



International Prevention Research Institute

**Spatio-colour Asplünd's metric
and
Logarithmic Image Processing
for
colour images (LIPC)**

Guillaume NOYEL – Michel JOURLIN
guillaume.noyel@i-pri.org

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Institute of
Global
Public
Health

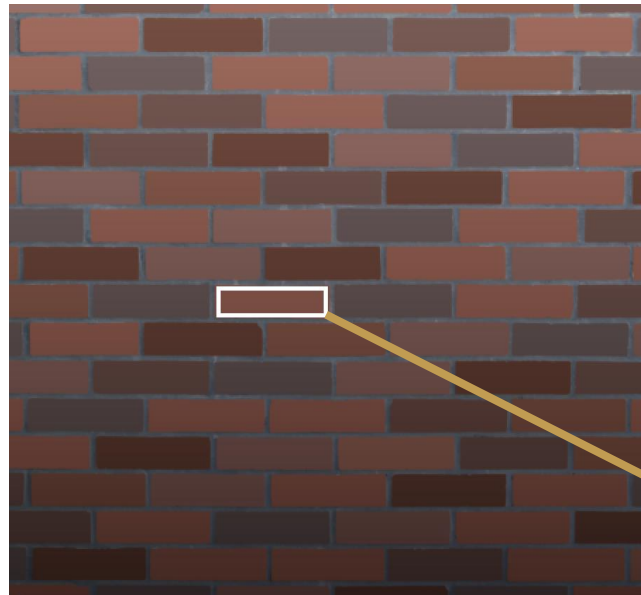


Laboratoire Hubert Curien, UMR CNRS 5516
St Etienne, France





How to detect all the bricks knowing a brick pattern?



Colour image

Colour probe or template
(magnified)



Correlation



Correlation map



Grey probe or template
(magnified)



Our approach

A large, light blue arrow pointing to the right, containing two rounded rectangular boxes. The left box is white with a blue border and contains the text 'A model of colour processing: LIPC'. The right box is solid blue with white text and contains 'A distance: Asplünd's distance'.

A model of colour
processing: LIPC

A distance:
Asplünd's distance



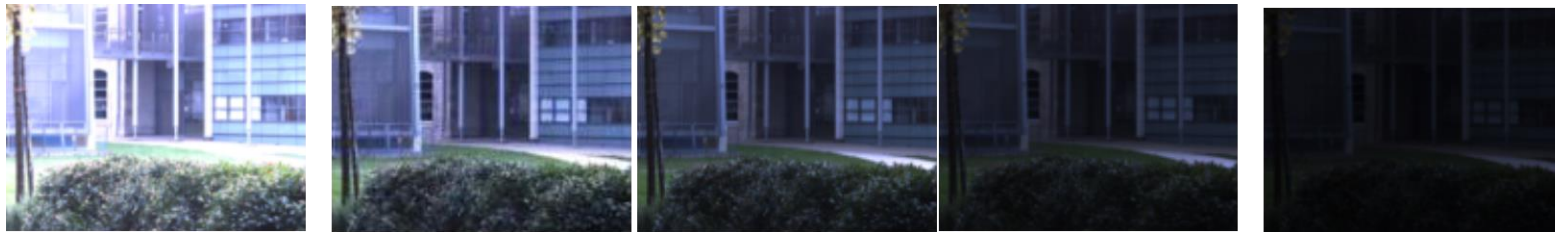
Background: colour LIPC Model & Asplünd's metric



Logarithmic Image Processing (colour)

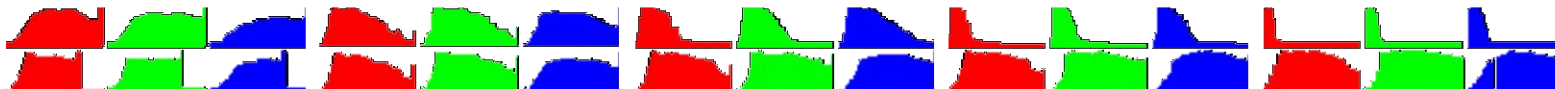
Each law of the model allows to brighten/darken images

Images
acquired with
a variable
aperture of
the camera



...

Corrected
images



Jourlin, M., et al. (2011). Advances in imaging and electron physics, Elsevier. **168**.



Images

A colour image \mathbf{f} :

$$\mathbf{f}: \begin{cases} D \subset \mathbb{R}^2 & \rightarrow \mathcal{T}^3 = [0, M]^3 \subset \mathbb{R}^3 \\ x & \rightarrow \mathbf{f}(x) = (f_R(x), f_G(x), f_B(x)) \end{cases}$$

For 8 bits images $M = 256$.

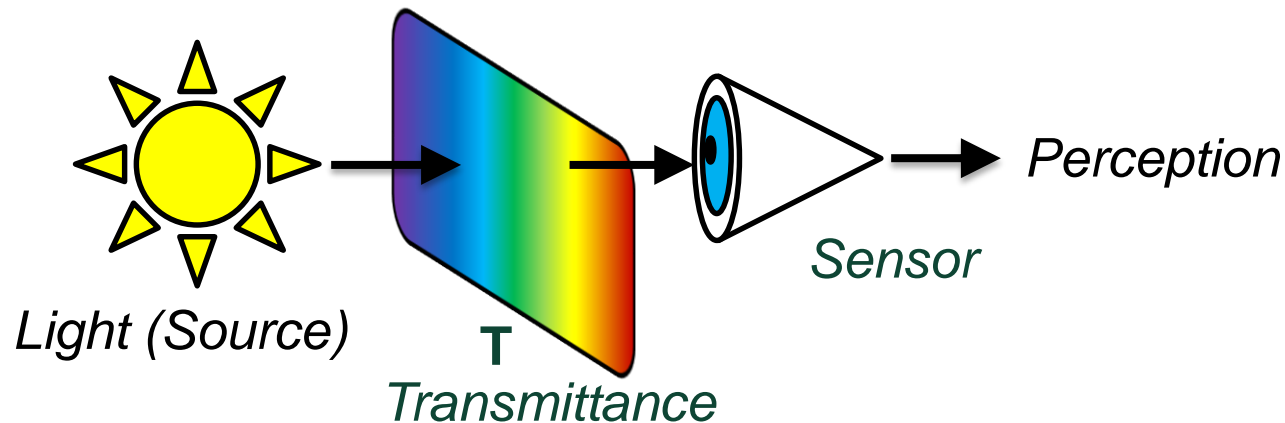
The matrix \mathbf{F} associated to the image

$$\mathbf{F} = \begin{bmatrix} f_R(x_1) & f_R(x_2) & \cdots & f_R(x_P) \\ f_G(x_1) & f_G(x_2) & \cdots & f_G(x_P) \\ f_B(x_1) & f_B(x_2) & \cdots & f_B(x_P) \end{bmatrix}$$





LIPC model



- Addition of two images $F \triangleleft_c G$: superposition of the obstacles (objects) generating F and G
- The transmittance
$$\mathbf{T}_F(x) = \frac{\text{outcoming flux}}{\text{incoming flux}}$$
- $\mathbf{T}_F(x)$: probability, for a particle of the source at x , to pass through the obstacle (to be seen by the sensor).
- LIPC model is also valid for reflected images



Some properties of the LIPC model

- LIPC is a model based on the Transmittance Law
 - it is adapted to process images
- LIPC is consistent with the Human Visual System*
 - it simulates the interpretation of an image by the human eye
- LIPC gives access to novel notions of Contrast, Metrics...
- There exists a grey level version of LIPC, namely the LIP model

*Braillean, J. C., et al. (1991). Evaluating the EM algorithm for image processing using a human visual fidelity criterion.

International Conference on Acoustics, Speech, and Signal Processing, 1991., Toronto



Physical and mathematical bases

- The Transmittance law:

$$\mathbf{T}_{\mathbf{F} \triangleleft_c \mathbf{G}} = \mathbf{T}_{\mathbf{F}} * \mathbf{T}_{\mathbf{G}} \quad (1)$$

- Link between $\mathbf{T}_{\mathbf{F}}(x)$ and the colour $\mathbf{F}(x)$:

$$\mathbf{T}_{\mathbf{F}}(x) = \mathbf{U}^{-1} \mathbf{K} \mathbf{F} \quad (2)$$

- (1) and (2) => the summation law

$$\mathbf{F} \triangleleft_c \mathbf{G} = \mathbf{K}^{-1} \mathbf{U} (\mathbf{U}^{-1} \mathbf{K} \mathbf{F} * \mathbf{U}^{-1} \mathbf{K} \mathbf{G}) \quad (3)$$

- (3) => the multiplication of an image by a positive real number α

$$\alpha \triangleleft_c \mathbf{F} = \mathbf{K}^{-1} \mathbf{U} (\mathbf{U}^{-1} \mathbf{K} \mathbf{F})^\alpha \quad (4)$$

- (3) and (4) are the two basic laws of the LIPC model
- \mathbf{K} and \mathbf{U} are the mixing matrices of the LIPC model.



LIPC operations



$3 \times_c F$



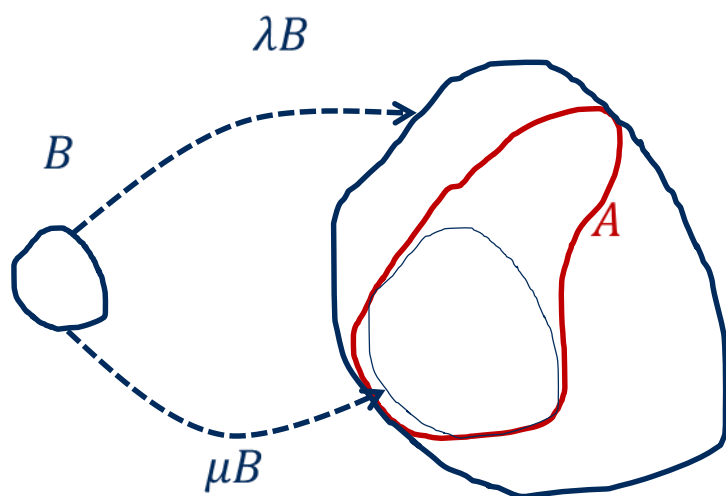
$0.5 \times_c F$





Asplünd's distance

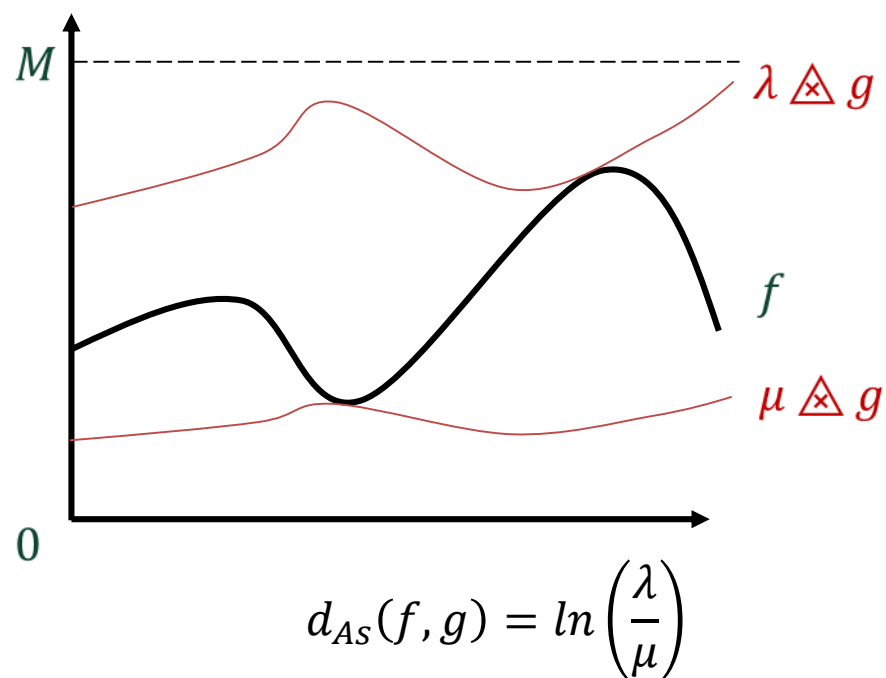
Binary shapes



$$d_{AS}(A, B) = \ln \left(\frac{\lambda}{\mu} \right)$$

- Asplünd, E. *Mathematica Scandinavica* 8, 171–180 (1960)
- Grünbaum, B. In: *Proceedings of Symposia in Pure Mathematics*. pp. 233–270 vol.7 (1963) (1963)

Functions (grey level images)

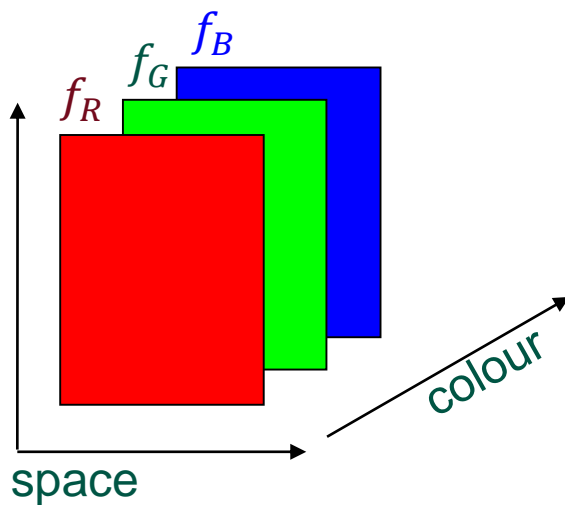


- Jourlin, M., et al. (2014). *Pattern Recognition* **47**(9): 2908-2924.

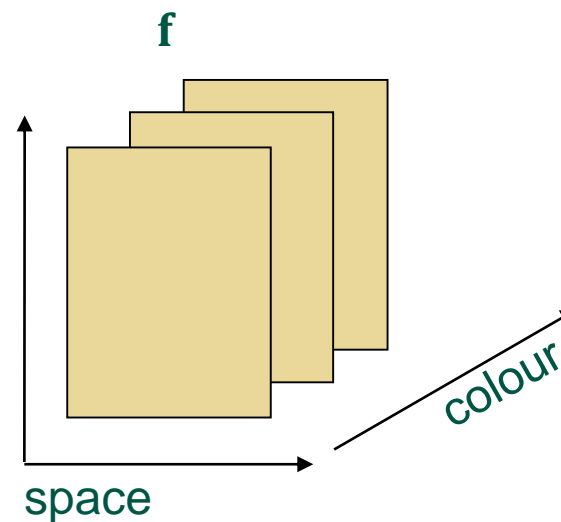


Generalisation to colour

Marginal approach



Spatio-colour approach



Noyel, G., Jourlin, M. In: Image Processing (ICIP), 2015 IEEE International Conference on. pp. 3921–3925



Spatio-colour Asplünd's metric



Asplünd's metric - LIPC

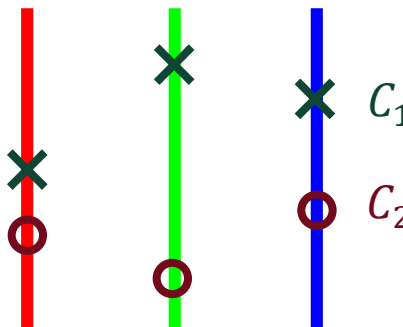
Marginal order between two colours $C_1 = (r_1, g_1, b_1)$, $C_2 = (r_2, g_2, b_2) \in \mathcal{T}^3$
 $C_1 \geq C_2 \Leftrightarrow \{r_1 \geq r_2 \text{ and } g_1 \geq g_2 \text{ and } b_1 \geq b_2\}$

Asplünd's distance (with LIPC) between colours

Given two colours C_1 and $C_2 \in \mathcal{T}^3$, their Asplünd's distance is equal to:

$$d_{AS}^{\Delta_c}(C_1, C_2) = \ln\left(\frac{\mu}{\lambda}\right)$$

$$\lambda = \inf\{k, k \Delta_c C_2 \geq C_1\} \quad \text{and} \quad \mu = \sup\{k, k \Delta_c C_2 \leq C_1\}$$





Asplünd's metric - LIPC

Asplünd's metric (with LIPC) between colour functions

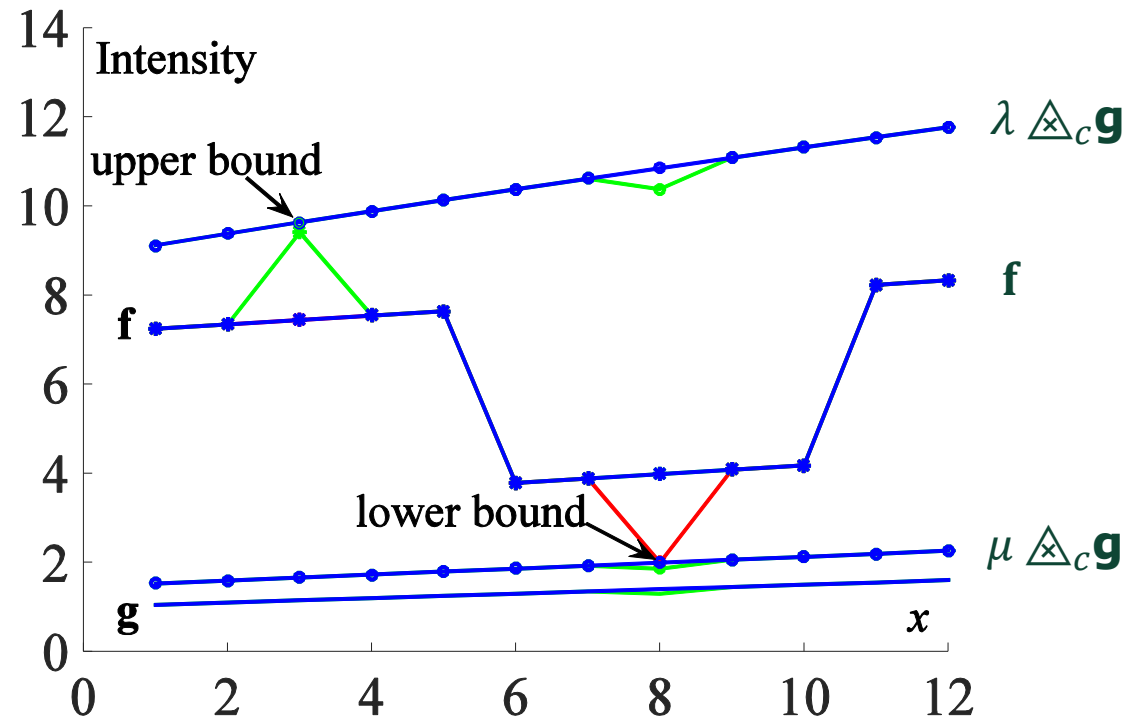
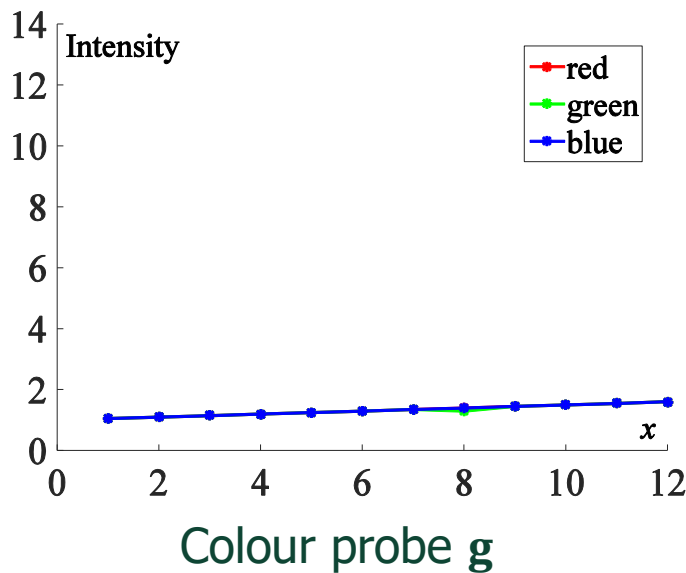
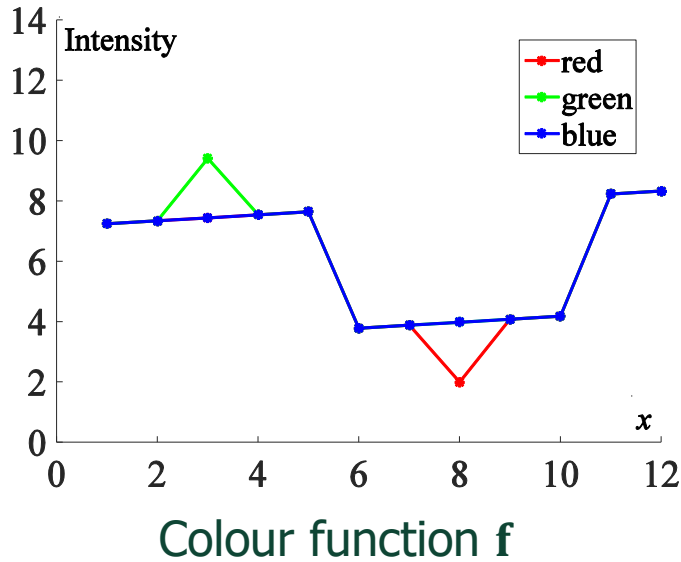
The Asplünd's metric between two colour images \mathbf{f} and \mathbf{g} on a region $Z \subset D$ is:

$$d_{As,Z}^{\Delta}(\mathbf{f}, \mathbf{g}) = \ln(\mu/\lambda)$$

$$\lambda = \inf \{k, \forall x \in Z, k \Delta_c \mathbf{g}(x) \geq \mathbf{f}(x)\} \quad \text{and} \quad \mu = \sup \{k, \forall x \in Z, k \Delta_c \mathbf{g}(x) \leq \mathbf{f}(x)\}$$



Asplünd's metric - LIPC



Lower (μ) and upper (λ) bounds

$$d_{As,Z}^{\Delta_c}(f, g) = 0.43$$



Asplünd's colour probing

Map of Asplünd's distances (with LIPC) - colour

Given a colour image \mathbf{f} defined on D and a colour probe \mathbf{t} defined on D_t , the map of Asplünd's distances is:

$$AS_t^{\triangle c} \mathbf{f} : \begin{cases} (\mathcal{J}^3)^D \times (\mathcal{J}^3)^D & \rightarrow (\mathbb{R}^+)^D \\ (\mathbf{f}, \mathbf{t}) & \rightarrow AS_t^{\triangle c} \mathbf{f}(x) = d_{AS, D_t}^{\triangle c}(\mathbf{f}|_{D_t(x)}, \mathbf{t}) \end{cases}$$

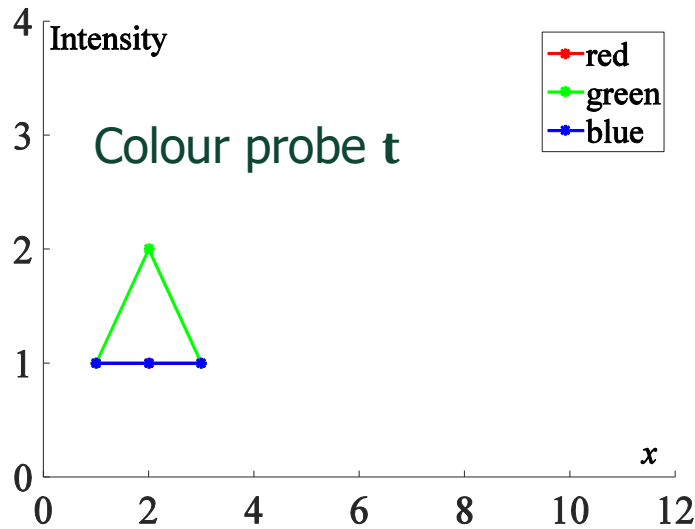
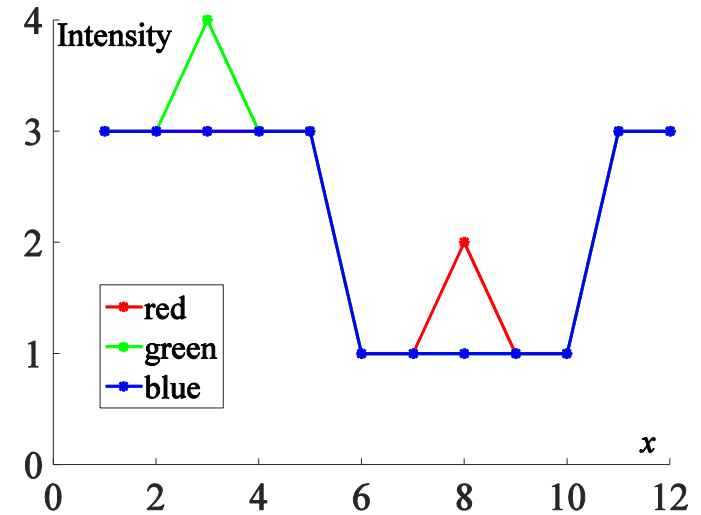
$D_t(x)$ is the neighbourhood D_t centred in x .

$\mathbf{f}|_{D_t}$ is the restriction of \mathbf{f} to D_t .

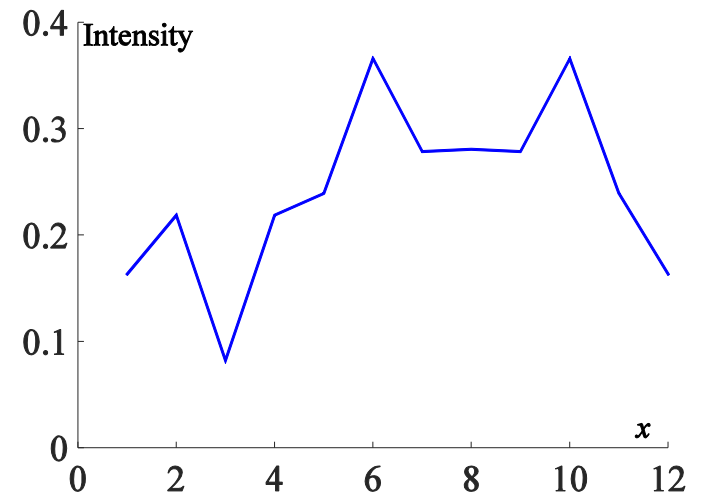


Asplünd's colour probing

Colour function f

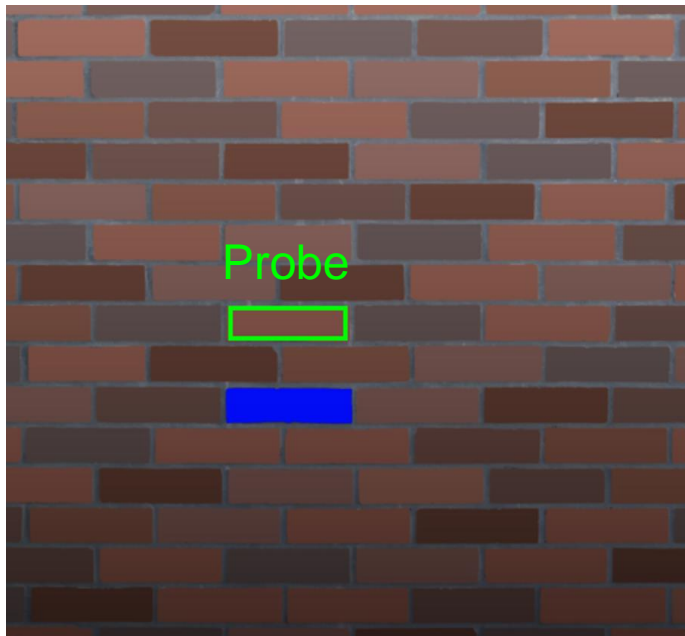


Map of Asplünd's distances $As_t^{\triangle c} f$

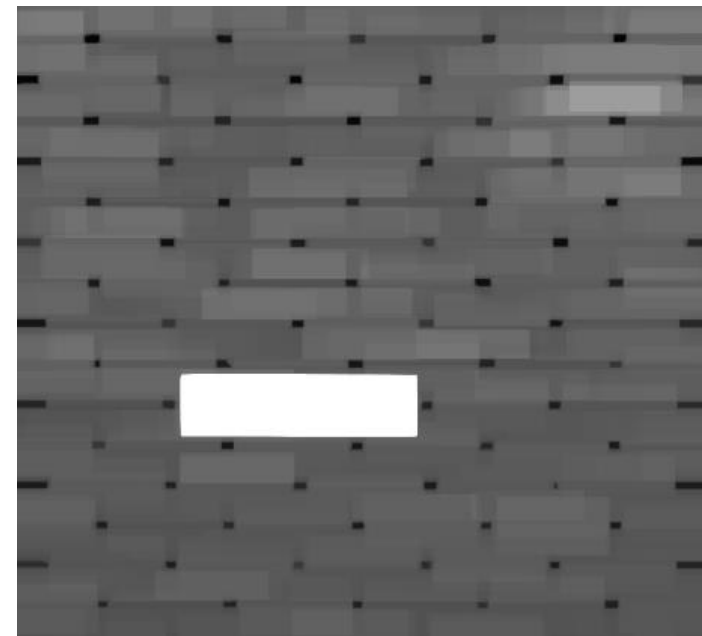




Asplünd's colour probing



Colour function \mathbf{f}

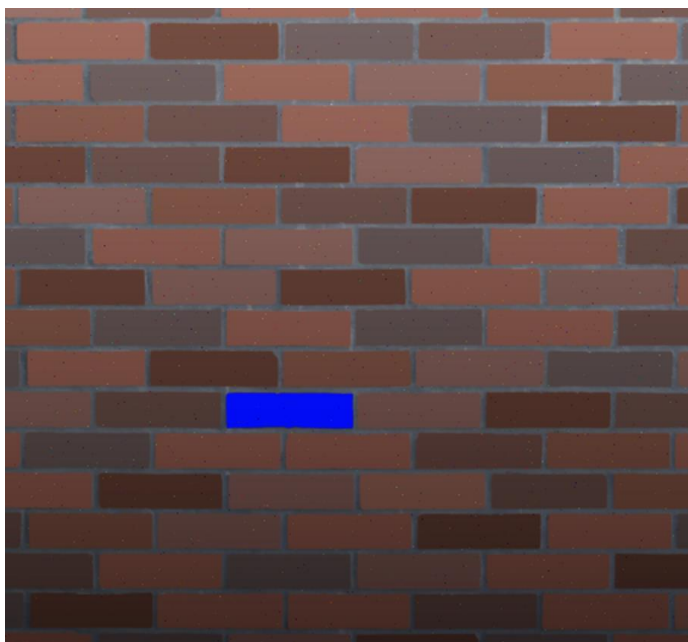


Map of Asplünd's
distances $A_{\mathbf{t}}^{\Delta c} \mathbf{f}$

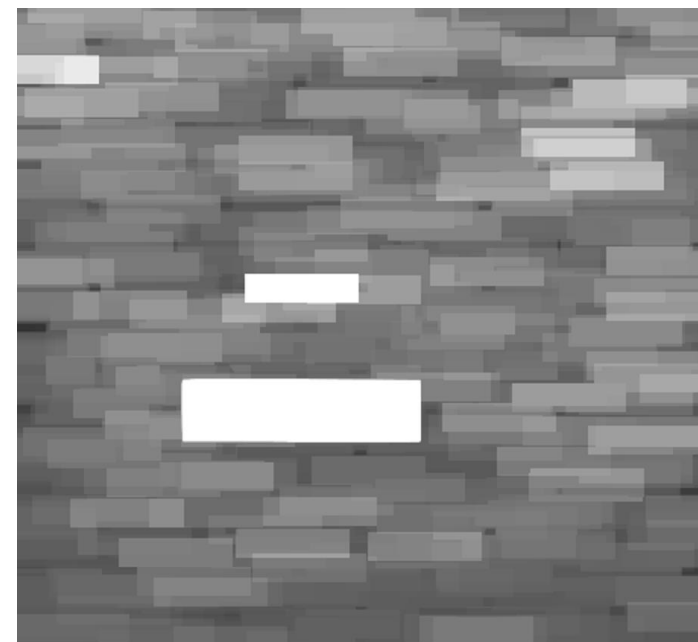


Asplünd's colour probing - Noise

A gaussian white noise with zero-mean, variance 2.6 and spatial density 1% is added to each channel of image f .



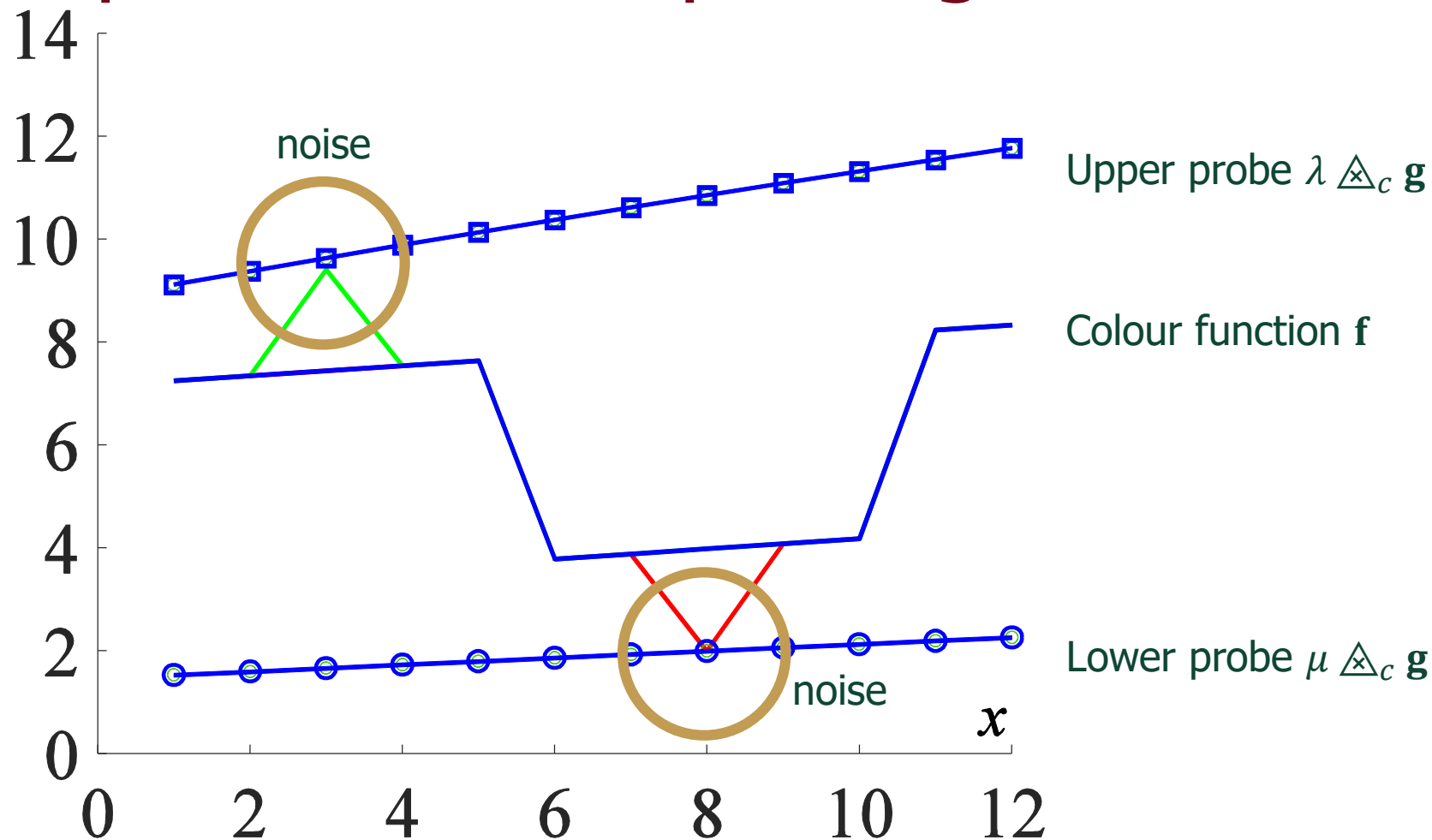
Colour function \tilde{f}
(with noise)



Map of Asplünd's
distances $A_{s_t}^{\Delta_c} \tilde{f}$



Asplünd's colour probing - Noise



Asplünd's distance is sensitive to noise because the probes are defined using extrema (*inf* and *sup*).



A solution to noise sensitivity

Solution: Measure metric

Given two colour images \mathbf{f} and \mathbf{g} and a metric d in the space of colour images, a neighbourhood $N_{P,d_{AS},\epsilon,p}$ of function \mathbf{f} may be defined thanks to an arbitrary small positive real number ϵ and a percentage p of points to be discarded :

$$N_{P,d_{AS},\epsilon,p}(\mathbf{f}) = \left\{ \mathbf{g} \mid \exists D' \subset D, \quad \underbrace{d_{AS}^{\Delta c}(\mathbf{f}|_{D'}, \mathbf{g}|_{D'})}_{\text{neighbours in subset } D' \subset D} < \epsilon \quad \text{and} \quad \underbrace{\frac{\#(D \setminus D')}{\#D}}_{\text{set of points to be discarded is small}} \leq p \right\}$$

$\mathbf{f}|_{D'}$ and $\mathbf{g}|_{D'}$ are neighbours in a subset $D' \subset D$

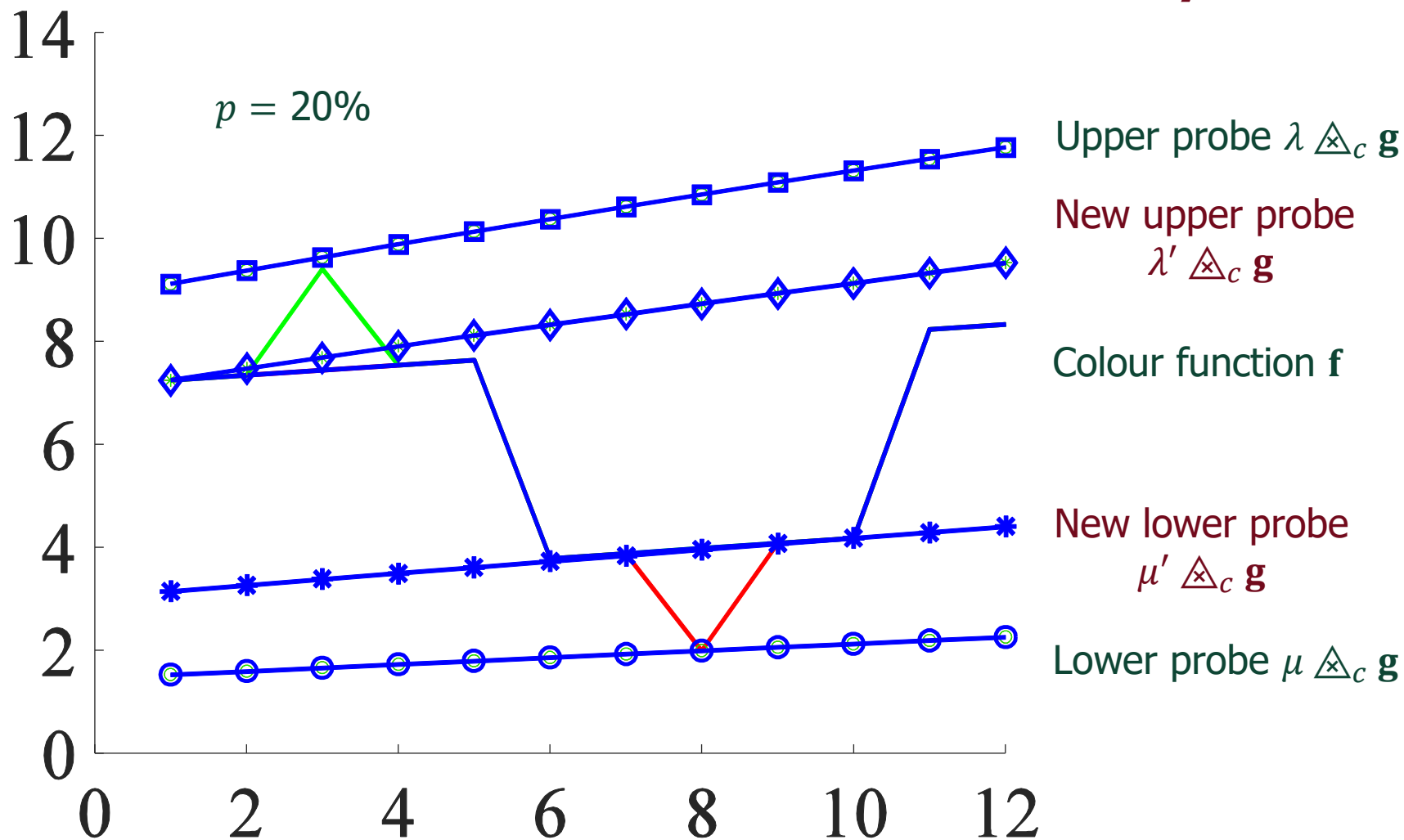
The set of points to be discarded $D \setminus D'$ is small sized when compared to D

p an acceptable percentage

$\#D$ the cardinal of D



A solution to noise sensitivity



$$d_{AS,Z}^{\triangle_c}(\mathbf{f}, \mathbf{g}) = 0.43 \text{ becomes } d_{AS,Z,p}^{\triangle_c}(\mathbf{f}, \mathbf{g}) = 0.21$$



Examples



Example

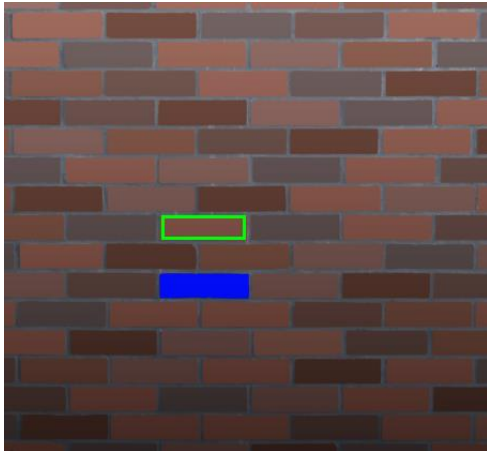
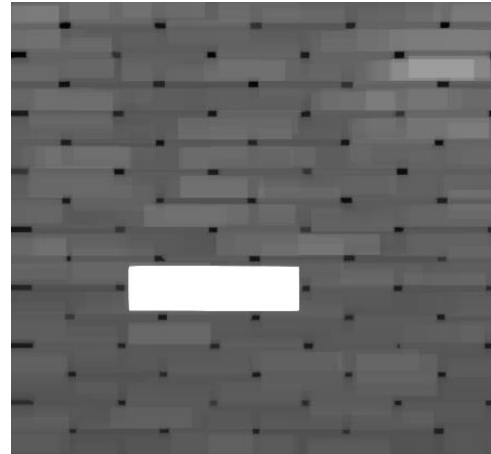


Image f and probe t



Map $A_{st}^{\Delta} f$



Colour probe t
(magnified)

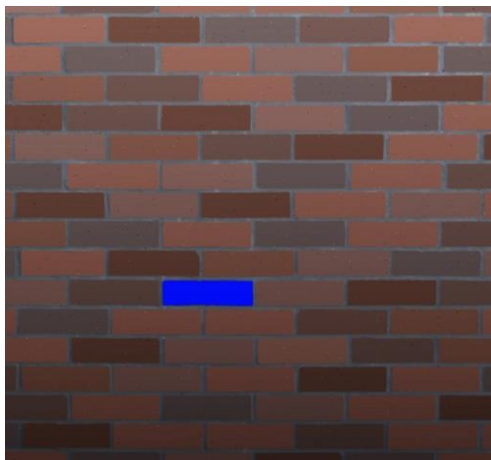
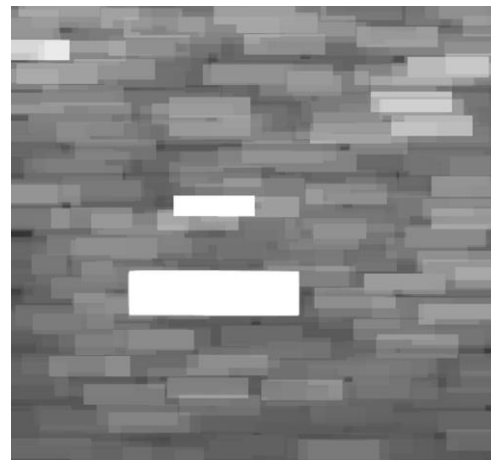


Image with noise \tilde{f}



Map $A_{st}^{\Delta} \tilde{f}$



Example

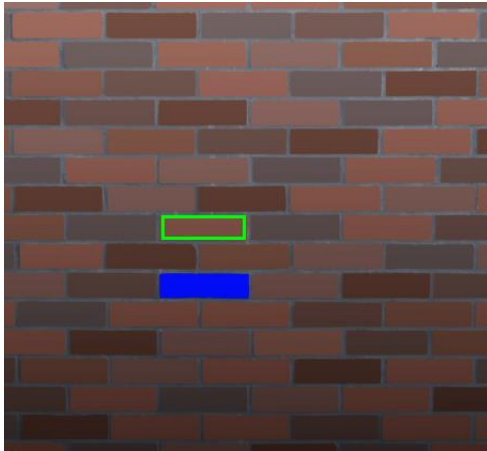
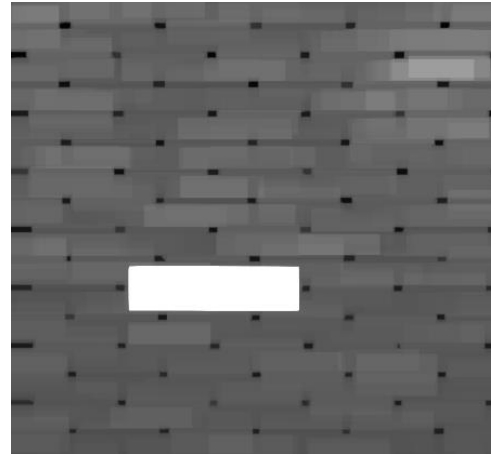


Image f and probe t



Map $A_{s_t}^{\Delta} f$



Colour probe t
(magnified)

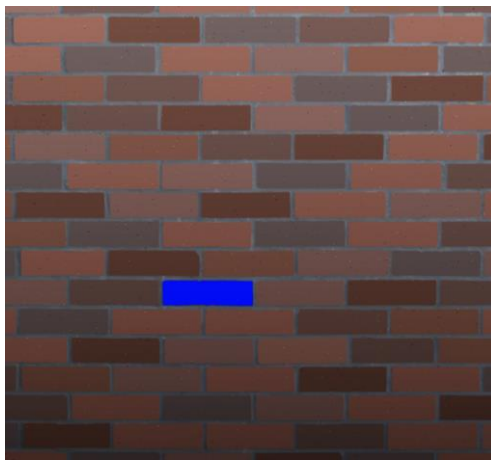
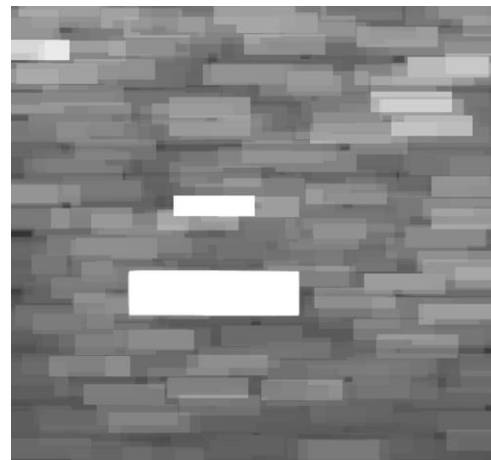
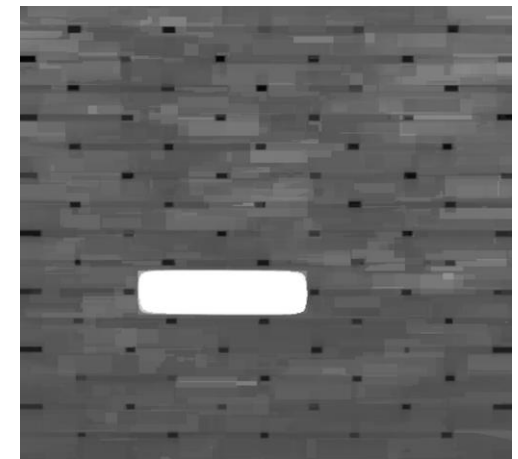


Image with noise \tilde{f}



Map $A_{s_t}^{\Delta} \tilde{f}$



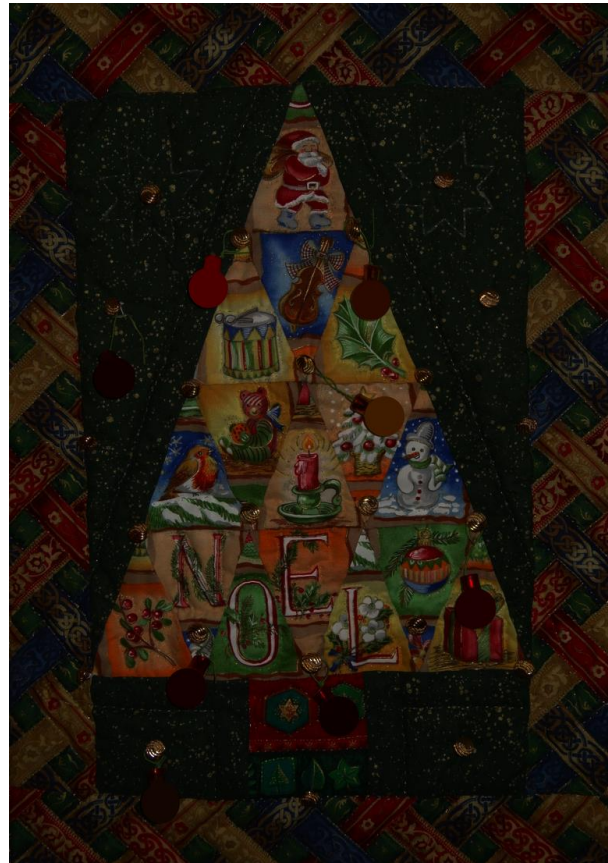
Map $A_{s_{t,p=2\%}}^{\Delta} \tilde{f}$



Pattern matching



Image f and probe t



Dark image \tilde{f}



Pattern matching

Local minima of the distances

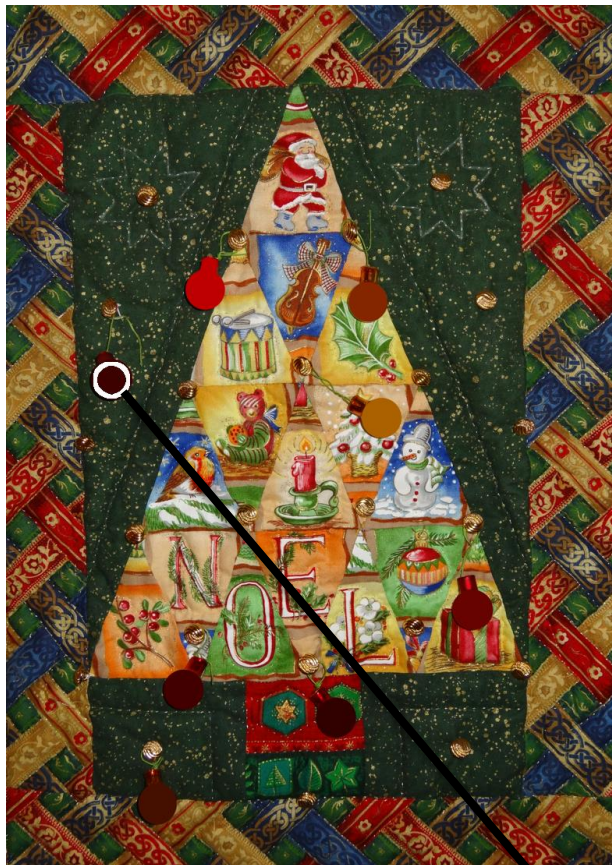
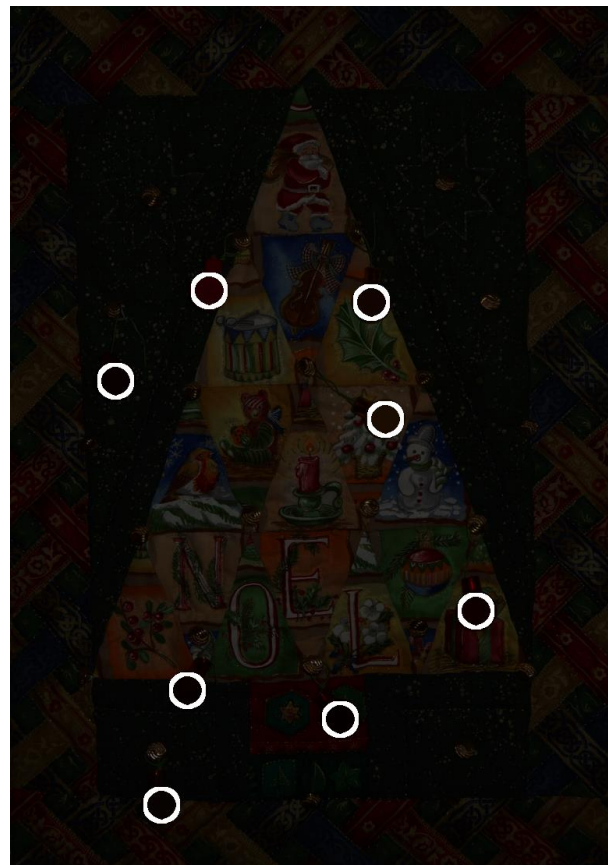
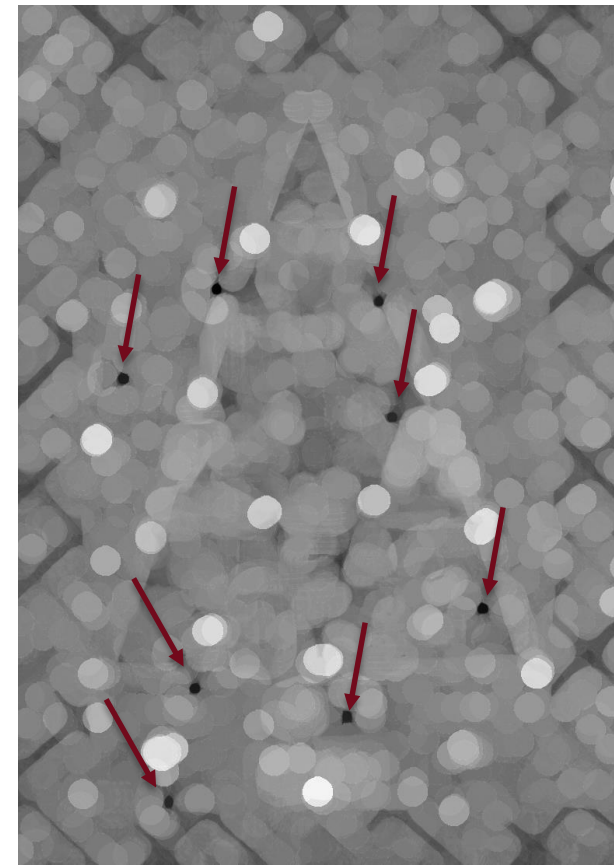


Image f and probe t



Dark image \tilde{f}
Balls detected



Map $A_{t^c}^{\Delta} f$



Comparison with other distances

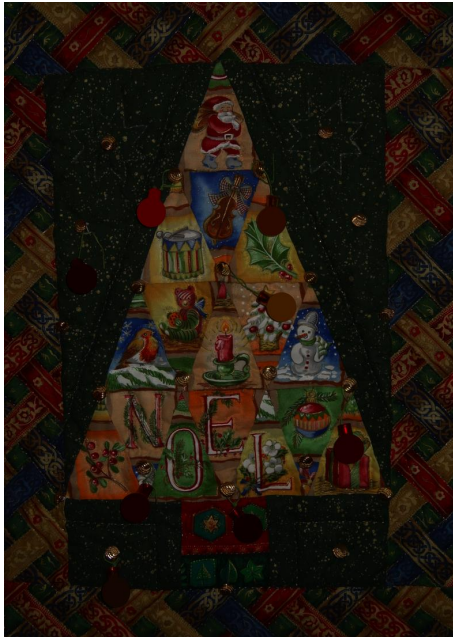
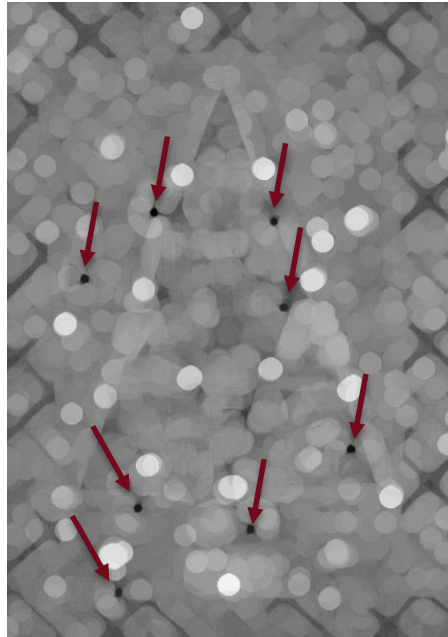


Image \tilde{f} and probe \mathbf{t}



Map $As_t^{\Delta_c f}$

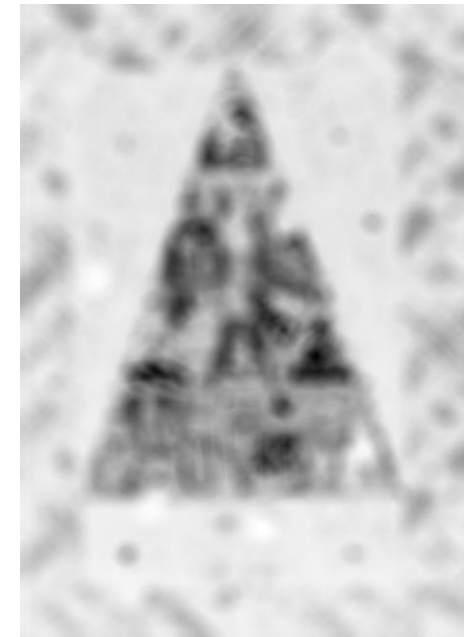
Asplünd + LIPC



Map

$$\sup\{d_{Lab}(\mathbf{f}|_{D_t(x)}, \mathbf{t})\}$$

L*a*b* CIE 1976



Map

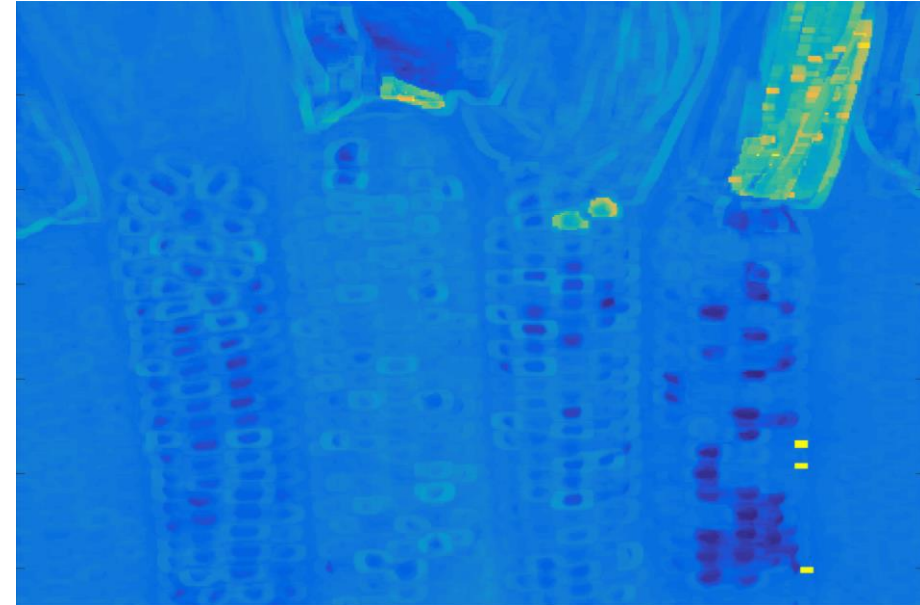
$$\sqrt{\frac{1}{N} d_{Lab}(\mathbf{f}|_{D_t(x)}, \mathbf{t})^2}$$



Pattern matching



Input Image f



Map $As_{t,p=5\%}^{\Delta c} f$



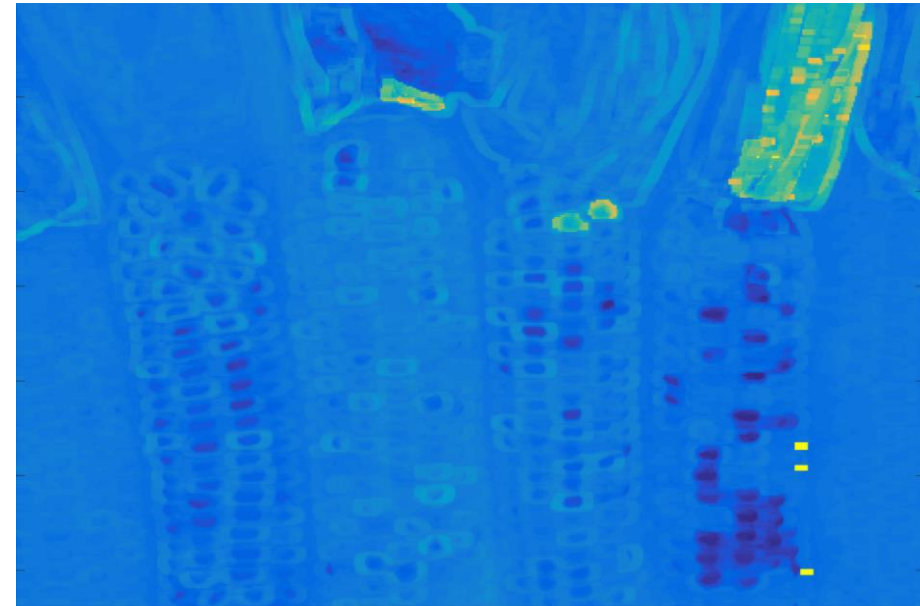
probe t (magnified)



Pattern matching



Corn grains detected with a similar colour



Map $As_{t,p=5\%}^{\Delta c} \mathbf{f}$



probe t (magnified)



Conclusion



Conclusion

- An efficient method for colour pattern matching
- The method is robust to lighting variations and to noise
- A new spatio-colour approach based on Asplünd's distance and the LIPC model