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Could Darboux's Forces be an alternative to the dark matter/energy

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Abstract: In this paper we study a classical alternative to Dark matter and Dark energy hypotheses. We assume that a harmonic potential is progressively added to the Newtonian's when acceleration decreases, such trajectories remains always conic. We show that with correct choice of constant this force leads to flat curve of rotation of spirals galaxies and can, under certain conditions, be repulsive.

Keywords: Central force; conic; dark matter; force of gravitation; galaxies; modified gravitation; PACS Number:04.70.Kd

1. Introduction

It is well known that an important problem of the contemporary physics is to describe the celestial mechanics for large distances. For example, it appears that the classical laws of gravitation failed to describe the "flat" curves of rotation of the spirals galaxies without add, inside these galaxies, an important amount of a mysterious matter, called "dark matter" [1-3]. Another important problem of the celestial mechanics is to explain the expansion, at an increasing rate, of the universe [4] because classical theories of gravitation are always attractive. To solve this contradiction, physicians suggest that another mysterious energy, called this time "dark energy", exists inside the Universe [5, 6]. Problem is that there is no proof of the reality of the "dark matter" and the "dark energy" despite several important experiments on the Earth [7]. Moreover, the most accurate study to try to detect them in the space seems also have failed [8].
Consequently, an important contemporary way of research is to modify the classical laws of Gravitation. Several alternative theories are in competition as for example, the $f(R)$ theories of gravity [9], the scalar-tensor gravity theories [10] or the Modified Newtonian dynamics (MOND) theory [11]. But it appears today that no one of these theories does unanimity, and it is one of the reason of this work.

We present in the paper the final version of an alternative and classical theory, which should be understood as the limiting case of a relativistic theory, to be build. Model is based on two original assumptions:

1. *"if a point-particule interacts with a force of Gravity due to a massive body, and without external disturbance its trajectory is conic (circle, ellipse, parabola or hyperbolae)".*
2. *"The force of Gravity is conservative, but not necessary central".*

These assumptions (which are discussed at the end of the paper) led us to write a theory where a harmonic potential is progressively added to the Newtonian's, when distance to the attractive body increases. Originality, due to assumption 1, is that the two centers of potential aren't necessary confused.
To present our results we have organized the paper as following: we began with a study of forces which lead to conic trajectories, using a method based on an extension of the Binet's equations. We

show in particular that the addition of a Harmonic potential to the Newtonian's, if it is located at the center of the conic, doesn't modify the trajectory of the point-particle, only its velocity.

In a second part we determine the exact form of the force we expect, using astronomical observations. At end we compare results with other models, such MOND or cosmological constant.
Note that in all the paper we consider the ideal case of a point-particle subjected to a gravitational potential. Energies are given per mass unity, Forces and Accelerations are confused.

A. Family of accelerations and central forces which lead to conics trajectories

A.1 Generalization of the Binet's equation

To obtain these accelerations we generalize the Binet's equation. Indeed, as distinguished from it we consider not only the radial acceleration but also the tangential acceleration. We present here this original method (at our knowledge) which has been published elsewhere [12].

As usual in celestial mechanics we will use the polar system of coordinate $(F;\vec{e}_R;\vec{e}_\theta)$ where F (foci of the conic) is the origin of this system, r is the radial distance to the origin with the relation

$$F\vec{M} = r.\vec{e}_R$$

And the angle θ is measured from the periapsis of the orbit. In this system of coordinate the acceleration is given by the classical relation

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{e}_R + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta$$

But the orbital shape is more concisely described by the reciprocal $u = \dfrac{1}{r}$ as a function of θ .

And by using the relations

$$\dot{r} = \frac{d}{dt}r = \frac{d}{dt}\frac{1}{u} = -\frac{\dot{u}}{u^2}$$

$$\dot{u} = \frac{d}{dt}u = \frac{d\theta}{dt}\frac{du}{d\theta} = \dot{\theta}.u'$$

$$\dot{r} = -\frac{\dot{\theta}.u'}{u^2}$$

$$\dot{u}' = \frac{d}{dt}u' = \frac{d\theta}{dt}\frac{d}{d^2\theta}u = \dot{\theta}.u''$$

$$\ddot{r} = -\frac{(\ddot{\theta}.u' + \dot{\theta}.\dot{u}')u^2 - 2\dot{u}u\dot{\theta}u'}{u^4} = -\frac{(\ddot{\theta}.u' + \dot{\theta}^2.u'')u^2 - 2u\dot{\theta}^2u'^2}{u^4}$$

We obtain a generalization of the Binet's equation.

$$\vec{a} = \frac{-u''u^2\dot{\theta}^2 - u^3\dot{\theta}^2 + 2u'^2\,u\dot{\theta}^2 - u'u^2\ddot{\theta}}{u^4}\vec{e}_R + \frac{-2u'u^2\dot{\theta}^2 + u^3\ddot{\theta}}{u^4}\vec{e}_\theta$$

By noticing that this equation can be written

$$\vec{a} = \left[\frac{-u''u^2\dot{\theta}^2 - u^3\dot{\theta}^2}{u^4} + \frac{u'}{u}\frac{2u'u^2\dot{\theta}^2 - u^3\ddot{\theta}}{u^4}\right]\vec{e}_R + \frac{-2u'u^2\dot{\theta}^2 + u^3\ddot{\theta}}{u^4}\vec{e}_\theta$$

We introduce two functions given by

$$Y(u) = \frac{-u''u^2\dot{\theta}^2 - u^3\dot{\theta}^2}{u^4}$$

And

$$Z(u) = \frac{-2u'u^2\dot{\theta}^2 + u^3\ddot{\theta}}{u^4}$$

And the acceleration becomes

$$\vec{a} = \left[Y(u) - \frac{u'}{u}Z(u)\right]\vec{e}_R + Z(u)\vec{e}_\theta$$

We can now write the system of equation

$$\begin{cases} a_R = Y(u) - \dfrac{u'}{u}Z(u) \\ a_\theta = Z(u) \end{cases}$$

We introduce a new function $f(u)$ definite by

$$Y(u) = -A.f(u)$$

Where A is constant. To obtain $r(\theta)$ as a conic, we have to solve a differential equation as

$$u''+u = B$$

Where B is a second constant. Consequently we have now to introduce a relation between u and $\dot{\theta}$. This relation is

$$\dot{\theta} = Cu\sqrt{f(u)}$$

Where C is a constant of the motion. Indeed with this relation we obtain

$$u''\dot{\theta}^2 - u\dot{\theta}^2 = -Au^2 f(u)$$

And

$$u''+u = B = \frac{A}{C^2}$$

This differential equation leads now to the classical solution

$$r(\theta) = \frac{p}{1+eCos\theta}$$

The parameter p of the conic is

$$p = a(1-e^2) = \frac{C^2}{A}$$

Where e is the eccentricity and a the semi major axis. Thus we obtain

$$C = \sqrt{A}\sqrt{a(1-e^2)}$$

We have now to determine the tangential component of the acceleration and by using

$$\ddot{\theta} = \frac{C.\dot{r}}{r^2\sqrt{f}}\left[\frac{1}{2}f'r - f\right]$$

Where

$$f' = \frac{d}{dr}f(r)$$

We obtain

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{1}{2}\frac{Cf'\dot{r}}{\sqrt{f}} + C\frac{\dot{r}}{r}\sqrt{f}$$

Consequently our family of force (per unit mass) is with respect for the Newton's law of dynamics

$$\vec{F} = -Af.\vec{e}_R + \dot{r}^2\left[\frac{1}{2}\frac{f'}{f} + \frac{1}{r}\right]\vec{e}_R + C\dot{r}\sqrt{f}\left[\frac{1}{2}\frac{f'}{f} + \frac{1}{r}\right].\vec{e}_\theta$$

Or, more simply

$$\vec{F} = -Af.\vec{e}_R + \dot{r}^2\left[\frac{1}{2}\frac{f'}{f} + \frac{1}{r}\right]\vec{e}_R + r\frac{dr}{d\theta}\dot{\theta}^2\left[\frac{1}{2}\frac{f'}{f} + \frac{1}{r}\right].\vec{e}_\theta \qquad (1)$$

Where A and C are two constants. Their physical dimensions depend on the choice of $f(r)$. Speed is given by

$$V = \sqrt{\dot{r}^2 + (r\dot{\theta})^2} = \sqrt{\frac{A}{a}r(2a-r)}\sqrt{f(r)} \qquad (2)$$

A.2. Limiting case: The newton's and the Hooke's

We know since the Bertrand' theorem [13] that the only two central and conservative forces which lead to closed trajectories are the Newton's and the Hooke's. These two cases can be easily obtained using previously equations.

To obtain central forces we introduce an unmovable point (we call it I) which is located between O and F (Figure 1).

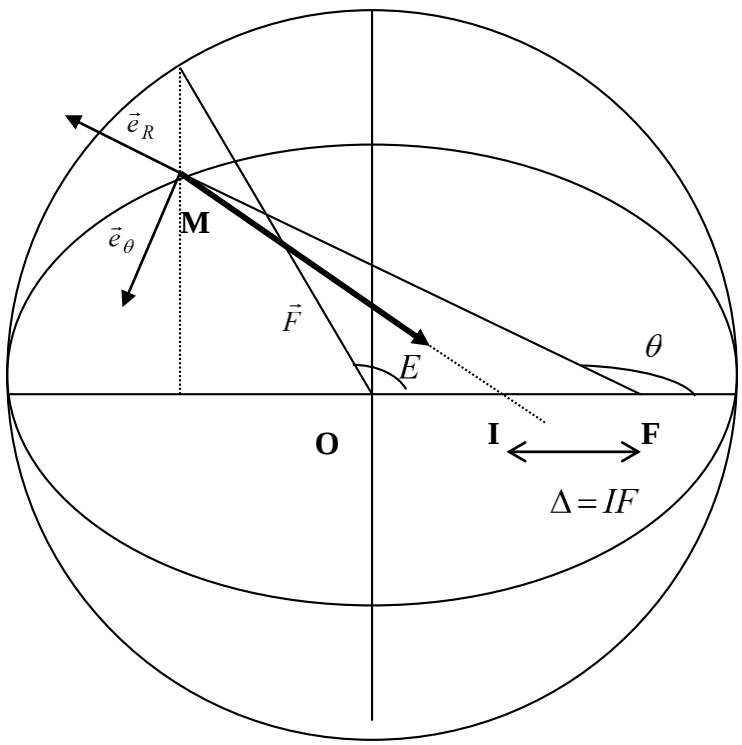

Figure 1. Representation of the central force

We call Δ the distance FI. If the force is directed to I then the vector product

$$\vec{IM} * \vec{F} = \vec{0}$$

Noting that, in our system of coordinate $F\vec{e}_R\vec{e}_\theta$ the vector $\vec{IM}$ is given by

$$\vec{IM}\left|\begin{matrix}\Delta Cos\theta + r \\ -\Delta Sin\theta\end{matrix}\right.$$

We obtain

$$\Delta Sin\theta.F_R + (\Delta Cos\theta + r)F_\theta = 0$$

With the specific relationships to the conics

$$r = \frac{a(1-e^2)}{1+eCos\theta} \quad \text{and } \dot{r} = \frac{eCSin\theta}{a(1-e^2)} r\sqrt{f(r)}$$

And using relations (5) and (6) result is

$$\Delta = \frac{ae(2fr + rf' + 2f)}{f'(r-a) + 2f(r-a) + 2f}$$

To obtain a central force we are looking for the family of functions $f(r)$ which leads to Δ as a constant. Consequently, we write the equation

$$\frac{d}{dr}\Delta = 0$$

This relation leads to

$$f(r) = \frac{1}{(C_A r + C_B)^2} \qquad (3)$$

Where C_A and C_B are two constant. Δ is simply given by

$$\Delta = \frac{eaC_B}{aC_A + C_B}$$

We can now obtain the mathematical expression of our force. Introducing relation (3) in equation (1) force becomes

$$\vec{F} = A\frac{ar(C_B - rC_A) - C_B(a^2(1-e^2) + r^2)}{ar(C_A r + C_B)^3}\vec{e}_R + A\frac{eC_B}{(C_A r + C_B)^3}Sin\theta.\vec{e}_\theta$$

Its magnitude is

$$\vec{F} = \frac{A}{a}\frac{\sqrt{2aC_AC_B(-ar + r^2 + a^2(1-e^2) + C_B{}^2(2a^2 - e^2a^2 + r^2 - 2ar) + C_A{}^2a^2r^2}}{(C_A r + C_B)^3}$$

Noting that the distance $IM = R$ is given by

$$R = \sqrt{(\Delta + r\cos\theta)^2 + (r\sin\theta)^2} = \sqrt{\Delta^2 + r^2 + 2r.\cos\theta}$$

After simplification

$$R = \frac{\sqrt{2aC_AC_B(-ar + r^2 + a^2(1-e^2) + C_B{}^2(2a^2 - e^2a^2 + r^2 - 2ar) + C_A{}^2a^2r^2}}{C_A r + C_B} \qquad (4)$$

We obtain

$$F = \frac{A}{a}\frac{C_A a + C_B}{(C_A r + C_B)^3}R$$

These forces are a part of the forces which have been discovered by Darboux in 1873 [14]. We deduce from this expression the central and conservative forces which leads to conic trajectories: Indeed, this kind of forces has to be only depending on the distance to the center of force: it exists two possibilities:
First,

$$C_A = 0$$

And force becomes the Hooke's given by

$$F = \frac{A}{a}\frac{1}{C_B{}^2}R$$

Which corresponds to

$$\Delta = ea$$

(force is located at the center of the conic) and function $f(r)$ is constant such

$$f(r) = \frac{1}{C_B{}^2}$$

And the force can be written

$$\vec{F} = -\frac{A}{a}\frac{1}{C_B{}^2}(r + aeCos(\theta))\vec{e}_R + \frac{A}{C_B{}^2}eSin\theta.\vec{e}_\theta$$

Or; simpler

$$\vec{F} = -\frac{A}{a}\frac{1}{C_B{}^2}O\vec{M}$$

With O center of the conic (see figure 1). Second possibility is given by $r = R$ consequently to (4)

$$C_B = 0$$

Force is the Newton's

$$F = \frac{1}{C_A{}^2}\frac{A}{r^2}$$

And $\Delta = 0$: Force is located at the foci of the conic. Associated function is

$$f(r) = \frac{1}{C_A{}^2}\frac{1}{r^2}$$

A3: Our model

To answer to the initial assumptions, it was interesting to see if a combination of these two fundamental forces lead to a conic trajectory.
To verify it we have choose simply

$$f(r) = \frac{1}{r^2} + k$$

Where k is a constant (m^{-2}).Associated force is given using (1) : force is simply

$$\vec{F} = -\frac{A}{r^2}\vec{e}_R - \frac{A}{a}k(r + aeCos(\theta))\vec{e}_R + AkeSin\theta.\vec{e}_\theta \qquad (5)$$

We see that it is the vector addition of the two fundamental forces. We deduce that the addition of a force of Newton and of Hooke, if they are located at two different points (in general case) leads to a

conic trajectory. Distance between the two force is the distance between the foci and the center of the conic. At our knowledge, this result seems original. Note that the Bertrand's theorem isn't violated, because the force isn't central.

A4: Potential

We can re-obtain previously force using potential: To do it we consider two distant centers of potential, such presented in figure 2. We introduce a polar system of coordinate, definite as previously. Harmonic potential is located at A, Newtonian at B, and OM = R.

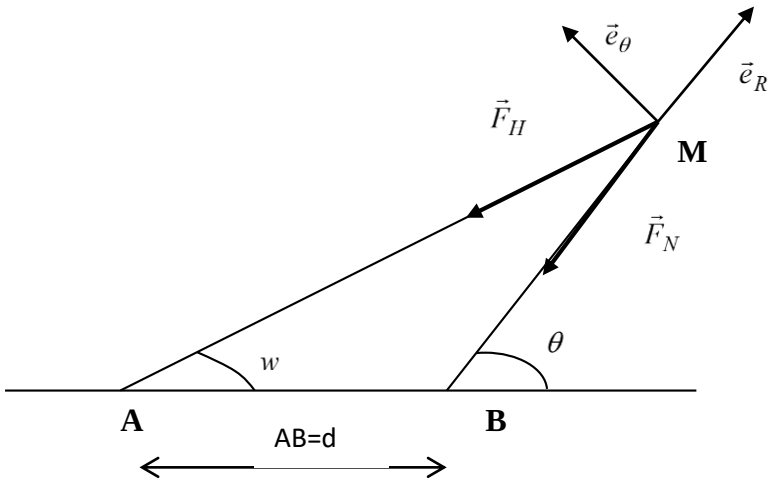

Figure 2 : Harmonic and Newtonian Potential

Total potential at the point M is thus given by

$$U(r,R) = -\frac{A}{r} + \frac{1}{2}KR^2$$

We can write the trigonometric relations

$$RCos(w) = d + RCos(\theta)$$
$$RSin(w) = rSin(\theta)$$

We obtain

$$R^2 = r^2 + d^2 + 2rd\cos(\theta)$$

And potential becomes

$$U(r,\theta) = -\frac{A}{r} + \frac{1}{2}K(r^2 + d^2 + 2rd\cos(\theta))$$

We deduce the associated force, using

$$\vec{F} = -grad(V(r,\theta))$$

And obtain

$$a_r = -\frac{A}{r^2} - K\left[r + dCos(\theta)\right]$$
$$a_\theta = KdSin(\theta)$$

We write the equality between this force and (5) :

$$-\frac{A}{r^2}-\frac{A}{a}k(r+aeCos(\theta))=-\frac{A}{r^2}-K\left[r+dCos(\theta)\right]$$
$$AkeSin\theta=KdSin(\theta)$$

We see that

$$K=\frac{A}{a}k$$
$$d=ae$$

Which are the correct expression : our force is well the derivative of the previously potential. Using (2) we obtain the kinetic energy

$$T=\frac{1}{2}V^2=\frac{1}{2}\frac{A}{a^2}(2a-r)(1+kr^2)$$

And the mechanical energy

$$E=T+U=-\frac{1}{2}\frac{A}{a}(1-ka^2(2-e^2))$$

B. Theory of gravitation

As we presented it, the theory is based on the idea that a harmonic potential is progressively added to the Newtonian, when the distance to the Newtonian center of force increases. Trajectory remains conic and isn't modified, but velocity increases.

To write it we introduce a coefficient $C_1(a)$ such $C_1(a)\in\left]0,1\right]$ and we have definited :

The usual coefficient of attraction as

$$A=GM_G$$

Where M_G is the gravitational mass of the attractive body, G the constant of gravitation.

And the function $f(r)$ as

$$f(r)=\frac{1}{r^2}+\frac{1-C_1(a)}{C_1(a)}\frac{1}{a^2}$$

Force becomes

$$\vec{F}=-\frac{GM_G}{r^2}\vec{e}_R-\frac{GM_G}{a^3}\frac{1-C_1(a)}{C_1(a)}\vec{OM}$$

Potential is given by

$$U(r,\theta)=-\frac{GM_G}{r}+\frac{1}{2}\frac{GM_G}{a^3}\frac{1-C_1(a)}{C_1(a)}OM^2$$

To determine the mathematical form of the coefficient $C_1(a)$ we have study parabolic motion :

Considering a point particle which is dropped out, whitout initial speed, at a distance $2a$ of the center of Newtonian force : eccentricity is equal to 1 and potential becomes

$$U(r,\theta)=-\frac{GM_G}{r}+\frac{1}{2}\frac{GM_G}{a^3}\frac{1-C_1(a)}{C_1(a)}(r-a)^2$$

To minimise the action (principle of least action) we want Potential accepts the less possible extremum.

Consequently we have solved

$$\frac{d}{da}U = 0$$

Which leads to two solutions

$$r_1 = a$$

And

$$r_2 = a\frac{\left[\frac{d}{da}C_1(a)\right]a + C_1(a) - 2(C_1(a))^2)}{-3(C_1(a))^2 + 3C_1(a) + a\frac{d}{da}C_1(a)}$$

We want this solution be equal to 0, such extremum be at the end of the motion. Thus we have writted

$$\left[\frac{d}{da}C_1(a)\right]a + C_1(a) - 2(C_1(a))^2) = 0$$

Solving is

$$C_1(a) = \frac{C_2}{C_2 + a}$$

Where C_2 is a constant (m). To determine it, we have this time used astronomical observations.

B.2. Periphery of galaxies: determination of coefficients

It is well known that the speed of the stars at the periphery of one spiral galaxy is constant and well described by the relation

$$V = [GM_G a_0]^{1/4}$$

Where M_G is the visible mass of the galaxy and a_0 the Millgrom's acceleration (around $a_0 \approx 1{,}2.10^{-10} ms^{-2}$)(note that expression (17) has been often verified, see for example references [11, 15] and is in agreement with Tully-Fisher law [16]).

To determine our coefficient we introduce a constant of the galaxy r_0 given by

$$r_0 = \sqrt{\frac{GM_G}{a_0}}$$

We have two limiting cases: Firstly, when the force is the Newton's we have relations

$$r_0 >> a$$

Secondly, when the force is the Hooke's we have the conditions

$$r_0 << a$$

We consider the circular motion : In our model the corresponding acceleration is given by

$$-\frac{GM_G}{a^2}\frac{1}{C_1(a)} = -\frac{GM_G}{a^2}\frac{C_2 + a}{C_2}$$

In the second limiting case this one has to be equal to

$$-\frac{V_\infty}{a^2}$$

We write thus

$$\frac{GM_G}{a^2}\frac{C_2+a}{C_2} \approx \frac{[GM_G a_0]^{1/2}}{a}$$

And

$$\frac{1}{a}\frac{C_2+a}{C_2} \approx \frac{1}{r_0}$$

In this limiting case $a >> r_0$ and consequently

$$C_2 = r_0$$

We obtain thus

$$C_1(a) = \frac{r_0}{a+r_0}.$$

Force Becomes

$$\vec{F} = -\frac{GM_G}{r^2}\vec{e}_R - \frac{GM_G}{a^2 r_0}\vec{OM}$$

Note that, if trajectory is circular, we obtain an interesting simplification

$$\vec{F} = -\frac{GM_G}{r}\left[\frac{1}{r}+\frac{1}{r_0}\right]\vec{e}_R$$

Where it is visible that the curve of rotation becomes flat.

C. Predictions

We study in this part several predictions of our model, in particular about the curve of rotation of galaxies and about the expansion of the Universe.

C.1. Curve of rotation of galaxies: comparison with MOND

MOND theory assumes that "the Newtonian acceleration g_N produced by the visible matter is linked to the true acceleration g by means of an interpolating function μ » [11] given by

$$\mu(\frac{g}{a_0})g = g_N$$

Where

$$\mu(x) = 1 \text{ if } x >> 1$$

$$\mu(x) = x \text{ if } |x| << 1$$

Force and acceleration are linked by

$$\vec{F} = m_I \mu(\frac{g}{a_0})\vec{g} \qquad (25)$$

Where F is the Newton's force. In his theory inertial and gravitational mass are equal and (25) becomes in the case of circular motion

$$\frac{GM_G}{a^2} = \mu(\frac{g}{a_0})\frac{V^2}{a} \qquad (26)$$

Several possible expressions of μ has been tested with success. The most popular choice was the "standard" μ function [11]

$$\mu_1 = \frac{x}{\sqrt{1+x^2}}$$

And the "simple" μ function [17]

$$\mu_2 = \frac{x}{1+x}$$

Note that these two functions allows to obtain excellent fit for $a_0 \approx 1{,}2.10^{-10} ms^{-2}$. We can compare this coefficient with our own coefficient $C_1(a)$. Indeed in our case acceleration is given (for circular motion) by

$$\frac{GM_G}{a^2}\frac{1}{C_1(a)} = \frac{V^2}{a}$$

Thus

$$C_1(a) = \mu$$

With

$C_1(a)$ becomes

$$C_1(a) = \frac{\sqrt{\frac{GM}{a_0}}}{\sqrt{\frac{GM}{gC_1(a)}} + \sqrt{\frac{GM}{a_0}}}$$

Thus

$$C_1(a) = \frac{\sqrt{x}}{\sqrt{\frac{1}{C_1(a)}} + \sqrt{x}}$$

$$\sqrt{x}C_1(a) + \sqrt{C_1(a)} - \sqrt{x} = 0$$

And

$$C_1(a) = \left[\frac{-1+\sqrt{1+4x}}{2\sqrt{x}}\right]^2$$

Mathematical expressions are proximate. We suggest thus to test our coefficient. If theory is correct fit should be better.

C.2. Friedmann's equations

The first equation of Friedmann is used for modeling an isotropic and homogeneous universe and is derived from the Einstein's field equations [18]. In this equation Einstein has inserted a factor called the cosmological constant. By taking care of this factor the first Friedmann's equation becomes

$$\dot{a}_1{}^2 - \frac{8\pi G}{3}\rho(t)a_1{}^2 - \frac{\Lambda c^2}{3}a_1{}^2 = kc^2$$

Where a_1 is the scale factor, G the Newton's gravitational constant, Λ the cosmological constant, c the speed of light in vacuum, $\rho(t)$ the density of mass of the universe and $\frac{k}{a_1^{\,2}}$ the spatial curvature in any time slice of the Universe. Considering $\rho(t) \propto a_1^{\,-3}$ [25] the equation of Friedmann becomes

$$\dot{a}_1^{\,2} - \frac{m}{a_1} - \frac{\Lambda c^2}{3} a_1^{\,2} = kc^2 \qquad (27)$$

Where m is constant.
The first term of this equation can be compared with a Kinetic energy. The second term with a Newtonian potential, the third with an isotropic harmonic oscillator potential.
The right-hand side of this equation is constant. Consequently this equation can be understood as the conservation of the Mechanical energy.
This equation is in agreement with our model. Indeed, we introduced in the theory a harmonic potential. A theoretical test should be to see if the potential we obtained is compatible with the cosmological constant.

C3. Possible test inside our solar system?

We can list several prediction or tests:

- We should detect small modification of the velocity of celestial bodies at the extreme periphery of it.
- Possible test is about the parabolic-hyperbolic motion, for example in the case of spatial aircraft: we should detect a small modification of the acceleration, due to the addition of the harmonic potential to the Newtonian.

However, in all the cases, we shouldn't detect modification of the trajectory, such modification of the advance of the perihelia, etc. for the reason that trajectories remain conic in the model.

D: Discussion

D1: Trajectory

A natural question about the model is about the initial assumptions: why, if two celestial bodies interact only with themselves, trajectories should be necessary conic?

We discuss this point as following: this simple assumption can naturally be understood as the consequence of classical theories of gravitation and of observational data. In particular, it is well known that a test-body moving under the influence of a Newtonian potential due to a central mass will describe a conic. Moreover, it is also well known that if we modify arbitrary the mathematical form of this potential we will induce perturbations on its trajectory (generally a modification of the perihelion precession). It is the reason for what attempts to modify Newtonian potential are generally strongly constraint with observational data in our solar system or simply failed. We can evoke attempts to build a theory with a logarithmic corrected Newtonian potential [19, 20] or with a Yukawa-like potential [17], MOND [17], Dark matter theory [21] or other examples [22, 23, 24].
Our idea was thus to consider the possibility the correct force of gravitation won't modify trajectories inside our solar system. Consequently, this force (as the Newton's) should lead to conic trajectories.

Naturally this assumption based on the trajectory limit drastically our choice of possible modified force of gravitation.

D2: Conservative force and Physical interpretation

Model is based on the assumption that the force of gravitation is conservative (and not necessary central). Reason is the following: With this condition it appears that building a relativistic theory should be possible. Indeed, all modern theories respect the corpus of classical Physics, in particular the principle of least action.

Moreover, its leads to a physical interpretation: A point particle, submitted to a Newtonian potential, describes a closed conic around the center of potential.
Progressively, or immediately, a second potential is added to the Newtonian, and is depending on the semi major axis of the trajectory. A part of the gravitational matter seems "diluted" inside a closed volume, and corresponding potential is modeled as a harmonic potential, located at the center of the conic. Trajectory isn't modified, but velocity of the point increases, until the curve of rotation becomes flat. The proportion between the two potentials is linked to a limiting acceleration, previously introduced by M. Millgrom, which should be a constant of the Universe.
If we consider now the same point particle describing this time a hyperbolic motion: it is possible under certain conditions that the force be repulsive, unlike actual theories of gravity. This possibility should perhaps explain the expansion of the Universe at an increasing rate. In this case, model could be an interesting alternative to the elusive Dark Matter/Energy.

E. Conclusion

In this paper we present a classical model as an alternative to Dark Matter and Dark Energy. We have considered that a harmonic potential is progressively added to the Newtonian when distance to the attractive body increases. Originality is that the two potential have not the same center, and lead to conic trajectories. Model is building with respect for the corpus of classical physics.

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