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# Is the market really a good teacher ?

Market selection, collective adaptation  
and financial instability.\*

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## Abstract

This paper proposes to model market mechanisms as a collective learning process for firms in a complex adaptive system, namely **Jame1**, an agent-based, stock-flow consistent macroeconomic model. Inspired by Alchian's (1950) "blanketing shotgun process" idea, our learning model is an ever-adapting process that puts a significant weight on exploration *vis-à-vis* exploitation. We show that decentralized market selection allows firms to collectively adapt their overall debt strategies to the changes in the macroeconomic environment so that the system sustains itself, but at the cost of recurrent deep downturns. We conclude that, in complex evolving economies, market processes do not lead to the selection of optimal behaviors, as the characterization of successful behaviors itself constantly evolves as a result of the market conditions that these behaviors contribute to shape. Heterogeneity in behavior remains essential to adaptation in such an ever-changing environment. We come to an evolutionary characterization of a crisis, as the point where the evolution of the macroeconomic system becomes faster than the adaptation capabilities of the agents that populate it, and the so far selected performing behaviors suddenly cease to be, and become instead undesirable.

**Keywords** – Evolutionary economics, Learning, Firms' adaptation, Business cycles.

**JEL classification codes** – B52, C63, D83, E32.

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# 1 Introduction

A market operates on a decentralized ground: it is a place where a collection of heterogeneous agents locally and constantly interact, without seeing the resulting whole picture. This property poses a challenge to the use of a representative agent with rational expectations, and raises the question of how to model agents' behaviors and learning in market economies. This paper proposes a decentralized adaptation model rooted in the functioning of the market itself: the selection mechanism operates through market competition, as firms that use non performing strategies are driven out of the market by bankruptcy.

The idea that market mechanisms determine the *aggregate* behavior of the system, by selecting appropriate behaviors and discarding inappropriate ones, without the need to model any rationality, foresight of adaptive behavior from the *individual* agents is originally due to Alchian (1950). Alchian (1950, p. 219) calls such a process the “blanketing shotgun process” (BSP hereafter): a multitude of agents randomly select strategies, without assuming any intentional decision making at the individual level, and the market selects the best-performing behaviors by excluding the unsuccessful ones. This process requires individual heterogeneity and market interactions, and postulates that the collective adaptation force of the system is superior to the one of the individual agents. This process also puts more emphasis on the *exploration* for potential strategies than on the *exploitation* of already discovered strategies. The BSP therefore appears particularly well-suited to represent adaptation of a population in an *ever-changing* environment. We believe that all these features, rather than referring to the principle of the survival of the fittest as a defense of profit maximization, bring simple but relevant principles that are reconcilable with, and even precursory of both the theory of bounded rationality (Simon 1961) and evolutionary economics (Nelson & Winter 1982), and may be useful for modeling behavior in macro ABMs.

In this paper, we introduce investment and capital depreciation in the `Jame1` model<sup>1</sup>, along with refinements in the banking sector. We then apply the principles of the BSP

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<sup>1</sup>`Jame1` stands for Java Agent-based Macroeconomic Laboratory, see Seppecher (2012*b,a*), Seppecher & Salle (2015).

to the determination of firms' leverage strategies. We choose this model as a playground because it is simple enough to get a grip on the emerging dynamics, while allowing for rich monetary and real interactions between agents, and especially between firms, in a fully stock-flow consistent framework. We choose the leverage strategies for testing the BSP because this decision is in several ways particularly challenging from the firms' perspective. First, in an ABM, not even the modeler would be able to identify an "optimal" solution. What is more, investment dynamics brings instability into the macroeconomic dynamics of the model compared to previous versions, and reinforces market competition. Leverage decisions amount specifically to solving a "growth-safety trade-off": a high indebtedness allows the firm to quickly gain market shares, but at the risk of an increased financial fragility; a low indebtedness may be insufficient to renew depreciating capital and may drive the firm out of the market. The debt behaviors of firms in turn collectively contribute to shape the macroeconomic environment, so that the environment constantly changes, and complex dynamics emerge. In such an hostile and selective environment, we let the leverage strategies of a collection of competing firms evolve on a completely random basis, and the only selection pressure comes from bankruptcies.

With the `Jame1` model as a playground, we perform a theoretical exercise that aims to assess to what extent the process of "natural" market selection constitutes a suitable adaptation model for agents in a complex system. This amounts to characterizing the dynamics that emerge from ever-adapting individual behaviors under the sole selection pressure of market conditions, that they in turn contribute to shape: Can the system settle down on an "equilibrium"? Otherwise, what are the emerging dynamics?

Our results are as follows. Decentralized market selection allows the firms to collectively adapt the overall leverage level to the changes in the macro environment in a way that the system can sustain itself. However, this regulation comes at the price of wild fluctuations and deep downturns. This emerging macro dynamics are caused by a clear alternating pattern between a sustained rise in indebtedness along the boom phase, that feeds back into the goods demand, and brutal deleveraging movements along the busts, once the financial fragility of firms, combined with increased interest rates and excess

production capacities, increases to the point where insolvency and bankruptcies are unavoidable. We conclude that, even if the “natural” market selection process allows for a certain resilience and adaptability of the system, it does not deliver any convergence towards an “optimal” equilibrium. Our conclusion stands in sharp contrast to the view, dating back to Friedman (1953), that systematically advocates market selection to justify full rationality assumptions and equilibrium reasoning.

Furthermore, we observe that debt behaviors that are rewarded along the boom turns out to be vicious along the busts, and firms need to constantly adapt along the different phases of the business cycles. We then make the point that heterogeneity of behaviors is essential to the adaptation process of a population in an unstable, and quickly evolving environment. The BSP allows us to make this heterogeneity endogenous and dynamic: it combines converging forces (market selection and imitation) with diverging forces (exploration), so that behaviors co-evolve with the macroeconomic dynamics that they contribute to shape. This is a major point, because it allows us to show that, while individual and aggregate behaviors appear to commonly self-reinforce each other, they can suddenly disconnect from each other. This observation leads us to suggest an evolutionary characterization of a *crisis*, as the point when the evolution of the macro system becomes faster than the adaptation capabilities of the agents that populate it. One particular strength of our framework is to be able to robustly account for such a phenomenon as a result of simple, random and individual adjustments.

The rest of the paper is organized as follows. Section 2 discusses the non-trivial problem of modeling individual behaviors in complex systems, Section 3 details the `Jame1` model and our implementation of the BSP, Section 4 presents the results from the numerical simulations, Section 4.3 discusses the characterization of a crisis in the model, and Section 5 concludes.

## 2 Modeling individual behaviour in macro ABMs: learning and adaptation

This section paves the way to the introduction of the adaptation process based on the BSP in the `Jame1` model. We first discuss the challenges posed by the modeling of agents' behavior in macro ABMs. We then define the concepts of adaptation and *learning* and stress their importance in this type of models. We finally contrast individual versus social learning by focusing on evolutionary models and discuss their limitations.

### 2.1 Modeling individual behavior in macro ABMs: particularities and challenges

**The “wilderness of bounded rationality”** The functioning of ABMs is rooted in a multitude of heterogeneous agents who repeatedly interact in a decentralized way. Those interactions generate *complexity*, in the sense that even the perfect knowledge of individual behavior is not enough to anticipate the resulting macroeconomic outcomes. In such a complex world, no agent is endowed with the whole picture of the economy, and uncertainty is both strategic and radical: because of the uncertainty regarding all other agents' behavior, there is no trivial probabilistic mapping between the entire set of possible actions of an agent and the resulting states of the world and associated pay-off. Neither the agents nor the modeler may be able to define what the fully rational/optimal decision is (Dosi et al. 2003). As a consequence, the use of the standard microeconomic maximization tools is not suited in ABMs, and there is a priori no trivial alternative. In such complex worlds, agents' rationality can only be bounded, in the sense of Simon (1955), i.e. *procedural* and *adaptive*, but the challenge is how to model this boundedly rational behavior. This is a challenge because the modeler has to cope with the so-called “wilderness of bounded rationality” (Sims 1980): while there is one single way of solving an optimization program, there are many ways of being boundedly rational, and the question is how to discriminate between the multitude of alternative behavioral rules. This

is a crucial question as the dynamics of the ABM, and the conclusions drawn from their analysis, are likely to depend on the behavioral rules that have been incorporated into it.

**Empirical observations as the main guideline** We argue that what we can observe from real-life behavior should be the main ground for modeling agents' behavior in artificial economies. Common sense and guesswork cannot be sufficient, as there is obviously no guarantee that they will result in mimicking real behavior (Farmer & Foley 2009). This idea was already present in Cohen (1960, p. 536):

Trying to formulate a detailed computer model of the actions of individual households or firms spotlights the kind of empirical information needed to obtain a better understanding of these activities.

Ever since, the growing amount of experimental evidence from controlled lab environments with human subjects in economics, sociology and psychology, as well as the increasing availability of survey data has fueled our knowledge of how agents actually behave under alternative environments. This collection of evidence constitutes an obvious ground for the derivation of behavioral rules in ABMs.

However, such an exercise is not straightforward: agents' behavior in the real world do not always find a clear-cut interpretation, they can be highly heterogeneous and vary between agents, and from one period to the next.<sup>2</sup> The behaviors of agents and the system as a whole can be then path-dependent (Bassi & Lang 2016). In other words, real agents' behaviors are *unstable*, and any attempt to summarize agents' reaction by a fixed behavioral rule derived from a sample of empirical observations may pose a problem of realism. Such an attempt could be acceptable if the behavior under study does not appear as a central issue for the research question that the model has been built to address, or if the model is only aimed at the analysis of very short-run dynamics, over which we can consider that agents' behavior is fixed. However, when it comes to the analysis of longer-run dynamics, this modeling strategy introduces an ad-hoc, exogenous stickiness

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<sup>2</sup>For instance, Lainé forthcoming shows the challenge posed by the heterogeneity of the observed investment behavior of firms if one seeks to derive a model of investment decisions.



in the model that may distort the conclusions. When it comes to policy analysis and the comparison of different model scenarios, this strategy does not allow to address the so-called Lucas critique: fixing behavioral rules amounts to performing *ceteris paribus* analysis, and ignoring that policy changes are likely to affect in turn micro behavior. This was also the criticism made by Keynes to Tinbergen’s macroeconomic models (Keuzenkamp 1995). What is more, we argue that this is a gross contradiction with the decentralized and autonomous nature of ABMs: in an ABM, agents should be autonomous and free to interact and adapt without any intervention of the modeler (Gaffeo et al. 2008, Delli Gatti et al. 2010).

**Modeling adaptation and learning** The alternative to the use of a fixed set of behavioral rules is to endow agents with a genuine ability to adapt or, in other words, to *learn* (Farmer & Geanakoplos 2009). Modeling learning shall be understood as designing behaviors that agents constantly and endogenously adapt as a reaction to the feedback that they receive from their environment. As stressed by Delli Gatti et al. (2010), modeling learning can combine heuristics based on empirical observations and adaptation:

The solution we advocate is a bottom-up approach: let us start from the analysis of the behaviour of heterogeneous constitutive elements (defined in terms of simple, observation-based behavioural rules) and their local interactions, and allow for the possibility that interaction nodes and individual rules change over time (adaptation). (Delli Gatti et al. 2010, p. 4)

This idea is also at the root of the heterogeneous agent literature in which agents endogenously switch between a fixed (Brock & Hommes 1997) or evolving (Anufriev et al. 2015) set of heuristics according to their relative pay-off performances.

Learning introduces an *additional layer of complexity* to ABMs in a twofold way. On the one hand, agents adapt their behavior as a result of the macro environment, so that the macro level feeds back into the micro level. On the other, there is an interdependence between individual learning behavior. This is precisely what March (1991, p. 81) defines as an “ecology of competition”:

External competitive processes pit organizations against each other in pursuit of scarce environmental resources and opportunities. . . . In these ecologies of competition, the competitive consequences of learning by one organization depend on learning by other organizations.

Learning induces an intricate co-evolution between the micro and the macro dynamics (Winter 1971). The environment in which agents interact cannot be considered as exogenous and is, on the contrary, *ever-changing*, as emphasized by Dosi et al. (2003, p. 30):

. . . in population-based adaptive frameworks, the systematic appearance of novelties implies also an ever-expanding payoff matrix, continuously deformed by the interaction with new events and strategies.

This idea is a crucial component of *complex adaptive systems* as discussed by Holland (1992). Because the environment is constantly changing, this type of systems cannot be comprehended in terms of fixed point analysis, in which the equilibrium of the system is the fixed point of the mapping between beliefs and realizations, as this is the case for rational expectations macro models.

## 2.2 Why social learning in ABMs?

Learning can be modeled at the individual level or the social level (Vriend 2000). Individual learning assumes that each agent is endowed with an evolving set of strategies that can be interpreted as his search capacities. Social learning envisions each agent as a single strategy and adaptation intervenes at the population level.

Individual learning can be understood as a trial-and-error process. On its own, it is certainly slow, as a time step is necessary to evaluate one strategy (unless the agent makes use of some foregone/“what-if” pay-off functions). By contrast, social learning allows the agents to parallelize the evaluation of the available strategies, so that the larger the population, the quicker the evaluation process. Allen & Carroll (2001) and

Palmer (2012) illustrate this difference within the simple framework of the buffer-stock consumption rule; see also Salle & Seppelcher (2016).

Most importantly, if the environment is itself ever-changing, as argued above in a complex adaptive system, a trial-and-error individual learning becomes unfeasible:

In a static environment, if one improves his position relative to his former position, then the action taken is better than the former one, and presumably one could continue by small increments to advance to a local optimum. . . . [in a changing environment] there can be no observable comparison of the result of an action with any other. Comparability of resulting situations is destroyed by the changing environment . . . the possibility of an individual's converging to the optimum activity via a trial-and-error process disappears. (Alchian 1950, p. 219)

For this reason, we show in this paper how social learning can instead be used to model adaptation in an ABM. We take social learning in a broad sense:

Social learning means all kinds of processes, where agents learn from one another. Examples for social learning are learning by imitation or learning by communication. (Riechmann 2002, p. 46)

Social learning in market economies is derived from the “Darwinian” archetype (Dosi et al. 2003, p. 62). This is also the “as if” interpretation of rational behavior (Friedman 1953): selection between individual strategies operates according to the principle of the survival of the fittest, so that the least performing strategies in terms of pay-off are eliminated from the population, and replaced by the best performing ones. Because of this Darwinian analogy, social learning in a decentralized economy is often represented by the means of evolutionary algorithms, such as genetic algorithms (GAs hereafter) – see Arifovic (2000) for a survey of GA in stylized macro models. GA learning dynamics is driven by two main forces: *innovation* that constantly introduces new behaviors in the system, and *selection* pressure that duplicates the best performing ones at the expense of the other.

However, GAs are not exempt of limitations. Their operators do not always find an easy economic interpretation (Chattoe 1998, Salle & Seppecher 2016). Most importantly, because they have been initially developed to find optima in complicated *static* problems (Holland 1975), they have been used in economics as a way for agents to learn how to maximize their profits or utility functions, and the focus has been put on the conditions under which agents end up coordinating on the optimal state of the model under GA learning (Arifovic 1990). In these set-ups, the mapping between strategies and pay-off is supposed to be time-invariant. In face of perpetually evolving environment, as in complex adaptive systems, GAs perform badly because they assimilate adaptation with convergence on an equilibrium and individual coordination (which implies a progressive loss of diversity in the strategy population). This is even sometimes obtained at the price of ad-hoc mechanisms such as an exogenous decrease in the innovation force of the algorithm (Arifovic et al. 2013). We believe that this is a major flaw of the macroeconomic learning literature: the neoclassical paradigm has contributed to reduce learning to convergence on a fixed optimum. On the contrary, an ABM is an ever-evolving system. We therefore argue that decentralized learning mechanisms and market selection can be represented in ABMs without the use of GAs, precisely because ABMs allow to model directly these mechanisms in a simpler and more realistic way.<sup>3</sup> The purpose of this paper is to provide such a proof-of-concept.

### 2.3 The “blanketing shotgun process”

We now develop a learning model based on the “blanketing shotgun process” of Alchian (1950, p. 219) because the BSP consists precisely in constantly and randomly covering the space of strategies, instead of modeling learning as an individual converging search. We support the idea that Alchian (1950) can be considered as a major precursor of the evolutionist/post-Schumpeterian school of thought because this author provided a precise description of the co-evolution between market selection and behavior adaptation.

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<sup>3</sup>As stressed by Dosi & Winter (2003, p. 396), nor are necessary aggregate/centralized interaction models like the replicator dynamics. We could make a similar point for the heuristic switching model à la Brock & Hommes (1997).

**Three operators** The BSP encompasses three operators, all inspired by the biological, Darwinian metaphor (Alchian 1950). First, profits stand for the natural *selection process*: firms with positive profits are considered successful and survive, while those with losses go bankrupt and disappear. We notice that Alchian stresses that positive, *not maximal* profits, are the success criterion:

Adaptive, imitative, and trial-and-error behavior in the pursuit of “positive profits” is utilized rather than its sharp contrast, the pursuit of “maximized profits.” (Alchian 1950, p. 211).

This fitness measure is also in line with the satisficing principle of Simon (1955). Second, *imitation* stands for heredity: operating characteristics (or “routines” in the terminology of Nelson & Winter (1982)) of successful firms are copied by others. Third, innovation (or mutation or individual experimentation) intervenes twice: once during the imitation process because the copy of the firm’s strategies is not exact<sup>4</sup>, and at any time, even in case of positive profits, during a “trial-and-error” process. This type of innovations maintains a constant diversity in the population of strategies, which ensures the ergodicity of the learning process, and can be quite drastic (Alchian 1950, p. 219). Trial-and-error processes may for instance represent internal organizational changes, whether voluntary or not. They may happen even if the firm is making profits (Winter 1964). We also refer here to the concept of “persistent search” in Winter (1971):

By “persistent search” is meant a search process that continues indefinitely, regardless of how satisfactory or unsatisfactory performance may be - although the search may be slow, sporadic, or both. (Winter 1971, p. 247)

**BSP versus GA** Even if, at a first glance, the three operators of the BSP seem to have a lot in common with those of a GA, there are important differences. In a GA, changes in behavior are triggered by exogeneously fixed probabilities, and the imitated

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<sup>4</sup>This can be because the firm’s operating characteristics are not perfectly observable by its competitors, or because the firm’s routines cannot exactly transferred to another firm. Alchian (1950, pp. 218-219) uses the concept of “rough-and-ready imitative rules”.

agents are selected in relation to their relative performances, e.g. through a tournament or roulette-wheel selection process. By contrast, in our implementation of the BSP, both the frequency of imitation and the firm to be imitated are *endogenous* and driven by the *market selection pressure*: this is a major, decentralized feature of our learning algorithm.<sup>5</sup> The market triggers the adaptation reaction of the firms and the imitation process: after if going bankrupt, a firm is taken over by a new management team, its operating characteristics disappear and are replaced by the ones of a *randomly chosen* firm in the population of surviving firms.

Furthermore, the BSP and GAs differ in the relative weight that they give to exploration versus exploitation. Indeed, any learning mechanism implies a trade-off between *exploitation* of existing, well-performing strategies, and *exploration* in search of potentially better ones (March 1991). A weak selective pressure favors exploration and allows for the survival of poor-performing strategies. Such systems may end of with “many undeveloped new ideas and too little distinctive competence” (March 1991, p. 1971). Conversely, a strong selection process puts much emphasis on exploitation at the expense of exploration, and exposes the system to the risk of a premature loss of diversity and homogenization of the strategies on poor ones. The adaptive process is then potentially self-destructive (March 1991, p. 85). Consequently, the ability of a system to adapt and survive relies heavily on the balance between exploitation and exploration. GA-based learning algorithms have been primarily designed to coordinate individual behaviors on a fixed optimal strategy. This coordination requires a progressive homogenization of the strategy population. GA-based learning therefore allows for the *adaptation* of the agents, and emphasizes exploitation over exploration. By contrast, the BSP favors exploration, by keeping a perpetual dispersion of the strategies, and therefore reinforces the *adaptability* of the system. In Section 4.2.1, we show that this dimension turns out to be crucial in shaping the emerging macroeconomic dynamics. We believe that this feature of the BSP is very much in line with the evolutionary metaphor of Darwinian selection:

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<sup>5</sup> We neither model any counter-part of the cross-over operator as we find little interpretation for it in economics, see Salle & Seppecher (2016) for further discussion.

Conversely, in the opposite “Darwinian” archetype, nobody learns and system dynamics is driven by selection operating upon blindly generated variants of e.g. behaviour, technologies, etc., (taken literally this is also the “as...if” interpretation of rational behaviour). (Dosi et al. 2003, p. 62)

We argue that this feature is most convincing in a dynamic market environment in which firms have to compete without being able to derive an optimal strategy. We now apply the BSP learning algorithm in a simple macro ABM – `Jame1` – and ask the question whether “the market is indeed a good teacher” (Day 1967, p. 303).

### 3 Learning and adaptation in a simple macro ABM

The first innovation of this paper is to model the firms’ leverage strategies through the BSP. We therefore introduce capital accumulation and depreciation in the model in the `Jame1` model. The size of the firms evolves endogenously as a result of their investment decisions. We also refine the specification of the banking sector. We intend to provide here a self-contained presentation of `Jame1`, and we pay a specific attention to the description and the explanation of the new features that this paper introduces. We refer the interested reader to Seppecher & Salle (2015) for an exhaustive discussion and justification of the rest of the assumptions of the model. Appendix B provides the pseudo-code of the model that makes the timing of events together with each equation explicit, and defines each variable and each parameter. We refer the reader to this appendix for the detail of the model design. The open source code (in java) as well as an executable demo are available on the corresponding author’s website at <http://p.seppecher.free.fr/jame1/>, as we believe that this is a necessary step for the transparency and credibility of the simulation results.

We now describe the main features of `Jame1`, then detail the firms’ behavior and finally the rest of the model.

### 3.1 The main features of Jame1

Jame1 exhibits two essential features: full decentralization and stock-flow consistency. *Decentralization* ensures that aggregates, such as prices and wages, stem from the local interactions in the markets: there is no planner, no auctioneer and all interactions are direct and individual. The resulting emerging patterns, such as income distribution, are therefore endogenous. *Stock-flow consistency* links all agents' balance sheets together and guarantees that micro behaviors are correctly aggregated (Godley & Lavoie 2007). In Appendix C, we provide the accounting identities of stocks and flows as well as the balance sheets matrices of all types of agents in the model (households, firms and the bank), so as to make the stock-flow consistent nature of the model explicit. Furthermore, for the sake of internal consistency, the design of the behavioral rules in Jame1 follows mostly a common general pattern: agents successively adjust their behavior by observing imbalances between their actual and some targeted, *satisficing* levels of their variables. These behavioral rules translate both a principle of reaction to stress and a principle of conservatism or “smoothing” (Cyert & March 1963). Those rules are also partly stochastic, so that even with the same values of their state variables and the same information, agents keep heterogeneous behavior. This persistent heterogeneity is crucial to account for adaptation of the system, as stressed in Section 2.

The economy is populated by  $h$  heterogeneous households (indexed by  $i = 1, \dots, h$ ),  $f$  heterogeneous firms (indexed by  $j, j = 1, \dots, f$ ) and one bank (indexed by  $b$ ). The firms produce homogeneous goods by using labor, supplied by households, and fixed capital, resulting from their investment decisions. Labor and capital are complementary production factors. One unit of fixed capital lasts for an exogenous and stochastic number of periods (this should be interpreted as the time before a machine breaks and becomes irreversibly unproductive). Both households, for consumption purposes, and firms, for investment purposes, purchase the goods. There is a capital accumulation dynamics through investment, but no technical progress, as the productivity of capital (parameter  $pr^k$  hereafter) remains fixed and common to all firms. The bank provides loans to the



firms to finance their production (wage bill and capital investment). The firms and the bank are assumed to be owned by households, who then receive dividends.

## 3.2 The firms

Firms have the most detailed behavior in the model. We now describe each of their decision rules.

### 3.2.1 Production

Each firm  $j$  is endowed with an integer  $k_{j,t}$  of fixed capital, that can be understood as its number of machines. Each machine can be used in combination with at most one unit of labor (one worker) in every period. One unit of labor increments the production process of the machine by one step in each period. Each machine needs  $d^p$  time steps to deliver an output and, after completion, this output represents  $d^p \cdot pr^k$  units of goods, and adds to the firm's inventories level, denoted by  $in_{j,t}$ . The firms allocate the workforce over the production processes, trying to smooth their output over time.

### 3.2.2 Goods supply

We assume that each firm maintains a fraction  $1 - \mu_F$  of its inventories  $in_{j,t}$  as a buffer to cope with unexpected variations of its demand and intends to put in the goods market the fraction  $\mu_F$ . We assume also that the maximum market capacity of each firm is proportional to the potential output of the firm:  $d^m \cdot pr^k \cdot k_{j,t}$ . Hence, in each period  $t$ , each firm  $j$ 's goods supply is given by:  $\max(\mu_F \cdot in_{j,t}, d^m \cdot pr^k \cdot k_{j,t})$

### 3.2.3 Labor demand

For the sake of parsimony, we assume that the targeted/normal level of inventories is the maximum market capacity, i.e  $in_{j,t}^T = d^m \cdot pr^k \cdot k_{j,t}$ . The firms take the variations in the level of their inventories as a proxy for the variations in the goods demand that is addressed to them: if their inventories  $in_{j,t}$  are lower (resp. higher) than their target

$in_{j,t}^T$ , this may be a sign of excess demand (resp. lack of demand), and firms are likely to increase (resp. decrease) their production and, hence, their labor demand  $n_{j,t}^T$ .<sup>6</sup> The firms proceed by small, stochastic adjustments in the corresponding direction. In the case where  $n_{j,t}^T > n_{j,t}$ , where  $n_{j,t}$  denotes the current workforce of firm  $j$ , it seeks to hire  $n_{j,t}^T - n_{j,t}$  workers. Otherwise, it fires  $n_{j,t} - n_{j,t}^T$  workers, on a first hired-first fired basis.

### 3.2.4 Price setting

Each firm increases (resp. decreases) its price in case of lower-than-targeted (resp. higher-than-targeted) level of inventories *and* if it was (resp. was not) able to sell all its supply during the last period. Each firm proceeds by *tâtonnement*, and keeps track of a floor price  $\underline{P}_{j,t}$  (that can be understood as a price thought to be lower than the market price), and a ceiling price  $\overline{P}_{j,t}$  (a price thought to be higher than the market price). The floor and the ceiling prices constitute the range in which the new price is randomly and uniformly picked up in case of adjustment. In case of a price increase,  $\underline{P}_{j,t}$  is set to the last price for which the firm was able to sell all its supply, and  $\overline{P}_{j,t}$  is increased by a factor  $(1 + \delta^P)$ . An exact symmetric routine operates in case of a price decrease. The idea is that the search area for the suitable price  $[\underline{P}_{j,t}, \overline{P}_{j,t}]$  increases when the firm keeps on adjusting its price in the same direction, and decreases when the firm reverts its price trend. Therefore, in a strong inflationary environment (resp. deflationary environment), the firm can quickly increase (resp. decrease) its price, and adapt in order to “catch-up” with the price level in the economy. Parameter  $\delta^P$  measures the strength of this adaptation, and hence price flexibility: the higher, the more flexible the prices (see Appendix B for the equations).

### 3.2.5 Wage offer

Each firm  $j$  updates its wage offer according to one of the two following routines. The first routine is the same mechanism as the price updating process just described. The firm computes its vacancy rate  $\rho_{j,t} \equiv \frac{n_{j,t-1}^T - n_{i,t-1}}{n_{j,t-1}^T}$ , and compares it to its targeted, normal level

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<sup>6</sup>We follow, *inter alia*, Cyert & March (1963) on the twofold role of inventories, both as a buffer and as a proxy for the variations in the goods demand that the individual firms face.

of vacancies, that is exogenously fixed to  $\rho^T$  for all firms. A higher-than-targeted (resp. lower) vacancy rate indicates an excess of labor demand (resp. supply) and leads to an increase (resp. decrease) in the offered wage  $W_{j,t}$ . The wage adjustment is computed in the same way as the price adjustment, a parameter  $\delta^W$  measures wage flexibility, and the firm keeps track of a floor and a ceiling wage levels. Such a routine is easy to implement in the case of prices, as firms interact with consumers and/or investors in the goods market in every period. However, firms go irregularly in the labor market (only in periods when they need to renew a contract or increase their workforce), so that the information that they collect by interacting with households is fragmented, and may be insufficient to set wages that are compatible with market conditions. Moreover, the vacancy level is indicative only if the firm's size is large enough, but is of little informational content for a small firm. For instance, in case of a single employee, this information is binary: either 0 or 100% of vacancies.

We therefore introduce a second wage setting routine that is akin to a convention or a norm in the wage determination: each firm observes a sample  $g'$  of other firms, and copies the wage offered by the first-observed bigger firm in the sample (i.e. the first-observed firm in the sample that employs more workers than it does). If there is no bigger firm, it follows the first wage setting procedure based on vacancy rate.<sup>7</sup>

The duration of an offered contract is set to  $d^w > 1$  periods, and the wage remains fixed for this whole period. Note that this is a maximum duration period, as the firms can fire workers in any period in case of a decrease in their workforce needs, on a first-hired-first-fired basis. This also implies that firms may pay different wages to their employees.

We shall stress that these pricing rules imply flexible and independently-fixed prices and wages. The only rigidity stems from the dependence on the previous price and wage levels. For the purpose of this paper, it appears to us important not to impose exogenous constraints such as menu costs, or fixed pricing rules, such as a mark-up procedure, on the firms, in order to let the market exert the only pressure on the firms.

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<sup>7</sup>This amounts to assuming that the smaller firms are wage-followers, while the bigger ones are wage-makers. Moreover, copying another firm's wage offer can be easily justified as every machine, and hence every worker, has exactly the same productivity.

### 3.2.6 Financial decisions and investment

**Payment of dividends** At the beginning of each period, the firm distributes to its owners a share of its equities  $E_{j,t}$  as dividend. The higher the level of its equity relative to its target, the higher the dividend, in the limit of  $\kappa_d$  of the equity.

**Borrowing** The firm may have to obtain loans from the bank. There are four types of loans. Short-run (non-amortized) loans allow the firm to finance wages if its available cash-on-hand is not enough to fully cover its expected wage bill. Short-run (amortized) loans partly finance its investment (see below), and investment is primary financed with (amortized) long-run loans. The bank also grants short-run loans as overdraft facility in case where a firm does not have enough cash-on-hand to cover any of its monthly repayment (see Sub-section 3.3.2 how the loans are granted).

**Investment decisions** The BSP endogenously determines the firms' investment financing strategies (i.e. their leverage/indebtedness strategies) and, indirectly, the pace and size of the investment decisions. We now present in detail the investment procedure, but as shown in the pseudo-code in Appendix B, this procedure is rather simple in its implementation.

Each firm has a targeted level of equity  $E_{j,t}^T \equiv (1 - \ell_{j,t}^T)A_{j,t}$ , where  $A_{j,t}$  denotes the total assets of the firm  $j$  in time  $t$ , and  $\ell_{j,t}^T \in [0, 1]$  its target debt ratio. Its equity target is the amount of its assets that the firm is not willing to finance by debt. Each firm compares its equity target to its actual level  $E_{j,t}$ . Only if  $E_{j,t} > E_{j,t}^T$  will the firm consider to invest.<sup>8</sup>

In this case, the firm computes the size of the investment by applying an expansion factor, or “*greediness*” factor  $\beta > 1$ , to its average past sales (in quantities and computed over the past *window* periods), denoted by  $\tilde{s}_{j,t}$ . Its sales expansion objective is therefore given by  $s_{j,t}^e = \beta \cdot \tilde{s}_{j,t}$ . Note that this investment objective includes *de facto* both the renewing of obsolete, aging machines and the purchase of new ones.

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<sup>8</sup>See e.g. Kalecki (2010), who stresses that the amount of the entrepreneurial equity is the main limitation to the expansion of a firm.

As there is no intermediate consumption good, the firms willing to invest buy and transform the homogeneous goods into machines. Firms need  $v^k$  goods to deliver a machine. Once purchased, we assume that those goods are transformed into machines immediately and at no cost.<sup>9</sup> We assume that each firm uses the net present value (NPV) analysis to choose the number of machines to purchase.<sup>10</sup> The firm randomly samples  $g$  sellers in the goods market to estimate the price of the investment goods. The  $NPV_m$  of an investment project  $m$ , i.e. buying  $m \geq 0$  machines is given by:

$$NPV_m \equiv \frac{CF_m}{r_t} \left( 1 - \frac{1}{r_t(1+r_t)^{d^k}} \right) - I_m \quad (1)$$

where  $I_m$  is the initial outlay (the price of the  $m$  new machines),  $r_t$  is the discount factor, which is equal to the risk-free interest rate of the bank  $i_t$  (see below) discounted by average past inflation  $\pi_t$  (over *window* periods),  $d^k$  is the average expected life-time of a machine, and  $CF_m$  is the expected cash-flow of the project, based the firm's current price and wage:<sup>11</sup>

$$CF_m = \min(s_{j,t}^e, m \cdot pr^k) \cdot P_{j,t} - m \cdot W_{j,t} \quad (2)$$

where the min term ensures that the future sales cannot exceed the maximum market capacity of the firms.

The firms reviews investment projects by starting from  $m = 0$  (i.e. buying 0 machine), then  $m = 1$ , etc. until the NPV of the project  $m + 1$  is less than the NPV of the project  $m$  previously considered. The firm then chooses the previously considered project  $m$ , and buys  $m$  machines.

Figure 1a shows the pace of investment decisions for an arbitrary chosen firm in the baseline simulation: only when the effective level of debt lies below its target can invest-

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<sup>9</sup>This is a simplifying assumption in order to avoid to complicate the model by introducing a second industrial sector. An upcoming version of the model does encompass two sectors.

<sup>10</sup>This is a quite standard procedure in corporate finance, this is the reason why we use it. However, other types of investment functions could be easily envisioned, and will be considered in further developments of the model.

<sup>11</sup>The price and wage could be computed in a more complicated way, such as a trend projection of past values over the next *window* periods. However, this would complicated the decision making of firms, without adding much to the qualitative simulation results.

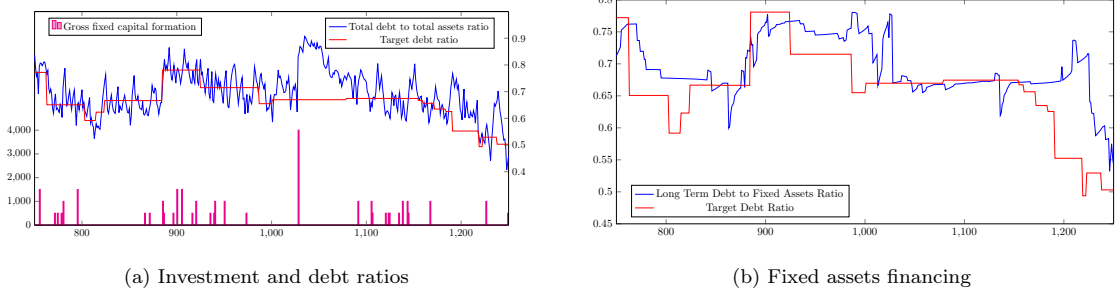


Figure 1: An individual example of a firm’s investment and financing behaviors from the baseline simulation: periods 750–1250

ment be performed, but this is not a sufficient condition though. The NPV also integrates expected demand, real interest rates and profitability considerations.

Once the firm decides to purchase  $m$  new machines, it computes the share  $\ell_{j,t}^T$  of the total price  $I_{j,t} = I_m$  that is financed using a long-run, *amortized* loan, for a total amount of  $\ell_{j,t}^T I_m$ . For simplification, we assume that the length of a long-run loan equals the average expected lifetime of the machines  $d^k$ . If the firm’s cash-on-hand is not enough to cover the share  $1 - \ell_{j,t}^T$  of the investment, the firm uses an *amortized short-run* loan. This procedure ensures that the firm is never constrained by insufficient cash-on-hand whenever it has decided to invest. The firm’s debt may temporary exceed its debt objective due to the additional short-run loan, but the gap progressively closes, as illustrated in Figure 1a for the same, arbitrary chosen firm in the baseline simulation.

Each new machine adds to the firm’s assets  $A_{j,t}$  at its purchasing price  $\frac{I_{j,t}}{m}$  (i.e. the price of the goods necessary to produce it), and is uniformly depreciated by a fraction  $\frac{1}{d^k}$  of its initial value in every period, unless it breaks down before  $d^k$  periods, and its value then falls to zero (see Appendix A). The fixed capital depreciation on the asset side of the balance sheet, together with the long run loan amortization on the liability side, allow the firms to roughly maintain the ratio between long run loans and fixed assets in line with their debt objective throughout the life of the machines (see Figure 1b).

### 3.2.7 Firms' adaptation through the BSP

Firms adapt their indebtedness strategy  $\ell_{j,t}^T$  through the BSP. We choose to make this behavior subject to the BSP because it is at the core of the market competition in the model. Indeed, it summarizes the “growth-safety trade-off” (Crotty 1990, 1992, 1993, Crotty & Goldstein 1992) that firms face between a continuous, debt-financed increase in market capacities, and financial safety, that preserves a low debt level, but at the risk of losing market shares. The higher the debt target, the more likely the investment to be realized and the quicker the market expansion, because the firm needs less of its own equity to finance it; but the higher the risk of insolvency and bankruptcy. This trade-off is made particularly complicated in the ABM, as the market conditions and, hence, the selection pressure, in turn depend on the firms' indebtedness strategies. Even for the modeler, there is no trivial optimal strategy to deal with this issue. Therefore, in light of what we discussed in Section 2, we believe that the debt behavior is a good playground for modeling adaptation through the BSP.

**Innovations and trial-and-error processes** In order to introduce a permanent trial-and-error innovation process, we follow here Alchian's “extreme” hypothesis by modeling a completely random, blind and unintended model of exploration (Alchian 1950, p. 211). In each period, with a given probability  $proba_{BSP}$ , firms perturb their debt objective  $\ell_{j,t}^T$  by a Gaussian noise, with the same standard deviation  $\sigma_{BSP}$  as the one applied during the imitation process. Those innovations constantly introduce heterogeneity in the firms' debt strategies, which allows for exploration. This heterogeneity is counteracted by an endogenous selection and imitation process, which allows for exploitation.

**Bankruptcy and imitation through the BSP** Firms can go out of business in two ways: bankruptcy by insolvency when negative profits exhaust their equity (i.e. their liabilities exceed their assets), and the loss of productive capacities, in the case where they do not succeed in investing to renew their aging machines. We simplify here the

entry-exit process of firms and assume that the failed firm does not disappear.<sup>12</sup> The firm is bailed out by the bank, its ownership is changed (see Subsection 3.3.2 for details), its management team is fired, and replaced by another team coming from a more successful firm. Concretely, its debt objective  $\ell_{j,t}^T$  is copied on a randomly chosen surviving firm. The copy is not exact though, as a (small) Gaussian noise is introduced (with the same standard deviation  $\sigma_{BSP}$  across all firms).

It should be noted that we do not claim that deliberate individual learning plays no role in the real world but, following Alchian, we abstract from it in this paper in order to focus on social learning stemming from regulation by market competition. At most, we model individual learning as blind individual experimentation (“random mutations”) that is on average ineffective (i.e. the average change in strategies is zero at the population level).

### 3.3 The rest of the model

#### 3.3.1 The households

In the labor market, each household  $i$  is endowed with a constant one-unit labor supply and a reservation wage  $W_{i,t}^r$ . If employed, his reservation wage equals his wage, i.e.  $W_{i,t}^r = W_{i,t}$ . If unemployed, his reservation wage is adjusted downward, depending on his unemployment duration  $d_{i,t}^u$ : the longer the unemployment period, the more likely the downward adjustment. After  $d^r$  periods, the adjustment is systematic.

Regarding consumption decisions, the households follow a buffer-stock rule *à la* Allen & Carroll (2001) to smooth their consumption in face of unanticipated income variations by building precautionary savings (at a zero-interest rate). Households cannot borrow and consumption is budget-constrained in every period.

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<sup>12</sup>Our assumption is in line with empirical evidence that suggests that a number of new firms replace a similar number of obsolete firms, without significantly affecting the total number of firms in the market (see, notably, Bartelsman et al. (2003)).



### 3.3.2 The bank

The functioning of the banking system is very stylized in our model. The bank hosts firms and households deposits at a zero-interest rate, and grants to firms short-run credits for a period of  $d^l$  months and long-run credits for  $d^L$  months. For simplification, we assume that the interest rate is the same for the two types of loans and is equivalent to the risk-free interest rate. The risk-free interest rate is set by a central bank according to a most simplified Taylor rule that aims to stabilize inflation  $\pi_t$  around a target  $\pi^T$  (assuming the natural rate is zero):  $i_t = \max(\phi_\pi(\pi_t - \pi^T), 0)$ .

At a first step, the bank is fully accommodative, and satisfies all the credit demands. However, when a firm is not able to pay off a loan in due terms, the firm receives a new short-term loan to cover its repayments (an overdraft facility) at a higher interest rate  $i_t + rp$ . Parameter  $rp > 0$  translates a risk premium, due to the higher risk of the loan, and is assumed to be the same for all firms. If a firm  $j$  becomes insolvent (if its liabilities exceed its assets,  $L_{j,t} > A_{j,t}$ ), it goes bankrupt and the bank starts a foreclosure procedure. The bank first recapitalizes the failed firm : it computes the targeted value of the failed firm,  $E_{j,t}^T = \kappa_s A_{j,t}$  and then erases the corresponding amount of debt:  $L_{j,t} - A_{j,t} + E_{j,t}^T$ , absorbing this loss through its own resources  $E_{b,t}$ . Then the bank attempts to resell the restructured firm at its new book value  $E_{j,t} = E_{j,t}^T$ , by soliciting households that hold more than a threshold fraction of the restructured firm value in cash-on-hand, and progressively decreasing this threshold if not enough funds can be raised. In the case where the capital of the bank is not enough to recapitalize the bankrupted firm, the bank goes bankrupt and the simulation breaks off.<sup>13</sup>

The bank also distributes dividends to its owners. We assume that it simply distributes its excess net worth, if any, compared to its targeted one  $E_{b,t}^T$  (a share  $\kappa_b^T$  of its total assets  $A_{b,t}$ ).

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<sup>13</sup>We document the frequency of this event in the simulations in Section 4, see Footnote 17. This is due to the very simplistic design of the banking sector in `Jame1`, a feature that is intended to be abandoned in future versions of the model.

### 3.3.3 Markets and aggregation

The markets operate through decentralized interactions based on a standard tournament selection procedure. In the labor market, each firm posts  $\max(n_{j,t}^T - n_{j,t}, 0)$  job offers at a given wage  $W_{j,t}$ . Each unemployed household  $i$  consults  $g$  job offers, and selects the one with the highest wage, provided that this wage is at least as high as its reservation wage  $W_{i,t}^r$ . Otherwise, it stays unemployed.

In the goods market, each firm  $j$  posts  $s_{j,t}^T$  goods at a price  $P_{j,t}$ , each household  $i$  enters with its desired level of consumption expenditures  $C_{i,t}^T$ , and each investing firm enters with an investment budget  $I_{j,t}$ . Firms first meet investor-firms, and then interact with households.<sup>14</sup> Each household selects a subset of  $g$  firms, and chooses to buy to the cheapest one. These processes are repeated until one side of the markets is exhausted.

As usual in ABMs, aggregate variables are computed as a straightforward summation of individual ones.

## 3.4 Simulation protocol

We use a baseline scenario of the model derived from the empirical validation exercise performed in Seppelcher & Salle (2015), but we do not attempt to statistically match empirical micro- or macroeconomic regularities in this paper. We use the model as a virtual macroeconomic playground to test the simple idea of adaptation through the BSP learning model. This playground is nevertheless qualitatively realistic in the following important dimensions for the purpose of our study: it is a complex, monetary and stock-flow consistent market economy. Regarding the new parameters that have been introduced, the lifetime  $d^k$  of the machines is a random draw in  $\mathcal{N}(120, 15)$ , and we set  $v^k = 500$ , where  $v^k$  represents the real cost of an investment/a machine. This positive cost of capital shall be counter-balanced by a decrease in the length of production of a machine  $d^p$  (that we now set at 4 instead of 8 periods in the previous versions with a fixed endowment of

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<sup>14</sup>This matching order ensures that the biggest purchasers first enter the market, which appears as reasonable. However, this order does not matter as all simulations show that households' rationing in the goods market remains a rare and negligible event, which would not be realistic otherwise.

free machines) to maintain a similar profit share; see Seppelcher (2014) for further discussion. We set the firms' greediness at  $\beta = 1.2$ , which translates into a intended 20% increase in productive capacities. This could appear ambitious at a first glance, but it is actually rather conservative: recall that this investment objective includes both the renewing of aging machines and the purchase of new ones. We fix the individual experimentation parameters of the BSP to small values ( $proba_{BSP} = \sigma_{BSP} = 0.05$ ) in order to keep a constant but small noise in individual strategies, as usual in the learning literature discussed in Section 2. We set the parameters of the Taylor rule to standard values ( $\phi_\pi = 2$  and  $\pi^T = 2\%$ ). We set  $\delta^P = 0.04$  and  $\delta^W = 0.02$ , which implies more flexible prices than wages. This relative wage rigidity is necessary to dampen, and even interrupt deflationary dynamics along the bust dynamics, so that the single bank does not go bankrupt (see Seppelcher & Salle (2015) for more detail).<sup>15</sup> For the same purpose, the risk premium  $rp$  on doubtful debt is set to 4% (monthly) and the recapitalization rate in case of bankruptcy is  $\kappa_s = 20\%$ . The number of wage observations is set to  $g' = 3$ . However, the qualitative dynamics of the simulation does not seem sensitive to these three specific values. Appendix A lists all parameter values used in the sequel, and the initialization of the model is described in Appendix B.

## 4 Numerical results

We now give a broad description of the cyclical dynamics that comes out as a robust pattern of the simulations, and then zoom on one cycle to highlight the mechanisms at play.

### 4.1 Overview of the macroeconomic dynamics

In a first step, Figure 2 reports typical time series of one run of the baseline scenario: demand and supply in the goods and the labor markets, the corresponding (downward

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<sup>15</sup>This firstly comes from our very stylized banking system and the absence of government intervention besides the Taylor rule that is ineffective in deflationary downturns.

sloping) Phillips and Beveridge curves, nominal and real interest rates, the level of firms' indebtedness, the number of firms' bankruptcies as well as financial fragility.<sup>16</sup> We refer as financial fragility as the ratio between the aggregate debt level and the aggregate net profits (i.e. the firms' profits minus the interests). It is clear from the dynamics of all aggregate variables displayed that the macroeconomic dynamics of the model is characterized by a *cyclical pattern*, with alternating periods of booms and busts.

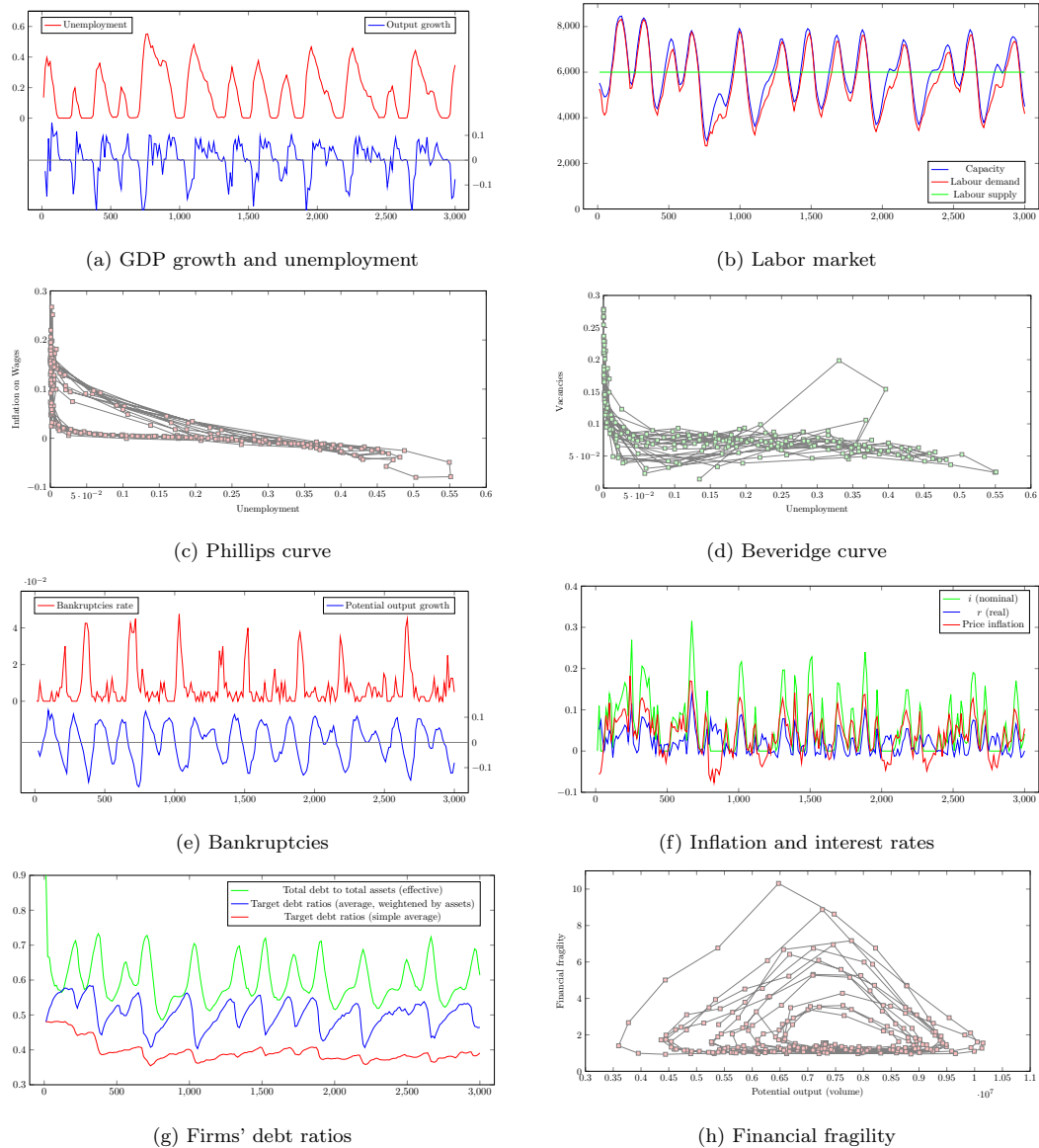


Figure 2: Baseline simulation

<sup>16</sup>The contribution of each figure to our argumentation will be presented throughout the whole section.

	mean	std. dev.	maximum	minimum
GDP growth rate	0.00226 (0.00092)	0.06493 (0.0028)	0.12335 (0.01408)	-0.21521 (0.01502)
Inflation rate	0.03852 (0.00547)	0.04709 (0.00283)	0.15261 (0.01128)	-0.06213 (0.0143)
Bankruptcy rate	0.0075 (0.00065)	0.01054 (0.00122)	0.0628 (0.01057)	0 (0)
Financial fragility	2.18919 (0.05951)	1.74851 (0.26648)	12.53134 (3.26673)	0.96359 (0.01974)
Firms' leverage	0.5976 (0.00621)	0.0551 (0.00334)	0.73687 (0.00969)	0.49978 (0.01391)
Investment growth rate	0.11017 (0.01198)	0.47834 (0.05258)	2.99064 (0.81132)	-0.60634 (0.06073)

Table 1: Average (and standard deviation between brackets) computed over all periods (discarding the first 500 periods) over 30 replications of the baseline scenario.

These cycles are a *robust feature* of our model, that we observed in all simulations that we have run, albeit irregular and of various amplitudes.<sup>17</sup> In order to show so, Table 1 presents descriptive statistics of the model outcomes over 30 replications of the baseline scenario with different seeds of the RNG. The similarity between the replications of the baseline scenario is clear from the low values of the standard deviations *between runs*, for all macroeconomic indicators that we report (see all numbers in brackets). As for the cyclical pattern, it is reflected by the particularly high values of the standard deviation of these indicators compared to their average values. For instance, on average between all runs, the GDP growth rate is 0.2%<sup>18</sup>, but with a standard deviation of 0.065. This clearly depicts a strong macroeconomic volatility.

The main conclusion that we can draw from our observations is that there seems to be *no such thing as equilibrium or collective optimization*, but the system exhibits some regularities and is sustainable. There is no explosive dynamics. The macroeconomic system survives and reproduces itself but at the price of a strong volatility. Market

<sup>17</sup> Because the model is randomly initialized and the single bank bears alone all the costs of firms' losses (see Appendix B), the required adjustments may be too drastic for the single bank to absorb firms' losses, and the simulation may break off at the beginning. We observe that this is the case in roughly 15% of the simulations. We do not report those runs in Table 1. However, once the economy survives this take-off period, we have always observed the same cyclical aggregate pattern.

<sup>18</sup> Recall that the model does not encompass any technological progress nor population changes. An average growth rate closes to zero is therefore an expected outcome.

pressure does work as a selection device between a multitude of randomly generated firms’ behaviors, but the market discipline is “brutal”, not stabilizing, as reflected by the pace of bankruptcies (Figure 2e).

Giving a closer look at the cycles that emerge from the model, we notice that the emerging boom and the bust phases differ in terms of both length and magnitude. For instance, in Figure 2, the recession around period 800 is the deepest in this simulation, while fluctuations between periods 1400 and 1800 are the most dampened of the simulation. This reflects the complex nature of the ABM. The timing as well as the size of the downturns are an *endogenous* product of the model, and result from the intricate relations between the collective adaptive behavior of firms and market selection. We now analyze in detail a cycle in order to shed light on this mechanism.

## 4.2 Analysis of a typical cycle

In this section, we zoom on a typical cycle (between periods 750 and 1250) of the baseline simulation displayed in Figure 2.

### 4.2.1 Firms’ adaptation

We now show that the very core mechanism at play in generating the cycles is the *alternating of two phenomena*: a sustained increase trend in firms’ indebtedness, followed by a brutal correction through a chain of bankruptcies. This is particularly clear from the evolution of the targeted debt ratio of firms weighted by their assets (blue curve in Figure 2g). To provide further insights into firms’ behavior over a business cycle, Figures 3 report the debt objectives  $\ell^T$  versus the sizes of the firms (in number of machines) at six different phases of a cycle, in the following order: the start of the downturn, the bust, the bottom of the bust, the beginning of the recovery, the boom and the top of the boom. We can draw the following insights.

It should be first recalled the growth-safety trade-off that the firms face in the model: the higher the financial risk (the further on the right side on the scatterplots of Figure

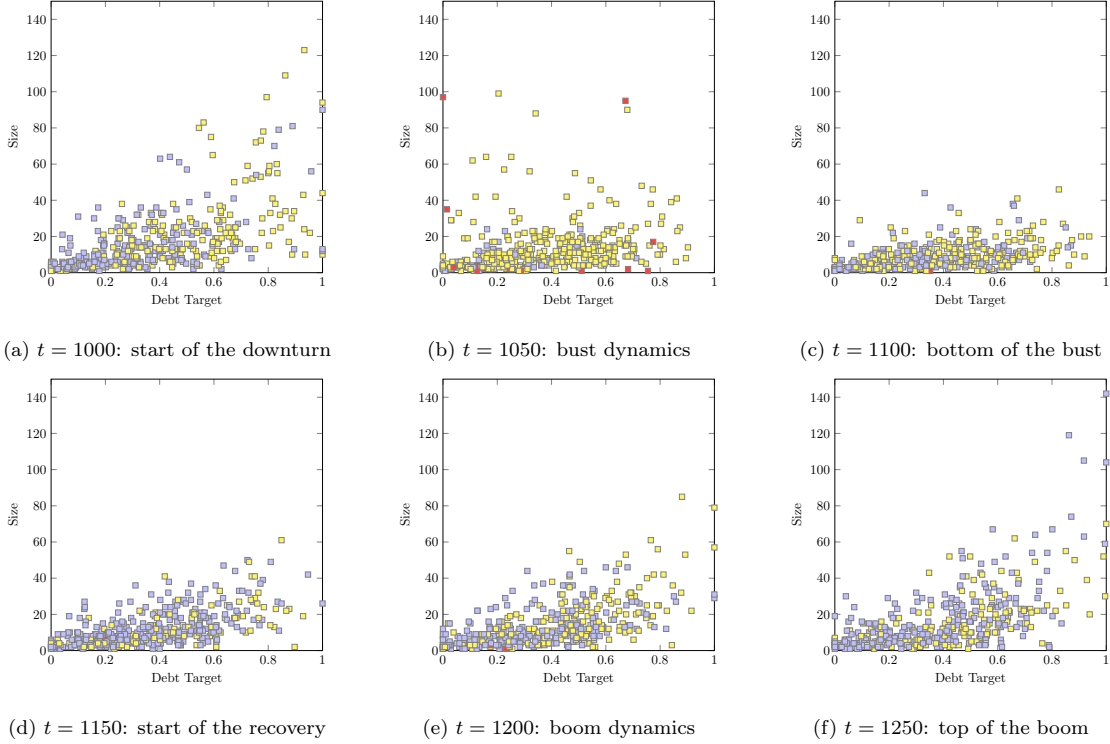


Figure 3: Firms’ size distribution, debt behaviour and income-debt relations in six phases of a business cycle. Scatterplots report,  $\forall j, \ell_j^T$  (debt target, x-axis) versus  $k_j$  (size as the number of machines, y-axis). Colors denote income-debt relations, according to the classification and terminology of Minsky (1986): blue for hedge, yellow for speculative, red for Ponzi financing firms.

3), the quicker the expansion of the firms (the further up on these same graphs). As a consequence, in the boom dynamics (Figures 3e-3f), we observe a *dispersion* towards the top-right corner of the plots (heavy debt and big size). This evolution is progressive, as a result of the small random but perpetual innovations in the adaptation process that determines the investment behavior of the firms. The “skittish” behaviors, that correspond to low debt strategies, run the risk of being eliminated if they are not enough to even renew the aging and obsolete machines, which would then drive the productive capacities to zero (i.e. towards the origin on the scatterplots). In this case, the firms go bankrupt and imitate another surviving firm. However, the top right corners of these plots are not densely populated because this area is competitive, and these behaviors are risky: only a few firms will end up cornering the market, but they all run the risk of unsold production, which would lead to a drop in profits and a risk of insolvency. The riskiness of

this behavior is also clear from the proportion of speculative, and even Ponzi firms in the top right corner of the figures. This risk is illustrated by the evolution of firms' positions on the scatterplots throughout the cycles (see Figures 3a-3c). Once the downturn starts, we observe a clear contraction of the firms towards the bottom of the scatterplot. This tightening phenomenon is the result of a twofold motion: the bankruptcies of the most indebted firms that massively and brutally drive out non-cautious high debt strategies (movements towards the left of the plot); and the decrease in capital due to the non-renewal of depreciating productive capacities (movement towards the bottom). As it is clear by comparing Figures 3f and 3c, economic crises endogenously produce an homogenization of firms' behavior because they first affect the few, but biggest firms which grew by heavily indebtedness (see how the population of speculative firms starts growing among the biggest firms first in Figure 3a). In the wake of the bust, the speculative, and even Ponzi-types of financing seem to affect every firm, not only the biggest ones (Figure 3b). Once the recovery starts (Figure 3d), indebtedness starts increasing again, and few firms start growing and cornering the market again (Figure 3e). This process repeats itself along each cycle (to see this, notice the striking similarity between Figures 3a and 3e).

Importantly, the market selection through bankruptcies along the bust dynamics is brutal (movements towards the bottom left of the scatterplots), and much quicker than the pace of the small-step innovations that progressively drive the system towards an increasing financial fragility along the boom dynamics (i.e. movements towards the top right). This difference explains why recoveries are slow and crises are severe. Deep crises as a brutal disciplining device have been part of the evolutionary economics ideas for a long time:

Severe depression eliminates large numbers of firms from the economy, but behavior patterns that would be viable under more normal conditions may be disproportionately represented in the casualty list. At the same time, behavior patterns that were in the process of disappearing under more normal conditions may suddenly prove viable... (Winter 1964, p. 266)



Our ABM provides a detailed micro-founded macro model of this mechanism.

From these observations, we draw the following conclusions. The adaptive model provided by the BSP collectively solves the growth-safety trade-off faced by the firms by eliminating the investment behaviors that are incompatible with current market conditions. The BSP is not an optimization process, but a process that ever creates heterogeneity in behaviors, with a strong emphasis on exploration. This heterogeneity is not random but is characterized by a salient emerging and recurring structure. This structure is endogenous, relatively stable from one cycle to the next, but quite importantly, *dynamic*: market conditions evolve along the cycle, and behaviors that were judged virtuous in a given phase of the cycle (audacious behavior in the boom) turn out to be vicious in another (during a bust). This heterogeneity provides to the system as a whole its ability to react and adapt. This simple simulation exercise shows that there is no such thing as an efficient or optimal behavior in this complex adaptive system, but the characterization of successful behaviors itself constantly evolves as a result of the market conditions *that these behaviors contribute to shape*.<sup>19</sup>

Last but not least, the BSP results in a *pro-cyclical leverage*, as clear from Figure 2g. We stress that this is an endogenous product of the adaptation process, not an ingredient of the model. We now show how this pro-cyclical leverage contributes to shape the emerging business cycles.

#### 4.2.2 Macroeconomic dynamics

Figure 4 zooms on the cycle between period 750 and 1250 of the baseline simulation. Figure 4d indicates that the building up and the collapse of *assets* of non-financial businesses (firms) seem to be the main force driving the adaptation of the system as a whole. On Figure 4d, the blue curve that depicts the average debt ratio weighted by assets moves faster than the red one, that reports the simple arithmetic average over firms. This

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<sup>19</sup>Brock & Hommes (1998) make a similar point by showing that “non-rational”, trend-chasing traders are not driven out by fundamental ones in a financial market model; but their relative share co-evolve in a non-linear way with the dynamics of the market that can display, as a result, very complicated, and even chaotic dynamics. See also Hommes (2006) for a related discussion.

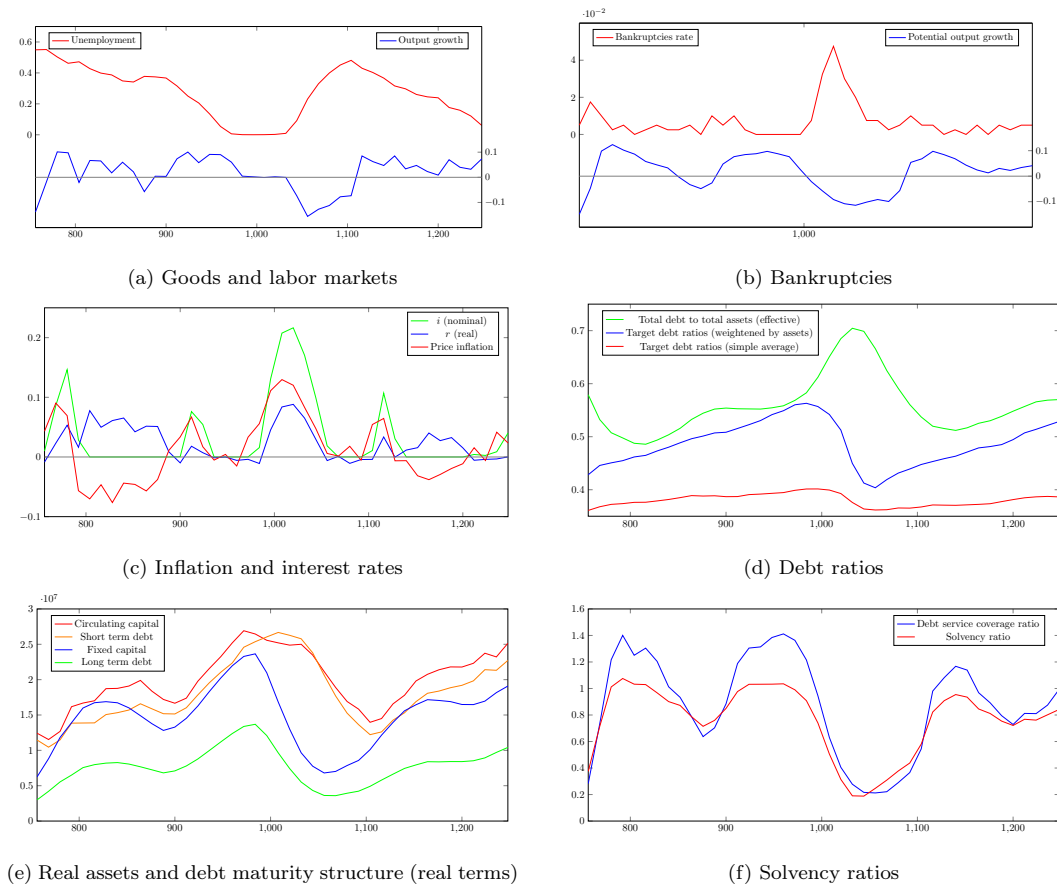


Figure 4: Zoom on one cycle of the baseline simulation: periods 750–1250

reflects the fact that during a boom, the aggregate amount of debt grows mostly as a result of few, big firms with high leverage strategies. We now explain how this financial instability interacts with the goods demand, and provokes the boom and bust cycles.

Along the boom phase of the cycle, the balance sheet of the firms becomes more fragile (Figure 4d). Investment feeds the demand for goods, which calls in turn for more expansion in market capacities (Figure 4a). This optimistic outlook of firms is self-reinforcing because it is followed by the bank, which is fully accommodative in our model. However, the lending interest rates rise in the boom phase<sup>20</sup> (Figure 4c). This rise generates a negative feedback between firms’ financial fragility, investment and goods demand that puts an end to this boom dynamics. Larger shares of firms’ cash-flow are absorbed by debt

<sup>20</sup>In our model, this raise stems from the Taylor rule that increases nominal rates along the boom. Another explanation is the increase in the bank’s risk premium in an attempt to control for the increasing borrowers’ financial fragility (Stockhammer & Michell 2014). For simplicity, we abstract here from modeling endogenous risk premiums.

services, especially for the biggest, and therefore more indebted, firms. This mechanism leads to a drop in profits and investment (see the evolution of potential output on Figure 4b), and a rise in unemployment result (Figure 4a). The BSP brutally adjusts firms' strategies towards more cautious debt behavior, as explained in Section 4.2.1. However, a phenomenon akin to a Fischerian debt-deflation sets in: we observe a sharp increase in indebtedness precisely when firms choose to deleverage (Figure 4d).<sup>21</sup>

This chain of events explains why financial fragility (as measured by the ratio between the total of debts and the total of assets) and potential output (as measured by the total amount of goods that can be produced by all the machines in the economy) interact along a strongly circular dynamics (Figure 2h). Along a business cycle, the simulations show that the economy follows an anti-clockwise motion in the output/fragility diagram, which indicates that output peaks before financial fragility; see Stockhammer & Michell (2014) for a detailed discussion.

We can also look at the building up and collapse of assets and the interaction with the goods demand through the balance sheets of the agents. Tables 2 and 3 report the balance sheet matrix just before (in period  $t = 1000$ ) and right after ( $t = 1050$ ) the downturn (see Appendix C how these matrices are constructed). Within these 50 periods, the overall value of the net worth (i.e. the sum of deposits and equities) has lost 30% of its real value. This loss stems from the collapse in investment which implies that depreciated capital is not replaced: the firms' capital represent almost half of the overall net worth before the downturn, and but only account for a quarter 50 periods later. By contrast, on the asset side of the firms, inventories represent 25% of the overall net worth in  $t = 1000$ , and more than 40% in  $t = 1050$ , which reflects the drop in goods demand and firms' sales. On the liabilities side of the firms, the drop in investment shows up in the drop of long-run loans (i.e. the loans that are only intended to finance investment), from 28 to 14% of the overall net worth. On the contrary, the share of the short-run loans increases from 54 to more than 70%, which translates the firms' liquidity problems as a result of the drop in their

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<sup>21</sup>As explained in Sepecher & Salle (2015), the relative wage rigidity that we assume, see Section 3.4, is the driving force that brings back the system on an increasing trend.

	Households	Firms	Banks	$\Sigma$
Work In Process		828809.29		828809.29
Inventories		766196.57		766196.57
Fixed Capital		1526549.43		1526549.43
Deposits	1413349.64	855523.67	-2268873.31	0
Short Term Loans		-1672184.92	1672184.92	0
Long Term Loans		-875731.31	875731.31	0
Equities	1708205.65	-1429162.72	-279042.92	0
$\Sigma$	3121555.28	0	0	3121555.28

Table 2: Balance sheet matrix, period 1000 (in real terms)

	Households	Firms	Banks	$\Sigma$
Work In Process		700091, 60		700091, 60
Inventories		878428, 60		878428, 60
Fixed Capital		586028, 52		586028, 52
Deposits	1039460, 42	603749, 48	-1643209, 89	0
Short Term Loans		-1529421, 24	1529421, 24	0
Long Term Loans		-312271, 74	312271, 74	0
Equities	1125088, 31	-926605, 22	-198483, 09	0
$\Sigma$	2164548, 73	0	0	2164548, 73

Table 3: Balance sheet matrix, period 1050 (in real terms)

sales.

Figure 4e allows us to give a similar reading. Along the bust phase, firms' fixed capital drops, which reflects the drop in productive capacities stemming from the non-renewal of depreciated capital. However, firms' circulating capital (which consists of the sum of finished and unfinished goods, and therefore measures firms' inventories) only drops with a lag and less dramatically than fixed capital, which indicates excess inventories. Figure 4e also illustrates the liabilities side of firms' balance sheets along the bust dynamics: the dramatic increase in inventories translates into firms' financial difficulties, and a strong rise in overdraft facilities/short-run loans (even above the amount of circulating capital). Figure 4f synthesizes the categorization of firms into the three Minskian financing types (hedge, speculative and Ponzi, see the blue curve that represents the ratio of revenues over debt services), and indicates the degradation of firms' solvency at the macroeconomic level.

This simple exercise stresses the usefulness of *stock-flow consistency* for macroeconomic modeling. SFC modeling provides both a disciplinary device in the design of the

financial behaviors and accounting relations between sectors, and an analysis tool to dissect dynamics emerging from the simulations. Finally, in our ABM, we note that the process of collective adaptation through the market selection pressure yields cyclical macroeconomic dynamics that look more in line with the “financial instability” hypothesis (Minsky 1986) than with the “as-if” hypothesis (Friedman 1953), which predicts a stabilization of the system around a socially desirable steady state by driving out inefficient behaviors.

### 4.3 Discussion

Our model touches upon two, somehow distinct, research areas – learning and agent-based modeling. This section makes the point that these areas should be more closely linked together in order to improve macroeconomic modeling and our understanding of macroeconomic dynamics.

Our exercise shows the interest of modeling learning, not as a process intended to converge towards a particular steady state, but as an *ever-changing, ever-adapting process*, as advocated in Section 2. In an adaptive complex environment, like the simple macroeconomy modeled in Section 3, and like the real world probably is, there is no such thing as an “optimal” or efficient behavior. To put our results in parallel with a quote from March (1991, p. 73), in our model, there is not a single efficient way for the firms of addressing the growth-safety trade-off:

- “What is good in the long run is not always good in the short run”: a cautious financial strategy (limiting the indebtedness of the firm) is desirable in a long-run perspective because these firms are more resilient to severe downturns, but impeding in the short-run, because it restrains their expansion and make them loose market shares in favor of more audacious firms.
- “What is good at a particular historical moment is not always good at another time”: high leverage strategies allow a virtuous expansion circle to set in in periods of output growth, while they turn into a vicious circle in downturns, when firms

unsuccessfully try to deleverage.<sup>22</sup>

- “What is good for one part of an organization is not always good for another part”: while the fast growth of capital is desirable from the production division viewpoint, it puts the financial department at risk by deteriorating the capital ratio of the firm.
- “What is good for an organization is not always good for a larger social system of which it is a part.”: in the wake of a downturn, firms individually pursue deleveraging strategies and downsizing of their investment to improve their financial situation and avoid insolvency, but this behavior has in turn dramatic effects on the macroeconomic system as a whole because it amplifies and deepens the recession. We note the proximity of this point with Keynes’s “no bridge” concept.

Our model shows that the occurrence of an economic downturn or *crisis* endogenously stems from the adaptation *and* the failure of adaptation of the agents in the system.<sup>23</sup> A crisis arises as a sudden, brutal event, when the pace of change of the economic context becomes faster than the adaptation capacity of the agents. The occurrence of a crisis results from the combination of the bounded rationality hypothesis and an ever-changing complex environment. Bounded rationality implies that adaptation of behaviors is gradual and inertial (Winter 1964). If the environment evolves only slowly, or has even a constant structure, agents are likely to be able to adapt, and crises are not likely to be an inherent, *endogenous* feature of the system. On the other extreme, if agents are fully rational and fully informed, so that they are able to be infinitely far-sighted, they can adapt *instantaneously* to any new condition, and crises could only result from exogenous shocks. As shown by our model, the crisis corresponds to the sudden moment when behaviors that were judged by the market successful and compatible with the environment suddenly appear unsuited and unsustainable from the firms’ financial perspective, and for the financial system as a whole. As explained by Gaffeo et al. (2008, p. 445):

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<sup>22</sup>On the deleveraging crisis and debt-deflation phenomenon, see notably Eggertsson & Krugman (2012). See Sepecher & Salle (2015) for an analysis within a simpler version of the `Jame1` model.

<sup>23</sup>On the phenomenon of economic crises as coordination failures, see also Clower (1965), Cooper & John (1988), Howitt (2001), Delli Gatti et al. (2008, 2010).

Adaptive and imitative behaviors give rise to stable and predictable aggregate configurations, as stability implies predictability and vice versa. Since it is sometimes safer to be wrong in the crowd than to be right alone, imbalances can now and then accumulate to the point that a bundle of chained bankruptcies becomes inevitable.

The interpretation of crises as brutal disconnections between individual behaviors and aggregate outcomes and reversal between what used to appear virtuous and what used to be considered as vicious have recently found some revival interests, in the wake of the Great Recession (Eggertsson & Krugman 2012, Blanchard 2014, Battiston et al. 2016). Modeling such a transition is a challenge though, and our paper shows how ABM can provide a micro-founded, fully decentralized, stock-flow consistent and endogenous approach to this question. The general interdependence of agents' balance-sheets and the interconnection between the financial and the real sectors provided by the stock-flow consistency constitute an essential channel through which imbalances can propagate and crises can emerge as contagion phenomena.

## 5 Conclusion

Learning models in market economies have been traditionally envisioned as processes converging on a particular fixed point of the system. This approach is not suited in complex systems because learning goes hand-to-hand with adaptation in an ever-changing environment. In fact, in a complex adaptive system, there is generally no such thing as an efficient or optimal behavior, but the characterization of successful behaviors itself constantly evolves as a result of the market conditions that these behaviors contribute to shape. In this paper, we propose to model market mechanisms as a collective learning process in a complex adaptive system, and ask the question whether the process of “natural” market selection constitutes a suitable adaptation model in this type of systems.

We develop an adaptation model based on the “blanketing shotgun process” (BSP) introduced by Alchian (1950, p. 219). This contribution provides us with simple and

useful principles for modeling learning behaviors in macro ABM that are fully in line with, and even precursors of, both the bounded rationality and the evolutionary economics approaches. We use a simple ABM of a complex, monetary and stock-flow consistent macroeconomy as a playground – the `Jame1` model augmented with investment. Firms use the BSP adaptation model to choose a debt strategy, and deal with the “growth-safety” trade-off.

Because the BSP puts a strong emphasis on exploration *vis-à-vis* exploitation of the space of strategies, the resulting heterogeneity in firms’ behaviors provides to the system as a whole its ability to react and adapt. We show that this heterogeneity evolves as a response to the changes in the macroeconomic environment that it contributes to provoke, which makes the BSP an appealing learning model within complex adaptive systems.

Our simulations further show that decentralized market selection provides the system as a whole enough flexibility and resilience to sustain itself. However, this relative stability comes at the cost of wild fluctuations and recurring deep downturns. The “natural” market selection process does not result in collective optimization or convergence on an optimal equilibrium, despite the fact that it is usually advocated to justify full rationality and equilibrium modeling assumptions.

The dynamics that we observed from the simulations leads us to suggest an evolutionary characterization of an economic crisis as the moment when individual behaviors suddenly turn out to be incompatible with the macroeconomic environment, while the two had been reinforcing each other previously. Stated differently, a crisis corresponds to the point where the evolution of the macro system becomes faster than the adaptation capabilities of the agents that populate it. It is a strength of the macroeconomic SFC/ABM approach to be able to account for genuine behavioral heterogeneity which, together with full decentralization, produce the resulting co-evolution between micro behaviors and macro outcomes, and the endogenous emergence of this type of crises.



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## A Parameter values

Parameter	Description	Baseline value
<b>Households</b>		
$h$	number	6,000
$d^r$	wage resistance	12 (months)
$g$	size of the market selection (same for firms)	10
$window$	memory (same for firms)	12 (months)
$\eta_H$	wage adjustment parameter	0.05
$\kappa_S$	targeted savings rate	0.2 (share)
$\mu_H$	rate of consumption of excess savings	0.5
<b>Firms</b>		
$f$	number	400
$d^k$	lifetime of the machines	$\mathcal{N}(120, 15)$ (months)
$d^l$	short-run credit length	12 (months)
$d^L$	long-run credit length (= average machine lifetime)	120 (months)
$d^m$	market capacity, also targeted proportion of inventories	2 (months of production)
$d^p$	length of the production process	4 (months)
$d^w$	length of employment contracts	$\mathcal{U}[6, 36]$ (months)
$g'$	number of wage observations	3
$pr^k$	productivity of the machines	100 (units)
$v^k$	value of a new machine in real terms (number of goods to produce a machine)	500 (units)
$\beta$	greediness in investment	1.2
$\delta^P$	price flexibility parameter	0.04
$\delta^W$	wage flexibility parameter	0.02
$\rho^T$	targeted level of vacancies	0.03
$\mu_F$	proportion of goods to be sold	0.5
$\kappa_d$	maximum share of equity to be distribute as dividends	0.2
$\nu_F$	production flexibility parameter	0.1
$\sigma_{BSP}$	size of individual innovations	0.05
$proba_{BSP}$	probability of individual innovations	0.05
<b>Bank</b>		
$\kappa_b^T$	capital adequacy ratio target	0.1
$rp$	risk premium on doubtful debt	0.04 (monthly)
$\kappa_s$	recapitalization rate (for insolvent firms)	0.2
$\phi_\pi$	reaction to inflation (Taylor rule)	2
$\pi^T$	inflation target	0.02/12 (monthly)
<b>Model</b>		
$d^S$	length of the simulations	3,000 (months)

Table 4: Baseline scenario. Random draws are performed at each period and for each agent.

## B Pseudo-code of Jamel

Initialization :

Variable	Description	Initial value ( $t = 0$ )
<b>Household <math>i</math></b>		
$C_{i,t}$	actual level of consumption expenditures	0
$C_{i,t}^T$	desired level of consumption expenditures (consumption budget)	0
$d_{i,t}^u$	unemployment duration	0
$FD_{i,t}$	dividends received	0
$M_{i,t}$	cash on hand (bank deposit held)	0
$W_{i,t}$	wage received	0
$W_{i,t}^r$	reservation wage	0
$Y_{i,t}$	monetary income ( $=W_{i,t}+FD_{i,t}$ )	0
<b>Firm <math>j</math></b>		
$A_{j,t}$	total assets (inventories, fixed capital and money)	0
$E_{j,t}$	shareholder's equity ( $=A_{j,t}-L_{j,t}$ )	0
$E_{j,t}^T$	target equity ( $= (1-\ell_{j,t}^T)A_{j,t}$ )	0
$F_{j,t}$	net profits ( $=E_{j,t}-E_{j,t-1}+FD_{j,t}$ )	0
$FD_{j,t}$	dividends paid to the owners	0
$i_{j,t}$	new fixed capital goods (investment) in number of machines	0
$I_{j,t}$	new fixed capital goods (investment) in nominal terms	0
$in_{j,t}$	inventories (finished goods) in real terms	0
$in_{j,t}^T$	inventories target in real terms $=d^m \cdot pr^k \cdot k_{j,t}$	0
$k_{j,t}$	number of machines, maximum number of jobs	15
$L_{j,t}$	total liabilities (bank loans)	0
$\ell_{j,t}^T$	target debt ratio	$\hookrightarrow \mathcal{U}(0, 0.9)$
$M_{j,t}$	cash on hand (money deposit held)	0
$n_{j,t}$	actual workforce, actual number of employees	0
$n_{j,t}^T$	demand for labour, workforce target	12
$P_{j,t}$	unit price of goods supplied	0
$s_{j,t}$	actual sales in real terms	0
$s_{j,t}^e$	sales expansion objective in real terms	0
$s_{j,t}^T$	goods supply (targeted sales) in real terms $=\max(\mu_F \cdot in_{j,t}, d^m \cdot pr^k \cdot k_{j,t})$	0
$W_{j,t}$	the wage offered in nominal terms	50
<b>Bank</b>		
$A_{b,t}$	total assets ( $=$ total outstanding loans to the firms)	0
$E_{b,t}$	shareholder's equity ( $=A_{b,t}-L_{b,t}$ )	0
$E_{b,t}^T$	capital requirement $=\kappa_b^T A_{b,t}$	0
$FD_{b,t}$	dividends paid to the owners of the bank	0
$L_{b,t}$	total liabilities ( $=$ sum of deposits held by households and firms)	0
$i_t$	rate of interest on bank loans (nominal) $=\max(\phi_\pi(\pi_t - \pi^T), 0)$	0
$r_t$	discount rate (real rate of interest on bank loans) $=i_t - \pi_t$	0
Equities ( $E_{j,0}$ ) of each firm and of the bank are divided in ten equal shares, and given to randomly drawn households.		
<b>Macroeconomic public data</b>		
$\pi_t$	price inflation rate	0

**Execution** In each period  $t$ ,  $t = 1, \dots, d^S$ :

1. **(Interest rate adjustment:)**

$$i_t = \max(\phi_\pi(\pi_t - \pi^T), 0) \quad (3)$$

where  $\pi_t$  is the price inflation computed over past *window* periods,  $\phi_\pi$  and  $\pi^T$  are parameters.

2. **(Fixed capital stock depreciation:)** Each machine  $m$  of each firm  $j$  is depreciated by  $\frac{I_{j,m}}{d^k}$  where  $I_{j,m}$ , is the initial value of the machine paid by  $j$  and  $d^k$  the expected life time of the machine (in months, straight-line depreciation method).

3. **(Payment of dividends:)**

**Each firm  $j$**  i) computes  $\tilde{F}_{j,t}$ , its average past net profits  $F_j$  over *window* periods, ii) computes the share of net profits to be distributed as  $\frac{E_{j,t}}{E_{j,t}^T}$ , and iii) distributes to its owners the amount  $FD_{j,t} = \min\left(\frac{E_{j,t}}{E_{j,t}^T}\tilde{F}_{j,t}, \kappa_d E_{j,t}\right)$ , in proportion to their relative share holding.

**The bank** distributes  $FD_{B,t} = \max(E_{B,t} - E_{B,t}^T, 0)$

Updating of the firms' and the bank's balance sheets.

4. **(Price:)**

$$\begin{aligned} & \text{if } (s_{j,t-1} = s_{j,t-1}^T \text{ and } in_{j,t} < in_{j,t}^T) && \begin{cases} \bar{P}_{j,t} = \bar{P}_{j,t-1}(1 + \delta^P) \\ \underline{P}_{j,t} = P_{j,t-1} \\ P_{j,t} \hookrightarrow \mathcal{U}(\underline{P}_{j,t}, \bar{P}_{j,t}) \end{cases} \\ & \text{else if } (s_{j,t-1} < s_{j,t-1}^T \text{ and } in_{j,t} > in_{j,t}^T) && \begin{cases} \bar{P}_{j,t} = P_{j,t-1} \\ \underline{P}_{j,t} = \underline{P}_{j,t-1}(1 - \delta^P) \\ P_{j,t} \hookrightarrow \mathcal{U}(\underline{P}_{j,t}, \bar{P}_{j,t}) \end{cases} \quad (4) \\ & \text{else} && \begin{cases} \bar{P}_{j,t} = \bar{P}_{j,t-1}(1 + \delta^P) \\ \underline{P}_{j,t} = \underline{P}_{j,t-1}(1 - \delta^P) \\ P_{t,j} = P_{j,t-1} \end{cases} \end{aligned}$$

with :

- $\bar{P}_{j,t}$ , the ceiling price,
- $\underline{P}_{j,t}$ , the floor price.

5. **(Wage offer:)** Each firm  $j$  observes a random sample of  $g'$  other firms. If the observed sample contains a firm  $k$  such that  $k_{k,t} > k_{j,t}$ , then:

$$\begin{cases} W_{j,t} = W_{k,t} \\ \bar{W}_{j,t} = W_{j,t}(1 + \delta^W) \\ \underline{W}_{j,t} = W_{j,t}(1 - \delta^W) \end{cases} \quad (5)$$

else:

$$\begin{aligned} \text{if } (\rho_{j,t-1} > \rho^T) & \quad \begin{cases} \overline{W}_{j,t} = \overline{W}_{j,t-1}(1 + \delta^W) \\ \underline{W}_{j,t} = \underline{W}_{j,t-1} \end{cases} \\ \text{else} & \quad \begin{cases} \overline{W}_{j,t} = \underline{W}_{j,t-1} \\ \underline{W}_{j,t} = \overline{W}_{j,t-1}(1 - \delta^W) \end{cases} \end{aligned} \quad (6)$$

$$\text{and then} \quad W_{j,t} \hookrightarrow \mathcal{U}(\underline{W}_{j,t}, \overline{W}_{j,t})$$

with:

- $\rho_{j,t-1} = \frac{n_{j,t-1}^T - n_{i,t-1}}{n_{j,t-1}^T}$ , the vacancy rate previously observed by the firm,
- $\overline{W}_{j,t}$ , the ceiling wage,
- $\underline{W}_{j,t}$ , the floor wage.

6. **(Labor demand:)**  $n_{j,t}^T$  (within the lower bound 0 and the upper bound  $k_{j,t}$ ):

$$n_{j,t}^T = (1 + \delta_{j,t}^h)n_{j,t-1}^T \quad (7)$$

where  $n_{j,t-1}^T$  is the labor demand of the firm in period  $t - 1$ , and  $\delta_{j,t}$  is the size of the adjustment, computed as:

$$\delta_{j,t}^h = \begin{cases} \alpha_{j,t}\nu_F & \text{if } 0 \leq \alpha_{j,t}\beta_{j,t} < \frac{in_{j,t}^T - in_{j,t}}{in_{j,t}^T}, \\ -\alpha_{j,t}\nu_F & \text{if } 0 \leq \alpha_{j,t}\beta_{j,t} < \frac{in_{j,t} - in_{j,t}^T}{in_{j,t}^T}, \\ 0 & \text{else.} \end{cases} \quad (8)$$

with  $\alpha_{j,t}, \beta_{j,t} \hookrightarrow \mathcal{U}(0, 1)$  and  $\nu_F > 0$ .

$$\begin{cases} \text{if } n_{j,t} > n_{j,t}^T & \text{fires } n_{j,t} - n_{j,t}^T \text{ (on a last-hired-first-fired basis)} \\ \text{else} & \text{posts } n_{j,t}^T - n_{j,t} \text{ job offers.} \end{cases} \quad (9)$$

7. **(Financing of current assets):** according to the existing job contracts, the workforce target  $n_{j,t}^T$ , and the new wage rate offered on the labor market  $W_{j,t}$ :

- computes the anticipated wage bill  $WB_{j,t}^T$ ;
- borrows  $\max(WB_{j,t}^T - M_{j,t}, 0)$  (non-amortized short-term loan).
- Updating of the firms' and the bank's balance sheets.

8. **(Reservation wages:)**

Each household  $i$  updates his reservation wage  $W_{i,t}^r$ .

- If  $i$  is unemployed:

$$W_{i,t}^r = W_{i,t-1}^r(1 - \delta_{i,t}^w) \quad (10)$$

where  $\delta_{i,t}^w \geq 0$  is the size of the downward adjustment, and is computed as:

$$\delta_{i,t}^w = \begin{cases} \beta_{i,t} \cdot \eta_H & \text{if } \alpha_{i,t} < \frac{d_{i,t}^u}{d^r} \\ 0 & \text{else.} \end{cases} \quad (11)$$



where  $\alpha_{i,t}$ ,  $\beta_{i,t}$  are  $\mathcal{U}(0,1)$ , and  $\eta_H > 0$  and  $d^w \geq 1$  are parameters.

- Else:

$$W_{i,t}^r = W_{i,t-1} \quad (12)$$

where  $W_{i,t-1}$  is the wage earned by household  $i$  at the previous period  $t - 1$ .

### 9. (Labor market :)

Each unemployed household i) consults a random sample of  $g$  job offers; ii) selects the job offer with the highest offered wage, denoted by  $W_{j,t}$ ; iii) if  $W_{j,t} \geq W_{i,t}^r$ , accepts the job for a duration of  $d^w$  months; else, remains unemployed for the period  $t$ .

10. **(Production):** Each firm distributes uniformly the hired workers on its machines (one per machine). Once a production process of a machine is completed (after  $d^p$  iterations by a worker), it adds  $pr^k$  goods to the firm's inventories  $in_{j,t}$ , whose value is then incremented by the production costs of  $pr^k$  goods.

This process updates i) firms' wage bills and vacancy rates, ii) production levels, and iii) households' cash-on-hand  $M_{i,t} = W_{i,t} + FD_{i,t} + M_{i,t-1}$  (where  $W_{i,t} + FD_{i,t}$  represents its income flow, made of  $FD_{i,t}$ , the dividends that household  $i$  may receive if it owns shares in the bank or a firm, see Step 1., and  $W_{i,t}$  its labor income, and  $M_{i,t-1}$  is its cash-on-hand transferred from  $t - 1$ ).

11. **(Goods supply):** Each firm  $j$  puts  $s_{j,t}^T$  goods in the goods market:

$$s_{j,t}^T = \max(\mu_F \cdot in_{j,t}, d^m \cdot pr^k \cdot k_{j,t}) \quad (13)$$

12. **(Individual experimentation :)** With a probability  $proba_{BSP}$ , for each firm  $j$ ,  $\ell_{j,t+1} \hookrightarrow \mathcal{N}(0, \sigma_{BSP})$ , else  $\ell_{j,t+1} = \ell_{j,t}$ .

### 13. (Investment decision):

- (a) selects a random sample of  $g$  suppliers (other firms);
- (b) if ( $k_{j,t} = 0$ ) buys  $m = 1$  new machine, for a value  $I_{j,t}$ ;
- (c) else if ( $E_{j,t}^T > E_{j,t}$ ):
  - i. computes the vector of the prices of each investment project  $\Pi_m$  ( $m$  the number of new machines to be bought), with  $m = 0, 1, 2, \dots$ ;
  - ii. computes  $\tilde{s}_{j,t}$ , average of the sales  $s_j$  over the past *window* periods;
  - iii. computes  $s_{j,t}^e = \beta \tilde{s}_{j,t}$ , its sales expansion objective;
  - iv. given its sales expansion objective  $s_{j,t}^e$ , the current price  $P_{j,t}$ , the current wage  $W_{j,t}$ , the real interest rate  $r$ , and the vector of prices of each investment project, computes the net present value  $NPV_m$  of each investment project  $m$  until  $NPV_{m+1} < NPV_m$ ;
  - v. chooses the project  $m$ , for a value  $I_{j,t}$ .

### 14. (Financing of fixed assets):

- (a) borrows (amortized long-run loan) the amount:  $\ell_{j,t}^T I_{j,t}$ ;

(b) borrows (amortized short-run loan) the amount:  $\max((1 - \ell_{j,t}^T)I_{j,t} - M_{j,t}, 0)$ ;

15. **(Saving/consumption plan:)** Each household computes

(a) his average monthly income flow over the last *window* periods, denoted by  $\tilde{Y}_{i,t}$ ;

(b) his cash-on-hand target  $M_{i,t}^T = \kappa_S \cdot \tilde{Y}_{i,t}$ ;

(c) is targeted consumption expenditures as:

$$C_{i,t}^T = \begin{cases} (1 - \kappa_S)\tilde{Y}_{i,t} & \text{if } M_{i,t} \leq M_{i,t}^T \\ \tilde{Y}_{i,t} + \mu_H(M_{i,t} - M_{i,t}^T) & \text{else.} \end{cases} \quad (14)$$

where  $\mu_H \geq 0$  is a parameter. The budget constraint always gives  $C_{i,t} \leq \min(C_{i,t}^T, M_{i,t})$ .

16. **(Goods market :):**

(a) matches first the firms' demand, then the households' demand with the firms' supply;

(b) goods bought by firms are transformed in new machines, while goods bought by households are consumed;

(c) updates the firms' inventories  $in_{j,t}$ , number of machines  $k_{j,t}$ , assets  $A_{j,t}$  and equities  $E_{j,t}$ , and the households' remaining cash-on-hand  $s_{i,t}$ .

17. **(Loans :)** The firms pay back part of their loans and the interests to the bank. Interest is due at each period. For an amortized loan, principal is repaid by equal fractions at each period, while for a non-amortized loan, the total principal is due at the term. If the cash-on-hand  $M_{j,t}$  of a firm  $j$  cannot fully cover the debt repayments, it benefits of an overdraft facility, ie a new short term loan at an higher rate including the risk premium of the bank ( $i_t + rp$ ).

18. **(Foreclosure :)** If a firm has become insolvent ( $A_{j,t} < L_{j,t}$ ), the bank starts the foreclosure procedure, a new  $\ell_{j,t}^T$  is copied from a surviving firm ( $+\mathcal{N}(0, \sigma_{BSP})$ ), the firm is recapitalized up to  $E_{j,t} = \kappa_s A_{j,t}$ , and new households become owners as follows: all households that have at least 20% of  $E_{j,t}$  as cash-on-hand are solicited for at most 50% of their wealth, and the firm's shares are distributed in proportion to their contribution. If the collected cash-on-hand is lower than  $E_{j,t}$ , the selection threshold is decreased to 10% of  $E_{j,t}$ . If the cash-on-hand on the solicited households is still not enough, the threshold is decreased to 4%, and then 2%. If this is still not enough to buy all the shares of the firm, the price of the shares is decreased by 10% until enough cash-on-hand can be collected. In case of more than 10 decreases, the simulation would stop.

19. This process starts all over again for a given length of  $d^S$  periods.

## C Stock-flow consistency

### C.1 Stock consistency

$M$	Money deposits supplied by banks
$M_f$	Money deposits held by firms
$M_h$	Money deposits held by households
$E$	Value of equities held by households
$E_b$	Value of equities issued by banks
$E_f$	Value of equities issued by firms
$NW$	Net worth of households
$IN$	Inventories of finished goods, at production cost
$WIP$	Work in process, at production cost
$K$	Value of fixed capital stock
$L$	Loans supplied by banks
$L_f$	Loans to firms

Table 5: Stocks

**Households:**  $M_h + E = NW$

**Firms:** <sup>24</sup>  $WIP + IN + K + M_f = L_f + E_f$

**Banks:**  $L = M + E_b$

**Deposits:**  $M = M_h + M_f$

**Loans:**  $L = L_f$

**Equities:**  $E = E_f + E_b$

**Closure:**  $NW = WIP + IN + K$

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<sup>24</sup> As the model does not encompass any financial market, firms are priced by households according to their shareholders' equity.

	Households	Firms	Banks	$\Sigma$
Work In Process		$WIP$		$WIP$
Inventories		$IN$		$IN$
Fixed Capital		$K$		$K$
Deposits	$M_h$	$M_f$	$-M$	0
Loans		$-L_f$	$L$	0
Equities	$E$	$-E_f$	$-E_b$	0
$\Sigma$	$NW$	0	0	$WIP + IN + K$

Table 6: Balance sheet matrix

## C.2 Flow consistency

$C$	Consumption goods sold by firms to households
$K^{dep}$	Fixed capital depreciation
$FD_b$	Dividends of banks
$FD_f$	Dividends of firms
$S$	Value of sales, at historic costs
$I$	New fixed capital goods
$INT$	Interest payments paid by firms
$L^{new}$	New loans
$L^{back}$	Repaid loans
$L^{np}$	Non performing loans
$PROD$	New finished goods valued at cost
$CAP$	Recapitalizations
$WB$	Wages paid to households

Table 7: Flows

**Households Deposits:**  $\Delta M_h = WB + FD_f + FD_b - C - CAP$

**Households Equities:**  $\Delta E = C + I - S - FD_f - FD_b - K^{dep} + CAP$

**Households net worth:**  $\Delta NW = WB + I - K^{dep} - S$

**Firms' work in progress:**  $\Delta WIP = WB - PROD$

**Firms' inventories:**  $\Delta IN = PROD - S$

**Firms' fixed capital:**  $\Delta K = I - K^{dep}$

**Firms' Deposits:**  $\Delta M_f = L^{new} - L^{back} + C - WB - INT$

**Firms' Debts:**  $\Delta L = L^{new} - L^{back} - L^{np}$

**Firms equities:**  $\Delta E_f = C + I - S - FD_f - K^{dep} - INT + L^{np}$

**Bank deposits:**  $\Delta M = L^{new} - L^{back} - INT - CAP$

**Bank loans:**  $\Delta L = L^{new} - L^{back} - L^{np}$

**Bank equities:**  $\Delta E_b = INT - FD_b - L^{np} + CAP$

	Households	Firms	Banks	$\Sigma$
Work In Process		$WB - PROD$		$WB - PROD$
Inventories		$PROD - S$		$PROD - S$
Fixed Capital		$I - K^{dep}$		$I - K^{dep}$
Deposits	$WB - C - CAP + FD_f + FD_b$	$L^{new} - L^{back} + C - WB - INT - FD_f$	$-L^{new} + L^{back} + INT + CAP - FD_b$	0
Loans		$-L^{new} + L^{back} + L^{sp}$	$L^{new} - L^{back} - L^{sp}$	0
Equities	$C + I - S - FD_f - FD_b - K^{dep} + CAP$	$-C - I + S + FD_f + K^{dep} + INT - L^{sp}$	$-INT + FD_b + L^{sp} - CAP$	0
$\Sigma$	$WB + I - S - K^{dep}$	0	0	$WB + I - S - K^{dep}$

Table 8: Flow of funds matrix