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# Comment on "Conjugate ESPRIT (C-SPRIT)"

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## Abstract

It is shown that in the above paper by N. Tayem and H.M. Kwon, the Esprit-like algorithm was derived from a rather unrealistic assumption, and therefore, this conjugate Esprit algorithm will have little utility in practice.

**Index terms:** Antenna array, direction of arrival (DOA) estimation, non circular signals.

In [1], the authors present an Esprit-like algorithm to estimate the directions of arrivals (DOA) from noncoherent one-dimensional (1-D) signal sources such as binary phase shift keying (BPSK) and M-ary amplitude shift keying (MASK). The proposed algorithm provides more precise DOA estimates and can detect more signals than well-known classical subspace-methods MUSIC and ESPRIT for 1-D signals. The paper has been read with great interest; however, it appears to be based on a key assumption that is never satisfied in practice. In this note, we explain the correct model that must be used for estimation of DOA of narrowband (1-D) signal sources.

The authors consider a uniform linear array composed of  $M$  elements that receives  $K$  noncoherent and narrowband 1-D signals from different DOAs  $\theta_k$ ,  $k = 1, \dots, K$ . The  $M \times 1$  received signal satisfies the following standard model

$$\mathbf{y}_t = \sum_{k=1}^K \mathbf{a}(\theta_k) s_{k,t} + \mathbf{n}_t,$$

where the first component of all the steering vectors  $\mathbf{a}(\theta_k)$  is equal to one, and where it is assumed in [1] that  $s_{k,t}$ ,  $k = 1, \dots, K$  are real-valued for 1-D signals such as BPSK and MASK. The two assumptions

$$[\mathbf{a}(\theta_k)]_1 = 1 \quad \text{and} \quad s_{k,t} = s_{k,t}^*, \quad \text{for } k = 1, \dots, K \quad (0.1)$$

are key assumptions that allow the authors to prove the following relation [1, rel. (15)]

$$\begin{pmatrix} \mathbf{y}_{t,1} \\ \mathbf{y}_{t,2} \end{pmatrix} = \begin{pmatrix} \mathbf{A} \\ \mathbf{A}\Phi^* \end{pmatrix} \mathbf{s}_t + \begin{pmatrix} \mathbf{n}_{t,1} \\ \mathbf{n}_{t,2} \end{pmatrix}$$

on which the proposed conjugate ESPRIT is devised.

We prove in the following that the two constraints (0.1) are not consistent for one-dimensional signals. In practice, the DOA estimation is made after frequency down conversion: The analog received BPSK or MASK modulated signals are bandpass filtered and after down-shifting the signal to baseband, the in-phase and quadrature components are matched-filtered, sampled and paired to obtain complex signals  $s_{k,t}$ . The output of the matched filter associated with the first element of the array is represented as a sum  $\sum_{k=1}^K s_{k,t}$  of 1-D signals with

$$s_{k,t} = \alpha_{k,t} e^{i\phi_k},$$

where  $\alpha_{k,t}$  are real-valued and where  $\phi_k$  are *arbitrary phase shifts* that can be different for each signal but constant with time. Consequently, the *amplitudes*  $s_{k,t}$  are *complex-valued*. This model has been used by numerous authors who have studied DOA estimation of non-circular sources (see e.g., [2, rels.(5),(7)], [3, rel.(28)], [4, rel. (2)], [5, Subsection II.A], [6, bottom of p.2]). Note that in these references all emphasize

that the phases  $\phi_k$  are arbitrary phase shifts (denoted natural phases in [2, 3]) that can be different for each signal.

The frequency offsets can be naturally neglected, but the phases are *intrinsic to the model if the first component of the steering vector is normalized to one*. The assumption that  $s_{k,t}$  are all real-valued would correspond to the case where all the  $K$  source signals with the same carrier frequency would be located at distances  $d_k$  from the reference element, which in turn would be a multiple of the wavelength associated with the carrier frequency: It is clear that this assumption is completely unrealistic.

This assumption cannot be circumvented because if phases were introduced into relations (12) of [1], the proposed algorithm would no longer be valid. Finally we note that this assumption (0.1) has been also used in very numerous papers of the authors dedicated to non-circular signals (see e.g., [7, 8, 9] for coherent non-circular sources for which the different signals are phase delayed amplitude weighted replicas on one of them), [10] for 2-D root MUSIC algorithms and [11] for Toeplitz based matrix pencil. Consequently all the nice results given in these papers are misleading because they do not apply in practice.

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