DNS of turbulent channel flow: can we imitate conjugate heat-transfer with a Robin boundary condition?

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Abstract – Turbulent heat fluxes, temperature variance and associated budgets obtained from the Direct Numerical Simulation of an incompressible turbulent channel flow with a Reynolds number of 150, based on the wall friction velocity, and a Prandtl number of 0.71 are presented and analysed for four cases : imposed temperature at the wall (Dirichlet), imposed heat flux (Neumann), heat exchange coefficient (Robin) and 3D conjugate heat transfer. Present results show that a Robin boundary condition can imitate most of the one point statistics obtained with conjugate heat-transfer accurately compared to the statistics obtained with an imposed temperature or heat flux at the wall.

1. Introduction

Most of the DNS of the turbulent channel flow performed with a passive scalar are based on an imposed temperature at the wall (Kasagi et al. 1992, Kawamura et al. 1998). When the temperature is imposed at the wall, there is a close similarity between thermal and momentum streaks (Abe et al. 2009). In a small number of DNS, a constant heat flux is imposed at the wall (Tiselj et al. 2001). However, it is widely recognized that neither isothermal nor isoflux boundary conditions can realistically mimic the actual heat transfer in real life where the wall has a thickness, especially when the thermal diffusivity and conductivity of the fluid and solid are of the same order of magnitude. When such a coupling is considered, it is referred as conjugate heat-transfer. Tiselj et al. (2001) were, to the authors' knowledge, the first to investigate by DNS the influence of the thermal boundary condition through direct comparison between conjugate heat transfer, imposed temperature and imposed heat flux at the wall.

In the present paper, the turbulent heat-fluxes, the temperature variance

and the associated budget obtained for three different boundary conditions (imposed temperature (isoT), imposed heat flux (isoQ), heat exchange coefficient (Robin)) and conjugate heat transfer (Conjug) are compared. In the present conjugate simulations, the ratio of thermal diffusivity and conductivity between the solid and the fluid can be equal to 2, 1 and 0.5: 9 conjugate simulations were performed (the heat flux is imposed at the outer wall of the solid domain).

2. Numerical setup

Present simulations are based on the open-source software Incompact3d (https://code.google.com/p/incompact3d/) developed at Université de Poitiers and Imperial College London by Laizet et al. (2011). The details of the numerical methods employed here are similar to the ones given in Flageul et al. (2014): high-order compact finite difference schemes in the fluid domain are combined with a mixed finite-difference/spectral discretization in the solid domain. The number of computational cells is equal to 256x193x256 and the domain size is [25.6,2,8.52]. For the conjugate heat transfer cases, the equation of evolution of the passive scalar is

$$\partial_t T + \partial_j (T u_j) = \alpha_f \partial^2_{jj} T \quad \text{in the fluid and} \quad \partial_t T_s = \frac{\alpha_f}{G_1} \partial^2_{jj} T_s \quad \text{in the solid}$$
$$T_f = T_s \quad \text{and} \quad \partial_y T_f = G_2 \partial_y T_s \quad \text{at the interface.} \qquad (1)$$

where G_1 is the ratio of thermal diffusivities α_f/α_s and G_2 is the ratio of thermal conductivities λ_s/λ_f . Based on those dimensionless numbers, the thermal activity ratio K can be defined with $1/K = G_2\sqrt{G_1}$ (Tiselj et al. (2012)). In this short version of the paper, 4 conjugate simulations are considered: two for $K = 1/\sqrt{2}$ $(G_1, G_2) = (2,1)$ and $(G_1, G_2) = (1/2,2)$ and two for $K = \sqrt{2}$ $(G_1, G_2) = (1/2,1)$ and $(G_1, G_2) = (2,1/2)$. For both values of K , a specific Robin boundary condition was designed to imitate the conjugate cases, as explained in the long version of the paper.

Theoretical analysis shows that a very low (resp. high) thermal conductivity in the solid domain is equivalent to an imposed heat-flux (resp. imposed temperature) at the fluid boundary. Therefore, for a given G_1, the lower G_2, the higher K, the closer to the isoQ case.

3. Results

In figure 1, the temperature variance obtained for the conjugate cases and the Robin ones are between the isoT and the isoQ one. There is a remarkable

agreement between the conjugate heat-transfer cases and the Robin ones. In figure 2, it is similar for the dissipation rate associated with the temperature variance, although the agreement is more qualitative. For the ratio of thermal properties considered in the present conjugate simulations, the temperature variance and the associated dissipation rate depends only on the thermal activity ratio K.
The correlation coefficient associated with the turbulent heat flux is $< u'_j T' > /(u_{jRMS} T_{RMS})$. In figure 3, the one associated with the streamwise turbulent heat-flux obtained with conjugate heat transfer and a Robin boundary condition lies between the isoT and isoQ cases. The Robin b. c. cases are very close to the isoT one. In addition, for a given thermal activity ratio, the correlation coefficients for the conjugate cases do not overlap.
The long version of the paper contains additional results: the correlation associated with the wall-normal heat flux, the autocorrelation of the temperature and of the wall-normal turbulent heat flux at the wall and a detailed analysis of thermal dissipation rate ε_θ at the fluid-to-solid interface.

5. Discussion

On the one hand, our results show that a Robin boundary condition can imitate accurately conjugate heat-transfer: the turbulent heat fluxes, the temperature variance and the associated budget are in fairly good agreement. On the other hand, some statistics show that there remains a difference between conjugate and non-conjugate heat-transfer, especially for the streamwise turbulent heat flux correlation coefficient as well as the two-point autocorrelations of the temperature and wall normal heat flux at the wall.
The analysis in the long version of the paper shows that in the case of conjugate heat transfer, there is a compatibility condition that connects the temperature and turbulent heat flux at the wall. It is a product in the spectral space, which is equivalent to a convolution in the physical space. Such a condition is non-local and cannot be imitated by a Robin boundary condition with constant coefficients.

6. Conclusion

As far as one point statistics or RANS models are concerned a Robin type boundary condition may economically replace conjugate heat transfer simulations. Second moments and their budgets are well reproduced by this condition using constant coefficients.
It is yet to demonstrate that this conclusion is still valid for ratio of thermal properties farther from unity than the ones considered here. When the Prandtl

number is 0.01, the temperature variance at the wall depends both on K and G_1 (Tiselj et al. (2012)).
From a LES perspective, however, there is a fundamental difference between conjugate and non-conjugate heat transfer visible on two point statistics. Small-scale thermal structures are more dominant for the Robin boundary conditions cases compared with the ones obtained in the conjugate case.

7. Acknowledgements

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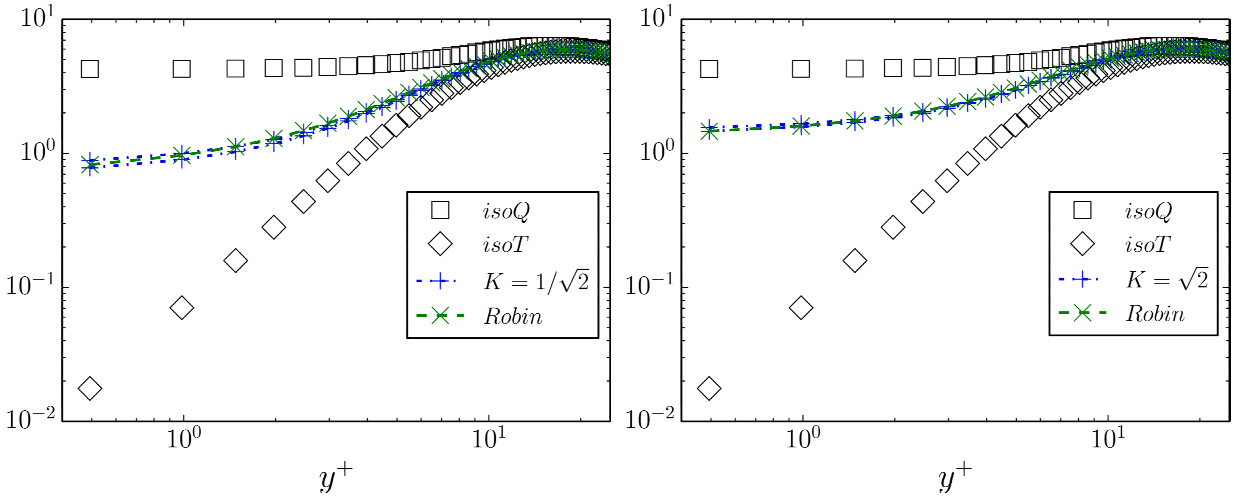

Figure 1: Temperature variance. Left: $K = 1/\sqrt{2}$. Right: $K = \sqrt{2}$. +: Conjugate. ×: Robin.

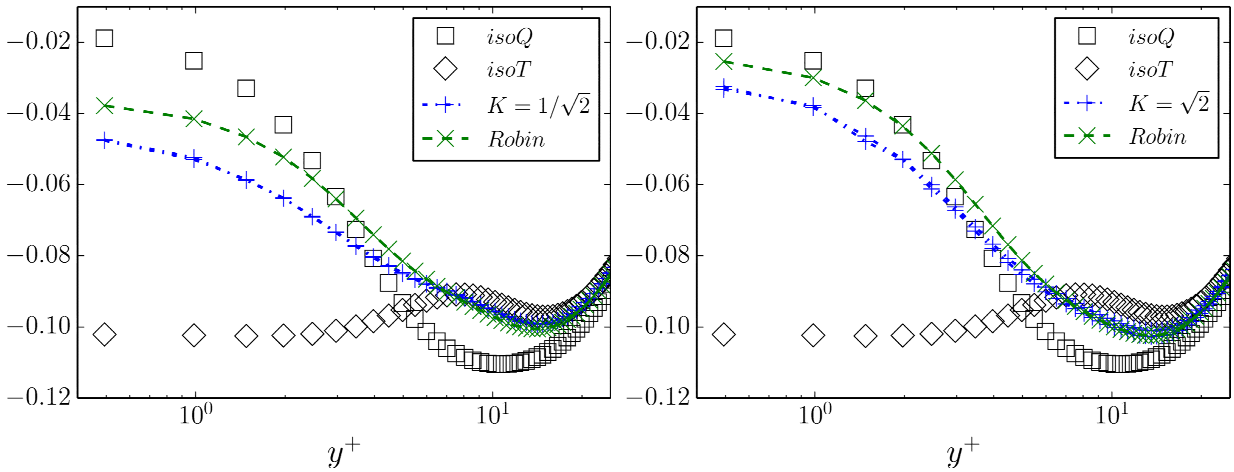

Figure 2: Thermal dissipation rate associated with the temperature variance. Left: $K = 1/\sqrt{2}$. Right: $K = \sqrt{2}$. +: Conjugate. ×: Robin.

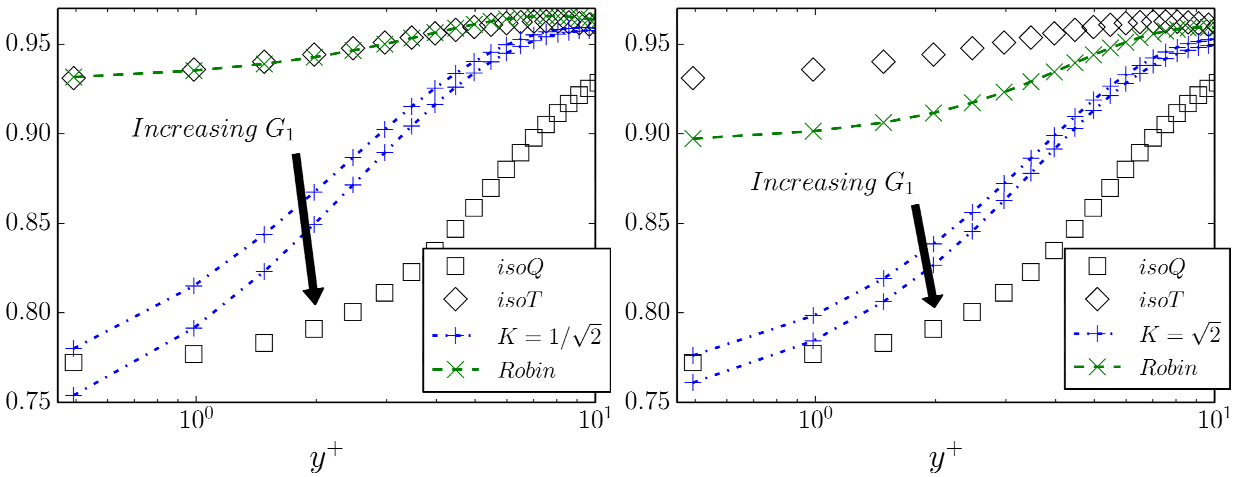

Figure 3: Correlation associated with the streamwise turbulent heat flux. Left: $K = 1/\sqrt{2}$. Right: $K = \sqrt{2}$. +: Conjugate. ×: Robin.