



HAL
open science

The Yoyo-Man

Jean-Paul Laumond, Mehdi Benallegue, Justin Carpentier, Alain Berthoz

► **To cite this version:**

Jean-Paul Laumond, Mehdi Benallegue, Justin Carpentier, Alain Berthoz. The Yoyo-Man. The International Journal of Robotics Research, 2017, 10.1177/ToBeAssigned . hal-01316032v2

HAL Id: hal-01316032

<https://hal.science/hal-01316032v2>

Submitted on 4 May 2017 (v2), last revised 15 May 2017 (v3)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution - ShareAlike 4.0 International License

Jean-Paul Laumond¹, Mehdi Benallegue², Justin Carpentier¹ and Alain Berthoz³

Abstract

The paper reports on two results issued from a multidisciplinary research action tending to explore the motor synergies of anthropomorphic walking. By combining biomechanics, neurophysiology and robotics perspectives, it is intended to better understand the human locomotion with the ambition to better design bipedal robot architectures.

The motivation of the research starts from the simple observation that humans may stumble when following a simple reflex-based locomotion on uneven terrains. The rationale combines two well established results in robotics and neuroscience respectively:

- Passive robot walkers, which are very efficient in terms of energy consumption, can be modelled by a simple rotating rimless wheel;
- Humans and animals stabilize their head when moving.

The seminal hypothesis is then to consider a wheel equipped with a stabilized mass on top of it as a plausible model of bipedal walking. The two results presented in the paper comfort the hypothesis:

- From a motion capture data basis of twelve human walkers, we show that the motions of the feet are organized around a geometric center, which is the center of mass, and surprisingly not the hip.
- After introducing a ground texture model that allows to quantify the stability performance of walker control schemes, we show how compass-like passive walkers are better controlled when equipped with a stabilized 2-degree-of-freedom moving mass on top of them.

CoM and head then play complementary roles that define what we call the Yoyo-Man. Beyond the two results presented in the paper, the Yoyo-Man model opens new perspectives to explore the computational foundations of anthropomorphic walking.

Keywords

Human Locomotion, Bipedal Locomotion, Humanoid Robotics, Synergies, Biomechanics, Passive Walker

1 Introduction: Legs versus Wheels

Goal oriented motion is a distinguished character of living beings. A stone does not move by itself. Within the living systems, displacement is what makes the difference between plants and animals. Animals make use of fins in the water and wings in the air. On land, apart from exceptions as crawling snakes, most of the animals are equipped with legs. Legged locomotion is based on rotating articulated limbs. The rotation of the limbs around the contact points on the ground transfers the body from a position to another one. Rotation then appears as a solution to translate an articulated body. If nature applies this principle to legged animals, it is surprising that it does not push this principle until the wheel discovery. Wheel has been invented and developed by humans*. Our cars are equipped with wheels and not with legs.

The paper reports on two results issued from a multidisciplinary research action tending to explore the motor synergies of anthropomorphic walking. By combining neurophysiology, biomechanics and robotics perspectives,

the objective is to better understand the human walking with the ambition to better design bipedal robot architectures.

Human legs are made of three rotating segments (foot, shank and thigh). The first question addressed in the paper deals with the quest of walking motion invariants: is there a *walking geometric center* to describe the motion of the feet independently from the motions of the shank and the thigh? On the other hand, from a neuroscience perspective,

¹ LAAS-CNRS, Université de Toulouse, CNRS, Toulouse, France

² National Institute of Advances Industrial Science and Technology (AIST), Tsukuba, Japan

³ Collège de France - 11, place Marcelin Berthelot, Paris, France

Corresponding author:

Jean-Paul Laumond, LAAS-CNRS, Université de Toulouse, CNRS, Toulouse, France

Email: jpl@laas.fr

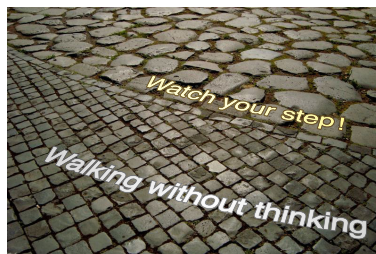
* The statement has to be nuanced: rotating engines exist at molecular scale and some insects are able to shape objects as spheres to move them.



(a) Walking without thinking. Reflex-based walking is stereotyped and robust enough to absorb slightly textured pavements.



(b) Two examples of stumble. Transiting from slightly to significantly textured pavements requires attention.



routes to continue exploring the computational foundations of human and humanoid walking.

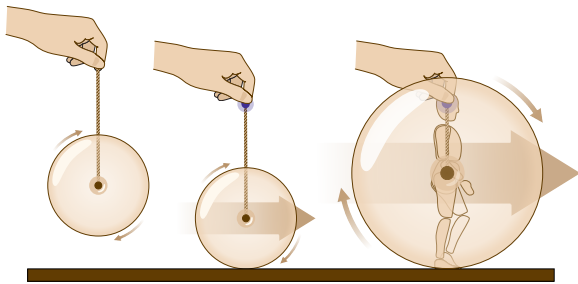


Figure 2. The Yoyo-Man: The magic of the wheel is to transform a rotational motion into a translational one as soon as the wheel touches the ground. The Yoyo-man is a human walker model made of the *geometric center* of a virtual rotating wheel together with a *control center* located at the head.

2 The Multiple Facets of Anthropomorphic Walking

By combining several perspectives on anthropomorphic walking, we intend to better understand the human walking with the ambition to better design bipedal robot architectures. This section presents the motivations and the rationales of our approach.

2.1 Three questions about human and humanoid walking

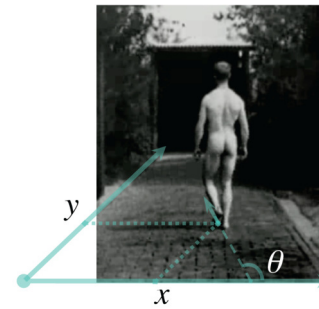
The motivation underlying the work presented in this paper is threefold: It gathers questions in neuroscience, biomechanics and robotics respectively.

Neuroscience Perspective: Watch your step! This is a well know fact that suddenly headless duck can continue walking for a short time. Some experiences have shown that cats with ruptured spinal cord move their rear legs according to natural walking gaits when trained on a treadmill (Jiang and Drew 1996). Neural architectures of locomotion control include medullary reflexes. From a neuroscience perspective we want to understand how simple reflex-based locomotion strategies make possible to walk without thinking and how robust are these strategies.

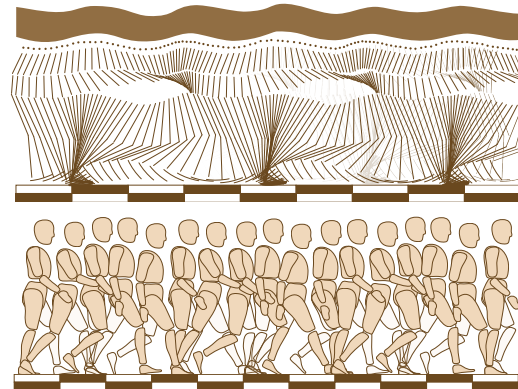
Do humans plan their steps in advance? Sometimes, they obviously do, when the ground is too uneven. However most of the time, they walk without thinking, i.e. without consciousness of any planning phase computing in advance where they have to place their feet. The pictures in Figure 1 have been taken in Roma behind the Arch of Constantine at the transition between two different pavements. A purely reflex-based walking allows to wander safely on the thinly textured pavement (Figure 1(a)). The same reflex-based gait fails when walking on the other pavement (Figure 1(b)). Walkers have to watch their steps, i.e. to change their walking mode to include footstep anticipation.

In which context do we start watching our steps? How to give a meaning to the notion of ground texture and to make it effective to compare walking modes? Section 4.4 addresses

those questions by introducing the notion of ground texture.



(a) The placement space of the body is defined by two position parameters and one orientation parameter



(b) Walking is a complex process involving the actuation and the motion coordination of many body segments. What does motion capture reveal on the underlying synergies?

Figure 3. From a picture and a chronophotographic image by E.J. Marey[‡]

Biomechanics Perspective: The quest of synergies. Anthropomorphic systems are made of a tree of articulated bodies linked together by rotational joints. This is true for all humanoid robots. This is also true for human at first glance, if we neglect mechanical scapula or kneecap subtleties. Joint angles define the system posture. The system configuration is made of all the joints together with the three placement parameters that give the position and the orientation of the system on the ground. From a control viewpoint, muscles or motors operate in the posture space. There is no direct control of the three placement parameters (see Figure 3). In that sense, humans and humanoid robots are underactuated systems. What is called locomotion is the process that modifies the posture of the system in such a way the reaction forces with the ground induce the variation of those placement parameters. Locomotion then appears as a process operating from a high dimensional motor space (i.e. more than 600 muscles for humans, around 30 motors for humanoids) to the 3-dimensional placement space. From a biomechanics perspective we want to explore the synergies of human walking. Motor synergies aim at distributing a

[‡]© Cinémathèque Française. Etienne-Jules Marey (1830 – 1904) was a French physiologist known for his pioneering work on human locomotion. With Eadweard Muybridge, he is one of the inventor of chronophotography.

task among all motor variables and ensure their coordination (see the pioneering work by Bernstein ([Bernshhtein 1967](#)) and the recent overview by Latash ([Latash 2008](#))).

How does motion capture reveal the walking body motion invariant beyond the well-known arm-leg coordination or the planar covariation of elevation angles of the leg segments ([Ivanenko et al. 2008](#))? In Section 3 we present a first original result showing that the motion of the feet describes an arc of a circle centered at the body center of mass, and not at the hip rotation center as it is often assumed.

Robotics Perspective: Anticipate or not anticipate foot placement? The question of footstep anticipation introduced from the above neuroscience perspective echoes two opposite paradigms addressing the locomotion control of humanoids. The most robust and popular one is based on the control of the so-called Zero Moment Point (ZMP) lying within the foot support on the ground ([Vukobratović 1972](#)) and the recent Capture Point concept ([Pratt et al. 2006](#)). Starting from the seminal work by Kajita et al ([Kajita et al. 2003](#)), most of these approaches require a preview control managing an anticipation of foot placements ([Wieber et al. 2015](#)). The second paradigm is based on clever mechanical designs that take advantage from the gravity: they are the so-called passive walkers ([Collins et al. 2005](#)). Passive walker locomotion is much less energy consuming; however it is very fragile with respect to the ground perturbations.

How to quantify walking robustness? How do walking features influence robustness against ground perturbations? Both questions are addressed in Section 4.

2.2 Related work grounding the Yoyo-Man model

This section gives an overview of the rationales underlying the approaches and methods developed in Section 3 and Section 4.

Neurophysiology basics in human walking: passiveness and head stabilization. One important property of the human steady gait dynamics is that it takes profit from the natural passive dynamics of the body. The passive dynamics is the dynamics of the body when no actuation is present, the robot is then only subject to gravity, external forces and passive elasticity and friction of the joints. The body morphology (especially the hip and knee joints ([Collins et al. 2001](#))) allows the emergence of most prominent features of walking dynamics. The benefits of this structure is to enable the generation of walking motion with high energy efficiency and low control frequency ([Alexander 2005](#)). Furthermore, the control of steady gait has been investigated to suggest that it happens in a very low level of the brain, at the spinal level, consisting in a combination of a simple rhythm generator and reflexes to external perturbations ([Dietz 2003](#)). The steady gait seems to require minimal muscular efforts and cognitive involvements: we walk without thinking about it.

On the other hand, neurophysiologists have observed that humans and animals stabilize their head when moving (see an illustration in Figure 4). By stabilization, we mean that the head tilt is controlled to remain relatively constant compared to other limbs of the body. Head stabilization is a task prone

to dissipate energy since it works almost always against the motion. So why do humans stabilize their head?

The head carries most of the sensory organs, and specifically the visuo-vestibular system, responsible for a great part of balance estimation, spatial localization and motion perception. It can be understood then that stabilizing the head facilitates the fusion of visual and vestibular information. Recent studies also show that head stabilization improves the accuracy of estimation of vertical direction by vestibular-like inertial sensor ([Farkhatdinov et al. 2013](#)). Head stabilization improves perturbation detection and safety supervision. Moreover, head tilt conservation offers a consistent and stable egocentric reference frame for perception and generation of motion in general ([Berthoz 2002](#)) and locomotion in particular ([Pozzo et al. 1990](#); [Hicheur et al. 2005](#)).

These explanations fit with clinical observations on humans. The unsteadiness and the loss of balance resulting from head-neck system sensorimotor disturbances have been widely documented ([Stokell et al. 2011](#); [Lajoie et al. 1996](#); [Bove et al. 2002](#); [Vuillerme et al. 2005](#)). It has even been suggested that the impairments in the neck somatosensory inputs and sensorimotor control are as important for balance as a lower-limb proprioception loss following a knee or an ankle injury ([Treleven 2008](#)).

In this paper we argue that head stabilization also contributes *mechanically* to the balance when walking. The head represents 7% of the total mass of the body, and occupies the top 12% of its height. That means a non-negligible inertia effect regarding to contact points is due to the head motion. Therefore, head stabilization which actively modifies the motion of the head, should have a noticeable impact on the dynamics of the gait. This effect may be negative, perturbing the walking dynamics and requiring the rest of the body to compensate for it. Alternatively it can be part of the desired dynamics, enhancing balance and improving coordination. In Section 4 we show that the head-stabilization by itself contributes to war effort against falling.



Figure 4. Sketch of the superimposition of walker positions in different phases of the cycle. The superimposition is achieved so that the head is in the same position. The head is stabilized to keep constant orientation displayed by the dotted blue line. (Inspired by a drawing in ([Pozzo et al. 1990](#)))

Mechanical basics of bipedal walking: the Poincaré map. Bipedal walking is a cyclic process sequencing two phases: single support when only one foot is touching the ground and double support when both feet are touching the ground. This physical description holds for all bipedal walking

systems. The cycle of locomotion is then made of four phases after which it starts again from (almost) the same starting posture. The stability of the locomotion is reflected by the attractiveness of a periodic orbit called limit cycle. It is captured by the so-called Poincaré map (Goswami et al. 1997). In our context, the Poincaré map is the intersection of the orbit of the periodic walking motion with the posture space at a same instant of the cycle, e.g., when the swing foot touches the ground (Figure 5).

Few metrics were designed to estimate and compare equilibrium robustness to perturbations for limit cycle walkers such as the volume of the basin of attraction (Schwab and Wisse 2001), the largest Floquet multiplier (McGeer 1990) or the Gait Sensitivity Norm (Hobbelen and Wisse 2007). But most of them do not study the whole state space and remain around the limit-cycle. Furthermore ground variations are generally not taken into account. (Byl and Tedrake 2009) present a metric which is particularly suitable to estimate the robustness of limit cycles in presence of external perturbations. This metric is derived from classical analysis of metastable systems. It will be used in Section 4 to introduce the notion of ground texture and to compare walking models.

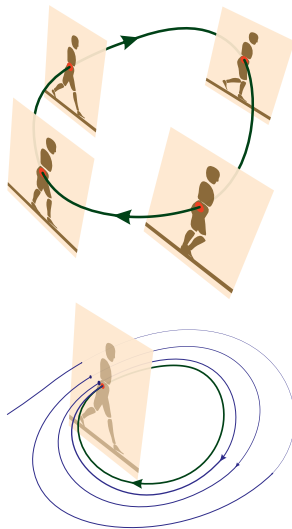


Figure 5. Locomotion cycle: Locomotion is a cyclic process sequencing the same postures alternatively (left). The stability of the underlying dynamical system is captured by the Poincaré map (right).

Basics in humanoid robot control: ZMP versus rimless wheel. At each phase of the locomotion cycle the pressure applied by the surface of the feet on the ground may be concentrated onto a single point: the center of pressure. When both the ground and the feet surfaces are flat, center of pressure and ZMP coincide. As soon as the ZMP remains strictly within the support surface, the system does not fall.

The property of the ZMP is at the origin of a popular locomotion control scheme. The ZMP and the center of mass (CoM) are linked together by nonlinear equations. The control of the CoM is easily derived from the control of the posture. So, in theory, it is possible to control the placement of the ZMP within the surface support. However the nonlinearities linking CoM and ZMP variables

make the problem computationally challenging. Under some hypothesis the equations are linear and the problem becomes easier. This is the case when the center of mass remains at the same altitude. Maintaining the CoM at the same altitude is made possible thanks to the redundancy of the anthropomorphic body. The hypothesis is at the origin of the cart-table model introduced in (Kajita et al. 2003) (Figure 6). The foundations of such control schemes are based on the knowledge of the foot steps to be performed. The literature refers to the so-called preview control (Wieber 2008): locomotion consists in planning the foot placement in advance.

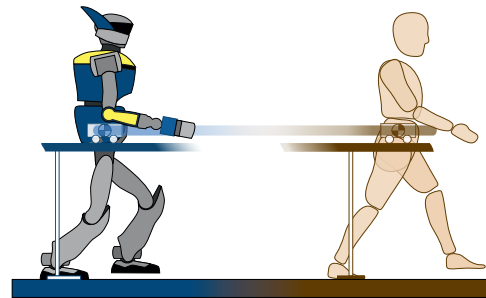


Figure 6. Cart-table: The cart-table model works under the hypothesis that the CoM moves on a horizontal plane. The hypothesis can be applied to control the locomotion of humanoid robots (left). Figure 3 suggests it does not hold for humans (right).

Passive walkers are designed from a completely different control perspective (Collins et al. 2005). They are minimally actuated. The mechanical design is devised to take advantage of the gravity and to convert potential energy into kinetic energy and vice versa. In its simplest version, the passive walker is made of two articulated legs connected to the hip (Collins et al. 2001). It can be modelled as a compass whose gaits induced a motion of the hip that is the same as the motion of the center of a rimless wheel. At that stage, it is noticeable that the motion of the center of a rimless wheel seems to be a rather good approximation of the hip motion in human walking (Figure 7). The analogy is part of the Yoyo-Man model rationale.

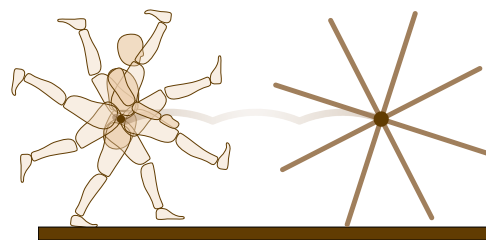


Figure 7. Rimless wheel: At a first glance, the center of a rolling rimless wheel roughly accounts for the motion the hip.

2.3 The Yoyo-Man: towards a mechanical and control co-design for bipedal walkers

The Yoyo-Man model (Figure 1) refers to a new perspective to design the control architectures for bipedal robots. It is not a new explicit walking control, such as ZMP-based or capture-point based controls. The Yoyo-Man model tends to extend the scope of the rimless wheel modeling simple compass-like passive walkers to more complex mechanical systems. It is derived from the two complementary results presented in the following sections. First, the observation of human walkers shows that the organization of foot motions is centered at the CoM, and surprisingly not at the hip. This reveals the presence of an embodied virtual rimless that concentrates walking synergies. The presence of such a virtual wheel comforts the principle that human walkers tend to optimize mechanical passiveness. It remains that the existing passive walkers exploiting a rimless wheel control scheme are fragile with respect to ground perturbations. Facing this drawback is the purpose of the second result (Section 4). We show that a compass-like walker equipped with a fixed mass on top of it is less robust to ground perturbations than the same walker equipped with the same but articulated and stabilized mass on top of it. So, designing a stabilizing control for this mass contributes to walking robustness. This comforts the intuition that the head can be considered as the control center of human walking. The center of the rimless wheel and the head are then the two main components of the Yoyo-Man. Beyond a control scheme, the model impacts also mechanical design: passive walkers are more robust when equipped with a stabilized human-like articulated head.

3 In search of a geometric center for the Yoyo-Man

This section brings to light the geometrical similarity between the rimless wheel and the human body during walking. While rolling on the floor, the center of the rimless wheel describes a sequence of circle arcs whose radius correspond to the stand beam. From a local point of view, this statement can be rephrased as follows: the contact point describes an arc of circle around the center of the rimless wheel during each supporting phase. In the case of human body, does there exist such a link between the foot touching the ground and some point that plays the role of the center of some rimless wheel? As far as we know, this question has never been addressed in human motion modeling.

At first glance, the articulation point between the thighbone and the pelvis, i.e. the hip center, would be a good candidate to play the role of the locomotion geometric center. This is not the case. In this section, we show both that the proposed rimless wheel model holds for human walkers, and that the center of the rimless wheel is the center of mass of the walking body. Our approach follows a standard empiricist methodology in biomechanics researches: after gathering a motion capture based data basis from several subjects walking barefoot according to a well-defined protocol, we make use of stochastic analysis to extract motion characteristics.

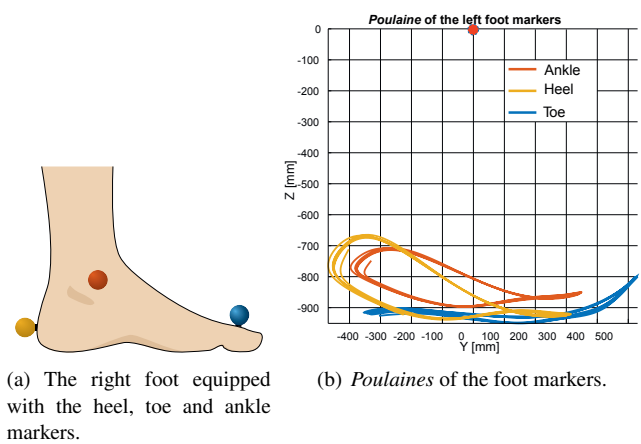


Figure 8. Illustration of the right foot equipped with the heel, toe and ankle markers and *Poulaines* of those markers along 8 steps. None of the *poulaines* describes a circular path relatively to the pelvis center.

3.1 Experimental setup

The experimental setup is based on an existing motion database used in (Olivier et al. 2011). It is composed of 12 participants (5 women and 7 men, 32.8 ± 5.9 years old, 1.71 ± 0.09 m, 65.3 ± 10.1 kg) who have been asked to walk straight at three different speeds three times each: natural, slow, and fast walking speed. To estimate the CoM trajectory, we used a standard approach from Biomechanics. It consists in equipping subjects with 41 optimal markers on each segment. The optimal markers are placed according to standard placements Dumas et al. (2007). The mass distribution is given by an anthropomorphic table Dumas et al. (2007). From the marker trajectories and the mass distribution, we estimate the whole-body CoM as the convex combination of all the CoM of the segments times the mass distribution. Finally, the segmentation of gait into simple and double support phases is achieved by using the methodology described in (Fusco and Crétual 2008).

In our study, we are interested by natural walking, i.e. what we refer to as reflex-based walking in Section 2.1. So, from the database we extracted the trials dealing with natural velocity. Then the total number of analyzed trajectories is $12 \times 3 = 36$.

3.2 Identification of the foot-CoM relationship

Poulaine[§] is a French word designating the trajectory of the anatomic feet markers (e.g. ankle, heel, toe) relatively to the geometric center of the pelvis and expressed in the world frame. For instance, Figure 8 illustrates the *poulaines* of the heel, toe and ankle markers.

At the first sight, none of the aforementioned anatomic markers describes a circular trajectory relatively to the pelvis center[¶]. At most, some *poulaines* have a temporally (i.e.

[§]We did not find the exact translation of this word in English.

[¶]In biomechanics, the pelvis center is considered as the root node from which the body segment tree is built.

during a short period) a constant curvature, but not during all the stance phase.

Our idea consists in moving the reference frame from the hip joint center to the CoM. We then show that a particular convex combination of the heel, ankle and toe markers of the stance leg describes a circular trajectory whose center is very close to the center of mass itself.

Choosing the CoM as the center of the reference frame and considering a convex combination of the toe, ankle and heel markers are supported by the following rationale. Firstly, the shift from the root marker to the center of mass allows us not to consider one precise segment (i.e. the root) but to take into account the overall movement of the human body. Secondly, by choosing a convex combination of the three aforementioned markers, we ensure that this particular point has a minimal velocity during the stance phase^{||}. It can therefore be treated as the pivot point of the rimless wheel.

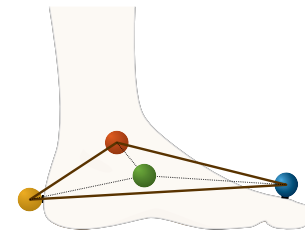
3.3 Methodology

Each walking trial is composed of 10 steps. We divided each of these trials into phases of single and double support phases. Then we introduce a virtual marker at the convex combination of toe, ankle and heel markers by selecting a particular convex combination for each subject, we fitted in the least-square sense the best circle passing through this virtual marker during 85% of the single support phase^{**}. On average, the root mean square error of the fitting part was around 2.5 mm. Figure 9 illustrates the procedure by showing the fitted circle having a center (yellow marker) very close to the CoM (red marker) and passing on average by the convex combination (in green). The other curves correspond to the anatomic markers of the foot, the hip joint center and the pelvis center.

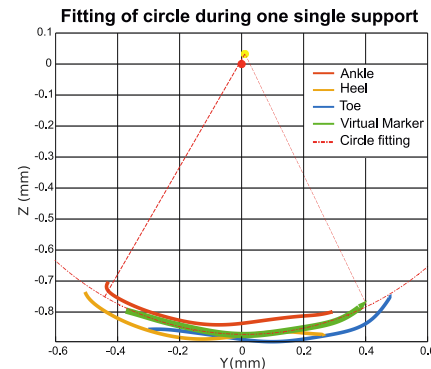
3.4 Results

For each subject, we computed the covariance matrix of the set of circle center positions relative to either the center of mass or the hip joint center. From the inverse of both covariance matrices, we define two distance metrics centered on the mean position of the circle centers and relative to the both reference points: the center of mass and the hip joint center. At the end, we obtained two dimensionless distances which discriminate if the two reference points belong to the circle center distributions or not.

Figure 10 summarizes the study over the 12 subjects. For the two metrics, the bar errors plotted at the top of each orange or blue boxes of Fig. 10 corresponds to the confidence interval $[-1; 1]$. While the height of the boxes corresponds to the dimensionless distance between either the center of mass or the hip joint center and the circle center distributions. We can remark that for all subjects, the CoM lives in the confidence interval of the circle center distributions. It is never the case concerning the hip joints center. Those observations allow us to conclude as following: first, there exists a similarity between the rimless wheel and humans during nominal walking gait and second, the center of this rimless does not correspond to the geometric pivot center (i.e. the hip joint center) but rather to the center of mass itself.



(a) The virtual marker as a convex combination of the anatomic foot markers.



(b) The virtual and anatomic marker trajectories and the fitted circle.

Figure 9. The virtual marker location and its trajectory relative to the CoM. The virtual marker (i.e. the convex combination of heel, toe and ankle markers) follows a circle whose center (yellow point) is close to the CoM (red point).

It is worth mentioning that our results hold only in the case of nominal gaits (i.e. walking gait with natural comfort velocity). Indeed, in the case of slow or fast walking velocities, we found that there is no convex combination of markers belonging to the stance foot which has a circular path. Some other studies have been focused on formulating a generic model describing the center of mass trajectory for a large class of walking speeds (Hayot et al. 2013). Nonetheless, the proposed model overestimates the vertical displacement of the center of mass while it fits well lateral motions.

Finally, the motion data basis considered in this study is made of straight line walking patterns. It is rightful to ask whether the same result would hold when walking along a curve. We guess this would be true. Indeed, goal oriented locomotion is known as obeying a nonholonomic constraint, i.e. when walking, the velocity of the body displacement belongs to the sagittal plane (Arechavaleta et al. 2008). It remains that a complete study would deserve to be conducted.

^{||}It is worth to mention at this stage that, due to the rolling of the foot on the ground, there is no zero velocity point which is fixed in the feet during the stance phase.

^{**}We only consider the starting 85% of the stance phase. This period does not take into account the part where the weight is mainly support by the toe and the deformation of the foot is maximal due to the toe actuation.

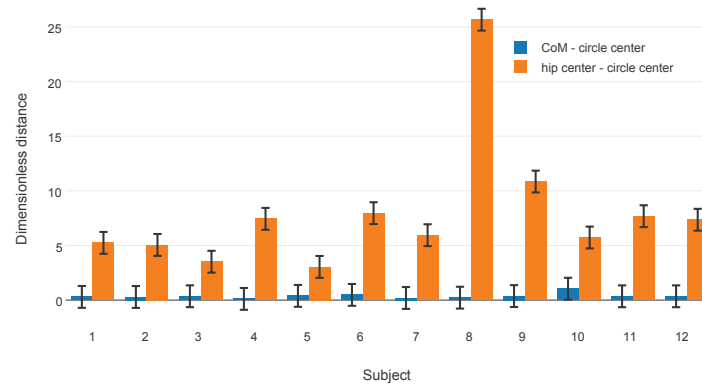


Figure 10. Dimensionless distance between the fitted circle centers and the CoM or the hip joint center. For all subjects, the center of mass belongs to the distribution of circle centers. This is not true in the case of the hip joint center.

4 In search of a control center for the Yoyo-Man

In this section we explore the mechanical contribution of the head stabilization to the balance when walking. The methodology is not based on observing human walkers as in the previous section. It is founded on the numerical simulation of two simple walking mechanical models.

We do not aim at modelling perfectly the human gait. Up to now, only simple dynamical models allow to reproduce locomotion gaits (Mombaur 2009). Precise dynamical modelling of human walking is out of reach of all current simulators. On the other hand, the rimless wheel model we are considering do not include articulated compliant legs (e.g. (Iida et al. 2009; Seyfarth et al. 2006)).

The following study is based on mechanical concepts derived from passive robot walkers (Collins et al. 2005; Byl and Tedrake 2009). It is noticeable that the energy efficiency of these robots, the low-frequency of their control and their natural limit-cycle dynamics are common characteristics with human locomotion (Alexander 2005; Goswami et al. 1997).

We introduce a walking simulation scheme where two walking control schemes are compared. The originality of these models includes improvements to classical compass-like walkers, by adding torso, interleg actuation, spring-damper at the feet, and rough terrains.

4.1 Walker mechanical model

Figure 11 illustrates the mechanical model we consider. It operates in the sagittal plane. It is made of five articulated rigid bodies: two bodies for the (knee-free) legs, one body for the torso, one for the neck and one for the head. Note that the neck is modelled as an articulated body and not as a simple joint. This setting reflects the property of the head-neck system to have two centers of rotation in the sagittal plane: one at the base of the neck and the other at ear level (Viviani and Berthoz 1975). The mass distribution and the limb lengths are anthropometric (Armstrong 1988) (see Table 1).

4.2 Two walking control schemes

To explore the mechanical effect of head stabilization to the walking balance, we introduce two original control schemes.

	Length (m)	Mass (Kg)
Head	$l_h=0.09$	$m_h = 5$
Neck	$l_n=0.07$	$m_n = 1$
Torso	$l_t=0.75$	$m_t = 55$
Leg	$l_{p,0}=1$	$m_l = 18$
Leg CoM	$l_l = 0.40$	

Table 1. Walker mass distribution

In the first one, the walker has a rigid neck and tends to stabilize the torso upright (Model A). In the second one the neck is modelled as a limb of two joints and the walker tends to maintain the head direction constant (Model B).

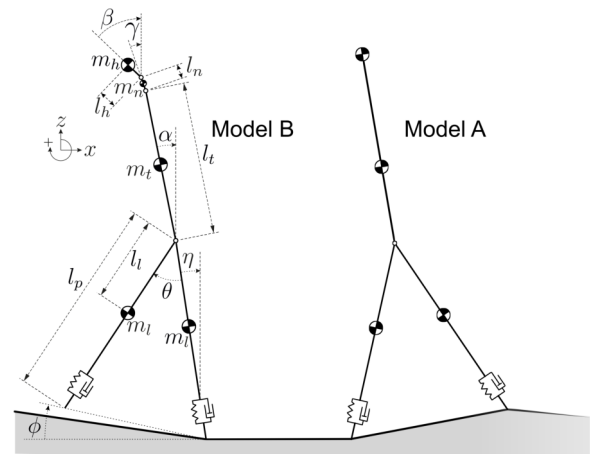


Figure 11. Representation of the two models we simulate on a rough terrain. Mass distribution of both models are the same. Model A has a rigid neck. Model B is equipped with an articulated neck.

In order to see the effect of head stabilization, we simulate the presented walker with two different actuation models. The first walker (Model A) we consider, has a rigid neck, i.e. the torso, the neck and the head constitute a single rigid body. The second walker corresponds to the model of head stabilization (Model B): the neck joints are controlled to maintain a zero tilt for the head regarding to the vertical direction.

Apart from the neck, both walkers have the same controls: the torso is actuated to be stabilized upright while a

lightweight controller actuates the inter-leg angle. Finally a velocity driven foot impulsion is given just before the swing phase. Except for toe off impulsion, the controllers for the robot are proportional-derivative (PD), each of them has two gain parameters. The gains for lower body are chosen to be lightweight, to approach the low energy consumption of human's steady walk^{††}. The weakness of the control of the lower limbs make them sensitive to perturbations, and their dynamics can differ according to upper-body control. The upper body has to guarantee a successful vertical stabilization for the trunk and the head, and has therefore stiffer actuators.

Toes: The exchange between the swing phase and the stance phase occurs at impacts of the swing leg with the ground. Several studies have addressed the complex dynamics of impulses in this context, e.g. (Ackerman and Seipel 2013; Akutsu et al. 2014; Asano and Kawamoto 2014). In this paper we consider a simplified model. Impacts are considered inelastic and contacts are considered perfect with no slipping. The toe of the stance leg has a spring-damper dynamics. The contact force follows the direction of the stance leg and its magnitude has a proportional-derivative (PD) expression:

$$f_t = -K_{toe,p}(l_p - l_{p,0}) - K_{toe,d}\dot{l}_p$$

where $K_{toe,p} = 50000$ N/m is the elasticity of the spring and $K_{toe,d} = 2000$ Ns/m is the damping factor. This force is applied only when it is positive because of the unilateral force constraint of the contact (the ground cannot pull the body).

When a leg is in a swing phase, its toe comes back instantly to the rest position $l_{p,0}$ of the spring, and remains constant until the end of the swing phase. We denote then simply by l_p the length of the stance leg.

The walkers loose a part of their mechanical energy at each impact. They require then to be actively fed with an equivalent source of energy. Therefore, at the instant of take-off of the stance leg, a velocity controlled impulsion is applied to the ground to give propulsion to the robot. The required force for this impulsion is f_t :

$$f_t = h(\dot{l}_{p,r})$$

where $\dot{l}_{p,r} = 1$ m.s⁻¹ is the desired velocity and h is the controller function.

Open Dynamics Engine provides virtual engine models which allow to generate the force to be applied during one simulation step in order to achieve a reference velocity. This reference velocity is introduced as a constraint in the Linear Complementary Program which also allows to generate contact forces and maintain kinematic constraints. The dynamic engine takes into account the state of the simulation and computes this force together with the resolution of the simulation step. This gives an automatic computation of h which has no closed form. We use it then to apply the force f_t during one time-step of simulation.

Interleg joint: The inter-leg joint is controlled by a PD pure torque generator toward a reference angle:

$$\tau_{hip} = -K_{hip,p}(\theta - \theta_r) - K_{hip,d}\dot{\theta}$$

where $K_{hip,p} = 10$ Nm/rad is the proportional gain, $\theta_r = 0.3$ rad is the reference angle and $K_{hip,d} = 1.5$ Nm s/rad is the derivative gain. We see that these values are small in order to reduce energy consumption and preserve the natural

dynamics of the legs.

Trunk On the other hand, we need to maintain the upper-body globally upright. There are two major approaches that enable passive walkers to control an upper-body. First, the bisector constrained walker, introduced by Wisse et al (Wisse et al. 2007), is a compass with an upper limb that is constrained to be in the midway angle of the two legs, a successful real design was presented using chains, and has the advantage to be entirely passive. However, beside the fact that it is not an accurate model to human walking, its bisecting constraint introduces an instability, especially in the presence of a heavy upper body (Asano and Luo 2008), which is the case for humans. The second method is to stabilize actively the upper body against the vertical, which better models the human gait (Winter 1991). The stabilization is achieved by applying torque on the stance leg similarly to what is done in (McGeer 1990). We choose this last solution to not disturb the passive swing motion. The trunk torque is actuated by a pure torque generator, controlled by a PD that brings back the trunk to vertical orientation (i.e. $\alpha_{desired} = 0$):

$$\tau_t = -K_{t,p}\alpha - K_{t,d}\dot{\alpha}$$

where τ_t is the trunk to stance-leg torque, $K_{t,p} = 300$ Nm/rad is the proportional gain and $K_{t,d} = 150$ Nms/rad is the derivative gain.

Model B: Head stabilization For the model B, there are two other controllers, which are the neck stabilization and head stabilization, they are also controlled by PD pure torque generators. Their torques expressions are the following:

$$\tau_n = -K_{n,p}\gamma - K_{n,d}\dot{\gamma}$$

$$\tau_h = -K_{h,p}\beta - K_{h,d}\dot{\beta}$$

where τ_n is the torque applied to the torso-neck joint, $K_{n,p} = 50$ Nm/rad is the neck proportional gain, $K_{n,d} = 0.6$ Nm s/rad is the neck derivative gain, τ_h is the torque applied to the neck-head joint, $K_{h,p} = 150$ Nm/rad is the head proportional gain and $K_{h,d} = 1$ Nm s/rad is the head derivative gain.

4.3 Ground texture: a measure of walker robustness

Due to difference in control schemes, the whole body dynamics of the walkers are different. However, both dynamics are balanced on a perfectly flat surface and converge to a stable limit cycle. Therefore both walkers can walk indefinitely on flat surface without falling and their difference does not appear.

The idea to highlight the difference between the control schemes is to perturb both systems to them make metastable. Metastability is the property of stochastic dynamic systems to keep a specific behavior for long periods, but being guaranteed to leave this state after a sufficiently long time (Talkner and Hänggi 1987). The system reaches then what is considered as a failed state (see Figure 12). The idea to use the concept of metastability to estimate the balance

^{††}For a better control of numerical sensitivity, we divided all masses in Table 1 by a factor 10. The control gains are presented accordingly. This scaling does not affect the results as the simulator is linear with the body masses. See (Benallegue et al. 2015) for details.

performance of a walker has been introduced by Byl and Tedrake (Byl and Tedrake 2009). It consists in considering that, under ground perturbation, the probability of falling tends to 1 as time goes to infinity, for any walking system. Indeed it is still today a challenging problem for passive-dynamics walkers to face uneven terrains. Therefore, one good way to evaluate the robustness of a walker is to evaluate the expectation of the number of steps that the robot can achieve before falling, which is also called the mean first-passage time (MFPT). However, the proposed method to compute this expectation is based on a Markov chain model, and requires a discretization of the state space. It is not easily applicable for high dimensional systems. This is why we introduced in (Benallegue and Laumond 2013) a new method that extends the scope of metastability concept to robust walking systems, while keeping reasonable simulation time.

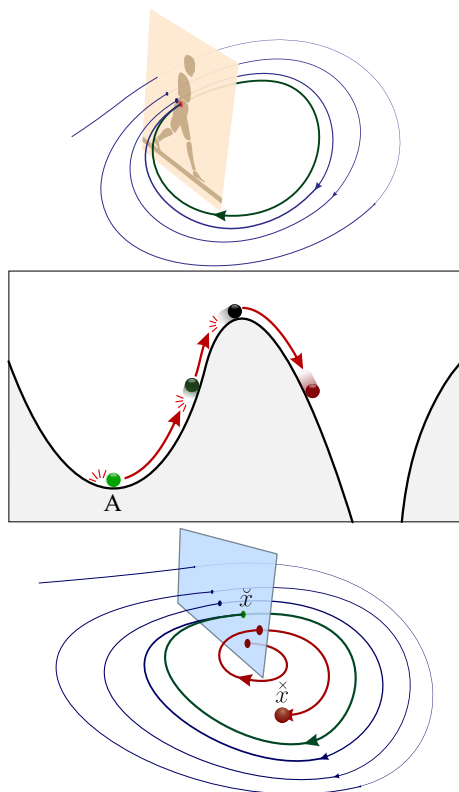


Figure 12. Making passive walkers metastable. Passive walkers rapidly converge to a stable limit-cycle (green curve) when walking on a flat surface (top). Introducing a stochastic ground perturbation imposes the walker to escape the attraction basin A with a probability tending to 1 when time tends to infinity (middle). When starting from a state in the basin of attraction of the limit-cycle, the state intersection with the Poincaré map converges to the limit cycle (blue curve converges to the green one in the picture on the bottom). When starting from a position outside this basin of attraction, there is a fall (red curve in the picture on the bottom). Pictures from (Benallegue and Laumond 2013).

We define a textured ground as a ground for which the unevenness follows a probability distribution. For our walkers, we model it by changing the ground inclination at each step (see Figure 11), following a centered Gaussian law. The standard deviation of the probability distribution define the degree of ground unevenness.

We computed MFPTs for both Model A and Model B on several ground textures. Results are presented in the next section.

4.4 Results

On flat terrain, and for both control models, it has not been possible to find an upper bound for MFPTs (see Figure 13). However walker performances greatly differ as soon as a slight texture change appears. The phenomenon can be seen from the example of 0.01 rad standard deviation. In this case, MFPT of the rigid neck model is 23 steps, while head stabilization guarantees MFPT of more than 3 million steps! This performance improvement persists as the ground texture increases, even if the difference declines. This is purely due to mechanical effects, i.e. to the contribution of the head motion to the balance of the gait.

These results may be seen differently. The head stabilization curve of Figure 13 can be seen as a shift to the right for the rigid neck curve. In other words, head stabilization enables to increase significantly the range of ground textures the walker can handle with the same balance performances.

At this level we may conclude that head stabilization may improve substantially the dynamic balance of walking systems. Head stabilization is an heuristic answer to the question of taking advantage of the head mobility during walking. Indeed, while it is likely not the optimal control of the neck regarding balance, it is a very simple control that produces a complex behavior with significant benefits. Additional explanations for the origin of this effect, including its impact on energy consumption can be found in (Benallegue et al. 2015).

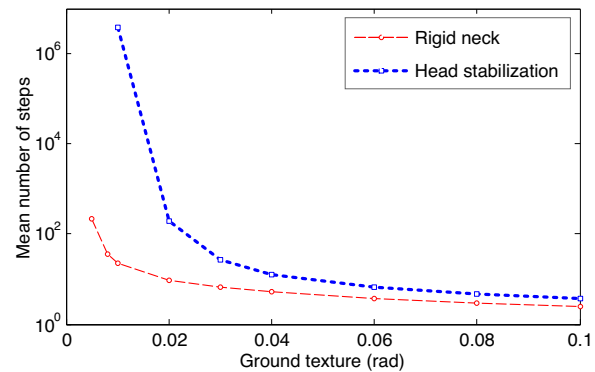


Figure 13. Mean number of steps for both Model A (rigid neck) and Model B (stabilized head) according to the texture of the ground. MFPTs are displayed in logarithmic scale. For high ground roughness, MFPT of both models drops such that they need to change their walking control: watching their step becomes necessary.

5 Discussions and Perspectives

Both results presented in this paper followed an empiricism approach that opens complementary research axis. We first observed that foot motions describe an arc of a circle centered at the center of mass when walking. Then numerical simulations showed that compass-like passive walkers are

better stabilized when considering a torso equipped with a stabilized mass on top of it.

The first result is surprising in the sense that it reveals the center of mass (CoM) as a critical point that concentrates the organization of leg motions, better than the center of the hip. This synergy has never been described in the literature. Now, the fact remains that the rimless wheel model is limited. If the CoM was the center of a rimless wheel, then its motion would be a sequence of arcs of a circle, separated by cusps. Yet this is not true. Intuition suggests that CoM motion is smooth and close (up to a vertical translation) to the motion followed by the head (see Figure 3). Therefore, a deeper observation of the CoM motion in the 3-dimensional space deserves to be pursued. CoM estimation depends on a very large number of parameters, including soft tissues shapes and densities. These parameters are classically reduced to articular angles coupled to a mass distribution model considering perfectly rigid limbs (Dumas et al. 2007). In (Carpentier et al. 2015) we recently introduced a new method combining motion capture and force sensor measurements and giving rise to CoM position estimation much more accurate than the estimators currently used in biomechanics. We then intend to refine the model the Yoyo-Man is suggesting.

Now, it remains that our proposition of model holds only for natural walking. The intuition may suggest that it could also hold for running. However, while walking is organized around sequences of simple and double supports, running is made of sequence of simple support and ballistic motions. The center of mass motion draws an arc of a parabola in the ballistic phase. This asks for an extension of the rimless wheel model underlying the Yoyo-Man model.

The second result is also surprising. It makes sense that the presence of a stabilized mass like the torso on top of a compass-like walker improves the stabilization of the global system. What is surprising is that the head significantly contributes to the stabilization while its mass accounts only for 7 percent of the total mass of the body. We may suspect that there is here a momentum effect that deserves to be deeper explored. In a more general perspective, our result is based on numerical simulation of a simple 2D model consisting of five limbs. It asks to be comforted by deeper mathematical analysis and generalization to more sophisticated walker models. But this study focuses on the specific issue of exploring the effect of head stabilization on the dynamics of gait, which is a recognized feature of human walking. In fact, the design and control of optimal models is the topic of our related research regarding simultaneous model design and control of robots in the same optimization loop Saurel et al. (2016).

After the contribution of the head stabilization in sensing (Farkhatdinov et al. 2011), the mechanical contribution of the head stabilization to bipedal walking enhances the role of the head in anthropomorphic action control. Furthermore, it is known that the head yaw angle anticipates body yaw (shoulder and trunk) and shift in locomotor trajectory (Hicheur et al. 2005; T. Imai and Cohen 2001). This behavior has been successfully implemented to steer the humanoid robot HRP2 by its head (Sreenivasa et al. 2009). However the implementation remains based on a classical preview control of the ZMP. Making humanoid machines "walking without thinking" challenges roboticists to devise

new locomotion controllers that would be free of any foot step anticipation. Taking inspiration of the Yoyo-Man model, a condition of the success is to consider new mechanical and control designs, which give the head and its sensors a place much more central than the current humanoid robot designs.

Finally, anthropomorphic locomotion is not reduced to walking. The Yoyo-Man is a reflex-based walking man. We have seen that the model clearly does not account for watch-your-step modes. Ground texture allows to quantify the application scope of the model. What happens when the model does not work anymore, i.e. for instance when facing highly textured ground? How do we evaluate that we have to watch our step? What happens in terms of motor control? Addressing the questions from a neurophysiology perspective requires to invent new experimental protocols that will tend to elucidate the motor control architecture of locomotion at large, including both reflex and deliberative levels, i.e. involving both medulla and central nervous systems.

The Yoyo-Man is a proposition of model for bipedal locomotion. The two results presented in this paper comfort the pertinence of the model for walking. As a human locomotion model, its plausibility raises challenging issues in biomechanics and neuroscience. As applied in robotics, the relevance of the Yoyo-Man model should be comforted by effective experiences and asks for the design of new robot architectures.

Acknowledgements

This paper benefits from the valuable comments by reviewers of a short version presented at the 17th International Symposium on Robotics Research (ISRR) in Sestri Levante in 2015.

We thank Armel Crétual and Anne-Hélène Olivier from M2S lab, University of Rennes 2, France, for providing the database of captured walking motion.

The work is supported by the European Research Council (ERC) through the Actanthrope project (ERC-ADG 340050) and by the European project KOROIBOT (FP7/611909).

References

- J. Ackerman and J. Seipel. Energy efficiency of legged robot locomotion with elastically suspended loads. *IEEE Transactions on Robotics*, 29(2):321–330, 2013.
- Y. Akutsu, F. Asano, and I. Tokuda. Passive dynamic walking of compass-like biped robot with dynamic absorbers. In *IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2014, pages 4855–4860. IEEE, 2014.
- R. M. Alexander. Walking made simple. *Science*, 308(5718):58–9, Apr 2005.
- G. Arachavaleta, J.P. Laumond, H. Hicheur, and A. Berthoz. On the nonholonomic nature of human locomotion. *Autonomous Robots*, 25(1-2):25–35, 2008.
- H. G. Armstrong. Anthropometry and mass distribution for human analogues. Technical report, Aerosp. Med. Res. Lab Wright-Patterson AFB Ohio, 1988.
- F. Asano and J. Kawamoto. Modeling and analysis of passive viscoelastic-legged rimless wheel that generates measurable

- period of double-limb support. *Multibody System Dynamics*, 31(2):111–126, 2014.
- F. Asano and Z.W. Luo. Pseudo virtual passive dynamic walking and effect of upper body as counterweight. In *IEEE/RSJ International Conference on Intelligent Robots and Systems, 2008*, pages 2934–2939, sept. 2008.
- M. Benallegue and J.-P. Laumond. Metastability for high-dimensional walking systems on stochastically rough terrain. In *Robotics Science and Systems*, 2013.
- M. Benallegue, J.-P. Laumond, and A. Berthoz. A head-neck-system to walk without thinking. LAAS-CNRS Technical Report, available at <https://hal.archives-ouvertes.fr/hal-01136826/>, 2015. URL <https://hal.archives-ouvertes.fr/hal-01136826>.
- N.A. Bernshstein. *The co-ordination and regulation of movements*. Pergamon Press, 1967.
- A. Berthoz. *The Brain's Sense of Movement*. Harvard University Press, 2002.
- M. Bove, G. Courtine, and M. Schieppati. Neck muscle vibration and spatial orientation during stepping in place in humans. *Journal of neurophysiology*, 88(5):223–241, Nov 2002.
- K. Byl and R. Tedrake. Metastable walking machines. *The International Journal of Robotics Research*, 28(8):1040–1064, 2009.
- J. Carpentier, M. Benallegue, N. Mansard, and J.P. Laumond. Center of Mass Estimation for Polyarticulated System in Contact - A Spectral Approach. *IEEE Transactions on Robotics*, August 2015.
- S. Collins, A. Ruina, R. Tedrake, and M. Wisse. Efficient bipedal robots based on passive-dynamic walkers. *Science*, 307(5712):1082–1085, February 2005.
- S. H. Collins, M. Wisse, and A. Ruina. A three-dimensional passive-dynamic walking robot with two legs and knees. *International Journal of Robotic Research*, 20:607–615, 2001.
- V. Dietz. Spinal cord pattern generators for locomotion. *Clinical Neurophysiology*, 114:1379–1389, August 2003.
- R. Dumas, L. Cheze, and J.-P. Verriest. Adjustments to mcconville et al. and young et al. body segment inertial parameters. *Journal of biomechanics*, 40(3):543–553, 2007.
- I. Farkhatdinov, V. Hayward, and A. Berthoz. On the benefits of head stabilization with a view to control balance and locomotion in humanoids. In *IEEE-RAS International Conference on Humanoid Robots, 2011*, pages 147–152, oct. 2011.
- I. Farkhatdinov, H. Michalska, A. Berthoz, and V. Hayward. Modeling verticality estimation during locomotion. In *Romansy 19 - Robot Design, Dynamics and Control*, volume 544, pages 359–366, 2013.
- N. Fusco and A. Crétaul. Instantaneous treadmill speed determination using subject's kinematic data. *Gait & Posture*, 28(4):663–667, 2008.
- A. Goswami, B. Espiau, and A. Keramane. Limit Cycles in a Passive Compass Gait Biped and Passivity-Mimicking Control Laws. *Autonomous Robots*, 4:273–286, 1997.
- C. Hayot, S. Sakka, V. Fohanno, and P. Lacouture. Biomechanical modeling of the 3d center of mass trajectory during walking. *Movement & Sport Sciences-Science & Motricité*, 2013.
- H. Hicheur, S. Vieilledent, and A. Berthoz. Head motion in humans alternating between straight and curved walking path: Combination of stabilizing and anticipatory orienting mechanisms. *Neuroscience Letters*, 383:87–92, 2005.
- D. G. E. Hobbelen and M. Wisse. A disturbance rejection measure for limit cycle walkers: The gait sensitivity norm. *IEEE Transactions on Robotics*, 23(6):1213–1224, Dec 2007.
- F. Iida, Y. Minekawa, J. Rummel, and A. Seyfarth. Toward a human-like biped robot with compliant legs. *Robotics and Autonomous Systems*, 57(2):139–144, 2009.
- Y. P. Ivanenko, A. d'Avella, R. E. Poppele, and F. Lacquaniti. On the origin of planar covariation of elevation angles during human locomotion. *Journal of neurophysiology*, 99(4):1890–1898, 2008.
- W. Jiang and T. Drew. Effects of bilateral lesions of the dorsolateral funiculi and dorsal columns at the level of the low thoracic spinal cord on the control of locomotion in the adult cat. i. treadmill walking. *Journal of Neurophysiology*, 76(2):849–866, 1996.
- H.L. Jin and M. Zacksenhouse. Oscillatory neural networks for robotic yo-yo control. *IEEE Transactions on Neural Networks*, 14(2):317–325, 2003.
- S. Kajita, F. Kanehiro, K. Kaneko, K. Fujiwara, K. Harada, K. Yokoi, and H. Hirukawa. Biped walking pattern generation by using preview control of zero-moment point. In *IEEE International Conference on Robotics and Automation*, volume 2, pages 1620–1626 vol.2, sept. 2003.
- Y. Lajoie, N. Teasdale, J. D. Cole, M. Burnett, C. Bard, M. Fleury, R. Forget, J. Paillard, and Y. Lamarre. Gait of a deafferented subject without large myelinated sensory fibers below the neck. *Neurology*, 47(1):10915, Jul 1996.
- M. L. Latash. *Synergy*. Oxford University Press, 2008.
- T. McGeer. Passive dynamic walking. *The International Journal of Robotics Research*, 9(2):62–82, 1990.
- K. Mombaur. Using optimization to create self-stable human-like running. *Robotica*, 27(3):321–330, May 2009.
- K. Mombaur and M. Sreenivasa. Inverse optimal control as a tool to understand human yoyo playing. In *International Conference Numerical Analysis and Applied Mathematics*, 2010.
- A.-H. Olivier, R. Kulpa, J. Pettré, and A. Crétaul. A step-by-step modeling, analysis and annotation of locomotion. *Computer Animation and Virtual Worlds*, 2011.
- T. Pozzo, A. Berthoz, and L. Lefort. Head stabilization during various locomotor tasks in humans. *Experimental Brain Research*, 82:97–106, 1990.
- Jerry Pratt, John Carff, Sergey Drakunov, and Ambarish Goswami. Capture point: A step toward humanoid push recovery. In *2006 6th IEEE-RAS international conference on humanoid robots*, pages 200–207. IEEE, 2006.
- Guilhem Saurel, Justin Carpentier, Nicolas Mansard, and Jean-Paul Laumond. A Simulation Framework for Simultaneous Design and Control of Passivity Based Walkers. In *2016 IEEE International Conference on Simulation, Modeling, and Programming for Autonomous Robots SIMPAR*, San Francisco, United States, December 2016. URL <https://hal.archives-ouvertes.fr/hal-01360450>.
- A.L. Schwab and M. Wisse. Basin of attraction of the simplest walking model. *Proceedings of ASME Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, 21363, 2001.

- A. Seyfarth, H. Geyer, R. Blickhan, S. Lipfert, J. Rummel, Y. Minekawa, and F. Iida. Running and walking with compliant legs. In *Fast motions in biomechanics and robotics*, pages 383–401. Springer, 2006.
- M. Sreenivasa, P. Soueres, J.-P. Laumond, and A. Berthoz. Steering a humanoid robot by its head. In *IEEE/RSJ International Conference on Intelligent Robots and Systems, 2009*, pages 4451–4456, oct. 2009.
- R. Stokell, A. Yu, K. Williams, and J. Treleaven. Dynamic and functional balance tasks in subjects with persistent whiplash: a pilot trial. *Manual therapy*, 16(4):3948, Aug 2011.
- T. Raphan T. Imai, S. T. Moore and B. Cohen. Interaction of the body, head and eyes during walking and turning. *Experimental Brain Research*, 136(136):1–18, 2001.
- P. Talkner and P. Hänggi. Discrete dynamics and metastability: Mean first passage times and escape rates. *Journal of Statistical Physics*, 48(1/2):231–254, 1987.
- J. Treleaven. Sensorimotor disturbances in neck disorders affecting postural stability, head and eye movement control. *Manual therapy*, 13(1):211, Mar 2008.
- P. Viviani and A. Berthoz. Dynamics of the head-neck system in response to small perturbations: analysis and modeling in the frequency domain. *Biological cybernetics*, 19(1):1937, Aug 1975.
- N. Vuillerme, N. Pinsault, and J. Vaillant. Postural control during quiet standing following cervical muscular fatigue: effects of changes in sensory inputs. *Neuroscience letters*, 378(3):1359, Apr 2005.
- M. Vukobratović. On the stability of anthropomorphic systems. *Mathematical Biosciences*, 15(1-2):1–37, 1972.
- P.-B. Wieber. Viability and Predictive Control for Safe Locomotion. In *IEEE-RSJ International Conference on Intelligent Robots & Systems, 2008*, Nice, France, 2008.
- P.-B. Wieber, S. Kuindersma, and R. Tedrake. *Handbook of Robotics, 2nd Edition*, chapter Modeling and Control of Legged Robots. Springer, 2015. To appear.
- D. A. Winter. *The Biomechanics and Motor Control of Human Gait*. Waterloo, ON, Canada: University of Waterloo Press, 1991.
- M. Wisse, D. G. E. Hobbelen, and A. L. Schwab. Adding an upper body to passive dynamic walking robots by means of a bisecting hip mechanism. *IEEE Transactions on Robotics*, 23(1):112–123, feb. 2007.