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# Contraction, propagation and bisection on a validated simulation of ODE<sup>1</sup>

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## Introduction

Many tools used for the validated simulation of initial value problem of ordinary differential equations (ODE) provide abstraction of the solution under the form of a list of boxes obtained by time discretization and local polynomial interpolation. Majority of them are based on Taylor methods (Vnode-LP or CAPD) or on Runge-Kutta methods (DynIbex). If some information on the system state at a given time are given (*e.g.*, from measurements, or guard intersection [1]), it is complex to take this information into account with a low computation cost. We propose a contraction/propagation algorithm to use this information in an elegant manner. This approach will allow one to avoid some costly steps which would appear in running a new simulation (time-step computation and Picard operator).

## Main idea

We consider an interval initial value problem of ODE of the form:

$$\dot{y} = f(y, p) \text{ with } y(0) \in [y_0] \text{ and } p \in [p] . \quad (1)$$

A validated simulation of (1) is then given in form of two lists of boxes:  
i) the *a priori* enclosures:  $\{[\tilde{y}_0], \dots, [\tilde{y}_N]\}$ , with  $y(t) \in [\tilde{y}_i] \forall t \in [t_i, t_{i+1}]$

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and ii) the tight enclosures  $\{[y_0], \dots, [y_N]\}$ , with  $y(t_i) \in [y_i]$ .

If an information provides  $y(t^*) \in [y^*]$ , then a contractor is used at  $t = t^*$  following the two steps

- add a  $k^{\text{th}}$  integration step to the time discretization:  
 $\{[y_0], \dots, [y_i], [y_k], [y_{i+1}], \dots, [y_N]\}$  s.t.  $y(t^*) \in [y_k]$  and  $t_k = t^*$
- apply the basic contractor  $[y_k] := [y_k] \cap [y^*]$

Then a Picard contractor [2] on  $[\tilde{y}_i]$  and a validated Runge-Kutta contractor [2] on  $[y_i]$  can be apply on each integration step  $i$ , in order to propagate (in forward for  $t > t^*$  and backward for  $t < t^*$ ) this information on the whole simulation, *i.e.*, on all the boxes in the lists.

**Remarks** The bisection can be seen as a copy followed by two contractions, then the bisection of a simulation w.r.t. a given time is available with our approach. A propagation of a contraction on one state is a contraction on a simulation [3]. It is easy to generalize to a contraction on parameters. We can also generalize to an interval of time during one we obtain information.

**Applications** Bisection on initial state to avoid an obstacle; attainability of an objective at a given time; parameter synthesis w.r.t. some constraints; etc.

## References

- [1] A. EGGERS, N. RAMDANI, N. S. NEDIALKOV, AND M. FRÄNZLE, Improving the SAT modulo ODE approach to hybrid systems analysis by combining different enclosure methods, *Software and System Modeling*, 2015.
- [2] J. ALEXANDRE DIT SANDRETTO, AND A. CHAPOUTOT, Validated Explicit and Implicit Runge-Kutta Methods, *Reliable Computing*, 2016. To appear.
- [3] A. BETHENCOURT, AND L. JAULIN, Solving Non-Linear Constraint Satisfaction Problems Involving Time-Dependant Functions, *Mathematics in Computer Science*, 2014.