  $1\mu\text{s}$  and with an integration time of  $130\text{ ms}$ .

As expected it appears in **Fig. 2** that the evolution of the PSLR of the correlation functions correspond to the theory (See **Tab. 2.**). It also appears that the evolution of the PSLR of the ambiguity functions looks like those of the AWGN, except for Small Kasami, No and TN which are  $\sim 4\text{ dB}$  lower.

**Table 2.** Upper Bound of the PSLR of the correlation functions

Family Name	Gold	Pseudo-Gold	Small Kasami	Large Kasami	Legendre & Weil	No	TN
PSLR	$t(n)/N$	$(t(n)-2)/N$	$s(n)/N$	$t(n)/N$	$(2\sqrt{N}+5)/N$	$s(n)/N$	$s(n)/N$

The simulation results show that all those spreading sequences can achieve very low side-lobe levels. Especially, the Small Set of Kasami, the No and the TN families offer better performances than all the other families studied here. Another advantage of those sequences is the constant envelope of the signal. This property means that the PAPR is one, allowing a better efficiency of the Power Amplifier.

Thus, those three families seem to be good candidates for an application in an active MSPSR system.

## 6. Conclusion

In this paper we present a comparison of the performances of different spreading codes for an application in active MSPSR system. With numerical simulations it is shown that the Small Set of Kasami, the No and the TN families exhibit the best performances, thus positioning them as possible candidates for active MSPSR waveforms. Future work will study in more details these pre-selected waveforms (mismatched filter, windowing), as well as other possible telecommunications waveforms.

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