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The Quantization of the Gravitational Charges At the Origin of the Gravitational Waves From a Binary Black Hole Merger (LIGO)

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Abstract

The Micro-black-holes particles produced at the horizon entry by quantum fluctuation at Reheating stay at the base of the origin and evolution of Universe. These adjust theirs dimensions by photons radiation but kipping someone else, to become electrons, others leptons (quarks) and uncharged particles (neutrinos).

During a merging process the resulting Black hole (BH) is supposed to contain inside the quantized gravitational charges from the collapsing stars matter (nucleons) where these were embedded during matter creation at Confinement (merging), and which in the following these were released as gravitons that deform the spacetime.

Only by using this model were obtained the parameters of gravitational wave which arrives at LIGO site.

1. Introduction

Recently [1], it was the first direct detection of gravitational waves and the first observation of a frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21} of binary black hole merger (BHmg) with $3.0^{+0.5}_{-0.5} M_{SUN} c^2 \approx 5.3 \times 10^{47} J$ radiated in gravitational waves.

Also in [1], the gravitational waves (GW) observations constrain the Compton wavelength of the graviton to be $\lambda_g > 10^{16} [m]$, which could be interpreted as a bound on the graviton mass $m_g < 1.2 \times 10^{-22} \ eV/c^2 \approx 2.15 \times 10^{-58} kg$, and $\varepsilon_g = 1.9 \times 10^{-41} J^{"}$, or $\hbar c/\lambda_g \approx 3 \times 10^{-42} J^{"}$.

In the following will be shown how that one obtain these parameters only if we adopt the gravity quantization in matter particles in inside the black holes.

2. The situation before BH formation-the quantization of gravity in matter particles

The electron

The average magnitude of the electric field inside the model electron is as from [2a], [4]:

$$\langle E \rangle = \sqrt{\frac{6\hbar c}{\pi \varepsilon_0 \lambda^4}}$$

, it results $\langle E \rangle = 5.7 \times 10^{17} [N/C]$, for electrons of $\lambda_c = \hbar/m_e c \approx 3.7 \times 10^{-13} [m]$

The graviton

In [2a], [2b], we have introduced the concept of primordial Micro-black-holes (μBH) which later decay into photons in t_P as the Hawking radiation where, the gravitational field is so strong that it causes the spontaneous production (absorbtion) of photon pairs (with black body energy distribution) and even of particle pairs.

Sure, only few free photons of photons dimension could remain embedded in ex-like-Micro-black-hole which adjusts its dimensions to electrons, other leptons, noncharged particles (neutrinos, Higgs) in order to equilibrate the gravitational charge $\sqrt{Gm_P}$ as it was explained following electron model ref. [4].

Very early in 1960 L.Motz ref. [3a]; [3b] has elaborated a new theory of the structure of fundamental particles, which introduces gravitational field into the interior of particles such as electrons to account for their stability.

Thus, $\sqrt{G}m_p$ is the gravitational charge defined by L. Motz as resulting from: $F_G = \frac{\sqrt{G}m_1\sqrt{G}m_2}{r^2}$, where it can say that the gravitational charge $\sqrt{G}m_1$ is the source of gravitational field $\sqrt{G}m_1/r^2$ at the distance r, and that this field is coupled to the gravitational charge $\sqrt{G}m_2$ at the position r (relative to the source of the field) via the product of the field strength and the charge. L. Motz derived the quantization condition on gravitational charge in a similar manner by noting that moving particle with velocity \vec{V} and with gravitational charge $\sqrt{G}m_p$ is coupled not only to the Newtonian gravitational fields of all other particles (the gravielectric field) in the usual way, but also to the Coriolis inertial force of field $2\vec{\omega}c/\sqrt{G}$ by means of the cross product $(\sqrt{G}m_p\vec{v}/c) \times (2\vec{\omega}c/\sqrt{G})$, see below the possible Motz's derivation (my guess), where $\vec{\omega}$ is an appropriate angular velocity. This cross product term give rice to an angular momentum ($L = rm_pv$; $v = r\omega$) component in the motion of the particle which is of the order of $L = 2m_p r^2\omega$ and is parallel to the field $\vec{\omega}$. Therefore, it is obtained the quantization condition of spin 2 (graviton) as:

$$L = 2m_P r^2 \omega = 2\hbar$$

Since it was interested in the quantization of fundamental gravitational charge, it is found a value for r and ω that must be associated with such charge. To do this, L.Motz considered the Universe to two such charges in gravitational equilibrium and resolving about each other in the first Bohr orbit. The radius of this orbit is just $r = \hbar^2/Gm_P^3$, $v \approx c$; $m_1 = m_2 = m_P$.

Such a coupling of the charge to gravimagnetic field $\frac{L}{\sqrt{G}m_P}$ is achieved,

 $\sqrt{G}m_P \frac{v}{c} \times \frac{2\varpi r^2}{\sqrt{G}}$, that becomes just the angular momentum which contains a *double*

moment: $L = 2m_P r^2 \omega$.

This will be so only if $v = \omega r \approx c$ is of the order of c since the centrifugal force (being charge times field) is just $(\sqrt{Gm}) \cdot (2\omega r^2/\sqrt{G})$. Introducing these two relationships into

the above quantization equation, is obtained $L = 2m_P cr = 2\hbar$, or, substituting for r as Bohr orbit when the electron is held in a circular orbit by electrostatic attraction.

$$\frac{m_e v^2}{r} = \frac{Zk_e e^2}{r^2} \quad ; \ k_e = \frac{1}{4\pi\varepsilon_0} = 4\pi G \; ; \ m = m_P \; ,$$
$$mvr = n\hbar \; , \ r = \frac{\hbar^2}{k_e e^2 m} \; , \ r = \frac{\hbar^2}{k_e e^2 m} = \frac{\hbar^2}{Gm^2 m} \; ; \text{ when } e \to \sqrt{G}m_P \; ,$$
$$2m_P c \; \frac{\hbar^2}{Gm_P^3} = \hbar \; , \text{ or } \frac{Gm_P^2}{c} = \hbar$$

In the case of an electron, the centrifugal force of gravity charge is equal with the the Lorentz force of the magnetic flux as been induced by the solenoidal current of the apparent charge which arises from the electric field of the confined photons (as above): $E_{i} = E_{i}$

$$F_G = F_{\gamma \text{-condensate}}$$

$$\frac{Gm_P^2}{r^2} = \langle E \rangle e \ ; \langle E \rangle = 5.7 \times 10^{17} [N/C]; \text{ Or, } 0.15 \ge 0.091 [N].$$

If the square of quantum charge is, indeed $\hbar c$, we can account for the stable electrically charged particles such as electrons by balancing the explosive positive electro-static energy (as due of the confined photons) with the negative binding energy of the gravitationally charge.

It is clear that a gravitational charge of magnitude $\hbar c$ will contribute negative gravitational binding energy of the order of mc^2 if this charge is distributed over a region whose dimensions are equal to the Compton wave length of the particle.

3. The spacetime inflation induced by the two merging BH

In our case, the resulting BH is supposed to contain inside these gravitational charges as a heritage of the collapsing star matter where these were embedded during matter creation. In the following during the merging process of two black-holes these are released as gravitons, the expression of the gravitational wave G-wave (as in case of photons and the light).

Therefore, it seems that only the merged part of the merged black hole is released as gravitons which wraps the space-time till today.

To estimate the horizon entry we use some derivations done in [2b]. In Inflation models [2a, 3b], the scale leaving the horizon at a given epoch is directly related to the number $N(\varphi)$ of *e*-folds of slow-roll space-time inflation that occur after the epoch of horizon exit. Indeed, since *H*-the Hubble length is slowly varying, we have

$$d \ln k = d(\ln(aH)) \cong d \ln a = \frac{adt}{a} = Hdt$$
. From the definition Eq. (38) of [2b] this gives

 $d \ln k = -dN(\varphi)$ as of eq.(46) from [2], and therefore $\ln(k_{end}/k) = N(\varphi)$, or,

 $k_{end} = ke^{N}[m]$ where k_{end} is the scale leaving the horizon at the end of slow-roll

inflation, or usually $k^{-1} << k_{end}^{-1}[m]$, the correct equation being $k = k_{end} e^{N}[m^{-1}]$. When the wavelength $(k^{-1}[m])$ is large compared to the Hubble length $(H^{-1}[m])$, the distance that G-wave can travel in a Hubble time becomes small compared to the wavelength, and

hence all motion is very slow and the pattern is essentially frozen in, but it will not be the case, as in the following.

Since, the FLRW metric of the universe must be of the form $ds^2 = a(t)^2 ds_3^2 - c^2 dt^2$; or where ds_3^2 is a three-dimensional metric that must be one of (a) flat space, (b) a sphere of constant positive curvature or (c) a hyperbolic space with constant negative curvature, or for small commoving time $dt = \frac{1}{aHc}$, we can consider the distance as $L = ds \cong a = a_{end}$. Here, the Hubble constant is defined as

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda c^2}{3}$$
$$\left(\frac{\dot{a}}{a}\right)^2 = -\frac{8\pi G}{c^2}p + \Lambda c^2$$

In Newtonian interpretation, the Friedmann equations are equivalent to this pair of equations:

$$\frac{\dot{a}^2}{2} = \frac{G\frac{4\pi a^3}{3}\rho}{a}$$

 $\rho [kg/m^3]$; energy density $p = -\rho c^2$

If we divide with a^2 we obtain for *outside* the object (BH, Universe, planets, stars etc.)

$$\frac{\dot{a}^2}{2a^2} = H^2[s^{-2}] = \frac{GM}{a^3} \Rightarrow \frac{GE}{c^2a^3} \Rightarrow \frac{GE}{c^4a^3} = \frac{G}{c^4a^3} \cdot n_g \cdot \varepsilon_g[m^{-2}]; \quad (1)$$

, where: $n_g = E/\varepsilon_g$; $n_{at-merging} = \frac{n_g}{(\varepsilon_P/\varepsilon_g)}; \quad M = \frac{4\pi a^3}{3}\rho$; $E = Mc^2; \quad M = n_{\mu BH}m_{\mu BH}$
 $R = H^{-1}.$

Or, for *inside* of these objects:

$$\frac{\dot{a}^2}{2a^2} = \frac{G \frac{4\pi a^3}{3} \rho}{a^3} [s^{-2}] \rightarrow \frac{4\pi G \rho}{3c^2} = \frac{4\pi G \frac{M_U}{n_{at-merging}} \cdot l_C^3}{3c^2} [m^{-2}]$$
(2)

During Universe evolution at *Electroweak epoch* or Reheating due of the quantum fluctuations [2a] a huge number of the micro-black holes as Planck particles n_P are generated, $m_{\mu BH} = m_P$; the graviton energy being at horizon leave ;

$$\varepsilon_{g} = \hbar c / a_{end} = 2 \times 10^{-26} J ; \text{ when } a_{leave} = k_{leave} / H_{leave} = 1, M_{U} = 2.2 \times 10^{53} kg ;$$

$$\lambda_{C} = \hbar c / m_{\mu BH} c^{2} ; n_{\mu BH} \approx 1 / H_{-1}^{3} \cdot V_{available} \approx n_{P} \approx 10^{60}, \text{ or } n_{P} = E_{U} / \varepsilon_{P} = 10^{70} J / 10^{9} J \text{ with}$$

$$\varepsilon_{\mu BH} = 10^{11} / a_{end-ee} = 10^{11} GeV \rightarrow 17.4J ; a_{end-ee} = 0.92 , \text{ it results with}$$

$$k_{leave}^{-1} = H_{leave}^{-1} = 10^{-27} [m] \text{ by iteration for } N = 16.2 ; \text{ in eq. } (2); k_{end} = 9.2 \times 10^{19} [m^{-1}] ;$$

$$H^{-1} = 10^{-20} [m]; t = 3.3 \times 10^{-29} s; R = 6.5 \times 10^{-20} [m]; l_{C} = l_{P} = 1.7 \times 10^{-27} [m];$$

$$a_{EW} = 0.92 ; \text{ where } n_{g} = \frac{\varepsilon_{P}}{\varepsilon_{g}} \cdot n_{P} \rightarrow 10^{96} \text{ as a "fix" number of gravitons escaped (an)}$$

;

inverse process of a black-hole) from micro-blacks holes created in Universe as the Planck particles during theirs *totally decaying*, and which deforms the spacetime. The number of gravitons that has been released following μBHs merging is only

$$n_{at-merging} = \frac{n_g}{\left(\varepsilon_{\mu BH} / 10^{-26}\right)} = \frac{10^{96}}{\left(17.4 / 10^{-26}\right)} \approx 5.7 \times 10^{68}$$
, and these generate the curvature

radius of the object R.

The necessary volume is $V_{necessary} = n_g \cdot \lambda_C^3 = 10^{-12} m^3$, and the available volume being $V_{available} = a_{end}^3 = 0.7m^3$ To mention that only this data set match the model.

4. The BH internal structure at the beginning of merging (inside)

Thus, the field for a gravity charge at time of quantum fluctuation (before BH) is as in section 3.

$$\varepsilon_g = \varepsilon_{g0} = \frac{GM_P^2}{a_{end_0}} = \frac{\hbar c}{a_{evHorizon_0}} \cong 2 \times 10^{-26} [J]; a_{EvHorizon_0} = 1$$
 is the scale factor at the

origin as it results from the inflation model [2a].

With $M \approx 3 \times M_{SUN} = 3 \times 1.98 \times 10^{30} kg$ as disruptive mass of the merged binary blackholes [1]; the energy being $E_{mBH} = Mc^2 = 5.3 \times 10^{47} J$, the Schwarzschild radius is $r_{Sch} = 2GM/c^2 = 8.7 \times 10^3 [m]$

, and the mass of graviton resulting as $m_{g0} = \frac{\mathcal{E}_{g0}}{c^2} = 10^{-43} [kg]$, here, the Planck mass is

 $M_{P} = 1.8 \times 10^{-8} kg$.

Numerically, the horizon leave is just the radius of the leaving of the *quarks tube string* as nucleons constituent together with gluons [6], when they firstly enter in BH, and when $a_{leave} = k_{leave} / H_{leave} = 1$, or $k_{leave}^{-1} = H_{leave}^{-1} = 10^{-17} [m],$

, as this leave is from the matter particles (nucleons) of the star collapsed. In other words it seems that a BH inside is constituted from quarks tubes (from) nucleons like a big

nucleus of density $\rho_{BH} = \frac{M_{BH}}{n_{at-merging}} l_C^3 \approx 5.1 \times 10^{18} [kg/m^3]$, that it simplifies to any atom's nucleus $\rho_n = n_n m_q / n_n l_c^3$, and equally with $\rho_{BH} = M_{BH} / r_{Sch}^3 \approx 5 \times 10^{18} [kg/m^3]$; where the mass of quarks tubes is $m_q = 6.6 \times 10^{-28} kg$; $l_c = 5 \times 10^{-16} m$; and $n_g = E_{BH} / \varepsilon_g = 5.3 \times 10^{47} / 10^{-26} \approx 5.3 \times 10^{73}$.

The number of gravitons that has been released following μBHs merging is only $n_{at-merging} = \frac{n_g}{\left(\epsilon_{mRH} / 10^{-26}\right)} = \frac{5.3 \times 10^{73}}{\left(6 \times 10^{-11} / 10^{-26}\right)} \cong 9 \times 10^{57}$, and that generate the curvature radius of the object ($\cong r_{Schw}$).

5

Now, the horizon-entry is when the wave length $k_{end} = k_{leave}e^{-N}$; $k_{end} = 2 \times 10^{-4} [m^{-1}]$; the scale factor arrives at $a_{end} = k_{end}/H_{end}$, the frequency is $v = c/k_{end}^{-1} = 5700Hz$, and the Compton length $\lambda_{C-g0} = \hbar/m_{nucleon_BH}c = 5 \times 10^{-16} [m]$, for $\varepsilon_{nucleons} \approx 10^{-10} J$ it results $\varepsilon_P = \varepsilon_{\mu BH} = \varepsilon_{nucleon}/a_{end} = 10^{-10}/a_{end} = 6 \times 10^{-11} \rightarrow \sigma_{strin_tube} = 0.4 GeV$; it results $a_{end_BH} \approx 1.7$, and from eq. (2) $H_{end}^{-1} \approx r_{Sch} \approx 8 \times 10^3 [m]$, $t_{end} = H_{end}^{-1}/c = 2.9 \times 10^{-5} s$, we found N = 47.7 to match the iterations cycle: $m_g \rightarrow \hbar v \rightarrow k_B T \rightarrow \varepsilon \rightarrow R \rightarrow H_{end}^{-1} \rightarrow a_{end} \rightarrow N$.

At BHs merging, the curvature radius *R* from eq. (2) with the number of merging nucleons during which release gravitons as $n_{merging}$, $R = H_{end}^{-1} = 8 \times 10^3 [m] \approx r_{Sch}$. The necessary volume of BH is $V_{necessary} = n_{at-merging} \cdot \lambda_C^3 = 10^{12} m^3$, and the available volume being

$$V_{available} = r_{Sch}^3 = 6.8 \times 10^{11} m^3$$
.

5. The gravitons release during BHs disruptive merging as a gravitational wave Therefore, the new horizon leave as gravitational wave (GW) is just the BH *escape* at Schwarzschild radius (the Universe is a viewed as an inverse big black hole) : $\frac{1}{2} = \frac{1}{2} =$

$$a_{leave} = k_{leave} / H_{leave} = 1$$
, or $k_{leave}^{-1} = H_{leave}^{-1} = r_{Schw} = 8.7 \times 10^{3} [m]$.

Now, the new horizon-entry is when the wave length $k_{end} = k_{leave} e^{-N}$;

 $k_{end} = 6 \times 10^{-8} [m^{-1}], k_{end}^{-1} = 1.6 \times 10^{7} [m]$; and the scale arrives factor at $a_{end} = k_{end} / H_{end}$, the Hubble length with Compton length $\lambda_{C-GW} = \hbar/m_{g_GW}c = 1.9 \times 10^{15} [m]$; from eq. (1) $H_{end}^{-1} = 10^{25} [m]$; it results $a_{end_GW} = 6.3 \times 10^{17}, t_{end} = H_{end}^{-1} / c = 3.35 \times 10^{16} s$, we found N = 7.5 to match the iterations cycle:

 $m_g \rightarrow \hbar v \rightarrow k_B T \rightarrow \varepsilon \rightarrow R \rightarrow H_{end}^{-1} \rightarrow a_{end} \rightarrow N$.

The energy of the graviton becomes at LIGO with the above value at merging

$$\varepsilon_{g_{GW}} = \frac{\varepsilon_{gBH}}{a_{end_{GW}}} = \frac{10^{-23}}{6.3 \times 10^{17}} = 1.6 \times 10^{-41} [J] \rightarrow 9.9 \times 10^{-32} \, GeV$$
 at BHs merging, and the

frequency is $v = c/k_{end}^{-1} = 18Hz$, the mass is $m_{g_{GW}} = 1.8 \times 10^{-58} kg$; the number of particles released as the gravitational wave (like the photons of the light wave) remains equally with $n_g = M_{BH}/m_{g0} = 5.9 \times 10^{73}$, the total energy initially released is $E_{GW} = E_{mBH} = \varepsilon_g \cdot n_g = M_{BH}c^2 = 5.3 \times 10^{47} [J]$, and the curvature radius it results from eq.(1) with the integral (which the starts at the outside of the disrupted BH) graviton energy $\varepsilon_g = 2 \times 10^{-26}/a_{end_{BH}}$, where $a_{end_{BH}} = 1.7$ from the above section (4.), with n_g , and with $a_{end} = a_{end_{GW}}$, as $R_{LIGO} = 7 \times 10^{24} [m]$. To note, the dark particles viewed as pure gravity charges have near the same gravitational charge, see [2a]. *The strain at LIGO site*

Now, based on eq. (1) we can derive for the G-wave effect in the deformation (strain) of the space-time between BH and LIGO site by using the gravitational pressure due of

gravity charges on the area of Schwarzschild radius r_{Schw} of BH, respectively with the values calculated in section 5, we have:

$$\left(\frac{r_{Schw}}{R}\right)^{2} = \frac{4\pi}{3} \frac{G \cdot \varepsilon_{g} \cdot n_{g} \cdot r_{Schw}^{2}}{c^{4} \cdot a_{end_GW}^{3}} = 1.36 \times 10^{-42}$$
where $\varepsilon_{g} = 2 \times 10^{-26} / a_{end_BH} = 10^{-26} J$ and $n_{g} = 5.9 \times 10^{73}$
Separately, with the LIGO site curvature radius as
 $R_{LIGO} = 1.3 \times 10^{9} Light_years \times c = 1.25 \times 10^{25} m$,
and the Schwarzschild radius
 $r_{Schw} = \frac{2G \cdot 3M_{SUN}}{2} = 8.7 \times 10^{3} [m]$, we have

$$\frac{r_{Schw}^2}{R_{LIGO}^2} = \frac{(8.7 \times 10^3)^2}{(1.25 \times 10^{25})^2} = 5 \times 10^{-43} \rightarrow r_{Schw} / R_{LIGO} = 7.2 \times 10^{-22}$$

, so, in both cases at (LIGO laser) the strain is around $\theta = r_{Schw}/R \approx 1.1 \times 10^{-21}$, that is near equally with the G-wave measured strain $\approx 10^{-21}$ given in [1].

Therefore, the present numerically analysis proves the concept of gravity charges which escape from the BHs during disruptive merging, as the corpuscle part of the G-wave. Therefore, at LIGO is obtained not only the deformation of space-time, but also the graviton properties, the inner structure of BH as a packing of quarks tubes of the nucleons, the photonic part (gluons) being released as radiation, and are verified the FLWR equations with these quantized masses. To mention that only the above datasets as being obtained by iterations its proves the model.

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