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Could the Darboux's forces be an alternative to the dark matter/energy?

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Abstract: In this paper we study the possibility that the Darboux's forces be a classical alternative to Dark matter and Dark energy hypotheses. Indeed these central forces, which has been discovered in 1877, admit the Newton's force as a limiting case. We study their second limiting case, the Hooke's, and we compare predictions of this model with the Friedman's equation and with MOND theory.

Keywords: Central force; conic; dark matter; force of gravitation; galaxies; modified gravitation; PACS Number:04.70.Kd

1. Introduction

It is well known that an important problem of the contemporary physics is to describe the celestial mechanics for large distances. For example, it appears that the classical laws of gravitation failed to describe the "flat" curves of rotation of the spirals galaxies without add, inside these galaxies, an important amount of a mysterious matter, called "dark matter" [1-3]. Another important problem of the celestial mechanics is to explain the expansion, at an increasing rate, of the universe [4] because classical theories of gravitation are always attractive. To solve this contradiction, physicians suggest that another mysterious energy, called this time "dark energy", exists inside the Universe [5, 6]. Problem is that there is no proof of the reality of the "dark matter" and the "dark energy" despite several important experiments on the Earth [7]. Moreover the most accurate study to try to detect them in the space seems also have failed [8].

Consequently, an important contemporary way of research is to modify the classical laws of Gravitation. Several alternative theories are in competition as for example, the $f(R)$ theories of gravity [9], the scalar-tensor gravity theories [10] or the Modified Newtonian dynamics (MOND) theory [11]. But it appears today that no one of these theories does unanimity, and it is one of the reason of this work.

Indeed we have studied an assumption which seems to have not been investigated. This one can be simply written: "*if a point-particule interacts with a center of force without external disturbance its trajectory is conic (circle, ellipse, parabola or hyperbolae)*". This simple assumption can naturally be understood as a consequence of classical theories of gravitation and of observational data. In particular it is well known that a test-body moving under the influence of a Newtonian potential due to a central mass will describe a conic. Moreover it is also well known that if we modify the mathematical form of this potential we will induce perturbations on its trajectory (generally a modification of the perihelion precession). It is the reason for what attempts to modify Newtonian potential are generally strongly constraint with observational data or simply failed. We can evoke attempts to build a theory with a logarithmic corrected Newtonian potential [12, 13] or with a Yukawa-like potential [14], MOND [14], Dark matter theory [15] or other examples [16, 17, 18].

Our idea was thus to consider the possibility that an alternative force of gravitation won't modify trajectories inside our solar system. Consequently this force (as the Newton's) should lead to conic

trajectories. Naturally this assumption based (as usual in the story of the Physics of Gravitation) on the trajectory limit drastically our choice of possible modified force of gravitation.

By a way of consequence we studied relations between conic trajectories and forces and we learned that important works has been done about this problem at the 19th century. In particular in 1873 the mathematician Bertrand published a theorem [19, 20] which prove that it exists only two central and conservative forces which lead to closed trajectories, the Newton's and the Hooke's. Moreover it is interesting to note that these trajectories are in the two cases conic. Difference is that the Newton's force is directed toward the foci of the conic and the Hooke's toward its center. Another important information was the works of the mathematicians Halphen and Darboux. Indeed they discovered in 1877 [21] a family of central forces which lead to conic trajectories and which admits two limiting cases, the Hooke's and the Newton's (other demonstrations has been published later [22, 23]). A consequence of this bibliography was to assume that these forces are perhaps the forces we are looking for. Firstly because one of their limiting case corresponds well to the force of gravitation in the case of small distances (as for example our solar system). Secondly because these forces are central and allow consequently to preserve the angular momentum of the system (we give the expressions in the paper). And thirsty because their second limiting case (the Hooke's) is often evoked has an alternative to the Dark Energy (for example in the Friedman equation) to describe the expansion of the Universe (see for example references [24, 25]).

The goal of the paper is thus to consider the possibility that the force of Newton becomes gradually a force of Hooke when distances increases while remaining central throughout its evolution. In other words to see if the Darboux's force could be a possible alternative force of gravitation. We present consequently results we obtained about curve of rotation of galaxies, dynamics of the Universe and equivalence principle, with comparison with other theories. We begin the paper with a study of accelerations and forces which lead to conic trajectories.

A. Family of accelerations and central forces which lead to conics trajectories

A.1 Generalization of the Binet's equation

To obtain these accelerations we generalize the Binet's equation. Indeed as distinguished from it we consider not only the radial acceleration but also the tangential acceleration. We present here this original method (at our knowledge) which has been published elsewhere [26].

As usual in celestial mechanics we will use the polar system of coordinate $(F; \vec{e}_R; \vec{e}_\theta)$ where F (foci of the conic) is the origin of this system, r is the radial distance to the origin with the relation

$$F\vec{M} = r.\vec{e}_R$$

And the angle θ is measured from the periapsis of the orbit. In this system of coordinate the acceleration is given by the classical relation

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{e}_R + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta$$

But the orbital shape is more concisely described by the reciprocal $u = \frac{1}{r}$ as a function of θ .

And by using the relations

$$\begin{aligned}\dot{r} &= \frac{d}{dt} r = \frac{d}{dt} \frac{1}{u} = -\frac{\dot{u}}{u^2} \\ \dot{u} &= \frac{d}{dt} u = \frac{d\theta}{dt} \frac{du}{d\theta} = \dot{\theta} u' \\ \dot{r} &= -\frac{\dot{\theta} u'}{u^2} \\ \dot{u}' &= \frac{d}{dt} u' = \frac{d\theta}{dt} \frac{d}{d^2\theta} u = \dot{\theta} u'' \\ \ddot{r} &= -\frac{(\ddot{\theta} u' + \dot{\theta} \dot{u}') u^2 - 2\dot{u} u \dot{\theta} u'}{u^4} = -\frac{(\ddot{\theta} u' + \dot{\theta}^2 u'') u^2 - 2u \dot{\theta}^2 u'^2}{u^4}\end{aligned}$$

We obtain a generalization of the Binet's equation.

$$\vec{a} = \frac{-u'' u^2 \dot{\theta}^2 - u^3 \dot{\theta}^2 + 2u'^2 u \dot{\theta}^2 - u' u^2 \ddot{\theta}}{u^4} \vec{e}_R + \frac{-2u' u^2 \dot{\theta}^2 + u^3 \ddot{\theta}}{u^4} \vec{e}_\theta$$

By noticing that this equation can be written

$$\vec{a} = \left[\frac{-u'' u^2 \dot{\theta}^2 - u^3 \dot{\theta}^2}{u^4} + \frac{u'}{u} \frac{2u' u^2 \dot{\theta}^2 - u^3 \ddot{\theta}}{u^4} \right] \vec{e}_R + \frac{-2u' u^2 \dot{\theta}^2 + u^3 \ddot{\theta}}{u^4} \vec{e}_\theta$$

We introduce two functions given by

$$Y(u) = \frac{-u'' u^2 \dot{\theta}^2 - u^3 \dot{\theta}^2}{u^4}$$

And

$$Z(u) = \frac{-2u' u^2 \dot{\theta}^2 + u^3 \ddot{\theta}}{u^4}$$

And the acceleration becomes

$$\vec{a} = \left[Y(u) - \frac{u'}{u} Z(u) \right] \vec{e}_R + Z(u) \vec{e}_\theta$$

We can now write the system of equation

$$\begin{cases} a_R = Y(u) - \frac{u'}{u} Z(u) \\ a_\theta = Z(u) \end{cases}$$

We introduce a new function $f(u)$ definite by

$$Y(u) = -A f(u)$$

Where A is constant. To obtain $r(\theta)$ as a conic, we have to solve a differential equation as

$$u'' + u = B$$

Where B is a second constant. Consequently we have now to introduce a relation between $Y(u)$ and $\dot{\theta}$. This relation is

$$\dot{\theta} = Cu \sqrt{f(u)}$$

Where C is a constant of the motion. Indeed with this relation we obtain

$$u'' \dot{\theta}^2 - u \dot{\theta}^2 = -A u^2 f(u)$$

And

$$u'' + u = B = \frac{A}{C^2}$$

This differential equation leads now to the classical solution

$$r(\theta) = \frac{p}{1 + e \cos \theta}$$

The parameter p of the conic is

$$p = a(1 - e^2) = \frac{C^2}{A}$$

Where e is the eccentricity and a the semi major axis. Thus we obtain

$$C = \sqrt{A} \sqrt{a(1 - e^2)}$$

We have now to determine the tangential component of the acceleration and by using

$$\ddot{\theta} = \frac{C \dot{r}}{r^2 \sqrt{f}} \left[\frac{1}{2} f' r - f \right]$$

Where

$$f' = \frac{d}{dr} f(r)$$

We obtain

$$a_\theta = r \ddot{\theta} + 2 \dot{r} \dot{\theta} = \frac{1}{2} \frac{C f' \dot{r}}{\sqrt{f}} + C \frac{\dot{r}}{r} \sqrt{f}$$

Consequently our family of force (per unit mass) is with respect for the Newton's law of dynamics

$$\vec{F} = -A f \cdot \vec{e}_R + \dot{r}^2 \left[\frac{1}{2} \frac{f'}{f} + \frac{1}{r} \right] \vec{e}_R + C \dot{r} \sqrt{f} \left[\frac{1}{2} \frac{f'}{f} + \frac{1}{r} \right] \cdot \vec{e}_\theta$$

Or, more simply

$$\vec{F} = -A f \cdot \vec{e}_R + \dot{r}^2 \left[\frac{1}{2} \frac{f'}{f} + \frac{1}{r} \right] \vec{e}_R + r \frac{dr}{d\theta} \dot{\theta}^2 \left[\frac{1}{2} \frac{f'}{f} + \frac{1}{r} \right] \cdot \vec{e}_\theta \quad (1)$$

Where A and C are two constants. Their physical dimensions depend on the choice of $f(r)$. Speed is given by

$$V = \sqrt{\dot{r}_2 + (r \dot{\theta})^2} = \sqrt{\frac{A}{a} r (2a - r) \sqrt{f(r)}} \quad (2)$$

A.2. Central force

In previously part we determine a family of acceleration which lead to conic trajectories. In this part we determine by using the Newton's second law of motion the central forces which lead to these trajectories.

$$\vec{F} = \frac{d}{dt} m_I \vec{V} \quad (3)$$

Where m_I is the inertial mass and \vec{V} the speed. If we consider the general case relation (3) becomes

$$\vec{F} = \dot{m}_I \vec{V} + m_I \vec{a}_C \quad (4)$$

Consequently we obtain a system of two equations

$$\begin{cases} F_R = \dot{m}_1 \dot{r} + m_1 a_{CR} \\ F_\theta = \dot{m}_1 r \dot{\theta} + m_1 a_{C\theta} \end{cases}$$

Noting that

$$\dot{m}_1 = \frac{d}{dt} m_1 = \frac{d}{dt} \frac{d}{dr} m_1 = \dot{r} \cdot m'_1$$

This system becomes

$$\begin{cases} F_R = m'_1 \dot{r}^2 + m_1 a_{CR} \\ F_\theta = m'_1 \dot{r} r \dot{\theta} + m_1 a_{C\theta} \end{cases} \quad (5)$$

Where inertial mass is a function of r . Note that acceleration given by equation (2) isn't necessary central and consequently that vectors of accelerations and forces aren't necessary parallel. However and in agreement with our initial assumption we are looking for a central force. To obtain it we introduce an unmovable point (we call it I) which is located between O and F (Figure 1).

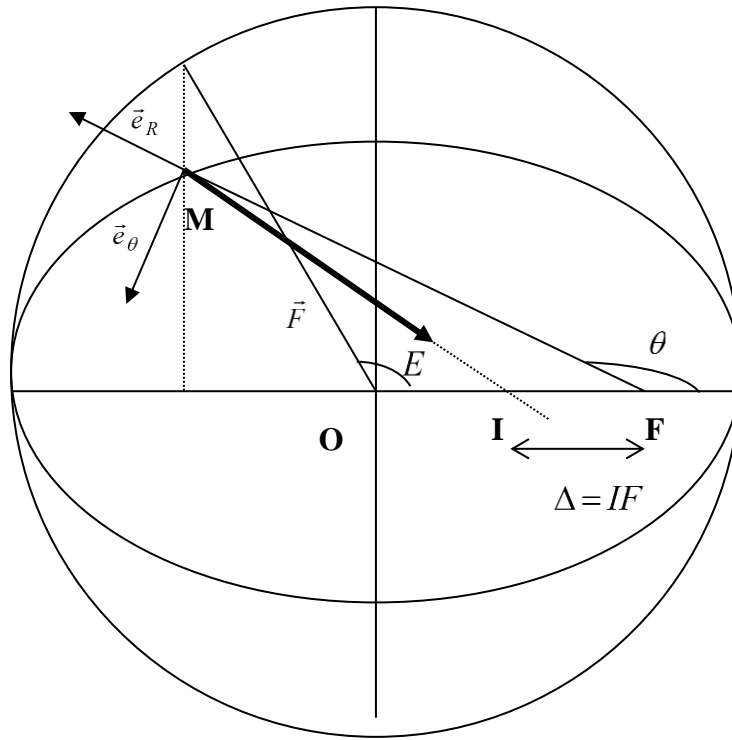


Figure 1. Representation of the central force

We call Δ the distance FI . If the force is directed to I then the vector product

$$\vec{IM} * \vec{F} = \vec{0}$$

Noting that, in our system of coordinate $F\vec{e}_R\vec{e}_\theta$ the vector \vec{IM} is given by

$$\vec{IM} \begin{cases} \Delta \cos \theta + r \\ -\Delta \sin \theta \end{cases}$$

We obtain

$$\Delta \sin \theta \cdot F_R + (\Delta \cos \theta + r) F_\theta = 0 \quad (6)$$

With the specific relationships to the conics

$$r = \frac{a(1-e^2)}{1+e\cos\theta} \quad \text{and} \quad \dot{r} = \frac{eC\sin\theta}{a(1-e^2)} r \sqrt{f(r)} \quad (7)$$

And using relations (5) and (6) result is

$$\Delta = \frac{ae(2fm_l' r + m_l r f' + 2m_l f)}{f' m_l (r-a) + 2fm_l' (r-a) + 2m_l f}$$

To obtain a central force we are looking for the family of functions $f(r)$ and $m_l(r)$ which leads to Δ as a constant. Consequently we write the equation

$$\frac{d}{dr} \Delta = 0$$

This relation leads to

$$f(r) = \frac{1}{m_l^2(r)(C_1' r + C_2')^2} \quad (8)$$

Where C_1' and C_2' are two constant (notations of X' will be used in all the paper for accelerations, X for forces). It is really interesting to note that Δ is independent from $m_l(r)$ simply given by

$$\Delta = \frac{eaC_2'}{aC_1' + C_2'} \quad (9)$$

We can now obtain the mathematical expression of our force. Introducing relation (8) in equations (2) and (5) force becomes

$$\vec{F} = A' \frac{ar(C_2' - rC_1') - C_2' (a^2(1-e^2) + r^2)}{m_l(r)ar(C_1' r + C_2')^3} \vec{e}_R + A' \frac{eC_2'}{m_l(r)(C_1' r + C_2')^3} \sin\theta \vec{e}_\theta$$

Its magnitude is

$$\vec{F} = \frac{A'}{a} \frac{\sqrt{2aC_1' C_2' (-ar + r^2 + a^2(1-e^2) + C_2'^2 (2a^2 - e^2 a^2 + r^2 - 2ar) + C_1'^2 a^2 r^2)}}{m_l(r)(C_1' r + C_2')^3}$$

Noting that the distance $IM = R$ is given by

$$R = \sqrt{(\Delta + r \cos \theta)^2 + (r \sin \theta)^2} = \sqrt{\Delta^2 + r^2 + 2r \cos \theta}$$

After simplification

$$R = \frac{\sqrt{2aC_1' C_2' (-ar + r^2 + a^2(1-e^2) + C_2'^2 (2a^2 - e^2 a^2 + r^2 - 2ar) + C_1'^2 a^2 r^2)}}{C_1' r + C_2'}$$

We obtain

$$F = \frac{A'}{a} \frac{1}{m_l(r)} \frac{C_1' a + C_2'}{(C_1' r + C_2')^3} R \quad (10)$$

A.3. Darboux's force and equivalence principle

The force given by (10) is a part of the Darboux's force. Indeed as we presented it in introduction mathematician Halphen and Darboux published simultaneous in 1877 a paper where they presented a family of central force which lead to conics trajectories. We have simply rediscovered them, using an original way. It is natural now to compare the different expressions. The second part of the Darboux's forces are given by [21]

$$F = \frac{\mu}{R^2 \left(\frac{1}{R} - a_1 \cos(w) - b_1 \sin(w) \right)^3}$$

Where a_1 , b_1 and μ are three constants and w the angle of rotation given by

$$w = \widehat{MIF}$$

It is possible to show [27] that if its center of force is located on the OF axis (see Figure 1.) this force can, be written

$$F = \frac{A}{a} \frac{C_1 a + C_2}{(C_1 r + C_2)^3} R \quad (11)$$

Where C_1 and C_2 are two constant (not necessary equal to C'_1 and C'_2). The center of force is given by

$$\Delta = IF = \frac{eaC_2}{aC_1 + C_2}$$

The force admits two limiting cases: If $C_2 = 0$ force is the Newton's. Constant A is thus given by

$$A = GM_G m_G$$

Where G is the constant of gravity, M_G the gravitational mass of the center of force and m_G of the body which orbits around it). We have also the relationship

$$C_1 = 1$$

Second limiting case is obtained when $C_1 = 0$ and is the Hooke's.

We can now write the equality of the expressions (10) and (11)

$$F = \frac{A'}{a} \frac{1}{m_l(r)} \frac{C'_1 a + C'_2}{(C'_1 r + C'_2)^3} R = \frac{A}{a} \frac{C_1 a + C_2}{(C_1 r + C_2)^3} R$$

After simplification we obtain

$$\frac{A'}{A} \frac{1}{m_l(r)} \frac{C'_1 a + C'_2}{(C'_1 r + C'_2)^3} = \frac{C_1 a + C_2}{(C_1 r + C_2)^3} \quad (12)$$

We can also write the equality of Δ

$$\frac{eaC_2}{aC_1 + C_2} = \frac{eaC'_2}{aC'_1 + C'_2}$$

This equation becomes

$$\frac{C_2}{C_1} = \frac{C'_2}{C'_1} \quad (13)$$

We solve now the system of equations given by {12,13}. Results is

$$m_l(r) = \frac{A'}{A} \frac{C_2^2}{C_2'^2}$$

Consequently $m_l(r)$ is constant. Force becomes

$$F = \frac{GM_G m_G}{a} \left[\frac{C'_2}{C_2} \right]^2 \frac{C'_1 a + C'_2}{(C'_1 r + C'_2)^3} R$$

And the law of dynamic is simply

$$F = m_l a_c$$

Acceleration and force are parallel and directed toward the same point. Acceleration is thus obtained by introducing

$$f(r) = \frac{1}{(C'_1 r + C'_2)^2}$$

In equation (1). It becomes

$$a_c = \frac{GM_G}{a} \frac{C'_1 a + C'_2}{(C'_1 r + C'_2)^3} R$$

We write the equality of the force

$$F = \frac{GM_G m_G}{a} \left[\frac{C'_2}{C_2} \right]^2 \frac{C'_1 a + C'_2}{(C'_1 r + C'_2)^3} R = \frac{GM_G m_l}{a} \frac{C'_1 a + C'_2}{(C'_1 r + C'_2)^3} R$$

We introduce now the ratio of masses η given by

$$\eta = \frac{m_G}{m_l}$$

This one is given by

$$\eta = \left[\frac{C_2}{C'_2} \right]^2 = \left[\frac{C_1}{C'_1} \right]^2 \quad (14)$$

It is interesting to that this ratio is constant for a given force of Darboux on all a conic trajectory.

A.4. Coefficient of Darboux's force

We saw that the force is given by

$$F = \frac{GM_G m_G}{a} \frac{C_1 a + C_2}{(C_1 r + C_2)^3} R$$

Is directed to I definite by

$$\Delta = IF = \frac{eaC_2}{aC_1 + C_2}$$

In this part we will add drastic conditions on its coefficient by studying the physical signification of the force. We begin by the limiting cases. These cases are three:

Firstly If $\Delta = 0$ the force is naturally the Newton's. The coefficients are given in this case by

$$C_1 = 1 \quad \text{and} \quad C_2 = 0$$

Secondly if $\Delta = ea$ the force is the Hooke's and coefficient C_1 is given by

$$C_1 = 0$$

Thirsty if $\Delta = 2ea$ the center of force is located at the second foci of the conic. Consequently force is also the Newton's. We write thus the equation

$$2ea = \frac{eaC_2}{aC_1 + C_2}$$

And obtain

$$C_2 = -2C_1 a$$

With

$$C_1 = \begin{matrix} + \\ - \end{matrix} 1$$

We can also consider the two body-problem. Indeed its solving add another drastic condition on the coefficients. We write thus that, if two bodies M_1 and M_2 interacts only with each other's the sum of the forces has to be equal to zero (by remembering that inertial masses are constant on all the trajectory)

$$m_{I1}\vec{\ddot{R}}_1 + m_{I2}\vec{\ddot{R}}_2 = \vec{0}$$

I is the center of mass given by

$$m_{I1}IM_1 = m_{I2}IM_2$$

Or

$$m_{I1}r_1 = m_{I2}r_2$$

Consequently as in the Newton's case the two body-problem can be reformulated as a one body-problem. Positions of M_1 and M_2 are thus located on two homothetic ellipses (Figure 2.)

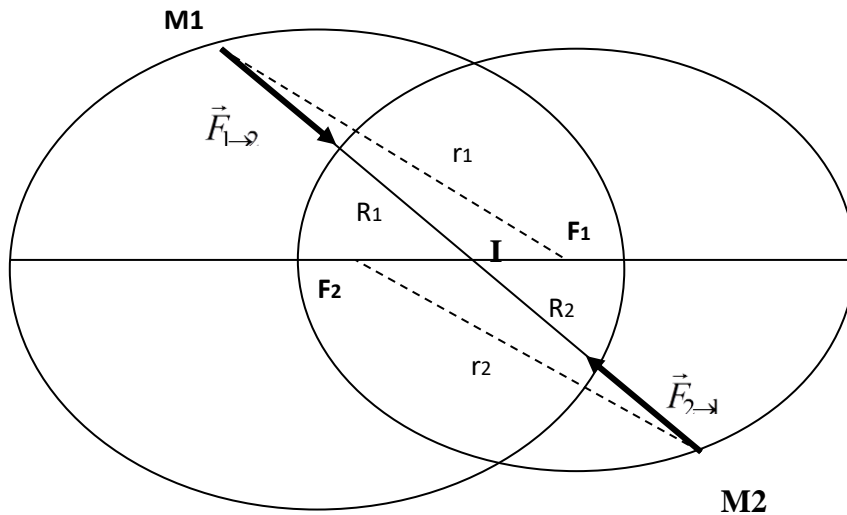


Figure 2. Representation of the two bodies-problem

We deduce that distances of ellipses are linked by

$$a_1 m_{I1} = a_2 m_{I2}$$

$$C_{21} m_{I1} = C_{22} m_{I2}$$

$$\Delta_1 m_{I1} = \Delta_2 m_{I2}$$

These relationships leads to

$$\frac{C_{21}}{a_1} = \frac{C_{22}}{a_2}$$

And

$$\frac{C_{21}}{a_1 C_{11}} = \frac{C_{22}}{a_2 C_{12}}$$

We deduce

$$C_{11} = C_{12}$$

Consequently to these observations we write

$$\begin{aligned} C_{21} &= a_1(1 - C_{11}) \\ C_{22} &= a_2(1 - C_{12}) \end{aligned}$$

General relationship is consequently

$$C_2 = a(1 - C_1) \quad (15)$$

We verify our initial conditions

$$\begin{aligned} \text{If } C_1 &= 1 & \text{then } C_2 &= 0 & \text{and } \Delta &= 0 \\ \text{If } C_1 &= 0 & \text{then } C_2 &= a & \text{and } \Delta &= ea \\ \text{If } C_1 &= -1 & \text{then } C_2 &= 2a & \text{and } \Delta &= 2ea \end{aligned}$$

It is interesting to note that the force admit now a great simplification

$$F = \frac{GM_G m_G}{(C_1(r - a) + a)^3} R \quad (16)$$

And distance Δ becomes simple

$$\Delta = ea(1 - C_1) \quad (17)$$

It is also interesting to note that if the motion is circular we have $r = R = a$. Force is equal to the Newton's and is consequently always attractive

$$\vec{F} = -\frac{GM_G m_G}{r^2} \vec{e}_R$$

But in the case of free fall motion we have $R = r - \Delta$. Force becomes

$$\vec{F} = -\frac{GM_G m_G}{(C_1 R + a(1 - C_1))^3} R \vec{e}_R$$

And can thus be attractive or repulsive. At end we can also have a brief look on the acceleration induced by the Darboux's force and given by

$$a_c = \frac{GM_G}{a} \frac{C'_1 a + C'_2}{(C'_1 r + C'_2)^3} R$$

With

$$\Delta = \frac{eaC'_2}{aC'_1 + C'_2}$$

For the same reasons we will obtain

If $C'_1 = 1$ and $C'_2 = 0$ acceleration becomes the Newton's and

If $C'_1 = 0$ it becomes the Hooke's.

Note that when force is the Newton's the ratio gravitational/inertial masses is given by

$$\eta = \left[\frac{C_1}{C'_1} \right]^2 = 1$$

The equivalence principle is respected in this limiting case. Note that this result can perhaps be considered as a demonstration of this principle.

B. Theory of gravitation

B.1. Periphery of galaxies: determination of coefficients

It is well known that the speed of the stars at the periphery of one spiral galaxy is constant and well described by the relation

$$V = [GM_G a_0]^{1/4} \quad (18)$$

Where M_G is the visible mass of the galaxy and a_0 the Millgrom's acceleration (around $a_0 \approx 1,2 \cdot 10^{-10} \text{ ms}^{-2}$) (note that expression (17) has been often verified, see for example references [11, 28] and is in agreement with Tully-Fisher law [29]). In this part we study if our model is compatible with this relation and we concluded about the principle of equivalence we expect.

To begin this study we introduce a constant r_0 given by

$$r_0 = \sqrt{\frac{GM_G}{a_0}} \quad (19)$$

We have two limiting cases: Firstly, when the force is the Newton's we have relations

$$r_0 \gg a$$

Secondly, when the force is the Hooke's we have the conditions

$$r_0 \ll a$$

We consider that this condition is satisfied at the periphery of the galaxies. We can consequently obtain information about C'_2 in this limiting case. To do it we approximate the trajectories of stars to circles as is customary [11, 26]. The acceleration given by equation (14) is

$$a_c = \frac{GM_G}{a} \frac{C'_1 a + C'_2}{(C'_1 r + C'_2)^3} R$$

We write in this case the equality between accelerations for circular motion (note that here $C'_1 = 0$ and $r = R = a$). Thus

$$a_c = \frac{GM_G}{C'^2_2} = \frac{V^2}{a}$$

And using (18) and (19) we obtain

$$C'_2 = \sqrt{a r_0}$$

We have a problem with this expression. Indeed if we write

$$C'_2 = C_2$$

It appears that we can't solve the two body-problem because

$$m_{I1} r_{01} \neq m_{I2} r_{01}$$

Consequently the two ellipses won't be homothetic because

$$m_{I1} \Delta_1 \neq m_{I2} \Delta_2$$

Moreover the force of gravitation will become

$$F = \frac{GM_G m_G}{a^2 r_0}$$

Which is inconsistent with the force of Hooke in the case of gravitation (see section E.1. of the paper). We have concluded that if we want use this force to describe the dynamics of stars, and if we want respect the classical physics, we have to abandon the equivalence principle. Indeed the ratio gravitation/inertial masses given by (14)

$$\eta = \left[\frac{C_1}{C'_1} \right]^2 = \left[\frac{C_2}{C'_2} \right]^2$$

Can't be here equal to 1 if the force is the Hooke's. But we can consider that the mathematical form of the two coefficients is proximate and by analogy with

$$C_2 = a(1 - C_1)$$

We assume that

$$C'_2 = \sqrt{a r_0} (1 - C'_1)$$

(Note that the boundaries conditions of acceleration are respected). Consequently we obtain

$$\eta = \frac{a}{r_0} \left[\frac{1 - C_1}{1 - C'_1} \right]^2 \quad (21)$$

And

$$\frac{C_1}{C'_1} = \sqrt{\frac{a}{r_0}} \left[\frac{1 - C_1}{1 - C'_1} \right]$$

Which leads to

$$C'_1 = \frac{C_1 \sqrt{r_0}}{C_1 (\sqrt{r_0} - \sqrt{a}) + \sqrt{a}} \quad (22)$$

We have here a second problem because we have only one equation for two unknowns. But it appears that we can use boundaries conditions to reduce the choice. Indeed we know that C_1, C'_1 and η are three continuous functions defined by

$$0 \leq C_1 \leq 1$$

$$0 \leq C'_1 \leq 1$$

$$1 \leq \eta \leq \frac{a}{r_0}$$

We know also that

If	$a \ll r_0$	then	$C_1 \rightarrow 1$
			$C'_1 \rightarrow 1$
			$\eta \rightarrow 1$

And

If	$a \gg r_0$	then	$C_1 \rightarrow 0$
			$C'_1 \rightarrow 0$
			$\eta \rightarrow \frac{a}{r_0}$

Moreover when $a = r_0$ force should be composed at an equal part of the Newton and the Hooke. For this reason we assume that in this case we have

$$\Delta = \frac{1}{2} e r_0$$

This means that point I is located at the same distance between O and F. we deduce that at this point

$$C_1 = \frac{1}{2} \quad \text{and} \quad C_2 = \frac{1}{2} r_0$$

We have tested several functions in order to see if they could respect our conditions. It appears that we obtained a family of simple and proximate relations which allows to respect all its conditions and could be consequently correct. It is the reason for what in this paper we suggest

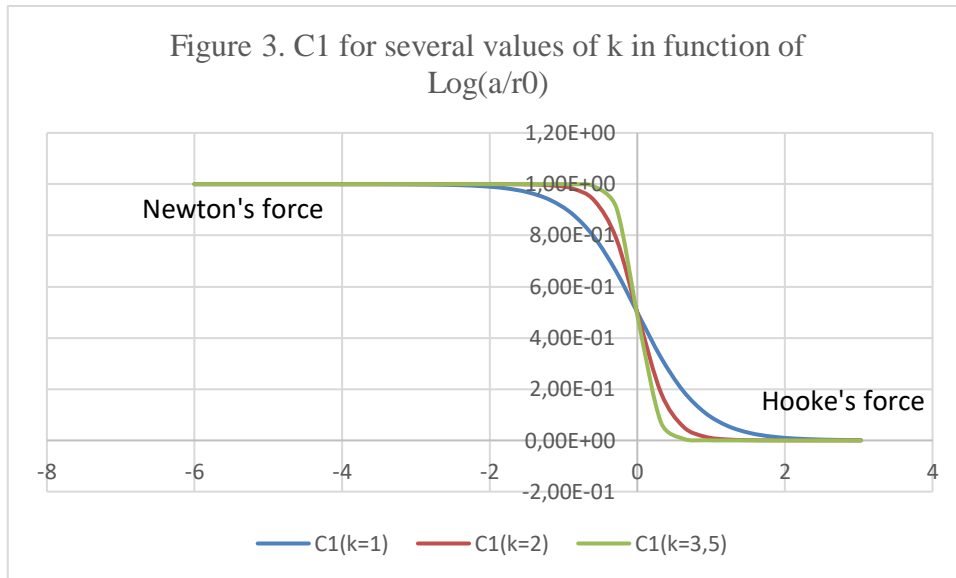
$$C_1 = \frac{r_0^k}{r_0^k + a^k}$$

Where k is a positive number such $k = n/2$. n is a natural integer. Relations between coefficients

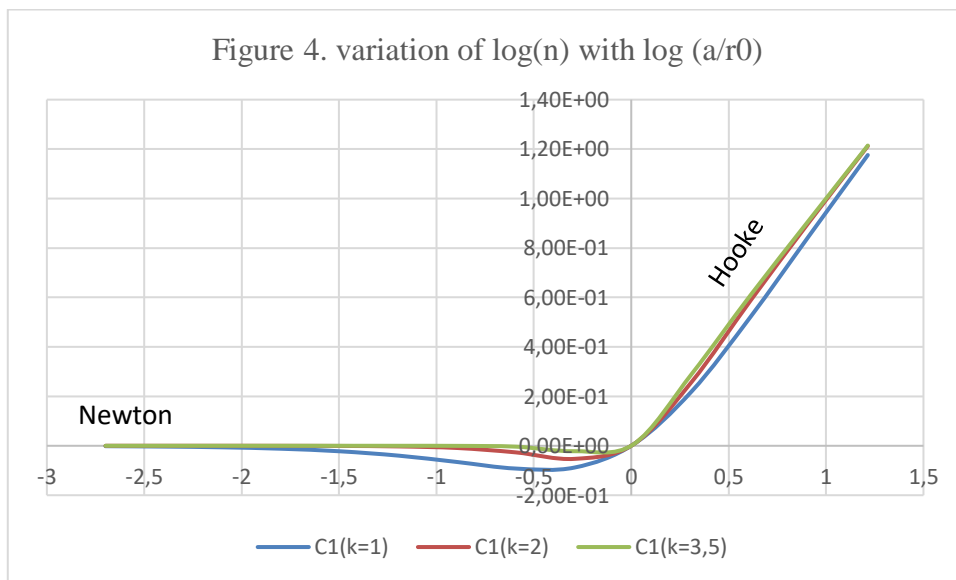
becomes

$$\left\{ \begin{array}{l} C_1 = \frac{r_0^k}{a^k + r_0^k} \\ C_2 = \frac{a^{k+1}}{a^k + r_0^k} \\ C'_1 = \frac{r_0^{k+1/2}}{a^{k+1/2} + r_0^{k+1/2}} \\ C'_2 = \frac{a^{k+1} \sqrt{r_0}}{a^{k+1/2} + r_0^{k+1/2}} \\ \eta = \frac{1}{r_0} \left[\frac{a^{k+1/2} + r_0^{k+1/2}}{a^k + r_0^k} \right]^2 \end{array} \right. \quad (22)$$

We see that coefficients have the same mathematical form and we think it is a cogent argument for this solution. Note also that the expression of the physical quantities are generally simple (see for example the angular momentum in section B.2.1). Moreover if we plot function $C_1(a)$ for several values of k (figure 3)



And the expected variations of $\eta(a)$ (Figure 4)



We see that the evolution seems correct whatever the value of k . At end note that a proximate relation given by

$$\eta = \sqrt{\frac{a_c + a_0}{a_c}}$$

has already been suggested in a relativistic theory [28].

B.2. Properties of the force

Magnitude of the force becomes now

$$F = \frac{GM_G m_G}{\left(\frac{r_0^k}{a^k + r_0^{kn}}(r-a) + a\right)^3} R = GM_G m_G \left[\frac{a^k + r_0^k}{r_0^k r + a^{k+1}} \right]^3 R$$

This force is located at I definite by

$$\Delta = IF = e \frac{a^{k+1}}{a^k + r_0^k}$$

B.2.1. Angular momentum

The angular momentum is the cross product of the particle's position vector and its momentum vector

$$\vec{p} = m_l \vec{V} = m_l (\dot{r} \vec{e}_R + r \dot{\theta} \vec{e}_\theta)$$

at F , foci of the conic we obtain

$$\vec{L}_F = m_l \cdot r \cdot \vec{e}_R * (\dot{r} \vec{e}_R + r \dot{\theta} \vec{e}_\theta) = m_l \cdot r^2 \dot{\theta} \vec{e}_Z$$

And by using

$$\dot{\theta} = \frac{C}{r} \sqrt{f(r)} = \frac{C}{r} \frac{1}{C'_1 r + C'_2} \quad (23)$$

we obtain

$$\vec{L}_F = m_l \frac{Cr}{C'_1 r + C'_2} \vec{e}_Z$$

The angular momentum at I is given by

$$\vec{L}_I = \vec{L}_F + I\vec{F} * (\dot{r} \vec{e}_R + r \dot{\theta} \vec{e}_\theta)$$

By noting that

$$I\vec{F} = \Delta (\cos \theta \vec{e}_R - \sin \theta \vec{e}_\theta)$$

\vec{L}_I becomes

$$\vec{L}_I = \vec{L}_F + \Delta m_l [r \dot{\theta} \cos \theta + \dot{r} \sin \theta] \vec{e}_Z$$

And

$$\vec{L}_I = m_l \frac{Cr}{C'_1 r + C'_2} + m_l \frac{aC_2 e}{C'_2 + aC'_1} [r \dot{\theta} \cos \theta + \dot{r} \sin \theta] \vec{e}_Z$$

By using the specific relationships to the conics (7) we obtain

$$\vec{L}_I = m_l \frac{aC}{C'_2 + aC'_1} \vec{e}_Z (m^2 s^{-1})$$

This vector is constant. Angular momentum becomes simply

$$\vec{L}_I = m_l \frac{aC}{\sqrt{ar_0} \left(1 - \frac{r_0^{k+1/2}}{a^{k+1/2} + r_0^{k+1/2}}\right) + a \frac{r_0^{k+1/2}}{a^{k+1/2} + r_0^{k+1/2}}} \vec{e}_Z = m_l \frac{C_1}{C'_1} \sqrt{aGM_G (1-e^2)} \vec{e}_Z$$

Angular momentum is well constant. It is also interesting to note that it can be written

$$\vec{L}_I = \sqrt{m_l m_G} \sqrt{aGM_G (1-e^2)} \vec{e}_Z$$

B.2.2. Equation of time

As usual, we write this equation by using the eccentric anomaly E (Figure 2). Indeed (in the case of an ellipse) we have the relation

$$\dot{\theta} = \dot{E} \frac{\sqrt{1-e^2}}{1-e\cos E}$$

By using equation (23)

$$\dot{E} \frac{\sqrt{1-e^2}}{1-e\cos E} = \frac{C}{r} \frac{1}{C'_1 r + C'_2}$$

And by using

$$r = a(1 - \cos E)$$

We obtain

$$\dot{E} [C'_1 a(1 - e\cos E) + C'_2] = \frac{C}{a\sqrt{1-e^2}} = \sqrt{\frac{A}{a}}$$

By an integration

$$C'_1 a(E - e\sin E) + C'_2 E = \sqrt{\frac{A}{a}} t$$

Consequently if trajectory is bounded we obtain

$$E(C'_1 a + C'_2) - eC'_1 a \sin E = \sqrt{\frac{A}{a}} t$$

$$(C'_1 a + C'_2) \left[E - e \frac{C'_1 a}{C'_1 a + C'_2} \sin E \right] = \sqrt{\frac{A}{a}} t$$

And

$$E - e \frac{C'_1 a}{C'_1 a + C'_2} \sin E = \frac{1}{(C'_1 a + C'_2)} \sqrt{\frac{A}{a}} t = mt$$

Thus the mean motion is given by

$$m = \frac{1}{(C'_1 a + C'_2)} \sqrt{\frac{A}{a}}$$

If the force is the Newton's we obtain the equation of time of Kepler

$$E - e \sin E = \sqrt{\frac{A}{a^3}} t$$

And if the force is the Hooke's we obtain simply

$$E = \sqrt{\frac{A}{aC_2'^2}} t$$

These results indicates that we have well a time invariance. In agreement with Noether's theorem Energy is conserved.

C. Predictions

We study in this part several predictions of our model, in particular about the curve of rotation of galaxies and about the expansion of the Universe.

C.1. Curve of rotation of galaxies: comparison with MOND

To obtain this curve we consider as usual the circular motion and we write in this case the fundamental equality of dynamics

$$m_l \frac{V^2}{r} = \frac{GM_G m_G}{(C_1(r-a) + a)^3} R$$

We have the relationships $r = a = R$. Speed is thus given by

$$V = \sqrt{\frac{GM_G}{a}} \sqrt{\eta} = \sqrt{\frac{GM_G}{ar_0}} \left[\frac{a^{k+1/2} + r_0^{k+1/2}}{a^k + r_0^k} \right] \quad (24)$$

Note that we can distinguish two limiting cases:

If $r_0 \gg a$ the speed becomes the Newton's given by

$$V = \sqrt{\frac{GM_G}{a}}$$

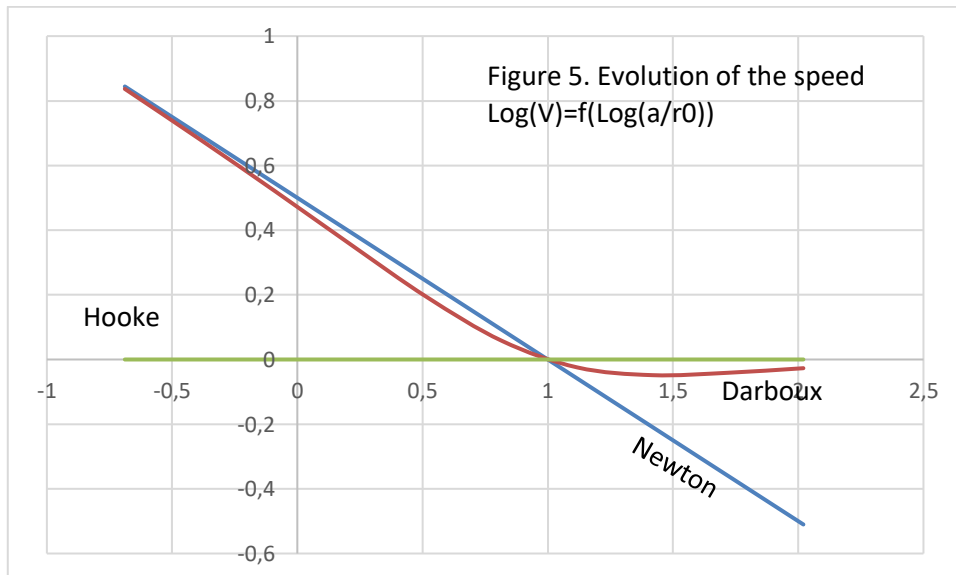
If $r_0 \ll a$ this speed becomes

$$V = \sqrt{\frac{GM_G}{r_0}} = [GM_G a_0]^{1/4}$$

We introduce a variable $y = \frac{a}{r_0}$ (24)

$$V = \sqrt{\frac{GM_G}{a}} \left[\frac{1 + y^{k+1/2}}{1 + y^k} \right]$$

We present the graph we obtained for $k = 3/2$ (Figure 5).



For comparison with our solar system Earth is located at $\text{Log}(a/r_0) \approx -5$ and pluton at $\text{Log}(a/r_0) \approx -3$:

Difference between the Newton's and Darboux's model are tiny. But we can expect modification of the dynamics at the extreme periphery of the solar system. Note also that the speed due to the model is inferior to the limital speed when $a \approx r_0$.

It is also interesting to compare the speed we have obtained with the speed due to MOND theory. Indeed it is well known that this theory allows to obtain very good fit of curve of rotation of spiral galaxies. If functions are proximate we will deduce that the model could likely be correct to describe these curves.

MOND theory assumes that "the Newtonian acceleration g_N produced by the visible matter is linked to the true acceleration g by means of an interpolating function μ » [11] given by

$$\mu\left(\frac{g}{a_0}\right)g = g_N$$

Where

$$\mu(x) = 1 \text{ if } x \gg 1$$

$$\mu(x) = x \text{ if } |x| \ll 1$$

Force and acceleration are linked by

$$\vec{F} = m_1 \mu\left(\frac{g}{a_0}\right) \vec{g} \quad (25)$$

Where F is the Newton's force. In his theory inertial and gravitational mass are equal and (25) becomes in the case of circular motion

$$\frac{GM_G}{a^2} = \mu\left(\frac{g}{a_0}\right) \frac{V^2}{a} \quad (26)$$

Several possible expressions of μ has been tested with success. The most popular choice was the "standard" μ function [11]

$$\mu_1 = \frac{x}{\sqrt{1+x^2}}$$

And the "simple" μ function [14]

$$\mu_2 = \frac{x}{1+x}$$

Note that these two functions allows to obtain excellent fit for $a_0 \approx 1,2 \cdot 10^{-10} \text{ms}^{-2}$. By using acceleration given by

$$g = \frac{V^2}{a}$$

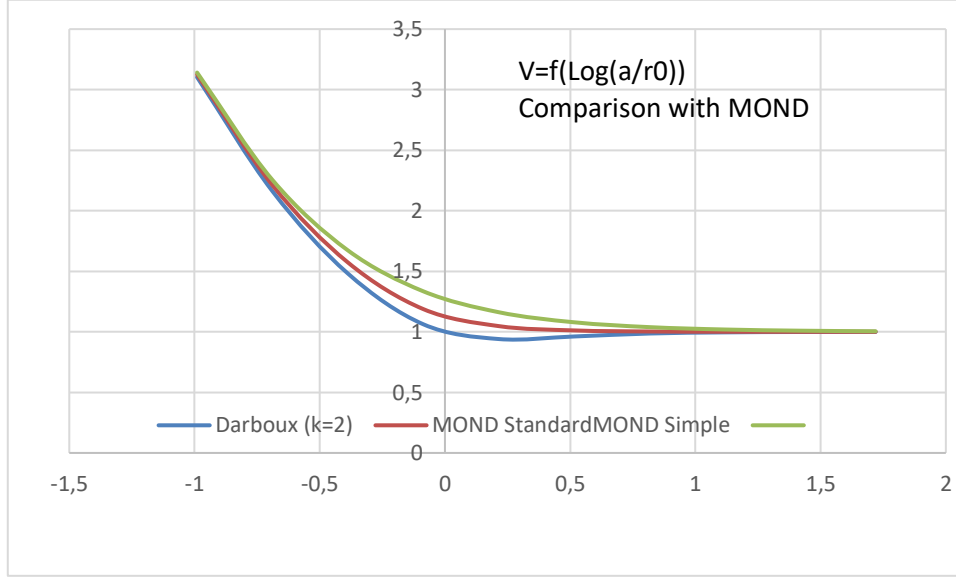
And by introducing this relation into (26) we obtain two expressions of the speed

$$V_1 = \sqrt{\frac{GM_G}{ar_0} \left[\frac{r_0^2 + \sqrt{r_0^4 + 4a^4}}{2} \right]^{1/4}} = \sqrt{\frac{GM_G}{a} \left[\frac{1 + \sqrt{1 + 4y^4}}{2} \right]^{1/4}}$$

And

$$V_2 = \sqrt{\frac{GM_G}{ar_0} \left[\frac{r_0^2 + \sqrt{r_0^2 + 4a^2}}{2} \right]^{\frac{1}{2}}} = \sqrt{\frac{GM_G}{a} \left[\frac{1 + \sqrt{1 + 4y^2}}{2} \right]^{\frac{1}{2}}}$$

We plot the functions on figure 6.



C.2. Comparison with “Mass Discrepancy Acceleration Relation”

Recently an empirical correlation (called MDAR”) between radial acceleration and distribution of baryon in a wide range of galaxies has been published [31]. We can thus compare our model with this result.

Correlation is given by

$$g_{OBS} = \frac{g_{BAR}}{1 - \exp\left(-\frac{g_{BAR}}{g_0}\right)}$$

Where g_{OBS} is the radial observed acceleration, g_{BAR} the acceleration due to visible baryonic matter and g_0 a constant acceleration proximate to the Millgrom's. Note that this function has already been used in MOND [32]. In our case radial acceleration is given (for circular motion) by

$$a_C = \frac{GM_G}{(C'_1 r + C'_2)^2}$$

With coefficients given by (22) and by using

$$g_{BAR} = \frac{GM_G}{r^2}$$

$$g_0 = \frac{GM_G}{r_0^2}$$

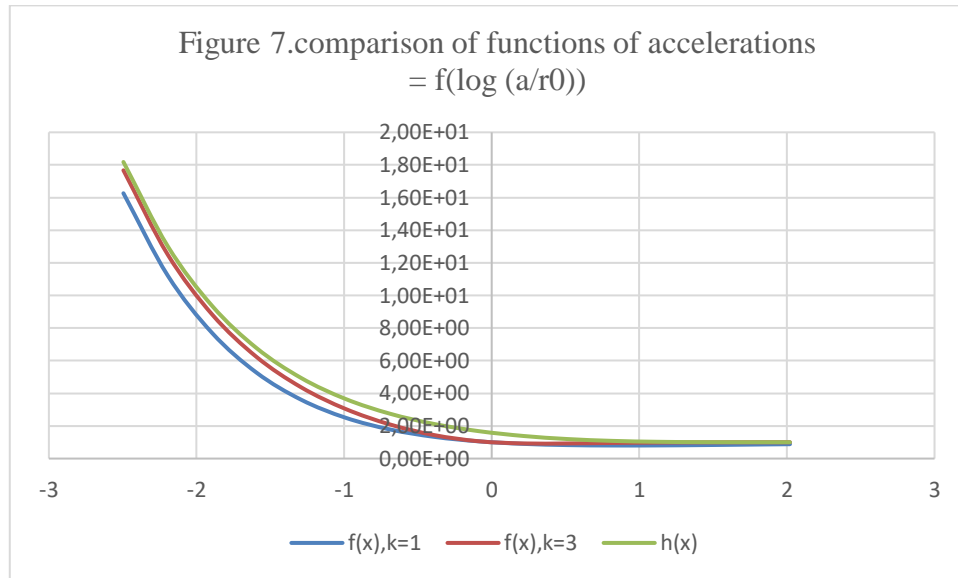
We obtain

$$a_C = g_{BAR} \sqrt{\frac{1}{x} \left[\frac{1 + x^{\frac{k/2 + 1/4}}}{1 + x^{k/2}} \right]^2} = g_{BAR} f(x)$$

With this notation MDAR becomes

$$g_{OBS} = g_{BAR} \frac{1}{1 - \exp(-x)} = g_{BAR} \cdot h(x)$$

It seems thus interesting to compare the functions $f(x)$ and $g(x)$. We plot the two functions on figure 7.



No that the nearest function we obtained was the “standard” interpolating function of MOND.

C.2. Expansion of the Universe

C.2.1. Consequence on the parabolic motion: dynamics of the Universe.

We see here the predictions of this model about the dynamics of the universe. To do it we study the limiting case of parabolic motion. The eccentricity is given by

$$e = 1$$

And in the simplest case the angular θ is determined by

$$\theta = \pi$$

And we have relation

$$R = r - a$$

Consequently our force becomes simply

$$\vec{F} = -\frac{GM_G m_G}{a^2} \left[\frac{r}{a} - 1 \right] \vec{e}_R = -\frac{GM_G m_G}{a^3} R \vec{e}_R$$

We can see that the sign of this force can be positive or negative, and is depending on the ratio $\frac{r}{a}$.

Indeed, if $r < a$ the force is repulsive. However, it also appears that this force can be attractive if $r > a$. Accelerations becomes

$$\vec{g} = -\frac{GM_G}{a^2} \left[\frac{r}{a} - 1 \right] \vec{e}_R$$

We can deduce the implication of the model about the dynamics of the Universe. We consider, for example, the interaction of two clusters of galaxies between themselves. We approximate their motion with a parabolic motion, and the force of gravitation is the force we present here.

If the clusters move away from one another it is possible that this force is repulsive. Consequently, their relative speed will increase, at least for a time. It appears that if our idea is correct this kind of interactions is actually majority inside the Universe. However when the distance between the clusters progresses this force becomes a day attractive. Consequently model can describe the actual expansion of the Universe at an increasing rate, and simultaneously can be compatible with the classical idea of "Big Bang / Big Crunch". It doesn't need "dark energy" hypothesis, because the Universe is considerate as a kind of harmonic oscillator.

Indeed for a closed homogeneous and isotropic universe, we can compare it to a kind of spring. The expansion of the universe is accelerated because this "spring" is today compressed. This point indicates that the free end of the spring hasn't reached its "relaxed" position. After this point the force becomes attractive and the rate of the expansion decreases.

We will now briefly compare the model with the Friedmann's equations.

C.2.2.. Comparison with the Friedmann's equations

The first equation of Friedmann is used for modeling an isotropic and homogeneous universe and is derived from the Einstein's field equations [24]. In this equation Einstein has inserted a factor called the cosmological constant. By taking care of this factor the first Friedmann's equation becomes

$$\dot{a}_1^2 - \frac{8\pi G}{3} \rho(t) a_1^2 - \frac{\Lambda c^2}{3} a_1^2 = kc^2$$

Where a_1 is the scale factor, G the Newton's gravitational constant, Λ the cosmological constant, c the speed of light in vacuum, $\rho(t)$ the density of mass of the universe and $\frac{k}{a_1^2}$ the spatial curvature in

any time slice of the Universe. Considering $\rho(t) \propto a_1^{-3}$ [25] the equation of Friedmann becomes

$$\dot{a}_1^2 - \frac{m}{a_1} - \frac{\Lambda c^2}{3} a_1^2 = kc^2 \quad (27)$$

Where m is constant.

The first term of this equation can be compared with a Kinetic energy. The second term with a Newtonian potential, the third with an isotropic harmonic oscillator potential.

The right-hand side of this equation is constant. Consequently this equation can be understood as the conservation of the Mechanical energy.

If we have a look on equation (27) we can deduce that the force we present here doesn't lead to a comparable equation. Indeed we can't write a potential which is only the addition of a Newtonian and harmonic oscillator potential, because in this case the trajectory won't be a conic.

However if $a_1 \rightarrow \infty$ we can consider that the Newtonian potential disappears in equation (27). It appears that in this case this equation possesses an interesting analogy with the model we study. Indeed, in the case of parabolic motion the force is given by

$$\vec{F} = -\frac{A}{a^3} \left[\frac{r}{a} - 1 \right] \vec{e}_R$$

In this limiting case the force is conservative. Consequently we can determine a potential which is only depending on r . This one is given by

$$V_p(r) = -\int -\frac{GM_G m_G}{a^2} \left[\frac{r}{a} - 1 \right] .dr = \frac{1}{2} \frac{A}{a^3} r^2 - \frac{A}{a^2} r + V_{p1}$$

Where V_{p1} is constant. By writing as usual that the potential has to be null when $r = a$ we obtain

$$V_{p1} = \frac{1}{2} \frac{A}{a}$$

And the potential becomes

$$V(r) = \frac{1}{2} \frac{GM_G m_G}{a^3} [r - a]^2$$

This force is located at O and the distance $OM = R$ is given by using the relation

$$R = \sqrt{a^2(1 - e^2) + r^2 + a^2 - 2ar}$$

With $e = 1$ and consequently

$$R = r - a$$

Consequently the potential (per mass unity) can be written

$$V(R) = \frac{1}{2} \frac{GM_G m_G}{a^3} R^2$$

The Kinetic energy is given by

$$E_C = \frac{1}{2} m_I V^2 = \frac{1}{2} m_I (\dot{r}^2 + (r\dot{\theta})^2)$$

Using equations (2)

$$E_C = \frac{1}{2} m_I \frac{(2a - r)r}{a^2 r_0} = \frac{1}{2} \frac{GM_G m_I}{a^2 r_0} (a^2 - R^2)$$

Thus the Mechanical energy is

$$E_M = E_C + V_p(R) = \frac{1}{2} \frac{GM_G m_I}{a^2 r_0} (a^2 - R^2) + \frac{1}{2} \frac{GM_G m_G}{a^3} R^2$$

And Mechanical energy per inertial mass unity

$$E_M = \frac{1}{2} \frac{GM_G}{a^2 r_0} (a^2 - R^2) + \frac{1}{2} \frac{GM_G}{a^3} \eta R^2 = \frac{1}{2} \frac{GM_G}{r_0}$$

And we obtain

$$\frac{1}{2} \dot{r}^2 + \frac{1}{2} \frac{GM_G}{a^2 r_0} R^2 = \frac{1}{2} \frac{GM_G}{r_0} \quad (28)$$

We can now compare equations (27) and (28). It appears that we can write two proportionality relations

$$-\frac{\Lambda c^2}{3} a_1^2 \propto \frac{1}{2} \frac{GM_G}{a^2 r_0} R^2$$

And

$$kc^2 \propto \frac{1}{2} \frac{GM_G}{r_0}$$

Where r_0 is given by

$$r_0 = \sqrt{\frac{GM_U}{a_0}}$$

It seems thus that if this analogy is correct the cosmological constant should be negative. Moreover this cosmological constant should be depending on the Millgrom's acceleration.

But as we wrote it previously it seems that it is only an analogy. However we can perhaps consider that the mathematical similarity between the two equations is an argument for the model.

D. Possible test inside our solar system?

As often in this domain of the Physics we are faced to the lake of experimental results. It is the case of the dominant approach (Dark Matter/ Energy approach) and of the majority of alternative theories. Indeed tests seem very difficult to implement when acceleration is so small [32]. However we can suggest tests inside our solar system to valid or invalid the model. Indeed it is perhaps possible to detect small perturbations around the Newton's law if it is correct. In particular we think that it could be interesting to test the parabolic or hyperbolic motion, for example with a spacecraft.

Indeed in the case of free fall motion, the acceleration (we have obtained it with a series when $r_0 \rightarrow \infty$) should be

$$\vec{a}_C = \left(-\frac{GM}{r^2} + \frac{GM}{r^3} \left[\frac{a}{r_0} \right]^{\frac{3}{2}} [4a - r] \right) \vec{e}_R$$

Which is consequently not absolutely equal to the Newton's.

Another test could be around the equivalence principle. Indeed model predicts that it isn't absolutely respected. For example if two test-body are dropped out from the same distance from the center of force the semi-major axis of their trajectory will be equal, but the ratio gravitational/inertial masses given by

$$\eta = \frac{m_G}{m_I} = \frac{1}{r_0} \left[\frac{a^2 + r_0^2}{a^{\frac{3}{2}} + r_0^{\frac{3}{2}}} \right]^2$$

Will be a little bit different because

$$r_0 = \sqrt{\frac{G(M_G + m_G)}{a_0}}$$

This constant isn't exactly the same for the two test-body. However generally difference should be a negligible amount, because masses of test-body are small in comparison with the mass of the Earth. For example mass of test bodies inside satellite Microscope [33] are around 0.5 and 1.5 Kg and Earth around 10^{24} Kg... Difference of r_0 and consequently of η should be in these conditions really small and likely undetectable.

E Discussion

E.1. About the force of Hooke

In this paper, we studied the possibility that the force of Hooke could perhaps be the force of gravitation valuable for large distances, by choosing correct coefficients. We think consequently it makes senses to ask ourselves if this idea can have a physical reality. It appears that we can find several argument in favor of this hypothesis.

Firstly, we can notice that the force of Hooke exist already in the gravitation. It is the force which interacts with a point-particle inside a sphere where the density of mass is uniform. This is the consequence of the theorem of Gauss. For example, in a sphere the force is given by

$$\vec{F} = -\frac{GM_{G_{\text{int}}}}{r^2} m_I \cdot \vec{e}_R$$

Where $M_{G_{\text{int}}}$ is the total gravitational mass contained inside the sphere of radius r . If the point particle is dropped out from a distance a from the center and if the mass density is uniform we obtain

$$\vec{F} = -\frac{GM_G}{a^3} m_I \cdot r \cdot \vec{e}_R$$

Where C is constant. We can compare this expression with force we obtained in a comparable motion

$$\vec{F} = -\frac{GM_G}{a^3} m_I \cdot r \cdot \vec{e}_R$$

We see that the expressions are identical. By using this analogy we can suggest a physical interpretation of our force: progressively, when the distance to the center of force increases, the point particle which orbits around the center of mass "considers" that this mass is progressively «diluted» inside a closed volume. This volume is depending on the semi major axis of its trajectory. At the end of this evolution the force becomes entirely the Hooke's. Note that this kind of interpretation was already used in another action-at-a-distance, to build the model of the atom of Thomson at the 19th century.

We can also add that this force is often used in chemistry for modeling molecular bonds. For this kind of reasons (and also for the analogy with the Friedman's equation we studied) our conclusion is that we can't absolutely exclude the possibility that the force of Hooke be a force of gravitation valuable for large distances.

E.2. Strengths and weakness of the model

We can list several strengths of the model we presented. In a first time it is interesting to note that it is in agreement with corpus of classical Physics. In particular, energy and angular momentum are conserved. Note that it is an important difference with other theories, in particular with MOND. Moreover model allows to solve the two body problem, at the cost of a modification of equivalence principle.

A second interesting point is that model seems describe correctly dynamics of Galaxies and Universe without Dark matter or Dark Energy. Curve of rotation of galaxies are well flat and actual expansion of Universe can be explained. It seems thus that this model present less difficulties than other which failed often to describe simultaneous these two facts.

Another important argument for this model is that the ratio inertial/gravitational mass is constant on all a trajectory: this indicates that it is perhaps possible that it becomes a day the limiting case of a relativistic theory.

Among the weak points we think to several difficulties. Firstly, the force is depending on the trajectory. And secondly, the model is non-relativistic and consequently can only be an approximation of reality. It is possible that a day these two difficulties are solved, if the model becomes a limiting case of a relativistic theory. In our opinion this theory should be based on the equivalence relations we give in the paper.

F. Conclusion

In this paper I present a classical model as an alternative to dark matter and dark energy. I used the Darboux's forces to see if these forces could be transitional forces between the Newton's and the Hooke's. Conclusion is that these forces should lead to curve of rotation of galaxies comparable to MOND theory, and that they could perhaps describe the actual dynamics of the Universe. Equivalence principle is violated (except in the Newton's case) but ratio of inertial/gravitational masses is constant on all a trajectory. Moreover model respect conservation of Energy and of Angular Momentum, and seems in agreement with corpus of classical physics. We don't expect modification on advance on perihelion of planets inside our solar system but a modification of the mean motion at the extreme periphery of it. Principal difficulty of the model is that the force isn't conservative and consequently is depending on the trajectory. However this difficulty is perhaps not unsurmountable, especially if we consider that it could be the limiting case of a relativistic theory. At end it seems interesting to test it on real curves of rotation of galaxies and on parabolic interactions between clusters of galaxies.

REFERENCES

- [1] Rubin V. C., Ford W. K. Jr., AJ, 159 (1970) pp. 379
- [2] Faber S.M., Jackson R.E., AJ, 204 (1976) pp. 668
- [3] Rubin V. C., Ford W. K. Jr., Thonnard N., AJ, 238 (1970) pp 471
- [4] A.G. Riess *et al*, AJ, 116, (1998) pp. 10
- [5] L. Patantonopoulos *The Invisible Universe, Dark Matter and Dark Energy* Springer (2007)
- [6] D. Huterer, M.S. Turner Phys. rev. D 60 (1999)
- [7] Akerid D.S. et al. Phys.Rev.Lett., 112 (2014) pp.091303
- [8] Moni Bidin C., Carraro G., Méndez R.A. ,van Altena W.F. *AJ. Lett.*, 724 (2010) L122
- [9] Sotiriou, Thomas P. at al. Rev.Mod.Phys., 82 (2010) pp. 451
- [10] Brans, Carl H, gr-qc/0506063, arXiv:gr-qc/0506063 v1, (2005)
- [11] Milgrom M. AJ, 270 (1983)
- [12] J.C.Fabris, J.Pereira Campos General Relativity & Gravitation. 41(1):93–104, Jan 2009
- [13] O.Ragos, I.Haranas and I.Gkigkitzis Astrophysics and Space Science 345, 1, 67-72 (2013)
- [14] M.Sereno , Ph.Jetzer, Mon.Not.R.Astron.Soc. 371:626-632 (2006)
- [15] I.B. Khriplovich, Int.J.Mod.Phys. D,16,1475 (2007)
- [16] Iorio L., arXiv:gr-qc/0608068 (2006)
- [17] Iorio L, arXiv:gr-qc/0608101 (2006)

- [18] L. Iorio, *Journal of High Energy Physics* 2007(10):041 · October 2007
- [19] Bertrand J.C.R. *Acad. Sci. Paris* (1873) 77 849
- [20] Santos F.C., V.Soares, A.C.Tort, *Phys. Rev. E* 78 (2009) 036605
- [21] Darboux G., *C.R.Acad. Sci. Paris*, 84, 760-762, 936-938 (1877)
- [22] Appell P. *Am. J. Phys*, Vol. 13, N°.2 153-158 (1891)
- [23] J.Suchar P.J. *N. A. Math.* 4ème série tome 6 (1906) p.532-546
- [24] Akad SB. *General Relativity and Gravitation*, Vol. 31, No. 12, 1999
- [25] Bianchi E., Rovelli C., arxiv:1002.3966v3 [astro-ph.CO]
- [26] Guiot.E. *Int.J.Mod.Phys.D* Vol.24, N°.13 (2015) 1550088
- [27] Guiot.E. <https://hal.archives-ouvertes.fr/hal-01322401> (2016)
- [28] I.Licata, C.Corda, E.Benedetto, *Grav.Cosmol.*22 (2016) n°1, 48-53
- [29] Tully R.B. , Fisher J.R., *A&A*, 54 n°3 (1977) pp. 661
- [30] G. Gentile, B. Famaey et W. J. G. de Blok, *A&A*, vol. 527, 1^{er} mars 2011, A76
- [31] S. Mc Gaugh, F. Celli and J.M. Schomberts arXiv :1609.05917 [astro-ph.GA]
- [32] M.Millgrom [arXiv:1609.06642](https://arxiv.org/abs/1609.06642) [astro-ph.GA]
- [32] De Lorenci V.A., Faundez-Abans M., Pereira J.P. *A&A* 503 L1 (2009)
- [33] P.Touboul, M.Rodriguez, *Class. Quantum Grav.*n,18 (2001) 2487-2498