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# NUMERICAL AND EXPERIMENTAL STUDY OF THE ENERGY PUMPING OF THE FIRST VIBRATION MODES OF A MECHANICAL SYSTEM USING BI-STABLE ATTACHMENT.

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## ABSTRACT

*In order to control the sound radiation by a structure, one aims to control vibration of radiating modes of vibration using “Energy Pumping” also named “Targeted Energy Transfer”. This principle is here applied to a simplified model of a double leaf panel. This model is made of two beams coupled by a spring. One of the beams is connected to a non-linear absorber. This non-linear absorber is made of a 3D-printing support on which is clamped a buckled thin small beam with a small mass fixed at its center having two equilibrium positions. Once attached onto a vibrating system to control, a one direction energy exchange from the primary vibrating system to the non-linear absorber can be achieved under special operation. Our experimentations have shown that this simple bi-stable device leads up to more than 10 dB attenuation for the first two vibration modes of the system.*

## 1 INTRODUCTION

Reducing noise and vibration is a very promising research topic because it allows, for example, improved fatigue resistance with a consequent reduction in maintenance costs and noise reduction resulting in increased comfort. Many active and passive devices have been developed to improve the vibroacoustic behaviour of mechanical assemblies such as double-leaf walls.

In the passive domain, for example, the absorption of acoustic waves is typically accomplished through the absorbent material placed on the domain walls. The effectiveness of the device depends strongly on the frequency of the waves to be absorbed. To mitigate structural vibration, the Frahm absorber, consisting of a mass-spring-damper system, tuned to the frequency of vibration

to eliminate is very efficient but has a limited frequency range of effectiveness. Passive non-linear Energy Pumping (EP) is a way to overcome such limitation. Since the seminal work by Gendelman *et al.* [1, 2], because of its various and numerous applications, the problem of passive non-linear energy pumping has become a subject of growing interest [3]. The simplest case requires consideration of a linear mechanical or acoustical system connected to a secondary oscillator having a strongly non linear stiffness (typically a cubic one). This attachment is usually termed as Non-linear Energy Sink (NES). This kind of non linearity corresponds to a resonance of the NES that varies with the amplitude of excitation. This enables a passive non linear energy transfer that is realized through resonance capture at high energy value [2]. Passive non linear energy transfer from the primary system to the NES occurs under resonance condition once the NES amplitude rises above a certain threshold; reverse energy flow from the NES to the primary system is prevented because of resonance escape due to the energy decrease induced by dissipation. The existence of such threshold in purely cubic or quintic NES can be viewed either as advantage either as disadvantage depending upon application. But the main feature of energy pumping lies in the fact that the higher the frequency of the primary linear system to control, the higher the amplitude for efficient non linear passive dissipation.

To date a wide variety of NESs have been proposed and tested: pure cubic spring in mechanical systems [3], membrane acting as cubic or quintic spring in acoustical systems [4], loudspeaker used as a suspended piston acting as an essentially non-linear oscillator [5]. A recent theoretical and numerical work by Manevitch *et al* [6] had shown that a bi-stable non-linear oscillator manifests significant advantages with respect to energy pumping efficiency. We have developed an experimental non-linear bi-stable absorber that provides improved robustness in frequency and range over existing passive devices by lowering the excitation threshold.

In order to control the sound radiation by a panel, one aims to control vibration of radiating modes of vibration using EP. This principle is here applied to a simplified model of a double leaf panel. This model is made of two beams coupled by a spring. One of the beams is connected to the non-linear resonator. This non-linear resonator is made of a 3D-printing support on which is clamped a buckled thin beam with a mass fixed at its middle. The main feature of this NES lies in the buckling that allows a bi-stable compartment easy to control. Our experimentations show that this simple device leads up to more than 10 dB attenuation for the first two vibration modes of the system.

An optimization made on a simplified model of the device by a parametric study of the influence of dissipation is conducted. We show that for a wide range of configurations with one non-linear dynamic absorbers, a reduction up to more than 10 dB of the vibration of the primary system around its first two resonances is obtained.

Section 2 is devoted to the description of the experiment. In section 3 a simplified model is established. In section 4 experimental and numerical results show the efficiency of the non-linear absorber to attenuate the vibration the primary linear system. Some comments conclude this short presentation.

## 2 EXPERIMENTAL FIXTURE

Since our aim was to describe the main feature of a double leaf wall close to its mass-air-mass resonance, we have chosen to make a simplified but representative experiment. A photograph of the fixture is given in figure 1 and a sketch of it is given in figure 2. Each panel is replaced by a cantilever viscously damped beam whose dimension had been chosen to recover the feature of the panel. Each beam is made of steel with Young modulus  $E_b = 185$  GPa, volume mass  $\rho_b = 7621$  kg/m<sup>3</sup> and viscous damping  $\mu_b = 0.1$  Kg/s. Its dimensions are given by thickness  $h_b = 4.2$  mm and height  $e_p = 2.52$  cm ; its length  $L = 35$  cm is comparable to the half size of a double leaf panel. The two beams are connected by a coupling spring with mass  $m_c = 6$  g and stiffness  $r_c = 2200$  Kg.s<sup>-2</sup> corresponding to the stiffness of the air gap separating two panels in

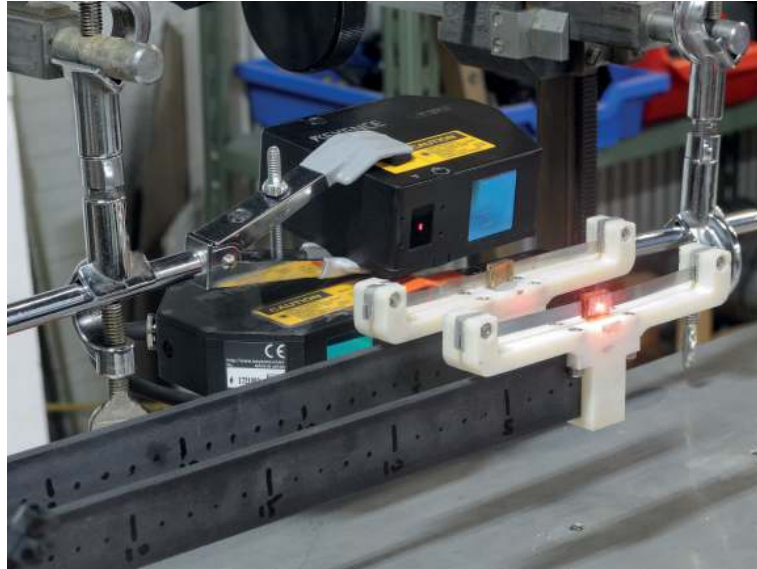


Figure 1: Photograph of the experiment. In that experiment, only one NES is active. The second remains fixed.

usual condition. This spring is located close to the free end of the beams at  $x_N = 34.5$  cm. The excitation is made by a non-contact device located at  $x_0 = 3.5$  cm of the clamped end of a beam. The NES consists in a small mass (here a mass  $m_0 = 2.6$  g had been chosen) fixed at the middle of very thin buckled viscously damped beam (steel  $E = 200$  GPa, viscous damping  $\mu = 0.18$  Kg/s, volume mass  $\rho = 7800$  Kg/m<sup>3</sup>, length  $\ell = 10$  cm, thickness  $h = 0.1$  mm, height is  $e = 5$  mm) as shown in figure 3. The buckled beam is rigidly fixed at its ends to an ABS (Acrylonitrile Butadiene Styrene) support made by a 3D-printer. This support of weight 32 g is fixed close to the end the cantilever beam at  $x_N = 34.5$  cm. To ensure a symmetry in the system a support is fixed at the end of each cantilever beam, but only one supports an active NES, the other remains blocked all along the experiment.

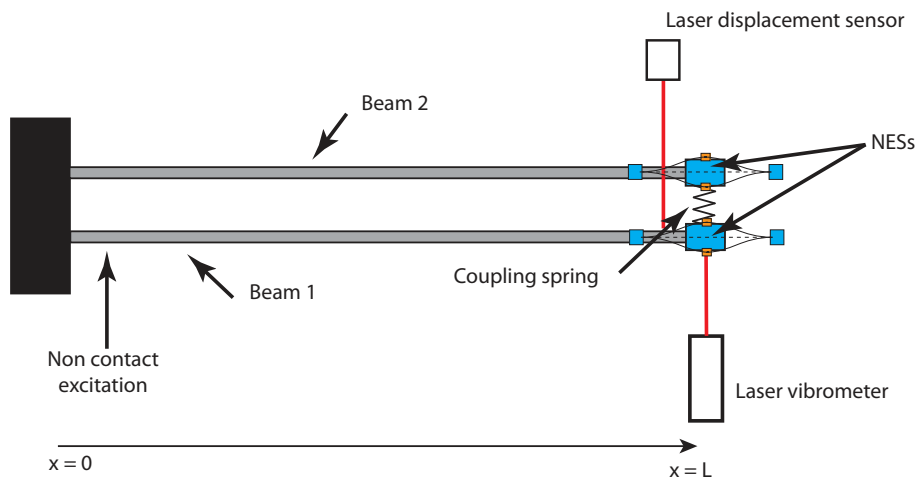


Figure 2. Sketch of the experiment. .

The first two modes of this system are obtained for the in-phase (close to 22 Hz) and the out-of-phase (close to 39 Hz) movement of the first mode of each cantilever beam. The displacement of the cantilever beam is measured by a Keyence CCD Laser Displacement Sensor LK-G 32. The displacement of the NES is measured by a a Keyence CCD Laser Displacement Sensor LK-G 82 and its velocity by Polytech Laser Doppler Vibrometer OVF-303.

### 3 SIMPLIFIED MODELISATION

#### 3.1 Approximation of the beams displacement

The two coupled beams dynamics is described by a simplified system with two degree of freedom. At low frequency, each displacement  $w_{1,2}(x, t)$  is expanded

$$w_{1,2}(x, t) = \phi_1(x)u_{1,2}(t), \quad (1)$$

with  $\phi_1(x)$  is the first mode of a cantilever beam given, with  $k_1L \approx 1.875$ , as

$$\phi_1(x) = \frac{1}{\sqrt{L}} \left( \cos(k_1x) - \cosh(k_1x) - \frac{\cos(k_1L) + \cosh(k_1L)}{\sin(k_1L) + \sinh(k_1L)} [\sin(k_1x) - \sinh(k_1x)] \right). \quad (2)$$

Using this approximation together with a Ritz reduction leads to a system of two coupled differential equations :

$$m_1\ddot{u}_1(t) + \mu_b\dot{u}_1(t) + k_1u_1(t) + k_c(u_1(t) - u_2(t)) = \phi_1^2(x_0)F(t) \quad (3)$$

$$m_1\ddot{u}_2(t) + \mu_b\dot{u}_2(t) + k_1u_2(t) - k_c(u_1(t) - u_2(t)) = 0, \quad (4)$$

$\dot{u}_i(t)$  is the time derivative of each component  $u_i(t)$ . It is worth noting that  $m_1 = \rho_b S_b + (m_N + \frac{m_c}{2}) \phi_1^2(x_N)$  is the total dynamic mass and  $k_c = r_c \phi_1^2(x_N)$  is the dynamic coupling stiffness.

#### 3.2 Approximation of the NES displacement

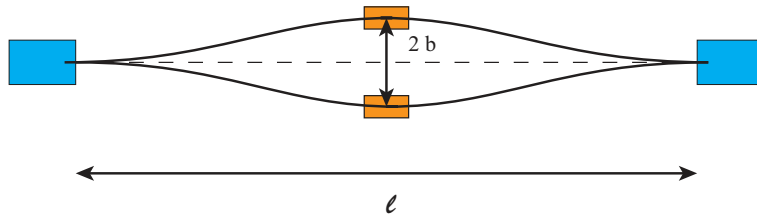


Figure 3. Sketch of the NES geometry.

A thin viscously damped beam with a small mass  $m_0$  fixed at its center when buckled under axial constraint  $\tilde{N}$  has a geometrical non linearity. Its displacement  $\tilde{w}(x, t)$  is solution of :

$$(\rho A + m_0 \delta_{\ell/2}(x)) \frac{\partial^2 \tilde{w}}{\partial t^2} + EI \frac{\partial^4 \tilde{w}}{\partial x^4} + \tilde{N} \frac{\partial^2 \tilde{w}}{\partial x^2} + \mu \frac{\partial \tilde{w}}{\partial t} - \frac{EA}{2\ell} \frac{\partial^2 \tilde{w}}{\partial x^2} \int_0^\ell \left( \frac{\partial \tilde{w}}{\partial x} \right)^2 dx = f(x) \sin(\omega t), \quad (5)$$

with  $I = eh^3/12$  and  $A = eh$ . To this equation one adds the usual boundary conditions for a clamped beam. Using the non dimensional quantities  $x = x/\ell$ ,  $r = \sqrt{I/A}$  is the radius of gyration of the cross section,  $w = w/r$ ,  $\omega_0 = 1/\ell^2 \sqrt{EI/(\rho A)}$ ,  $t = \omega_0 t$ ,  $N = \tilde{N} \ell^2/(EI)$ . Now using  $b = \tilde{b}/r = \sqrt{4(N - N_c)/\pi^2}$ ,  $N_c = 4\pi^2$  the non-dimensional post buckling deflection, the displacement of the clamped buckled beam is written as

$$w(x, t) = w_0(x) + v(x, t), \quad \text{with } w_0(x) = \frac{1}{2}b(1 - \cos 2\pi x). \quad (6)$$

Now let us approximate the dynamic deflection around equilibrium position  $v(x, t)$  using only the first buckling mode as

$$v(x, t) = w_0(x)q(t). \quad (7)$$

This approximation allows us to describe the change of equilibrium position but do not give access to a fine description of the buckled beam movement [7], particularly to its linear resonance. A Ritz reduction leads to the following non-linear equation for the NES displacement

$$(3/8 + \beta)\ddot{q}(t) + \frac{3}{8}\mu\dot{q}(t) + \frac{b^2\pi^4}{4}(q(t) + \frac{3}{2}q(t)^2 + \frac{1}{2}q(t)^3) = \frac{1}{b^2} \int_0^1 F_{eq}\Psi(x) dx \sin \Omega t, \quad (8)$$

with  $\beta \approx 7$  defined as the ratio of the small mass  $m_0$  to the NES beam mass. It is easy to show that this equation has three equilibrium points. Two are stable (0 and -2) corresponding to each buckled position one is unstable (-1). Then this NES is a bi-stable one. But since a one-mode approximation had been made, it is not possible to describe the whole dynamic of the NES. Then its physical parameter (stiffness and damping) are estimated from a measure of its first linear resonance  $f_{N1} \approx 31$  Hz. Then returning to the physical parameter, one obtains the following non-linear differential equation for the NES movement  $\tilde{q}(t)$

$$m_N\ddot{\tilde{q}}(t) + c_N\dot{\tilde{q}}(t) + k_N\mathcal{F}(\tilde{q}(t)) = 0, \quad (9)$$

where  $m_N = (3/8\rho Al + m_0)$  is the dynamic mass,  $c_N = 3/8\ell\mu_N$  is the identified dynamic damping and  $k_N = (2\pi f_{N1})^2 m_N$  is the identified dynamic stiffness. In this equation, the non linear stiffness is given by  $\mathcal{F}(\tilde{q}(t)) = (\tilde{q}(t) - \tilde{b}) + 3/(2\tilde{b}) (\tilde{q}(t) - \tilde{b})^2 + 1/(2\tilde{b}^2) (\tilde{q}(t) - \tilde{b})^3$ .

### 3.3 The full system

The complete system corresponding to the two coupled primary beams (with displacement  $u_1(t)$  and  $u_2(t)$ ) solution of equations 3 and 4) with two NESs fixed at each primary beam end (with displacement  $\tilde{q}_1(t)$  and  $\tilde{q}_2(t)$ , solution of equation 9) is given by

$$\begin{aligned} m_1\ddot{u}_1(t) + \mu_1\dot{u}_1(t) + k_1u_1(t) + k_c(u_1(t) - u_2(t)) \\ - c_N (\dot{\tilde{q}}_1(t) - \phi_1(x_N)\dot{u}_1(t)) - k_{1N}\mathcal{F}(\tilde{q}_1(t) - \phi_1(x_N)u_1(t)) = A\phi_1^2(x_0)F(t) \end{aligned} \quad (10)$$

$$\begin{aligned} m_1\ddot{u}_2(t) + \mu_1\dot{u}_2(t) + k_1u_2(t) - k_c(u_1(t) - u_2(t)) \\ - c_N (\dot{\tilde{q}}_2(t) - \phi_1(x_N)\dot{u}_2(t)) - k_{2N}\mathcal{F}(\tilde{q}_2(t) - \phi_1(x_N)u_2(t)) = 0 \end{aligned} \quad (11)$$

$$m_N\ddot{\tilde{q}}_1(t) + c_N (\dot{\tilde{q}}_1(t) - \phi_1(x_N)\dot{u}_1(t)) + k_{1N}\mathcal{F}(\tilde{q}_1(t) - \phi_1(x_N)u_1(t)) = 0 \quad (12)$$

$$m_N\ddot{\tilde{q}}_2(t) + c_N (\dot{\tilde{q}}_2(t) - \phi_1(x_N)\dot{u}_2(t)) + k_{2N}\mathcal{F}(\tilde{q}_2(t) - \phi_1(x_N)u_2(t)) = 0 \quad (13)$$

with  $F(t) = A \sin(\omega t)$ . When a NES is not active as in the experimental results presented below, only the NES fixed on the excited beam is active while the other remains blocked, the system of four coupled non-linear differential equations given by equations 10,11,12 and 13 is simply reduced to a three equations system given by equations 10,12 and 11 in which the non linear term  $k_{2N}\mathcal{F}(\tilde{q}_2(t) - \phi_1(x_N)u_2(t))$  had been deleted.

The solutions  $u_1(t)$ ,  $u_2(t)$  and  $\tilde{q}_1(t)$  of this system are calculated without any particular difficulty under the Mathematica[8] software by using the built-in numerical differential equation solving function ‘‘NDSolve’’.

## 4 RESULTS

As already said, in the experiments reported here, only the NES fixed on beam 1 is active. Around each mode of the primary system (that is 22.2 Hz and 39.3 Hz), a set of beam 1 displacement frequency response (FR) had been measured with a stepped sine source at constant amplitude. The sinusoidal forcing signal had a duration of 30 s, enough to reach stable movement for the primary system at a given amplitude and frequency. The lowest excitation amplitude had been fixed in



order to ensure a linear comportment of the whole system (coupled beams and NES). The highest excitation amplitude chosen ensures a linear comportment of the coupled beams (in that case, the maximum amplitude at their free ends remains much smaller than their thickness). In the results presented in figure 4 for the first mode around 22 Hz and in figure 5 for the second mode around 39 Hz, only excitation frequency and amplitude range had been changed. The various curves are obtained for a NES that remains unchanged.

In these figures the difference between two successive thin black lines corresponds to a change in level by 5 dB. Each point of the surface correspond to an experiment of duration 20 s length, the quantity plotted is the ratio of the RMS value calculated on the last 3 seconds of the signal of the displacement and of the excitation, that is  $20 \log U_{RMS}/A$ , where  $U_{RMS} = \sqrt{1/3 \int_{t_1}^{t_1+3} u_1^2(t) dt}$  is the measured beam 1 displacement and  $A = \sqrt{1/3 \int_{t_1}^{t_1+3} A^2(t) dt}$  is the measured excitation amplitude, the time  $t_1$  correspond to the time 3 second before the source switch off. These results show that when the NES activates, the EP lowers the response of the primary system up to 10 dB.

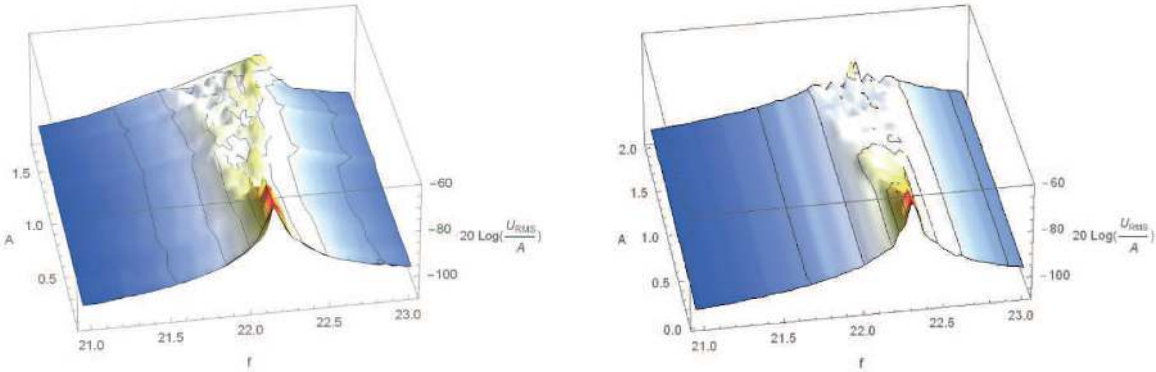


Figure 4: Surface plot of beam 1 displacement frequency response around the first mode. Left : measure, right : model

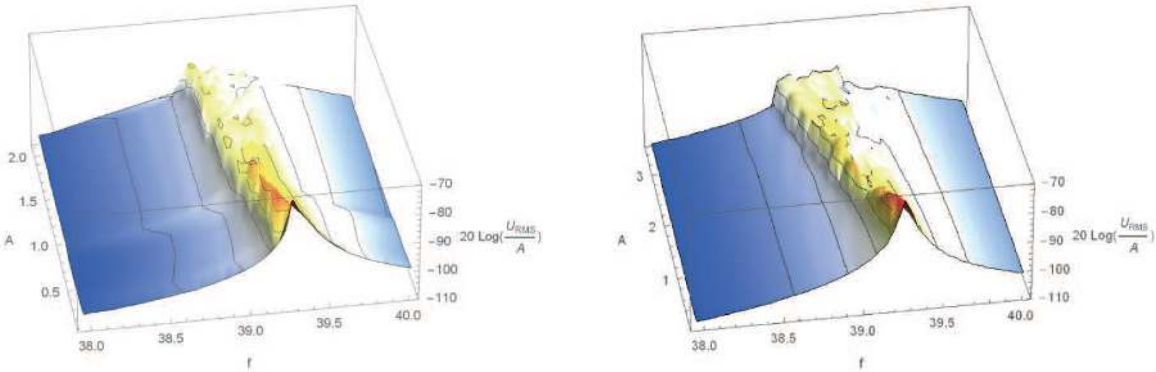


Figure 5: Surface plot of beam 1 displacement frequency response around the second mode. Left : measure, right : model

More, in that case, the higher the excitation amplitude the lower the relative response of the primary system. These curves show also that, despite all hypothesis, the simplified model is able

to recover most of the feature of the system : small shift of the frequency of the firsts two modes of the primary system, spreading and lowering of the FR up to 10 dB. It is worth noting that the results for the second beam are similar.

To see it more clearly, one presents in figure 6 for the first mode around 22 Hz and in figure 7 for the second mode around 39 Hz the ridge curves for these result. The ridge curve is defined as the curve connecting the maxima of each frequency response, each point showing the maximal frequency response of beam 1 displacement observed for a given excitation amplitude. In these figures, the red line corresponds to the ridge curve of the associated linear system which is obviously a straight line. The experimental linear ridge curve is estimated by drawing a straight horizontal line from the maximum FR obtained from the lowest amplitude. The numerical one is obtained in a similar way ; the linear FR is obtained by cancelling the non-linearity in the NES equation. While details are not perfectly recovered, in particular the first mode attenuation is a bit overestimated, on the whole most of the feature of EP is obtained. It is worth noting that one of the difficulty of such an experiment is ensuring its long term stability since each experiment take about 20 hours. Its also worth noting that the fixture is very robust since all along these experiments, the system shows a very good repeatability.

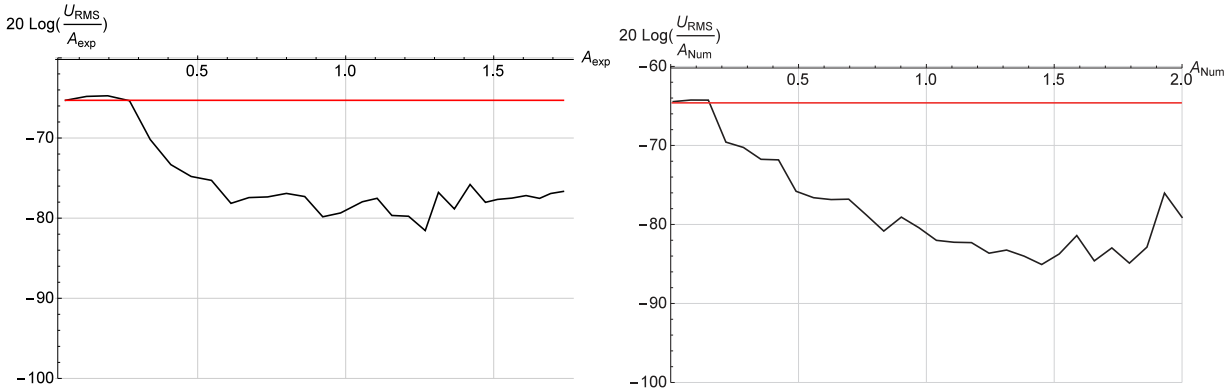


Figure 6: Ridge curve of beam 1 displacement frequency response around the first mode. Left : measure, right : model

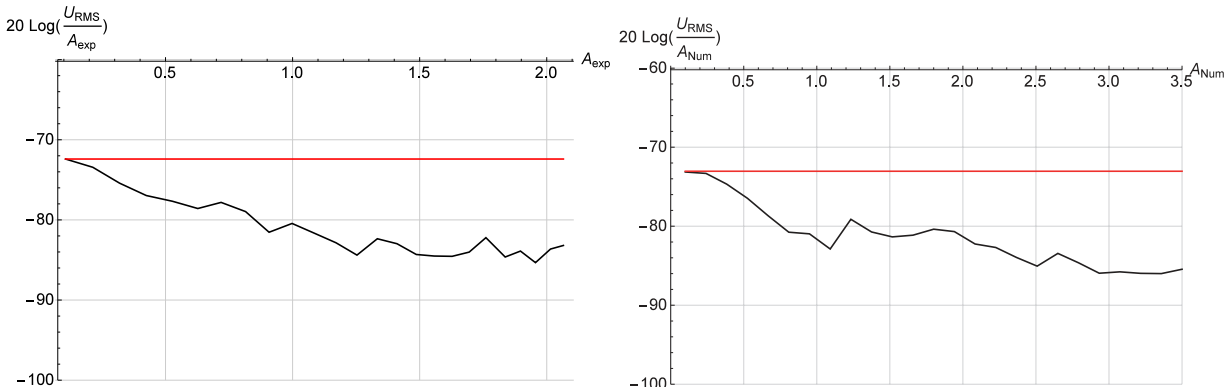


Figure 7: Ridge curve of beam 1 displacement frequency response around the second mode. Left : measure, right : model

An example of parametric study is given in figures 8 and 9 to study the influence of the damping of the NES. Each curve is obtained within about 5 minutes on a eight cores computer using Mathematica’s parallelization ability [8]. It is worth noting from these results that the damping of the NES must be carefully chosen to obtain a better attenuation. In the case considered here, the NES damping must be of the same order than primary system’s one.



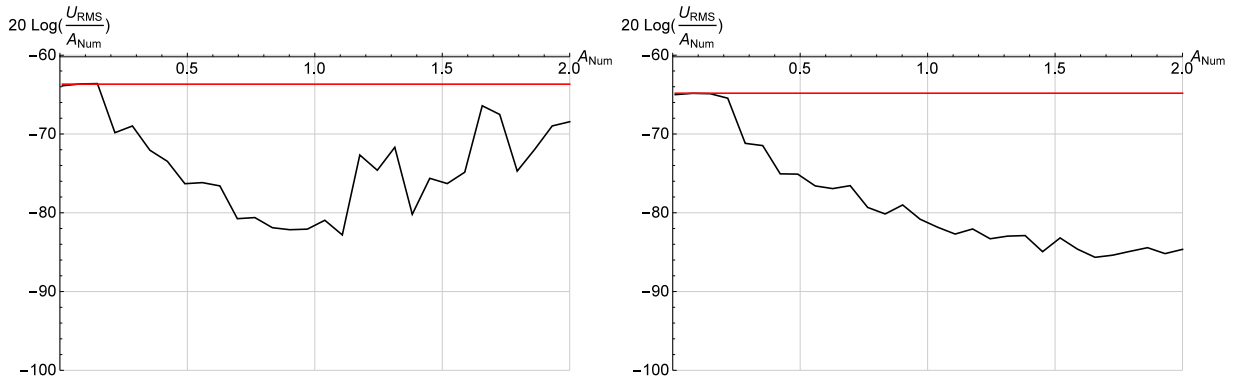


Figure 8: Calculated ridge curve of beam 1 displacement frequency response around the first mode for different NES damping. Left : viscous damping  $\mu = 0.08$  Kg/s, right : viscous damping  $\mu = 0.28$  Kg/s

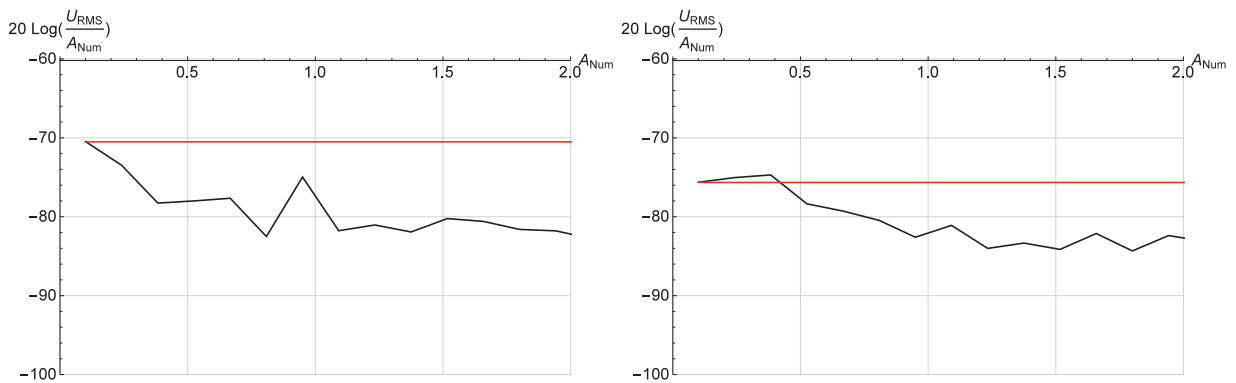


Figure 9: Calculated ridge curve of beam 1 displacement frequency response around the first mode for different NES damping. Left : viscous damping  $\mu = 0.08$  Kg/s, right : viscous damping  $\mu = 0.28$  Kg/s

## 5 CONCLUSION

The results presented here, both experimental and numerical, show that a very simple non-linear bi-stable NES is able to strongly reduce the amplitude of a primary system with multiple resonance. It is worth noting that the weight of the NES is small compared to the primary system weighing 500 g since the weight of the NES itself is less than 3 g and that of the support is about 30 g. Our recent experiments, still in progress, made on a thin plate excited by acoustic sound waves show similar ability and then confirm that this bi-stable NES is a promising way the reduce sound transmission.

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