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Calcul de pi par la méthode originale des carrés inscrits

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Calcul de pi par la méthode originale des carrés inscrits

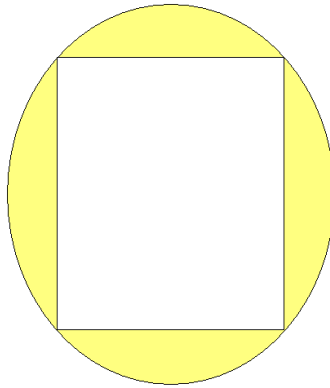
Introduction: définitions et notations

Soit le cercle de diamètre D . Sa surface S vaut par définition: $S=\pi D^2/4$, ce qui donne pour pi la relation suivante: $\pi=4S/D^2$.

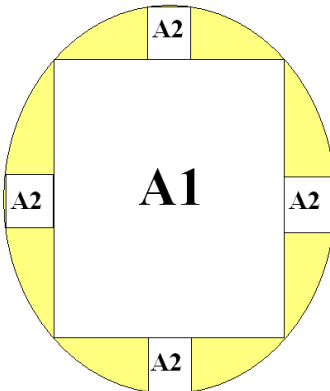
Il nous suffirait donc de déterminer une manière originale de définir S afin d'obtenir en conséquence une expression précise de pi. Ce défi constitue précisément l'objet de cette étude.

Ajoutons que pour rendre la lecture plus fluide et élégante ainsi que pour éviter des migraines chez le lecteur, ne seront développés ici que les résultats principaux, plus pertinents.

Amusons-nous dans un premier temps à inscrire un carré dans un cercle de diamètre $D=AB$ auquel nous attribuons par convention la valeur $D=1$. Dans chacun des quatre espaces vides identiques créés (colorés en jaune dans la figure suivante), on insère le plus grand carré possible.



On place progressivement dans chaque espace libre (non occupé par un carré) le plus grand carré possible.



En répétant patiemment l'opération une infinité de fois, nous couvrons par nos carrés une surface qui tend à égaler celle du cercle.

Pour y voir plus clair dans cette grande invasion des carrés, mettons-y un peu d'ordre.

On définit les zones suivantes:

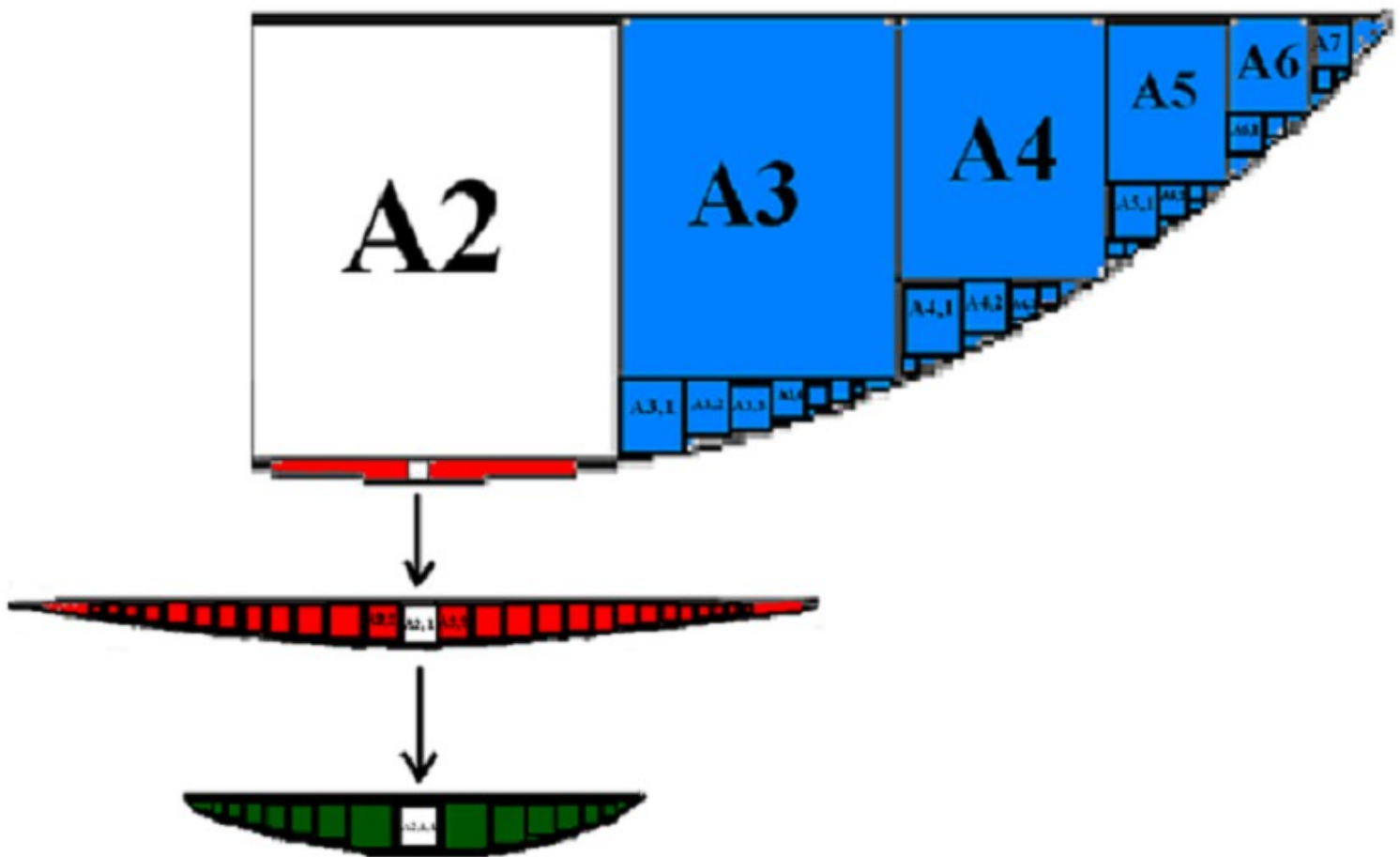
Zone C (centrale): colorée en blanc, regroupe les carrés centraux. Le carré le plus grand, au centre, est nommé A1. L'un en dessous de l'autre (par rapport au centre du cercle) nous avons successivement A2 ; A2,1 ; A2,1,1 et ainsi de suite. On rajoute une virgule à la notation pour chaque carré de niveau inférieur.

Zone N=1: en bleu, le plus gros carré est A3

Zone N=2: en rouge, le plus gros carré est A2,1

Zone N=3: en vert, le plus gros carré est A2,1,1

Ainsi de suite, on observe une similarité pour chaque zone (hors zone C), placée latéralement et symétriquement de chaque côté d'un carré blanc central.



Notations (pour un nombre d'indices ≥ 2)

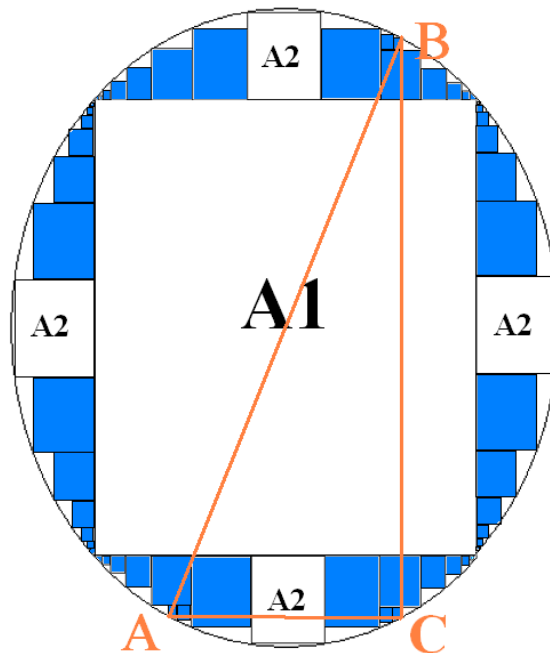
La notation de chaque carré reprend celle du carré de référence placé juste au dessus (toujours par rapport au centre de cercle) en y ajoutant un indice supplémentaire après une virgule. Le dernier chiffre de l'indice représente l'ordre, à partir du carré blanc central, du carré, parmi tous

les carrés sous celui de référence. Par exemple, le carré A3,2,3 est le troisième des carrés placés sous le carré A3,2, lui-même étant le deuxième sous le carré A3. La figure plus haut illustre bien le principe.

Pour un carré de notation A2,1,1,...,n_x (l'indice n_x signifie indice d'ordre x et de valeur n, exemple: pour le carré A2135, nous avons quatre indices, pour x=4 nous avons la valeur n₄=5, et ainsi de suite: n₃=3, n₂=1 et n₁=2.

Calcul de la dimension de la surface d'un carré

Le théorème de Pythagore (figure ci-dessous) nous donne aimablement la valeur du côté de chaque carré:



Pour la zone C (carré de la forme A2,1,...,1N-1,nN, de côté = a2,1,...,1N-1,nN)

$$AB^2 = AC^2 + BC^2$$

$$1 = (a_{2,1,\dots,1N-1,nN})^2 + (a_1 + 2 \times (a_2 + a_{2,1,\dots,1N-1,nN}))^2, N \geq 1$$

Pour le carré A1, on calcule facilement,

$$1 = (a_1)^2 + (a_1)^2 \rightarrow a_1 = 1/\sqrt{2} \rightarrow \text{aire A1} = (a_1)^2 = 1/2$$

Pour les autres valeurs d'aires, on trouve successivement, après calculs et simplifications:

$$A_2 = \frac{1}{2 \times 5^2}, \quad A_{2,1} = \frac{397 - 28\sqrt{201}}{2 \times 5^4}, \quad A_{2,1,1} = \frac{8809 + 84\sqrt{201} - \sqrt{(5397 - 28\sqrt{201})(28 + 8\sqrt{201})}}{2 \times 5^6}, \dots$$

On obtient la formule générale suivante, qui est valable pour les aires de tous les carrés de la zone C, excepté A1.

$$A_{2,1,\dots,1_{N-1},n_N} = 25^{N-1} + 372 \left(\sum_{p=2}^{p=N} (-1)^{p-2} 3^{p-2} 25^{N-p} \right) \\ + \left(\sum_{p=2}^{p=N} (-1)^{p+1} 28.3^{p-2} R_{2,1,\dots,n_{N-(p-2)}} \right) + \left(\sum_{p=3}^{p=N} (-1)^p 8.3^{p-3} R_{2,1,\dots,n_{N-(p-3)}} \right) (R_{21} + \dots + R_{2,1,\dots,n_{N-(p-3)-1}}) / 2.5^{2N}$$

Avec $R_{2,1,\dots,1_{N-1},n_N}$, racine caractéristique de $A_{2,1,\dots,1_{N-1},n_N}$ ($N \geq 2$), de formule générale

$$R_{2,1,\dots,1_{N-1},n_N} = 201 \times 25^{N-2} + 372 \left(S_{p=3/N} (-1)^{p-1} 3^{p-3} 25^{N-p} \right) \\ + \left(S_{p=3/N} (-1)^p 28.3^{p-3} R_{2,1,\dots,n_{N-(p-2)}} \right) + \left(S_{p=4/N} (-1)^{p+1} 8.3^{p-4} R_{2,1,\dots,n_{N-(p-3)}} \right) \\ (R_{21} + \dots + R_{2,1,\dots,n_{N-(p-3)-1}})$$

En sommant les différentes aires jusqu'à l'infini, tout en tenant compte de la symétrie (4 sous-zones identiques auxquelles il convient de rajouter le carré central A1), on trouve la surface totale de la zone C.

On observe que lorsque N tend vers l'infini, on a $R_{2,1,\dots,1_N} \rightarrow 2.5^{N-1} \sqrt{2}$,

Soit $X_{2,1,\dots,1_N} = R_{2,1,\dots,1_N} - 2.5^{N-1} \sqrt{2}$, la différence entre $R_{2,1,\dots,1_N}$ et sa limite à l'infini

On pose donc $R_{21,1,\dots,1_N} = 5^{N-1} \cdot 2\sqrt{2} + X_{21,1,\dots,1_N}$

Surface de la zone C =

$$\frac{16}{5} - \frac{26\sqrt{2}}{125} - \frac{52\sqrt{402}}{875} - \frac{2\sqrt{201}}{25} \\ - \frac{4\sqrt{2}}{175} (X_{211} + (1/5)X_{2111} + (1/5)^2 X_{21111} + \dots + (1/5)^{N-3} X_{21,1,\dots,1_N}) \\ - (4/4375) \left[((7/2) + X_{21})X_{211} + (1/25)((7/2) + X_{21} + X_{211})X_{2111} + \dots + (1/25)^{N-3} ((7/2) + X_{21} + X_{211} + \dots + X_{21,1,\dots,1_{N-1}}) X_{21,1,\dots,1_N} \right]$$

Pour la zone $N=1$ (carré de la forme A_{n_1,n_2,\dots,n_x} , de côté = a_{n_1,n_2,\dots,n_x})

$$AB^2 = AC^2 + BC^2$$

$$1 = \left[a_2 + 2 \times \left((a_3 + a_4 + \dots + a_{n_1-1}) + (a_{n_1,1} + a_{n_1,2} + \dots + a_{n_1,n_2-1}) + \dots + (a_{n_1,n_2,\dots,1_x} + a_{n_1,n_2,\dots,2_x} + \dots + a_{n_1,n_2,\dots,n_x}) \right) \right]^2 + \left[a_1 + 2 \times (a_{n_1} + a_{n_1,n_2} + \dots + a_{n_1,n_2,\dots,n_x}) \right]^2$$

On trouve successivement, après calculs et simplifications, les aires suivantes:

$$A_3 = \frac{15-3\sqrt{21}}{2^2 \times 5^2}, \quad A_4 = \frac{30+10\sqrt{21} - \sqrt{(14\sqrt{21}+30)(3+\sqrt{21})}}{2^4 \times 5^2}, \dots$$

On trouve la formule générale suivante, valable pour les aires de tous les carrés de la zone N=1. Chacun des exposants E1 , E2, ..., E7 indique que le terme auquel il est rattaché possède une exception particulière qui remplace ce terme, et dont la formule est donnée plus bas.

$$\begin{aligned}
& An_{1,n_2,\dots,n_x} = \left[5.2^{\binom{n_1+\dots+n_x}{x}-3} (R_2+\dots+R_{n_1-2}) + (5.2^{\binom{n_1+\dots+n_x}{x}-3} - 3.2^{\binom{n_2+\dots+n_x}{x}}) (R_{n_1,1} + \dots + \right. \\
& R_{n_1,n_2-2}) + \dots + \left. \left(5.2^{\binom{n_1+\dots+n_x}{x}-3} - 3(2^{\binom{n_2+\dots+n_x}{x}} + 2^{\binom{n_3+\dots+n_x}{x}} + \dots + 2^{\binom{n_{x-1}+n_x}{x}}) \right) \left(\right. \right. \\
& An_{1,\dots,n_{x-2},1_{x-1}+\dots+An_{1,\dots,n_{x-2},n_{x-1}-2} \left. \right) \left. + \left[(15.2^{\binom{n_1+\dots+n_x}{x}-3} - (3+(R_3+\dots+R_{n_1-2})) \right) \right. \right. \\
& \left. \left. 2^{\binom{n_2+\dots+n_x}{x}+2} \right) E1 \text{ et } E2 R_{n_1-1} + (15.2^{\binom{n_1+\dots+n_x}{x}-3} - 9(2^{\binom{n_2+\dots+n_x}{x}}) - (3+(R_{n_1,1}+\dots+R_{n_1,n_2-2})) \right. \right. \\
& \left. \left. 2^{\binom{n_3+\dots+n_x}{x}+2} \right) R_{n_1,n_2-1} + \dots + (15.2^{\binom{n_1+\dots+n_x}{x}-3} - 9(2^{\binom{n_2+\dots+n_x}{x}} + 2^{\binom{n_3+\dots+n_x}{x}} + \dots + 2^{\binom{n_{x-1}+n_x}{x}}) \right. \right. \\
& \left. \left. - (3+(R_{n_1,\dots,n_{x-2},1}+\dots+R_{n_1,\dots,n_{x-1}-2})) 2^{\binom{n_x}{x}+2} \right) E3 R_{n_1,\dots,n_{x-1}-1} \right] - \left[(5.2^{\binom{n_1+2n_2+\binom{n_3+\dots+n_x}{x}}{x}-3} - \right. \\
& (3+(R_3+\dots+R_{n_1-1})) 2^{2n_2+\binom{n_3+\dots+n_x}{x}} - 5.2^{\binom{n_1+\dots+n_x}{x}-3}) R_{n_1} + (5.2^{\binom{n_1+n_2+2n_3+\binom{n_4+\dots+n_x}{x}}{x}-3} - \\
& 3.2^{\binom{n_2+2n_3+\binom{n_4+\dots+n_x}{x}}{x}} - (3+(R_{n_1,1}+\dots+R_{n_1,n_2-1})) 2^{2n_3+\binom{n_4+\dots+n_x}{x}} - 5.2^{\binom{n_1+\dots+n_x}{x}-3} + 3(2^{\binom{n_2+\dots+n_x}{x}} \\
& \left. \left. \right) R_{n_1,n_2} + \dots + (5.2^{\binom{n_1+\dots+n_x}{x}-3} + 2^{2n_x-3} - 3(2^{\binom{n_2+\dots+n_x}{x}} + 2^{2n_x} - (R_{n_1,\dots,n_{x-2},1}+\dots \\
& +R_{n_1,\dots,n_{x-1}-1}) 2^{2n_x} - 5.2^{\binom{n_1+\dots+n_x}{x}-3} + 3(2^{\binom{n_2+\dots+n_x}{x}} + \dots + 2^{\binom{n_{x-1}+n_x}{x}})) R_{n_1,\dots,n_{x-1}} \right] \\
& + \left[\left(- (R_3+\dots+R_{n_1-1}) \left[(2^{\binom{n_2+\dots+n_x}{x}} (R_{n_1,1}+\dots+R_{n_1,n_2-2}) + (3.2^{\binom{n_2+\dots+n_x}{x}} + 2^{\binom{n_3+\dots+n_x}{x}+2}) \right. \right. \right. \\
& R_{n_1,n_2-1} - (2^{\binom{n_2+2n_3+\binom{n_4+\dots+n_x}{x}}{x}} + 2^{2n_3+\binom{n_4+\dots+n_x}{x}} - 2^{\binom{n_2+\dots+n_x}{x}}) R_{n_1,n_2} \left. \right) + \left((2^{\binom{n_2+\dots+n_x}{x}} + \right. \\
& 2^{\binom{n_3+\dots+n_x}{x}}) (R_{n_1,n_2,1}+\dots+R_{n_1,n_2,n_3-2}) + (3(2^{\binom{n_2+\dots+n_x}{x}} + 2^{\binom{n_3+\dots+n_x}{x}}) + 2^{\binom{n_4+\dots+n_x}{x}+2}) \\
& R_{n_1,n_2,n_3-1} - (2^{\binom{n_2+n_3+2n_4+\binom{n_5+\dots+n_x}{x}}{x}} + 2^{2n_3+2n_4+\binom{n_5+\dots+n_x}{x}} + 2^{2n_4+\binom{n_5+\dots+n_x}{x}} - (2^{\binom{n_2+\dots+n_x}{x}} + \\
& 2^{\binom{n_3+\dots+n_x}{x}})) R_{n_1,n_2,n_3} \left. \right) + \dots + \left((2^{\binom{n_2+\dots+n_x}{x}} + \dots + 2^{\binom{n_{x-1}+n_x}{x}}) (R_{n_1,\dots,n_{x-2},1}+\dots+R_{n_1,\dots,n_{x-1}-2}) \right. \\
& \left. + (3(2^{\binom{n_2+\dots+n_x}{x}} + 2^{\binom{n_{x-1}+n_x}{x}}) + 2^{n_x+2}) E4 R_{n_1,\dots,n_{x-1}-1} - (2^{\binom{n_2+\dots+n_x}{x}+2n_x} + \dots + 2^{2n_x} - \right. \\
& \left. (2^{\binom{n_2+\dots+n_x}{x}} + \dots + 2^{\binom{n_{x-1}+n_x}{x}}) R_{n_1,\dots,n_{x-1}}) \right] + \left[\text{termes identiques à ceux du crochet précédent,} \right. \\
& \left. \text{sauf que les } + \text{ deviennent des } - \text{ et l'exception E4 est remplacée par E5} \right] \left. \right) \\
& + \dots + \left(-(R_{n_1,\dots,n_{x-3},1}+\dots+R_{n_1,\dots,n_{x-2}-1}) \left[2^{\binom{n_{x-1}+n_x}{x}} (R_{n_1,\dots,n_{x-2},1}+\dots+R_{n_1,\dots,n_{x-1}-2}) + \right. \right. \\
& \left. \left. (3.2^{\binom{n_{x-1}+n_x}{x}} + 2^{n_x+2}) E6 R_{n_1,\dots,n_{x-1}-1} - (2^{\binom{n_{x-1}+2n_x}{x}} + 2^{2n_x} - 2^{\binom{n_{x-1}+n_x}{x}}) R_{n_1,\dots,n_{x-1}} \right] + \right. \\
& R_{n_1,\dots,n_{x-2}} \left[\text{termes identiques à ceux du crochet précédent, sauf que les } + \text{ deviennent des } - \text{ et} \right. \\
& \left. \text{l'exception E6 est remplacée par E7} \right] \left. \right) \\
& + (R_{n_1,\dots,n_{x-1},1}+\dots+R_{n_1,\dots,n_{x-1}}) \left[(5.2^{\binom{n_1+\dots+n_x}{x}-3} - 3(2^{\binom{n_2+\dots+n_x}{x}} + \dots + 2^{\binom{n_x}{x}}) + (-2^{\binom{n_2+\dots+n_x}{x}} \right. \\
& \left. + \dots + 2^{\binom{n_x}{x}}) (R_3+\dots+R_{n_1-1}) + (2^{\binom{n_2+\dots+n_x}{x}} - \dots - 2^{\binom{n_x}{x}}) R_{n_1} \right) + \dots + (-2^{n_x} (R_{n_1,\dots,n_{x-2},1}+\dots \\
& +R_{n_1,\dots,n_{x-1}-1}) + 2^{n_x} R_{n_1,\dots,n_{x-1}}) \left. \right]
\end{aligned}$$

$$- R_{n_1, \dots, n_x} \left[(R_2 + \dots + R_{n_1}) + (R_{n_1, 1} + \dots + R_{n_1, n_2}) + \dots + (R_{n_1, \dots, n_{x-1}, 1} + \dots + R_{n_1, \dots, n_x - 1}) \right] / 25.2^{2(n_1 + \dots + n_x) - 4}$$

Exceptions de la formule de A_{n_1, n_2, \dots, n_x} :

E1:

Soit n_b , le premier indice de valeur ≥ 2 dans la notation de A_{n_1, \dots, n_x} . On a $b \geq 2$.

Exemple: pour $A_{3, 1, 1, 5, 2, 3}$, $b=4$ (quatrième indice) de valeur $n_b=5$

*Pour $n_{b+1}, \dots, n_x \geq 1$, $3 \leq b \leq x-1$, $[(5.2^{n_1-4} - 3) 2^{2b} - (5.2^{n_1-4} - 6) 2^b] 2^{(n_b + \dots + n_x) - 1}$,
 $[3.2^{2b} + 3.2^{b-1}] 2^{(n_b + \dots + n_x) - 1}$, pour $n_1 = 3$

Pour $n_{b+1}, \dots, n_x \geq 1$, $b=2$, termes notés dans la formule ci dessus. Pour $n_1 = 3$, on a $9.2^{(n_2 + \dots + n_x)}$
 $R_2 = 27.2^{(n_2 + \dots + n_x)}$, $R_2 = 3$

*Pour $n_2, \dots, n_{x-1} = 1$, $n_x \geq 1$, $(5.2^{2x+n_1-4} - 5.2^{x+n_1-4} - 3.2^{2x} + 3.2^{x+1}) 2^{n_x-1}$,
 $(3.2^{2x} + 3.2^{x-1}) 2^{n_x-1}$ pour $n_1 = 3$

E2:

* $n_{b+1}, \dots, n_x \geq 1$, $2 \leq b \leq x-1$, $(2^{2b+(n_b + \dots + n_x) - 1} - 2^{b+(n_b + \dots + n_x)}) (R_3 + \dots + R_{n_1-2}) R_{n_1-1}$

* Pour $n_2, \dots, n_{x-1} = 1$, $n_x \geq 1$, $(2^{2x} - 2^{x+1}) 2^{n_x-1} (R_3 + \dots + R_{n_1-2}) R_{n_1-1}$

E3:

Soit n_k , le dernier indice de valeur ≥ 2 dans la notation de A_{n_1, \dots, n_x} . Le coefficient placé devant

$R_{n_1, \dots, n_{k-1}}$ est une exception qui obéit à la règle E3

Exemple: pour $A_{3, 5, 2, 1, 3, 1, 1}$, $k=5$ (cinquième indice) de valeur $n_k=3$, ainsi, le coefficient placé devant $R_{3, 5, 2, 1, 2}$ est une exception

$$[5.2^{(n_1 + \dots + n_k) + 2(x-k-1)} - 3(2^{(n_2 + \dots + n_k)} + 2^{(n_3 + \dots + n_k)} + \dots + 2^{(n_k)}) (2^{2(x-k)+1} - 2^{x-k}) - (3 + (R_{n_1, \dots, n_{k-1}, 1} + \dots + R_{n_1, \dots, n_{k-2}})) (2^{2(x-k+1)} - 2^{x-k+2}) - 5.2^{(n_1 + \dots + n_k) + x - k - 3}] R_{n_1, \dots, n_{k-1}}$$

E4:

$$[(2^{(n_2 + \dots + n_k)} + 2^{(n_3 + \dots + n_k)} + \dots + 2^{(n_k)}) (2^{2(x-k)+1} - 2^{x-k}) + (2^{2(x-k+1)} - 2^{x-k+2})] R_{n_1, \dots, n_{k-1}} (R_3 + \dots + R_{n_1-1})$$

E5:

[termes identiques à ceux du crochet précédent, sauf que les + deviennent des -] $R_{n_1, \dots, n_{k-1}}$

$1 \times R_{n_1}$

E6:

$$[(2^n_k)(2^{2(x-k)+1} - 2^{x-k}) + (2^{2(x-k+1)} - 2^{x-k+2})] R_{n_1, \dots, n_{k-1}} (R_{n_1, \dots, n_{k-2}}, 1 + \dots + R_{n_1, \dots, n_{k-1-1}})$$

E7

[termes identiques à ceux du crochet précédent, sauf que les + deviennent des -] R_{n_1, \dots, n_k}
 $1 \times R_{n_1, \dots, n_{k-1}}$

Soit R_{n_1, n_2, \dots, n_x} , la racine caractéristique de A_{n_1, n_2, \dots, n_x} tel que $R_2=3$, $R_3=\sqrt{21}$, $R_4=\sqrt{(14\sqrt{21}+30)}$, et ainsi de suite. Nous avons la formule générale suivante:

$$R_{n_1, n_2, \dots, n_x} = \left[5.2^{(n_1 + \dots + n_x)-3} (R_2 + \dots + R_{n_1-2}) + (5.2^{(n_1 + \dots + n_x)-3} - 3.2^{(n_2 + \dots + n_x)}) (R_{n_1, 1} + \dots + R_{n_1, n_2-2}) + \dots + (5.2^{(n_1 + \dots + n_x)-3} - 3(2^{(n_2 + \dots + n_x)} + 2^{(n_3 + \dots + n_x)} + \dots + 2^{(n_{x-1} + n_x)})) (A_{n_1, \dots, n_{x-2}, 1_{x-1} + \dots + A_{n_1, \dots, n_{x-2}, n_{x-1-2}}) \right] + \left[(15.2^{(n_1 + \dots + n_x)-3} - (3 + (R_3 + \dots + R_{n_1-2})) 2^{(n_2 + \dots + n_x)+2})^{E1 \text{ et } E2} R_{n_1-1} + (15.2^{(n_1 + \dots + n_x)-3} - 9(2^{(n_2 + \dots + n_x)}) - (3 + (R_{n_1, 1} + \dots + R_{n_1, n_2-2})) 2^{(n_3 + \dots + n_x)+2}) R_{n_1, n_2-1} + \dots + (15.2^{(n_1 + \dots + n_x)-3} - 9(2^{(n_2 + \dots + n_x)} + 2^{(n_3 + \dots + n_x)} + \dots + 2^{(n_{x-1} + n_x)}) - (3 + (R_{n_1, \dots, n_{x-2}, 1} + \dots + R_{n_1, \dots, n_{x-1-2}})) 2^{(n_x)+2})^{E3} R_{n_1, \dots, n_{x-1-1}} \right] - \left[(5.2^{n_1 + 2n_2 + (n_3 + \dots + n_x)-3} - (3 + (R_3 + \dots + R_{n_1-1})) 2^{2n_2 + (n_3 + \dots + n_x)} - 5.2^{(n_1 + \dots + n_x)-3}) R_{n_1} + (5.2^{n_1 + n_2 + 2n_3 + (n_4 + \dots + n_x)-3} - 3.2^{n_2 + 2n_3 + (n_4 + \dots + n_x)} - (3 + (R_{n_1, 1} + \dots + R_{n_1, n_2-1})) 2^{2n_3 + (n_4 + \dots + n_x)} - 5.2^{(n_1 + \dots + n_x)-3} + 3(2^{(n_2 + \dots + n_x)})) R_{n_1, n_2} + \dots + (3 \times 2^{-1} \times 5.2^{(n_1 + \dots + n_{x-1}) + 2n_x - 3} - 3 \times 2^{-1} \times (2^{(n_2 + \dots + n_{x-1}) + 2n_x} + \dots + 2^{2n_x}) - 3 \times 2^{-1} \times (R_{n_1, \dots, n_{x-2}, 1} + \dots + R_{n_1, \dots, n_{x-1-1}}) 2^{2n_x} - 5.2^{(n_1 + \dots + n_x)-3} + 3(2^{(n_2 + \dots + n_x)} + \dots + 2^{n_{x-1} + n_x})) R_{n_1, \dots, n_{x-1}} \right] + \left[(- (R_3 + \dots + R_{n_1-1}) \left[(2^{(n_2 + \dots + n_x)} (R_{n_1, 1} + \dots + R_{n_1, n_2-2}) + (3.2^{(n_2 + \dots + n_x)} + 2^{(n_3 + \dots + n_x)+2}) R_{n_1, n_2-1} - (2^{n_2 + 2n_3 + (n_4 + \dots + n_x)} + 2^{2n_3 + (n_4 + \dots + n_x)} - 2^{(n_2 + \dots + n_x)}) R_{n_1, n_2} \right) + (2^{(n_2 + \dots + n_x)} + 2^{(n_3 + \dots + n_x)}) (R_{n_1, n_2}, 1 + \dots + R_{n_1, n_2, n_3-2}) + (3(2^{(n_2 + \dots + n_x)} + 2^{(n_3 + \dots + n_x)}) + 2^{(n_4 + \dots + n_x)+2}) R_{n_1, n_2, n_3-1} - (2^{n_2 + n_3 + 2n_4 + (n_5 + \dots + n_x)} + 2^{n_3 + 2n_4 + (n_5 + \dots + n_x)} + 2^{2n_4 + (n_5 + \dots + n_x)} - (2^{(n_2 + \dots + n_x)} + 2^{(n_3 + \dots + n_x)})) R_{n_1, n_2, n_3} \right) + \dots + (2^{(n_2 + \dots + n_x)} + \dots + 2^{(n_{x-1} + n_x)}) (R_{n_1, \dots, n_{x-2}, 1} + \dots + R_{n_1, \dots, n_{x-1-2}}) + (3(2^{(n_2 + \dots + n_x)} + 2^{(n_{x-1} + n_x)}) + 2^{n_x+2})^{E4} R_{n_1, \dots, n_{x-1-1}} - (3 \times 2^{-1} \times (2^{(n_2 + \dots + n_{x-1}) + 2n_x} + \dots + 2^{2n_x}) - (2^{(n_2 + \dots + n_x)} + \dots + 2^{n_{x-1} + n_x})) R_{n_1, \dots, n_{x-1}}) \right] + \left[\text{termes identiques à ceux du crochet précédent, sauf que les + deviennent des - et l'exception E4 est remplacée par E5} \right])$$

$$\begin{aligned}
& + \dots + \left(-(R_{n_1, \dots, n_{x-3}, 1} + \dots + R_{n_1, \dots, n_{x-2}, 1}) \left[2^{n_{x-1} + n_x} (R_{n_1, \dots, n_{x-2}, 1} + \dots + R_{n_1, \dots, n_{x-1}, 2}) + \right. \right. \\
& \left. \left. (3 \cdot 2^{n_{x-1} + n_x} + 2^{n_x + 2})^{E6} R_{n_1, \dots, n_{x-1}, 1} - (3 \cdot 2^{-1} \times (2^{n_{x-1} + 2n_x} + 2^{2n_x}) - 2^{n_{x-1} + n_x}) R_{n_1, \dots, n_{x-1}} \right] \right. \\
& \left. + R_{n_1, \dots, n_{x-2}} \left[\text{termes identiques à ceux du crochet précédent, sauf que les } + \text{ deviennent des } - \right. \right. \\
& \left. \left. \text{et l'exception E6 est remplacée par E7} \right] \right) \\
& + (R_{n_1, \dots, n_{x-1}, 1} + \dots + R_{n_1, \dots, n_{x-2}}) \left[(5 \cdot 2^{(n_1 + \dots + n_x) - 3} - 3(2^{(n_2 + \dots + n_x)} + \dots + 2^{(n_x)})) + (-2^{(n_2 + \dots + n_x)}) \right. \\
& \left. + \dots + 2^{(n_x)} (R_3 + \dots + R_{n_1 - 1}) + (2^{(n_2 + \dots + n_x)} - \dots - 2^{(n_x)}) R_{n_1} \right) + \dots + (-2^{n_x} (R_{n_1, \dots, n_{x-2}, 1} + \dots \\
& + R_{n_1, \dots, n_{x-1}, 1}) + 2^{n_x} R_{n_1, \dots, n_{x-1}}) \\
& + 2R_{n_1, \dots, n_{x-1}} \left[(5 \cdot 2^{(n_1 + \dots + n_x) - 3} - 3(2^{(n_2 + \dots + n_x)} + \dots + 2^{(n_x)}) - 3) + (-2^{(n_2 + \dots + n_x)} + \dots + 2^{(n_x)} + 1) \right. \\
& \left. (R_3 + \dots + R_{n_1 - 1}) + (2^{(n_2 + \dots + n_x)} - \dots - 2^{(n_x)} + 1) R_{n_1} \right) + \dots + (-2^{n_x - 1} (R_{n_1, \dots, n_{x-2}, 1} + \dots \\
& + R_{n_1, \dots, n_{x-1}, 1}) + 2^{n_x} R_{n_1, \dots, n_{x-1}}) \\
& - 2R_{n_1, \dots, n_{x-1}} (R_{n_1, \dots, n_{x-1}, 1} + \dots + R_{n_1, \dots, n_{x-2}})
\end{aligned}$$

La formule pour R_{n_1} est singulière, car pour $x=1$, le premier terme est 3 et non 1 ce qui a pour conséquence que la formule générale ne s'applique plus parfaitement sans modification. La formule appropriée est la suivante.

$$\begin{aligned}
R_{n_1} &= (R_2 + \dots + R_{n_1 - 2}) (5 \cdot 2^{n_1 - 3}) + 2R_{n_1 - 1} (5 \cdot 2^{n_1 - 3} - 3) - 2R_{n_1 - 1} (R_3 + \dots + R_{n_1 - 2}) \\
&\text{avec } R_3 = \sqrt{21} \text{ (exception à la formule ci-dessus)}
\end{aligned}$$

Exceptions de la formule de R_{n_1, n_2, \dots, n_x} :

E1:

Soit n_b , le premier indice de valeur ≥ 2 dans la notation de R_{n_1, \dots, n_x} . On a $b \geq 2$.

Exemple: pour $R_{3, 1, 1, 5, 2, 3}$, $b=4$ (quatrième indice) de valeur $n_b=5$

$$\begin{aligned}
& * \text{Pour } n_{b+1}, \dots, n_x \geq 1, 3 \leq b \leq x-1, [(5 \cdot 2^{n_1 - 4} - 3) 2^{2b} - (5 \cdot 2^{n_1 - 4} - 6) 2^b] 2^{(n_b + \dots + n_x) - 1}, \\
& [3 \cdot 2^{2b} + 3 \cdot 2^{b-1}] 2^{(n_b + \dots + n_x) - 1}, \text{ pour } n_1 = 3
\end{aligned}$$

Pour $n_{b+1}, \dots, n_x \geq 1, b=2$, termes notés dans la formule ci-dessus. Pour $n_1 = 3$, on a $9 \cdot 2^{(n_2 + \dots + n_x)}$
 $) R_2 = 27 \cdot 2^{(n_2 + \dots + n_x)}, R_2 = 3$

$$\begin{aligned}
& * \text{Pour } n_2, \dots, n_{x-1} = 1, n_x \geq 2, (5 \cdot 2^{2x+n_1-4} - 5 \cdot 2^{x+n_1-4} - 3 \cdot 2^{2x} + 3 \cdot 2^{x+1}) 2^{n_x - 1}, \\
& (3 \cdot 2^{2x} + 3 \cdot 2^{x-1}) 2^{n_x - 1} \text{ pour } n_1 = 3
\end{aligned}$$

$$\begin{aligned}
& * \text{Pour } n_2, \dots, n_x = 1, (3 \cdot 2^{-1} \times 5 \cdot 2^{2x+n_1-4} - 5 \cdot 2^{x+n_1-4} - 3 \cdot 2^{-1} \times 3 \cdot 2^{2x} + 3 \cdot 2^{x+1}), \\
& (3 \cdot 2^{-1} \times 3 \cdot 2^{2x} + 3 \cdot 2^{x-1}) \text{ pour } n_1 = 3
\end{aligned}$$

E2:

$$*n_{b+1}, \dots, n_x \geq 1, 2 \leq b \leq x-1, (2^{2b+(n_b+\dots+n_x)-1} - 2^{b+(n_b+\dots+n_x)})(R_3+\dots+R_{n_1-2})R_{n_1-1}$$

$$* \text{ Pour } n_2, \dots, n_{x-1} = 1, n_x \geq 2, (2^{2x} - 2^{x+1})2^{n_x-1}(R_3+\dots+R_{n_1-2})R_{n_1-1}$$

$$* \text{ Pour } n_2, \dots, n_x = 1, (3 \times 2^{-1} \times 2^{2x} - 2^{x+1})2^{n_x-1}(R_3+\dots+R_{n_1-2})R_{n_1-1}$$

E3:

* Soit n_k , le dernier indice de valeur ≥ 2 dans la notation de R_{n_1, \dots, n_x} . Le coefficient placé devant $R_{n_1, \dots, n_{k-1}}$ est une exception qui obéit à la règle E3

Exemple: pour $R_{3,5,2,1,3,1,1}$, $k=5$ (cinquième indice) de valeur $n_k=3$, ainsi, le coefficient placé devant $R_{3,5,2,1,2}$ est une exception

$$[3 \times 2^{-1} \times 5.2^{(n_1+\dots+n_k)+2(x-k-1)} - 3(2^{(n_2+\dots+n_k)} + 2^{(n_3+\dots+n_k)} + \dots + 2^{(n_k)}) (3 \times 2^{-1} \times 2^{2(x-k)+1} - 2^{x-k}) - (3+(R_{n_1, \dots, n_{k-1}, 1} + \dots + R_{n_1, \dots, n_{k-2}})) (3 \times 2^{-1} \times 2^{2(x-k+1)} - 2^{x-k+2}) - 5.2^{(n_1+\dots+n_k)+x-k-3}] R_{n_1, \dots, n_{k-1}}$$

E4:

$$[(2^{(n_2+\dots+n_k)} + 2^{(n_3+\dots+n_k)} + \dots + 2^{(n_k)}) (3 \times 2^{-1} \times 2^{2(x-k)+1} - 2^{x-k}) + (3 \times 2^{-1} \times 2^{2(x-k+1)} - 2^{x-k+2})] R_{n_1, \dots, n_{k-1}}(R_3+\dots+R_{n_1-1})$$

E5:

$$[\text{termes identiques à ceux du crochet précédent, sauf que les } + \text{ deviennent des } -] R_{n_1, \dots, n_{k-1}} \times R_{n_1}$$

E6:

$$[(2^{n_k}) (3 \times 2^{-1} \times 2^{2(x-k)+1} - 2^{x-k}) + (3 \times 2^{-1} \times 2^{2(x-k+1)} - 2^{x-k+2})] R_{n_1, \dots, n_{k-1}}(R_{n_1, \dots, n_{k-2}, 1} + \dots + R_{n_1, \dots, n_{k-1}-1})$$

E7

$$[\text{termes identiques à ceux du crochet précédent, sauf que les } + \text{ deviennent des } -] R_{n_1, \dots, n_{k-1}} \times R_{n_1, \dots, n_{k-1}}$$

En sommant les différentes aires jusqu'à l'infini, tout en tenant compte de la symétrie (8 sous-zones identiques), on trouve la surface totale de la zone $N=1$.

Soient $x=n_1=n_2=\dots=n_x$, tendent vers l'infini

Dans la formule suivante, certaines parenthèses et crochets seront colorés, cela afin de mieux visualiser les différentes parties et rendre ainsi la présentation globale plus esthétique.

Pour la formule ci-dessous, nous posons l'opérateur $S(\)$ qui symbolise des sommes de sommes (ou de simples sommes). Le premier terme, R_x , dans la parenthèse représente la partie variable de la somme. Chaque sous-parenthèse représente l'opération appliquée à chaque terme successif de la somme, ainsi que le dernier terme. Plusieurs sous-parenthèses successives (séparées par une virgule) représentent une somme dont chaque terme est la somme définie par la parenthèse précédente (de gauche à droite).

exemples:

$$\text{somme simple: } R_3 + (1/2)R_4 + \dots + (1/2^{n_1-5})R_{n_1-2} = S(R_3, (1/2, n_1-2))$$

$$\text{somme de somme: } [R_{31} + (1/2)R_{32} + \dots + (1/2^{n_2-3})R_{3n_2-2}] + (1/2)[R_{41} + (1/2)R_{42} + \dots + (1/2^{n_2-3})R_{4n_2-2}] + \dots + (1/2^{n_1-3})[R_{n_11} + (1/2)R_{n_12} + \dots + (1/2^{n_2-3})R_{n_1n_2-2}] = S(R_{31}, (1/2, n_2-2), (1/2, n_1))$$

cas particuliers:

$$R_3R_4 + (1/2)R_4R_5 + \dots + (1/2^{n_1-4})R_{n_1-1}R_{n_1} = S(R_3R_4, (1/2, n_1))$$

$$R_3R_4 + (1/2)(R_3+R_4)R_5 + \dots + (1/2^{n_1-4})(R_3+R_4+\dots+R_{n_1-1})R_{n_1} = S(R_3R_4, (1, n_1-1)(1/2, n_1))$$

Surface de la zone $N=1$ =

$$8 \left[3x(x+4)/50 + \left(\left[(x-1/5.2^3) S(R_3, (1/2, n_1-2)) - (x-1/5.2^{n_1-1}) S(R_3, (1, n_1-2)) \right] + \left[(x-2/5.2^4) S(R_{31}, (1/2, n_2-2), (1/2, n_1)) - (x-2/5.2^{n_2+2}) S(R_{31}, (1, n_2-2), (1/2, n_1)) \right] - 3 \left((x-2/25.2^4) S(R_{31}, (1/2, n_2-2), (1/4, n_1)) - (x-2/25.2^{n_2+2}) S(R_{31}, (1, n_2-2), (1/4, n_1)) \right) \right] + \dots + \left[(1/5.2^{x+1}) S(R_{311\dots 1_{x-1}}, (1/2, n_{x-1}-2), (1/2, n_{x-2}), \dots, (1/2, n_1)) - (x-2/5.2^{n_{x-1}+x+2}) S(R_{311\dots 1_{x-1}}, (1, n_{x-1}-2), (1/2, n_{x-2}), \dots, (1/2, n_1)) \right] - 3 \left((1/25.2^{x+1}) S(R_{311\dots 1_{x-1}}, (1/2, n_{x-1}-2), (1/2, n_{x-2}), \dots, (1/4, n_1)) - (1/25.2^{n_{x-1}+x-1}) S(R_{311\dots 1_{x-1}}, (1, n_{x-1}-2), (1/2, n_{x-2}), \dots, (1/4, n_1)) \right) + \left((1/25.2^{x+2}) S(R_{311\dots 1_{x-1}}, (1/2, n_{x-1}-2), (1/2, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1)) - (1/25.2^{n_{x-1}+x}) S(R_{311\dots 1_{x-1}}, (1, n_{x-1}-2), (1/2, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1)) \right) + \dots + \left((1/25.2^{2x-2}) S(R_{311\dots 1_{x-1}}, (1/2, n_{x-1}-2), (1/4, n_{x-2}), \dots, (1/4, n_1)) - (1/25.2^{n_{x-1}+2x-4}) S(R_{311\dots 1_{x-1}}, (1, n_{x-1}-2), (1/4, n_{x-2}), \dots, (1/4, n_1)) \right) \right] + \left((1/5.2^4)(2x^2-2) S(R_3, (1/2, n_1-1)) - 3(1/25.2^3)(x^2-x) S(R_3, (1/4, n_1-1)) - (1/25.2^5)(x^2-x) S(R_4R_3, (1/4, n_1-1)(1, n_1-2)) \right] + \left[(1/5.2^5)(2(x-1)^2-2) S(R_{31}, (1/2, n_2-1), (1/2, n_1)) - 3 \left((1/25.2^5) (2(x-1)^2-2) S(R_{31}, (1/2, n_2-1), (1/4, n_1)) + (1/25.2^5)((x-1)^2-(x-1)) S(R_{31}, (1/4, n_2-1), (1/4, n_1)) \right) \right] - (1/25.2^7)((x-1)^2-(x-1)) S(R_{32}R_{31}, (1/4, n_2-1)(1, n_2-2), (1/4, n_1)) \right] + \dots + \left[(1/5.2^{x+2})(2(2)^2-2) S(R_{311\dots 1_{x-1}}, (1/2, n_{x-1}-1), (1/2, n_{x-2}), \dots, (1/2, n_1)) - 3 \left((1/25.2^{x+2})(2(2)^2-2) \left(S(R_{311\dots 1_{x-1}}, (1/2, n_{x-1}-1), (1/2, n_{x-2}), \dots, (1/2, n_2), (1/4, n_1)) + (1/2) S(R_{311\dots 1_{x-1}}, (1/2, n_{x-1}-1), (1/2, n_{x-2}), \dots, (1/2, n_3), (1/4, n_2), (1/4, n_1)) \right) + \dots + (1/2^{x-3}) S(R_{311\dots 1_{x-1}}, (1/2, n_{x-1}-1), (1/4, n_{x-2}), \dots, (1/4, n_1)) \right) \right] + (1/25.2^{2x-1})((2)^2-2) S(R_{311\dots 1_{x-1}}, (1/4, n_{x-1}-1), (1/4, n_{x-2}), \dots, (1/4, n_1)) - (1/25.2^{2x+1})((2)^2-2) S(R_{311\dots 2_{x-1}} R_{311\dots 1_{x-1}}, (1/4, n_{x-1}-1)(1, n_{x-1}-2), (1/4, n_{x-2}), \dots, (1/4, n_1)) \right]]$$

$$\begin{aligned}
& - \left(\left[\left((n2-1)/5.2^2 \right) S(R3, (1/2, n1)) - 3(n2/25.2^2) S(R3, (1/4, n1)) - (n2/25.2^4) S(R4R3, (1/4, n1) \right. \right. \\
& (1, n1-1) \left. \right] (x-1) + \left[\left((n3-1)/5.2^3 \right) S(R31, (1/2, n2), (1/2, n1)) - 3 \left(\left((n3-1)/25.2^3 \right) S(R31, (1/2, n2), \right. \right. \\
& (1/4, n1)) + (n3/25.2^4) S(R31, (1/4, n2), (1/4, n1)) \left. \right] - (n3/25.2^6) S(R32R31, (1/4, n2)(1, n2-1), \\
& (1/4, n1)) \left. \right] (x-2) + \dots + \left[\left((nx-1)/5.2^x \right) S(R311\dots 1_{x-1}, (1/2, nx-1), (1/2, nx-2), \dots, (1/2, n1)) - \right. \\
& 3 \left(\left((nx-1)/25.2^x \right) \left(S(R311\dots 1_{x-1}, (1/2, nx-1), (1/2, nx-2), \dots, (1/2, n2), (1/4, n1)) + (1/2) \right. \right. \\
& S(R311\dots 1_{x-1}, (1/2, nx-1), (1/2, nx-2), \dots, (1/2, n3), (1/4, n2), (1/4, n1)) + \dots + (1/2^{x-3}) S(R311\dots 1_{x-1}, \\
& (1/2, nx-1), (1/4, nx-2), \dots, (1/4, n1)) \left. \right] + (nx/25.2^{2x-2}) S(R311\dots 1_{x-1}, (1/4, nx-1), (1/4, nx-2), \dots, \\
& (1/4, n1)) \left. \right] - (nx/25.2^{2x}) S(R311\dots 2_{x-1} R311\dots 1_{x-1}, (1/4, nx-1)(1, nx-1-1), (1/4, nx-2), \dots, (1/4, n1)) \\
& \left. \right] (1) \\
& + \left(\left[- \left(\left[(1/25.2^6) S(R3R41, (1/2, n2-2), (1, n1-1)(1/4, n1)) - (1/25.2^{n2+4}) S(R3R41, (1, n2-2), \right. \right. \right. \\
& (1, n1-1)(1/4, n1)) - \left(\left((n3-1)/25.2^5 \right) S(R3R41, (1/2, n2), (1, n1-1)(1/4, n1)) + (n3/25.2^6) \right. \\
& S(R3R41, (1/4, n2), (1, n1-1)(1/4, n1)) \left. \right] (x-2) + \dots + \left[(1/25.2^{x+3}) \left(S(R3R411\dots 1_{x-1}, (1/2, nx-1-2), \right. \right. \\
& (1/2, nx-2), \dots, (1/2, n2), (1, n1-1)(1/4, n1)) + (1/2) S(R3R411\dots 1_{x-1}, (1/2, nx-1-2), (1/2, nx-2), \dots, \\
& (1/4, n2), (1, n1-1)(1/4, n1)) + \dots + (1/2^{x-3}) S(R3R411\dots 1_{x-1}, (1/2, nx-1-2), (1/4, nx-2), \dots, (1/4, n2), \\
& (1, n1-1)(1/4, n1)) \left. \right] - (1/25.2^{nx-1+x+1}) \left(S(R3R411\dots 1_{x-1}, (1, nx-1-2), (1/2, nx-2), \dots, (1/2, n2), (1, \right. \\
& n1-1)(1/4, n1)) + (1/2) S(R3R411\dots 1_{x-1}, (1, nx-1-2), (1/2, nx-2), \dots, (1/4, n2), (1, n1-1)(1/4, n1)) + \dots \\
& + (1/2^{x-3}) S(R3R411\dots 1_{x-1}, (1, nx-1-2), (1/4, nx-2), \dots, (1/4, n2), (1, n1-1)(1/4, n1)) \left. \right] - \left((nx- \right. \\
& 1)/25.2^{x+2} \left. \right) \left(S(R3R411\dots 1_{x-1}, (1/2, nx-1), (1/2, nx-2), \dots, (1/2, n2), (1, n1-1)(1/4, n1)) + \right. \\
& (1/2) S(R3R411\dots 1_{x-1}, (1/2, nx-1), (1/2, nx-2), \dots, (1/4, n2), (1, n1-1)(1/4, n1)) + \dots + (1/2^{x- \\
& 3}) S(R3R411\dots 1_{x-1}, (1/2, nx-1), (1/4, nx-2), \dots, (1/4, n2), (1, n1-1)(1/4, n1)) \left. \right] + (nx/25.2^{2x}) \\
& S(R3R411\dots 1_{x-1}, (1/4, nx-1), (1/4, nx-2), \dots, (1/4, n2), (1, n1-1)(1/4, n1)) \left. \right] (1) \\
& + \left((1/25.2^7) (2(x-1)^2-2) S(R3R41, (1/2, n2-1), (1, n1-1)(1/4, n1)) + (1/25.2^7) ((x-1)^2-(x-1)) \right. \\
& S(R3R41, (1/4, n2-1), (1, n1-1)(1/4, n1)) + \dots + \left[(1/25.2^{x+4}) (2(2)^2-2) \left(S(R3R411\dots 1_{x-1}, (1/2, \right. \right. \\
& nx-1-1), (1/2, nx-2), \dots, (1/2, n2), (1, n1-1)(1/4, n1)) + (1/2) S(R3R411\dots 1_{x-1}, (1/2, nx-1-1), (1/2, nx-2) \\
& \dots, (1/4, n2), (1, n1-1)(1/4, n1)) + \dots + (1/2^{x-3}) S(R3R411\dots 1_{x-1}, (1/2, nx-1-1), (1/4, nx-2), \dots, (1/4, \\
& n2), (1, n1-1)(1/4, n1)) \left. \right] + (1/25.2^{2x+1}) ((2)^2-2) S(R3R411\dots 1_{x-1}, (1/4, nx-1-1), (1/4, nx-2), \dots, (1/4, \\
& n2), (1, n1-1)(1/4, n1)) \left. \right] (1) \\
& + \left(\left[(1/25.2^4) S(R3R31, (1/2, n2-2), (1/4, n1)) - (1/25.2^{n2+2}) S(R3R31, (1, n2-2), (1/4, n1)) - \right. \right. \\
& \left(\left((n3-1)/25.2^3 \right) S(R3R31, (1/2, n2), (1/4, n1)) - (n3/25.2^4) S(R3R31, (1/4, n2), (1/4, n1)) \right. \left. \right] (x-2) \\
& + \dots + \left[(1/25.2^{x+1}) \left(S(R3R311\dots 1_{x-1}, (1/2, nx-1-2), (1/2, nx-2), \dots, (1/2, n2), (1/4, n1)) - \right. \right. \\
& (1/2) S(R3R311\dots 1_{x-1}, (1/2, nx-1-2), (1/2, nx-2), \dots, (1/4, n2), (1/4, n1)) - \dots - (1/2^{x-3}) S(R3R311\dots 1_{x- \\
& 1, (1/2, nx-1-2), (1/4, nx-2), \dots, (1/4, n2), (1/4, n1)) \left. \right] - (1/25.2^{nx-1+x-1}) \left(S(R3R311\dots 1_{x-1}, (1, nx-1- \\
& 2), (1/2, nx-2), \dots, (1/2, n2), (1/4, n1)) - (1/2) S(R3R311\dots 1_{x-1}, (1, nx-1-2), (1/2, nx-2), \dots, (1/4, n2), \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{1}{4}, n_1 \right) \dots - \left(\frac{1}{2}^{x-3} \right) S(R3R311 \dots 1_{x-1}, (1, n_{x-1-2}), (1/4, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1)) - \left(\left(\frac{n_x-1}{25 \cdot 2^x} \right) \left(S(R3R311 \dots 1_{x-1}, (1/2, n_{x-1}), (1/2, n_{x-2}), \dots, (1/2, n_2), (1/4, n_1)) - (1/2) S(R3R311 \dots 1_{x-1}, (1/2, n_{x-1}), (1/2, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1)) \right) \dots - \left(\frac{1}{2}^{x-3} \right) S(R3R311 \dots 1_{x-1}, (1/2, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1)) \right) - \left(\frac{n_x}{25 \cdot 2^{2x-2}} \right) S(R3R311 \dots 1_{x-1}, (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1)) \right) \Big] (1) \\
& + \left(\left(\frac{1}{25 \cdot 2^5} \right) (2(x-1)^2-2) S(R3R31, (1/2, n_2-1), (1/4, n_1)) - \left(\frac{1}{25 \cdot 2^5} \right) ((x-1)^2-(x-1)) S(R3R31, (1/4, n_2-1), (1/4, n_1)) \right) \dots - \left[\left(\frac{1}{25 \cdot 2^{x+2}} \right) (2(2)^2-2) \left(S(R3R311 \dots 1_{x-1}, (1/2, n_{x-1-1}), (1/2, n_{x-2}), \dots, (1/2, n_2), (1/4, n_1)) - (1/2) S(R3R311 \dots 1_{x-1}, (1/2, n_{x-1-1}), (1/2, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1)) \right) \dots - \left(\frac{1}{2}^{x-3} \right) S(R3R311 \dots 1_{x-1}, (1/2, n_{x-1-1}), (1/4, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1)) \right) - \left(\frac{1}{25 \cdot 2^{2x-1}} \right) ((2)^2-2) S(R3R311 \dots 1_{x-1}, (1/4, n_{x-1-1}), (1/4, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1)) \Big] \Big] \text{ (crochets roses ci-contre = terme pour } x=3, \text{ on a } R311 \dots 2_{x-2} 1_{x-1} = R4 \text{ pour } x=3)
\end{aligned}$$

$$\begin{aligned}
& + \dots + \\
& \left[- \left(\left[\left(\frac{1}{25 \cdot 2^{2x}} \right) S(R311 \dots 1_{x-2} R311 \dots 2_{x-2} 1_{x-1}, (1/2, n_{x-1-2}), (1, n_{x-2-1})(1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) - \left(\frac{1}{25 \cdot 2^{n_{x-1}+2x-2}} \right) S(R311 \dots 1_{x-2} R311 \dots 2_{x-2} 1_{x-1}, (1, n_{x-1-2}), (1, n_{x-2-1})(1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) \right) - \left(\left(\frac{n_x-1}{25 \cdot 2^{2x-1}} \right) S(R311 \dots 1_{x-2} R311 \dots 2_{x-2} 1_{x-1}, (1/2, n_{x-1}), (1, n_{x-2-1})(1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) \right) + \left(\frac{n_x}{25 \cdot 2^{2x}} \right) S(R311 \dots 1_{x-2} R311 \dots 2_{x-2} 1_{x-1}, (1/4, n_{x-1}), (1, n_{x-2-1})(1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) \right) \Big] (1) \right) \\
& + \left(\left(\frac{1}{25 \cdot 2^{2x+1}} \right) (2(2)^2-2) S(R311 \dots 1_{x-2} R311 \dots 2_{x-2} 1_{x-1}, (1/2, n_{x-1-1}), (1, n_{x-2-1})(1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) + \left(\frac{1}{25 \cdot 2^{2x+1}} \right) ((2)^2-2) S(R311 \dots 1_{x-2} R311 \dots 2_{x-2} 1_{x-1}, (1/4, n_{x-1-1}), (1, n_{x-2-1})(1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) \right) \Big] \\
& + \left(\left[\left(\frac{1}{25 \cdot 2^{2x-2}} \right) S(R311 \dots 1_{x-2} R311 \dots 1_{x-2} 1_{x-1}, (1/2, n_{x-1-2}), (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) - \left(\frac{1}{25 \cdot 2^{n_{x-1}+2x-4}} \right) S(R311 \dots 1_{x-2} R311 \dots 1_{x-2} 1_{x-1}, (1, n_{x-1-2}), (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) \right) - \left(\left(\frac{n_x-1}{25 \cdot 2^{2x-3}} \right) S(R311 \dots 1_{x-2} R311 \dots 1_{x-2} 1_{x-1}, (1/2, n_{x-1}), (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) - \left(\frac{n_x}{25 \cdot 2^{2x-2}} \right) S(R311 \dots 1_{x-2} R311 \dots 1_{x-2} 1_{x-1}, (1/4, n_{x-1}), (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) \right) \Big] (1) \right) \\
& + \left(\left(\frac{1}{25 \cdot 2^{2x-1}} \right) (2(2)^2-2) S(R311 \dots 1_{x-2} R311 \dots 1_{x-2} 1_{x-1}, (1/2, n_{x-1-1}), (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) - \left(\frac{1}{25 \cdot 2^{2x-1}} \right) ((2)^2-2) S(R311 \dots 1_{x-2} R311 \dots 1_{x-2} 1_{x-1}, (1/4, n_{x-1-1}), (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) \right) \Big] \Big] \\
& + \left(\left[\left(\frac{1}{5 \cdot 2^2} \right) S(R3, (1/2, n_1-1)) - \left(\frac{1}{5 \cdot 2^{n_1-1}} \right) S(R3, (1, n_1-1)) \right] + \left[\left(\frac{1}{5 \cdot 2^3} \right) S(R31, (1/2, n_2-1), (1/2, n_1)) - \left(\frac{1}{5 \cdot 2^{n_2+2}} \right) S(R31, (1, n_2-1), (1/2, n_1)) \right) \right] - 3 \left(\frac{1}{25 \cdot 2^3} \right) S(R31, (1/2, n_2-1), (1/4, n_1)) - \left(\frac{1}{25 \cdot 2^{n_2+2}} \right) S(R31, (1, n_2-1), (1/4, n_1)) \right) \Big] + \dots + \left[\left(\frac{1}{5 \cdot 2^{x+1}} \right) S(R311 \dots 1_x, (1/2, n_{x-1}), (1/2, n_x) \right)
\end{aligned}$$

$$\begin{aligned}
& i), \dots, (1/2, n_i)) - (1/5.2^{n_x + x}) S(R311\dots 1_x, (1, n_x), (1/2, n_{x-1}), \dots, (1/2, n_1)) - 3((1/25.2^{x+1}) \\
& S(R311\dots 1_x, (1/2, n_{x-1}), (1/2, n_{x-1}), \dots, (1/4, n_1)) - (1/25.2^{n_x + x}) S(R311\dots 1_x, (1, n_{x-1}), (1/2, n_{x-1}), \dots, \\
& (1/4, n_1)) + ((1/25.2^{x+2}) S(R311\dots 1_x, (1/2, n_{x-1}), (1/2, n_{x-1}), \dots, (1/4, n_2), (1/4, n_1)) - (1/25.2^{n_x} \\
& + x + 1) S(R311\dots 1_x, (1, n_{x-1}), (1/2, n_{x-1}), \dots, (1/4, n_2), (1/4, n_1)) + \dots + ((1/25.2^{2x-1}) S(R311\dots 1_x, \\
& (1/2, n_{x-1}), (1/4, n_{x-1}), \dots, (1/4, n_1)) - (1/25.2^{n_{x-1} + 2x - 2}) S(R311\dots 1_x, (1, n_{x-1}), (1/4, n_{x-1}), \dots, (1/4, \\
& n_1))) \Big) \Big)
\end{aligned}$$

$$\begin{aligned}
& + \Big(\Big[- \Big[(1/25.2^5) S(R3R41, (1/2, n_2-1), (1, n_1-1)(1/4, n_1)) - (1/25.2^{n_2+4}) S(R3R41, (1, n_2-1), (1, \\
& n_1-1)(1/4, n_1)) \Big] + \Big[(1/25.2^3) S(R3R31, (1/2, n_2-1), (1/4, n_1)) - (1/25.2^{n_2+2}) S(R3R31, (1, n_2-1), \\
& (1/4, n_1)) \Big] \Big)
\end{aligned}$$

$$\begin{aligned}
& + \Big(\Big(- \Big[(1/25.2^6) S(R3R411, (1/2, n_3-1), (1/2, n_2), (1, n_1-1)(1/4, n_1)) - (1/25.2^{n_3+5}) S(R3R411, \\
& (1, n_3-1), (1/2, n_2), (1, n_1-1)(1/4, n_1)) \Big] + \Big[(1/25.2^7) S(R3R411, (1/2, n_3-1), (1/4, n_2), (1, n_1-1) \\
& (1/4, n_1)) - (1/25.2^{n_3+6}) S(R3R411, (1, n_3-1), (1/4, n_2), (1, n_1-1)(1/4, n_1)) \Big] + \Big(\Big[(1/25.2^4) \\
& S(R3R311, (1/2, n_3-1), (1/2, n_2), (1/4, n_1)) - (1/25.2^{n_3+3}) S(R3R311, (1, n_3-1), (1/2, n_2), (1/4, \\
& n_1)) \Big] - \Big[(1/25.2^5) S(R3R311, (1/2, n_3-1), (1/4, n_2), (1/4, n_1)) - (1/25.2^{n_3+4}) S(R3R311, (1, n_3-1), \\
& (1/4, n_2), (1/4, n_1)) \Big] \Big) + \Big(- \Big[(1/25.2^7) S(R31R321, (1/2, n_3-1), (1, n_2-1)(1/4, n_2), (1/4, n_1)) - \\
& (1/25.2^{n_3+6}) S(R31R321, (1, n_3-1), (1, n_2-1)(1/4, n_2), (1/4, n_1)) \Big] + \Big[(1/25.2^5) S(R31R311, (1/2, \\
& n_3-1), (1/4, n_2), (1/4, n_1)) - (1/25.2^{n_3+4}) S(R31R311, (1, n_3-1), (1/4, n_2), (1/4, n_1)) \Big] \Big) \Big)
\end{aligned}$$

$$\begin{aligned}
& + \dots + \\
& \Big(\Big(- \Big[(1/25.2^{x+3}) S(R3R411\dots 1_x, (1/2, n_{x-1}), (1/2, n_{x-1}), \dots, (1, n_1-1)(1/4, n_1)) - (1/25.2^{n_x+x+2}) \\
& S(R3R411\dots 1_x, (1, n_{x-1}), (1/2, n_{x-1}), \dots, (1, n_1-1)(1/4, n_1)) \Big] + \Big[(1/25.2^{x+4}) S(R3R411\dots 1_x, (1/2, \\
& n_{x-1}), (1/2, n_{x-1}), \dots, (1/4, n_2), (1, n_1-1)(1/4, n_1)) - (1/25.2^{n_x+x+3}) S(R3R411\dots 1_x, (1, n_{x-1}), (1/2, \\
& n_{x-1}), \dots, (1/4, n_2), (1, n_1-1)(1/4, n_1)) \Big] + \dots + \Big[(1/25.2^{2x+1}) S(R3R411\dots 1_x, (1/2, n_{x-1}), (1/4, n_{x-} \\
& i), \dots, (1/4, n_2), (1, n_1-1)(1/4, n_1)) - (1/25.2^{n_x+2x}) S(R3R411\dots 1_x, (1, n_{x-1}), (1/4, n_{x-1}), \dots, (1/4, \\
& n_2), (1, n_1-1)(1/4, n_1)) \Big] + \Big(\Big[(1/25.2^{x+1}) S(R3R311\dots 1_x, (1/2, n_{x-1}), (1/2, n_{x-1}), \dots, (1/4, n_1)) - \\
& (1/25.2^{n_x+x}) S(R3R311\dots 1_x, (1, n_{x-1}), (1/2, n_{x-1}), \dots, (1/4, n_1)) \Big] - \Big[(1/25.2^{x+2}) S(R3R311\dots 1_x, \\
& (1/2, n_{x-1}), (1/2, n_{x-1}), \dots, (1/4, n_2), (1/4, n_1)) - (1/25.2^{n_x+x+1}) S(R3R311\dots 1_x, (1, n_{x-1}), (1/2, n_{x-} \\
& i), \dots, (1/4, n_2), (1/4, n_1)) \Big] - \dots - \Big[(1/25.2^{2x-1}) S(R3R311\dots 1_x, (1/2, n_{x-1}), (1/4, n_{x-1}), \dots, (1/4, n_2), \\
& (1/4, n_1)) - (1/25.2^{n_x+2x-2}) S(R3R311\dots 1_x, (1, n_{x-1}), (1/4, n_{x-1}), \dots, (1/4, n_2), (1/4, n_1)) \Big] \Big) + \Big(- \\
& \Big(\Big[(1/25.2^{x+4}) S(R31R321\dots 1_x, (1/2, n_{x-1}), (1/2, n_{x-1}), \dots, (1, n_2-1)(1/4, n_2), (1/4, n_1)) - \\
& (1/25.2^{n_x+x+3}) S(R31R321\dots 1_x, (1, n_{x-1}), (1/2, n_{x-1}), \dots, (1, n_2-1)(1/4, n_2), (1/4, n_1)) \Big] +
\end{aligned}$$

$$\begin{aligned}
& \left[(1/25.2^{x+5}) S(R31R321...1x, (1/2, nx-1), (1/2, nx-1), \dots, (1/4, n3), (1, n2-1)(1/4, n2), (1/4, n1)) - \right. \\
& (1/25.2^{nx+x+4}) S(R31R321...1x, (1, nx-1), (1/2, nx-1), \dots, (1/4, n3), (1, n2-1)(1/4, n2), (1/4, n1)) \left. \right] \\
& + \dots + \left[(1/25.2^{2x+2}) S(R31R321...1x, (1/2, nx-1), (1/4, nx-1), \dots, (1/4, n3), (1, n2-1)(1/4, n2), (1/4, \right. \\
& n1)) - (1/25.2^{nx+2x+1}) S(R31R321...1x, (1, nx-1), (1/4, nx-1), \dots, (1/4, n3), (1, n2-1)(1/4, n2), (1/4, \\
& n1)) \left. \right] + \left(\left[(1/25.2^{x+2}) S(R31R311...1x, (1/2, nx-1), (1/2, nx-1), \dots, (1/4, n2), (1/4, n1)) - \right. \right. \\
& (1/25.2^{nx+x+1}) S(R31R311...1x, (1, nx-1), (1/2, nx-1), \dots, (1/4, n2), (1/4, n1)) \left. \right] - \left[(1/25.2^{x+3}) \right. \\
& S(R31R311...1x, (1/2, nx-1), (1/2, nx-1), \dots, (1/4, n3), (1/4, n2), (1/4, n1)) - (1/25.2^{nx+x+2}) \\
& S(R31R311...1x, (1, nx-1), (1/2, nx-1), \dots, (1/4, n3), (1/4, n2), (1/4, n1)) \left. \right] - \dots - \left[(1/25.2^{2x}) \right. \\
& S(R31R311...1x, (1/2, nx-1), (1/4, nx-1), \dots, (1/4, n3), (1/4, n2), (1/4, n1)) - (1/25.2^{nx+2x-1}) \\
& S(R31R311...1x, (1, nx-1), (1/4, nx-1), \dots, (1/4, n3), (1/4, n2), (1/4, n1)) \left. \right]) + \dots + \left(- \left(\right. \right. \\
& \left. \left[(1/25.2^{2x+1}) S(R311...1x-1R31...2x-11x, (1/2, nx-1), (1, nx-1-1)(1/4, nx-1), ((1/4, nx-2), \dots, (1/4, \right. \right. \\
& n1)) - (1/25.2^{nx+2x}) S(R311...1x-1R31...2x-11x, (1, nx-1), (1, nx-1-1)(1/4, nx-1), ((1/4, nx-2), \dots, \\
& (1/4, n1)) \left. \right] + \left(\left[(1/25.2^{2x-1}) S(R311...1x-1R31...1x-11x, (1/2, nx-1), (1/4, nx-1), ((1/4, nx-2), \dots, \right. \right. \\
& (1/4, n1)) - (1/25.2^{nx+2x-2}) S(R311...1x-1R31...1x-11x, (1, nx-1), (1/4, nx-1), ((1/4, nx-2), \dots, (1/4, \\
& n1)) \left. \right]) \left. \right) \left. \right) \\
& - \left(\left[(3/25.2^2) S(R3, (1/4, n1)) + (1/25.2^4) S(R3R4, (1, n1-1)(1/4, n1)) \right] + \left[(3/25.2^4) S(R31, \right. \\
& (1/4, n2), (1/4, n1)) + (1/25.2^4) (S(R3R31, (1/4, n2), (1, n1)(1/4, n1))) + (1/25.2^6) S(R31R32, (1, \\
& n2-1)(1/4, n2), (1/4, n1)) \left. \right] + \dots + \left[(3/25.2^{2x}) S(R311...1x, (1/4, nx), \dots, (1/4, n1)) + (1/25.2^{2x}) (\right. \\
& S(R3R311...1x, (1/4, nx), \dots, (1, n1)(1/4, n1)) + S(R31R311...1x, (1/4, nx), \dots, (1, n2)(1/4, n2), \\
& (1/4, n1)) R311...1x, (1/4, nx), \dots, (1/4, n1)) + \dots + S(R311...1x-1 R311...1x, (1/4, nx), (1, nx-1)(1/4, \\
& nx-1), \dots, (1/4, n2), (1/4, n1)) \left. \right) + (1/25.2^{2x+2}) S(R311...1x R311...2x, (1, nx-i)(1/4, nx), (1/4, nx-1), \\
& \dots, (1/4, n2), (1/4, n1)) \left. \right]) \left. \right]
\end{aligned}$$

Pour une zone $N \geq 2$ (carré de la forme $A2,1, \dots, 1N-1, nN, nN+1, \dots, nX$,

de coté = $a2,1, \dots, 1N-1, nN, nN+1, \dots, nX$): attention les yeux!

$$AB^2 = AC^2 + BC^2$$

$$\begin{aligned}
1 = & \left[a2,1, \dots, 1N + 2 \times \left((a2,1, \dots, 1N-1, 2N + a2,1, \dots, 1N-1, 3N + \dots + a2,1, \dots, 1N-1, nN-1) + \right. \right. \\
& (a2,1, \dots, 1N-1, nN, 1N+1 + a2,1, \dots, 1N-1, nN, 2N+1 + \dots + a2,1, \dots, 1N-1, nN, nN+1-1) + \dots + \\
& (a2,1, \dots, 1N-1, nN, nN+1, \dots, 1X + a2,1, \dots, 1N-1, nN, nN+1, \dots, 2X + \dots + a2,1, \dots, 1N-1
\end{aligned}$$

$$1, n_N, n_{N+1}, \dots, n_x) \Big]^2 + \left[a_{1+2} \times (a_2 + a_{2,1} + \dots + a_{2,1}, \dots, 1_{N-1}) + a_{2,1}, \dots, 1_{N-1}, n_N + a_{2,1}, \dots, 1_{N-1}, n_N, n_{N+1} + \dots + a_{2,1}, \dots, 1_{N-1}, n_N, n_{N+1}, \dots, n_x) \right]^2$$

On trouve la formule générale suivante, valable pour les aires de tous les carrés de la zone $N \geq 2$. Chacun des exposants F1, F2 ..., F5 et E3, E2, ..., E7 (très similaires aux E correspondants, de la zone 1) indique que le terme auquel il est rattaché possède une exception particulière qui remplace ce terme, et dont la formule est donnée plus bas.

$$A_{2,1}, \dots, n_N, n_{N+1}, \dots, n_x =$$

$$\begin{aligned} & \left(\frac{-3.5^{2N} - 25.31(-3)^{N-1}}{7} 2^{2n_N - 2} + \frac{6.5^{2N} - 15.31(-3)^{N-1}}{7} 2^{n_N} \right)^{F1} 2^{(n_{N+1} + \dots + n_x)} \\ & - \left[(175.2^{2n_N} - 105.2^{n_N + 2}) 2^{(n_{N+1} + \dots + n_x)} \right]^{F2} \left[(R_{2,1}, \dots, 1_{N-1} + (-3)R_{2,1}, \dots, 1_{N-2} + \dots + (-3)^{N-3} R_{2,1} \right. \\ & \left. + (2/7)(R_{2,1}, \dots, 1_{N-1}(R_{2,1} + \dots + R_{2,1}, \dots, n_N, n_{N-2}) + (-3)R_{2,1}, \dots, 1_{N-2}(R_{2,1}, \dots, n_N, 1 + \dots + R_{2,1}, \dots, n_{N-3}) \right. \\ & \left. + \dots + (-3)^{N-4} R_{2,1,1} R_{2,1}) \right] + 35.2^{(n_N + \dots + n_x)} (R_{2,1}, \dots, 1_N + \dots + R_{2,1}, \dots, n_{N-2}) \\ & + (105.2^{(n_N + \dots + n_x)} - 21.2^{(n_{N+1} + \dots + n_x)^3})^{F3} R_{2,1}, \dots, n_{N-1} (1 + (2/7)(R_{2,1} + R_{2,1,1} + \dots + R_{2,1}, \dots, 1_{N-1})) \\ & - \left[(35.2^{n_N - 1} - 21) 2^{2n_{N+1} + (n_{N+2} + \dots + n_x)^3 + 1} - 35.2^{(n_N + \dots + n_x)} \right] R_{2,1}, \dots, n_N (1 + (2/7) \\ & (R_{2,1} + R_{2,1,1} + \dots + R_{2,1}, \dots, 1_{N-1})) \\ & + 5.2^{(n_N + \dots + n_x)^3 + 1} (R_{2,1} + R_{2,1,1} + \dots + R_{2,1}, \dots, 1_{N-1}) (R_{2,1}, \dots, 1_N + \dots + R_{2,1}, \dots, n_{N-2}) \\ & - (2^{(n_{N+1} + \dots + n_x)^3})^{F4} R_{2,1}, \dots, n_{N-1} (R_{2,1}, \dots, 1_N + \dots + R_{2,1}, \dots, n_{N-2}) \\ & + (2^{2n_{N+1} + (n_{N+2} + \dots + n_x)^3 + 1})^{F4} R_{2,1}, \dots, n_N (R_{2,1}, \dots, 1_N + \dots + R_{2,1}, \dots, n_{N-1}) \\ & 4 \left[(5.2^{(n_N + \dots + n_x)^3 - 1} - 3.2^{(n_{N+1} + \dots + n_x)^3}) (R_{2,1}, \dots, n_N, 1 + \dots + R_{2,1}, \dots, n_N, n_{N+1} - 2) + \dots + \right. \\ & \left. (5.2^{(n_N + \dots + n_x)^3 - 1} - 3(2^{(n_{N+1} + \dots + n_x)^3} + 2^{(n_{N+2} + \dots + n_x)^3} + \dots + 2^{(n_{x-1} + n_x)^3})) (R_{2,1}, \dots, n_N, \dots, n_{x-2}, 1_{x-1} \right. \\ & \left. + \dots + R_{2,1}, \dots, n_N, \dots, n_{x-2}, n_{x-1} - 2) \right] \left[(R_{2,1} + \dots + R_{2,1}, \dots, 1_{N-1}) + 7/2 \right] + \left[(4(15.2^{(n_N + \dots + n_x)^3 - 1} - \right. \\ & \left. 9.2^{(n_{N+1} + \dots + n_x)^3} - 3.2^{(n_{N+2} + \dots + n_x)^3}) \left[(R_{2,1} + \dots + R_{2,1}, \dots, 1_{N-1}) + 7/2 \right] - 2(R_{2,1}, \dots, n_N, 1 + \dots + R_{2,1}, \dots, n_N, n_{N-2}) \right. \\ & \left. 2^{(n_{N+2} + \dots + n_x)^3} \right) R_{2,1}, \dots, n_N, n_{N-2} - 1 + \dots + (4(15.2^{(n_N + \dots + n_x)^3 - 1} - 9(2^{(n_{N+1} + \dots + n_x)^3} \\ & + 2^{(n_{N+2} + \dots + n_x)^3} + \dots + 2^{(n_{x-1} + n_x)^3}) - 3.2^{(n_x)^3}) \left[(R_{2,1} + \dots + R_{2,1}, \dots, 1_{N-1}) + 7/2 \right] - 2(R_{2,1}, \dots, n_N, \dots, n_{x-2}, 1 + \dots + R_{2,1}, \dots, n_N, \dots, n_{x-1} - 2) \right]^{E3} R_{2,1}, \dots, n_N, \dots, n_{x-1} - 1 \Big] - \left[(4(5.2^{n_N + n_{N+1} + 2n_{N+2} + (n_{N+3} + \dots + n_x)^3 - 1} - 3.2^{2n_{N+2} + (n_{N+3} + \dots + n_x)^3} - 5.2^{(n_N + \dots + n_x)^3 - 1} + 3(2^{(n_{N+1} + \dots + n_x)^3})) \left[(R_{2,1} + \dots + R_{2,1}, \dots, 1_{N-1}) + 7/2 \right] - 2(R_{2,1}, \dots, n_N, 1 + \dots + R_{2,1}, \dots, n_{N+1} - 1) \right]^{2n_{N+3} + (n_{N+4} + \dots + n_x)^3} R_{2,1}, \dots, n_N, n_{N+1} + \dots + (4(5.2^{(n_N + \dots + n_x)^3 - 1} + 2n_x - 1 - 3(2^{(n_{N+1} + \dots + n_x)^3}) + 2n_x + \dots + \end{aligned}$$

$$\begin{aligned}
& 2^{2n_x} - 5 \cdot 2^{\binom{n}{N+\dots+n_x}-1} + 3(2^{\binom{n}{N+1+\dots+n_x}} + \dots + 2^{\binom{n}{x-1+n_x}}) - 2(R_{2,1,\dots,n_N,\dots,n_{x-2},1+\dots} \\
& + R_{2,1,\dots,n_N,\dots,n_{x-1}-1}) 2^{2n_x} R_{2,1,\dots,n_N,\dots,n_{x-1}} \Big] \\
& + 2 \Big[\left(- (R_{2,1,\dots,1N+\dots} + R_{2,1,\dots,n_{N-1}}) \left[(2^{\binom{n}{N+1+\dots+n_x}} (R_{2,1,\dots,n_N,1+\dots} + R_{2,1,\dots,n_N,n_{N+1}-2}) \right. \right. \right. \\
& + (3 \cdot 2^{\binom{n}{N+1+\dots+n_x}} + 2^{\binom{n}{N+2+\dots+n_x}+2}) R_{2,1,\dots,n_N,n_{N-1}} - (2^{\binom{n}{N+1}+2n_{N+2}+\binom{n}{N+3+\dots+n_x}} + 2^{2n_{N+2}+} \\
& \left. \left. \left. \binom{n}{N+3+\dots+n_x} - 2^{\binom{n}{N+1+\dots+n_x}} \right) R_{2,1,\dots,n_N,n_{N+1}} \right) + \left((2^{\binom{n}{N+1+\dots+n_x}} + 2^{\binom{n}{N+2+\dots+n_x}}) \right. \right. \\
& \left. \left. (R_{2,1,\dots,n_N,n_{N+1},1+\dots} + R_{2,1,\dots,n_N,n_{N+1},n_{N+2}-2}) + (3(2^{\binom{n}{N+1+\dots+n_x}} + 2^{\binom{n}{N+2+\dots+n_x}}) + \right. \right. \\
& \left. \left. 2^{\binom{n}{N+3+\dots+n_x}+2}) R_{2,1,\dots,n_N,n_{N+1},n_{N+2}-1} - (2^{\binom{n}{N+1}+n_{N+2}+2n_{N+3}+\binom{n}{N+4+\dots+n_x}} + 2^{\binom{n}{N+2}+2n_{N+3}+} \right. \right. \\
& \left. \left. \left. \binom{n}{N+4+\dots+n_x} + 2^{2n_{N+3}+\binom{n}{N+4+\dots+n_x}} - (2^{\binom{n}{N+1+\dots+n_x}} + 2^{\binom{n}{N+2+\dots+n_x}}) \right) \right) R_{2,1,\dots,n_N,n_{N+1},n_{N+2}} \right) \\
& + \dots + \left((2^{\binom{n}{N+1+\dots+n_x}} + \dots + 2^{\binom{n}{x-1+n_x}}) (R_{2,1,\dots,n_N,\dots,n_{x-2},1+\dots} + R_{2,1,\dots,n_N,\dots,n_{x-1}-2}) \right. \\
& \left. + (3(2^{\binom{n}{N+1+\dots+n_x}} + 2^{\binom{n}{x-1+n_x}}) + 2^{n_x+2}) E^4 R_{2,1,\dots,n_N,\dots,n_{x-1}-1} - (2^{\binom{n}{N+1+\dots+n_x}} + 2^{2n_x+\dots} + \right. \\
& \left. 2^{2n_x} - (2^{\binom{n}{N+1+\dots+n_x}} + \dots + 2^{\binom{n}{x-1+n_x}}) R_{2,1,\dots,n_N,\dots,n_{x-1}}) \right) \Big] + \left[\text{termes identiques à ceux du} \right. \\
& \left. \text{crochet précédent, sauf que les + deviennent des - et l'exception E4 est remplacée par E5} \right] \Big) \\
& + \dots + \left(- (R_{2,1,\dots,n_N,\dots,n_{x-3},1+\dots} + R_{2,1,\dots,n_N,\dots,n_{x-2}-1}) \left[2^{\binom{n}{x-1+n_x}} (R_{2,1,\dots,n_N,\dots,n_{x-2},1+\dots} \right. \right. \\
& \left. \left. + R_{2,1,\dots,n_N,\dots,n_{x-1}-2}) + (3 \cdot 2^{\binom{n}{x-1+n_x}} + 2^{n_x+2}) E^6 R_{2,1,\dots,n_N,\dots,n_{x-1}-1} - (2^{\binom{n}{x-1}+2n_x} + 2^{2n_x} - \right. \right. \\
& \left. \left. 2^{\binom{n}{x-1+n_x}}) R_{2,1,\dots,n_N,\dots,n_{x-1}} \right] + R_{2,1,\dots,n_N,\dots,n_{x-2}} \left[\text{termes identiques à ceux du crochet} \right. \right. \\
& \left. \left. \text{précédent, sauf que les + deviennent des - et l'exception E6 est remplacée par E7} \right] \right) \Big] \\
& + (R_{2,1,\dots,n_N,\dots,n_{x-1},1+\dots} + R_{2,1,\dots,n_N,\dots,n_{x-1}}) \left[\left((5 \cdot 2^{\binom{n}{N+\dots+n_x}-1} - 3(2^{\binom{n}{N+1+\dots+n_x}} + \dots + 2^{\binom{n}{x}}) \right) \right. \\
& \left. \left((R_{2,1+\dots} + R_{2,1,\dots,1N-1}) + 7/2 \right) + 2 \left(- (2^{\binom{n}{N+1+\dots+n_x}} + \dots + 2^{\binom{n}{x}}) (R_{2,1,\dots,1N+\dots} + R_{2,1,\dots,n_{N-1}} \right. \right. \right. \\
& \left. \left. \left. 1) + (2^{\binom{n}{N+1+\dots+n_x}} - \dots - 2^{\binom{n}{x}}) R_{2,1,\dots,n_N} \right) \right) + \dots + 2 \left(- 2^{\binom{n}{x}} (R_{2,1,\dots,n_N,\dots,n_{x-2},1+\dots} \right. \right. \\
& \left. \left. + R_{2,1,\dots,n_N,\dots,n_{x-1}-1}) + 2^{\binom{n}{x}} R_{2,1,\dots,n_N,\dots,n_{x-1}} \right) \right] \\
& - 2R_{2,1,\dots,n_N,\dots,n_x} \left[21 + 6(R_{21} + \dots + R_{2,1,\dots,n_{N-1}}) + (R_{2,1,\dots,n_N,1+\dots} + R_{2,1,\dots,n_N}) + \dots + \right. \\
& \left. (R_{2,1,\dots,n_N,\dots,n_{x-1},1+\dots} + R_{2,1,\dots,n_N,\dots,n_{x-1}}) \right] / 5^{2N} \cdot 2^{2\binom{n}{N+\dots+n_x}+1}
\end{aligned}$$

Exceptions de la formule de $A_{2,1,\dots,n_N,n_2,\dots,n_x}$:

Soit n_b , le premier indice de valeur ≥ 2 dans la notation de $A_{2,1,\dots,n_N,\dots,n_x}$, avec $N+1 \leq b \leq x$

Pour $n_{N+1}, \dots, n_x = 1$, on considère que $b=x$

Exemple: pour $A_{2,1,1,3,1,1,5,2,3}$, $b=7$ (septième indice) de valeur $n_b = 5$

F1:

Pour $n_N = 2$, on a la relation

$$\frac{3 \cdot 5^{2N} - 31(-3)^N}{28} 2^{2(b-N+2)} + 31(-3)^{N-1} 2^{b-N+2} \binom{n_b + \dots + n_x}{b}^{-1}$$

F2:

$$\text{Pour } n_N = 2, -(21 \cdot 2^{2(b-N+2)} + 49 \cdot 2^{b-N+4}) \binom{n_b + \dots + n_x}{b}^{-1}$$

F3:

$$[(35 \cdot 2^{n_N-2} - 21) 2^{2(b-N)+3} - (35 \cdot 2^{n_N-3} - 21) 2^{b-N+3}] \binom{n_b + \dots + n_x}{b}^{-1}$$

F4:

$$-(2^{2(b-N)+3} - 2^{b-N+3}) \binom{n_b + \dots + n_x}{b}^{-1}$$

E3:

Soit n_k , le dernier indice de valeur ≥ 2 dans la notation de $A_{2,1,\dots,n_N,\dots,n_X}$. Le coefficient placé devant $R_{2,1,\dots,n_N,\dots,n_{k-1}}$ est une exception qui obéit à la règle E3

Exemple: pour $A_{2,1,1,3,5,2,1,3,1,1}$, $k=8$ (huitième indice) de valeur $n_k=3$, ainsi, le coefficient placé devant $R_{2,1,1,3,5,2,1,2}$ est une exception

$$[4(5 \cdot 2^{\binom{n_N + \dots + n_k}{N} + 2(x-k-1)} - 3(2^{\binom{n_{N+1} + \dots + n_k}{N+1}} + 2^{\binom{n_{N+2} + \dots + n_k}{N+2}} + \dots + 2^{\binom{n_k}{k}})(2^{2(x-k)+1} - 2^{x-k}) - 3(2^{2(x-k)+1} - 2^{x-k+2}) - 5 \cdot 2^{\binom{n_N + \dots + n_k}{N} + x - k - 3}) - 2(R_{2,1,\dots,n_N,\dots,n_{k-1},1} + \dots + R_{2,1,\dots,n_N,\dots,n_{k-2}})] R_{2,1,\dots,n_N,\dots,n_{k-1}}$$

E4:

$$[(2^{\binom{n_{N+1} + \dots + n_k}{N+1}} + 2^{\binom{n_{N+2} + \dots + n_k}{N+2}} + \dots + 2^{\binom{n_k}{k}})(2^{2(x-k)+1} - 2^{x-k}) + (2^{2(x-k)+1} - 2^{x-k+2})] R_{2,1,\dots,n_N,\dots,n_{k-1}}(R_{2,1,\dots,1_N+\dots+R_{2,1,\dots,n_N-1}})$$

E5:

$$[\text{termes identiques à ceux du crochet précédent, sauf que les } + \text{ deviennent des } -] R_{n_1,\dots,n_{k-1}} 1 \times R_{n_1}$$

E6:

$$[(2^{\binom{n_k}{k}})(2^{2(x-k)+1} - 2^{x-k}) + (2^{2(x-k)+1} - 2^{x-k+2})] R_{n_1,\dots,n_{k-1}}(R_{2,1,\dots,n_N,\dots,n_{k-2},1} + \dots + R_{2,1,\dots,n_N,\dots,n_{k-1}-1})$$

E7:

$$[\text{termes identiques à ceux du crochet précédent, sauf que les } + \text{ deviennent des } -] R_{2,1,\dots,n_N,\dots,n_{k-1}} \times R_{2,1,\dots,n_N,\dots,n_{k-1}}$$

Les racines ont pour expression,

$$R_{2,1,\dots,n_N,n_{N+1},\dots,n_X} =$$

$$\begin{aligned} & \left(\frac{-3.5^{2N} - 25.31(-3)^{N-1} 2^{2n_N-2} + 6.5^{2N} - 15.31(-3)^{N-1} 2^{n_N}}{7} \right)^{F1} 2^{\binom{n_{N+1} + \dots + n_X}{x}} \\ & - \left[(175.2^{2n_N} - 105.2^{n_N+2}) 2^{\binom{n_{N+1} + \dots + n_X}{x}} \right]^{F2} \left[(R_{2,1,\dots,1_{N-1}} + (-3)R_{2,1,\dots,1_{N-2}} + \dots + (-3)^{N-3} R_{2,1}) \right. \\ & + (2/7)(R_{2,1,\dots,1_{N-1}}(R_{2,1} + \dots + R_{2,1,\dots,n_N,n_{N-2}}) + (-3)R_{2,1,\dots,1_{N-2}}(R_{2,1,\dots,n_N,1} + \dots + R_{2,1,\dots,n_{N-3}}) \\ & + \dots + (-3)^{N-4} R_{2,1,1} R_{2,1}) \left. \right] + 35.2^{\binom{n_{N+1} + \dots + n_X}{x}} (R_{2,1,\dots,1_N} + \dots + R_{2,1,\dots,n_{N-2}}) \\ & + (105.2^{\binom{n_{N+1} + \dots + n_X}{x}} - 21.2^{\binom{n_{N+1} + \dots + n_X}{x}+3})^{F3} R_{2,1,\dots,n_{N-1}} (1 + (2/7)(R_{2,1} + R_{2,1,1} + \dots + R_{2,1,\dots,1_{N-1}})) \\ & - \left[(35.2^{n_N-1} - 21) 2^{2n_{N+1} + \binom{n_{N+2} + \dots + n_X}{x}+1} - 35.2^{\binom{n_{N+1} + \dots + n_X}{x}} \right] R_{2,1,\dots,n_N} (1 + (2/7) \\ & (R_{2,1} + R_{2,1,1} + \dots + R_{2,1,\dots,1_{N-1}})) \\ & + 5.2^{\binom{n_{N+1} + \dots + n_X}{x}+1} (R_{2,1} + R_{2,1,1} + \dots + R_{2,1,\dots,1_{N-1}}) (R_{2,1,\dots,1_N} + \dots + R_{2,1,\dots,n_{N-2}}) \\ & - (2^{\binom{n_{N+1} + \dots + n_X}{x}+3})^{F4} R_{2,1,\dots,n_{N-1}} (R_{2,1,\dots,1_N} + \dots + R_{2,1,\dots,n_{N-2}}) \\ & + (2^{2n_{N+1} + \binom{n_{N+2} + \dots + n_X}{x}+1})^{F4} R_{2,1,\dots,n_N} (R_{2,1,\dots,1_N} + \dots + R_{2,1,\dots,n_{N-1}}) \\ & 4 \left[(5.2^{\binom{n_{N+1} + \dots + n_X}{x}-1} - 3.2^{\binom{n_{N+1} + \dots + n_X}{x}}) (R_{2,1,\dots,n_N,1} + \dots + R_{2,1,\dots,n_N,n_{N+1}-2}) + \dots + \right. \\ & \left. (5.2^{\binom{n_{N+1} + \dots + n_X}{x}-1} - 3(2^{\binom{n_{N+1} + \dots + n_X}{x}} + 2^{\binom{n_{N+2} + \dots + n_X}{x}} + \dots + 2^{\binom{n_{x-1} + n_x}{x}})) (R_{2,1,\dots,n_N,\dots,n_{X-2},1_{X-1}} \right. \\ & \left. + \dots + R_{2,1,\dots,n_N,\dots,n_{X-2},n_{X-1}-2}) \right] \left[(R_{2,1} + \dots + R_{2,1,\dots,1_{N-1}}) + 7/2 \right] + \left[(4(15.2^{\binom{n_{N+1} + \dots + n_X}{x}-1} - 9.2^{\binom{n_{N+1} + \dots + n_X}{x}} - 3.2^{\binom{n_{N+2} + \dots + n_X}{x}+2}) \right. \\ & \left. [(R_{2,1} + \dots + R_{2,1,\dots,1_{N-1}}) + 7/2] - 2(R_{2,1,\dots,n_N,1} + \dots + R_{2,1,\dots,n_N,n_{N-2}}) 2^{\binom{n_{N+2} + \dots + n_X}{x}+2} \right] R_{2,1,\dots,n_N,n_{N-1}} + \dots + \\ & (4(15.2^{\binom{n_{N+1} + \dots + n_X}{x}-1} - 9(2^{\binom{n_{N+1} + \dots + n_X}{x}} + 2^{\binom{n_{N+2} + \dots + n_X}{x}} + \dots + 2^{\binom{n_{x-1} + n_x}{x}}) - 3.2^{\binom{n_x}{x}+2}) \left[(R_{2,1} + \dots + R_{2,1,\dots,1_{N-1}}) + 7/2 \right] - 2(R_{2,1,\dots,n_N,\dots,n_{X-2},1} + \dots + R_{2,1,\dots,n_N,\dots,n_{X-1}-2}) 2^{\binom{n_x}{x}+2} \right]^{E3} R_{2,1,\dots,n_N,\dots,n_{X-1}-1} \left. \right] - \left[(4(5.2^{n_N + n_{N+1} + 2n_{N+2} + \binom{n_{N+3} + \dots + n_X}{x}-1} - 3.2^{n_{N+1} + 2n_{N+2} + \binom{n_{N+3} + \dots + n_X}{x}} - 3.2^{2n_{N+2} + \binom{n_{N+3} + \dots + n_X}{x}} - 5.2^{\binom{n_{N+1} + \dots + n_X}{x}-1} + 3(2^{\binom{n_{N+1} + \dots + n_X}{x}})) \right. \\ & \left. [(R_{2,1} + \dots + R_{2,1,\dots,1_{N-1}}) + 7/2] - 2(R_{2,1,\dots,n_N,1} + \dots + R_{2,1,\dots,n_1,n_{N+1}-1}) 2^{2n_3 + \binom{n_4 + \dots + n_x}{x}} \right] R_{2,1,\dots,n_N,n_{N+1}} + \dots + 3 \times 2^{-1} \times (4(5.2^{\binom{n_{N+1} + \dots + n_{x-1}}{x}} + 2n_x^{-1} - 3(2^{\binom{n_{N+1} + \dots + n_{x-1}}{x}}) + 2n_x + \dots + 2^{2n_x}) - 5.2^{\binom{n_{N+1} + \dots + n_x}{x}-1} + 3(2^{\binom{n_{N+1} + \dots + n_x}{x}} + \dots + 2^{n_{x-1} + n_x})) - 2(R_{2,1,\dots,n_N,\dots,n_{X-2},1} + \dots + R_{2,1,\dots,n_N,\dots,n_{X-1}-1}) 2^{2n_x} \left. \right] R_{2,1,\dots,n_N,\dots,n_{X-1}} \left. \right] \\ & + 2 \left[\left(- (R_{2,1,\dots,1_N} + \dots + R_{2,1,\dots,n_{N-1}}) \left[(2^{\binom{n_{N+1} + \dots + n_x}{x}}) (R_{2,1,\dots,n_N,1} + \dots + R_{2,1,\dots,n_N,n_{N+1}-2}) \right. \right. \right. \\ & \left. \left. + (3.2^{\binom{n_{N+1} + \dots + n_x}{x}} + 2^{\binom{n_{N+2} + \dots + n_x}{x}+2}) R_{2,1,\dots,n_N,n_{N-1}} - (2^{n_{N+1} + 2n_{N+2} + \binom{n_{N+3} + \dots + n_x}{x}} + 2^{2n_{N+2} + \binom{n_{N+3} + \dots + n_x}{x}} - 2^{\binom{n_{N+1} + \dots + n_x}{x}}) R_{2,1,\dots,n_N,n_{N+1}} \right) \right. \\ & \left. + (2^{\binom{n_{N+1} + \dots + n_x}{x}} + 2^{\binom{n_{N+2} + \dots + n_x}{x}}) (R_{2,1,\dots,n_N,n_{N+1}} + \dots + R_{2,1,\dots,n_N,n_{N+1},n_{N+2}-2}) + (3(2^{\binom{n_{N+1} + \dots + n_x}{x}} + 2^{\binom{n_{N+2} + \dots + n_x}{x}}) + \right. \\ & \left. (R_{2,1,\dots,n_N,n_{N+1}} + \dots + R_{2,1,\dots,n_N,n_{N+1},n_{N+2}-2}) + (3(2^{\binom{n_{N+1} + \dots + n_x}{x}} + 2^{\binom{n_{N+2} + \dots + n_x}{x}}) + \right. \end{aligned}$$

$$\begin{aligned}
& 2^{\binom{n}{N+3} + \dots + \binom{n}{x} + 2} R_{2,1,\dots,n_N,n_{N+1},n_{N+2}-1} - (2^{\binom{n}{N+1} + \binom{n}{N+2} + 2\binom{n}{N+3} + \binom{n}{N+4} + \dots + \binom{n}{x}} + 2^{\binom{n}{N+2} + 2\binom{n}{N+3} + \binom{n}{N+4} + \dots + \binom{n}{x}} \\
& (\binom{n}{N+4} + \dots + \binom{n}{x} + 2^{2\binom{n}{N+3} + \binom{n}{N+4} + \dots + \binom{n}{x}} - (2^{\binom{n}{N+1} + \dots + \binom{n}{x}} + 2^{\binom{n}{N+2} + \dots + \binom{n}{x}})) R_{2,1,\dots,n_N,n_{N+1},n_{N+2}} \\
& + \dots + \left((2^{\binom{n}{N+1} + \dots + \binom{n}{x}} + \dots + 2^{\binom{n}{x-1} + \binom{n}{x}}) (R_{2,1,\dots,n_N,\dots,n_{X-2},1} + \dots + R_{2,1,\dots,n_N,\dots,n_{X-1}-2}) \right. \\
& \left. + (3(2^{\binom{n}{N+1} + \dots + \binom{n}{x}} + 2^{\binom{n}{x-1} + \binom{n}{x}}) + 2^{\binom{n}{x} + 2}) E^4 R_{2,1,\dots,n_N,\dots,n_{X-1}-1} - (3 \times 2^{-1} \times (2^{\binom{n}{N+1} + \dots + \binom{n}{x-1}} + 2^{\binom{n}{x} + \dots + 2^{\binom{n}{x}}} - (2^{\binom{n}{N+1} + \dots + \binom{n}{x}} + \dots + 2^{\binom{n}{x-1} + \binom{n}{x}})) R_{2,1,\dots,n_N,\dots,n_{X-1}}) \right) \Big] + \left[\text{termes identiques à ceux du crochet précédent, sauf que les } + \text{ deviennent des } - \text{ et l'exception } E4 \text{ est remplacée par } E5 \right. \\
& \left. \Big] \right) \\
& + \dots + \left(-(R_{2,1,\dots,n_N,\dots,n_{X-3},1} + \dots + R_{2,1,\dots,n_N,\dots,n_{X-2}-1}) \left[2^{\binom{n}{x-1} + \binom{n}{x}} (R_{2,1,\dots,n_N,\dots,n_{X-2},1} + \dots + R_{2,1,\dots,n_N,\dots,n_{X-1}-2}) + (3 \cdot 2^{\binom{n}{x-1} + \binom{n}{x}} + 2^{\binom{n}{x} + 2}) E^6 R_{2,1,\dots,n_N,\dots,n_{X-1}-1} - (3 \times 2^{-1} \times (2^{\binom{n}{x-1} + 2\binom{n}{x}} + 2^{2\binom{n}{x}} - 2^{\binom{n}{x-1} + \binom{n}{x}}) R_{2,1,\dots,n_N,\dots,n_{X-1}}) \right] + R_{2,1,\dots,n_N,\dots,n_{X-2}} \left[\text{termes identiques à ceux du } \right. \right. \\
& \left. \left. \text{crochet précédent, sauf que les } + \text{ deviennent des } - \text{ et l'exception } E6 \text{ est remplacée par } E7 \right] \right) \Big] \\
& + (R_{2,1,\dots,n_N,\dots,n_{X-1},1} + \dots + R_{2,1,\dots,n_N,\dots,n_{X-2}}) \left[\left(4(5 \cdot 2^{\binom{n}{N} + \dots + \binom{n}{x} - 1} - 3(2^{\binom{n}{N+1} + \dots + \binom{n}{x}} + \dots + 2^{\binom{n}{x}})) \right) \left[(R_{2,1,\dots,n_N,\dots,n_{X-1}} + 7/2) + 2(-2^{\binom{n}{N+1} + \dots + \binom{n}{x}} + \dots + 2^{\binom{n}{x}}) (R_{2,1,\dots,n_N,\dots,n_{X-1}} + \dots + R_{2,1,\dots,n_N,\dots,n_{X-2}}) \right. \right. \\
& \left. \left. + (2^{\binom{n}{N+1} + \dots + \binom{n}{x}} - \dots - 2^{\binom{n}{x}}) R_{2,1,\dots,n_N} \right) \right] + \dots + 2(-2^{\binom{n}{x}} (R_{2,1,\dots,n_N,\dots,n_{X-2},1} + \dots + R_{2,1,\dots,n_N,\dots,n_{X-1}-1}) + 2^{\binom{n}{x}} R_{2,1,\dots,n_N,\dots,n_{X-1}}) \Big] \\
& + 2R_{2,1,\dots,n_N,\dots,n_{X-1}} \left[\left(4(5 \cdot 2^{\binom{n}{N} + \dots + \binom{n}{x} - 1} - 3(2^{\binom{n}{N+1} + \dots + \binom{n}{x}} + \dots + 2^{\binom{n}{x}}) - 3) \right) \left[(R_{2,1,\dots,n_N,\dots,n_{X-1}} + 7/2) + 2(-2^{\binom{n}{N+1} + \dots + \binom{n}{x}} + \dots + 2^{\binom{n}{x}} + 1) (R_{2,1,\dots,n_N,\dots,n_{X-1}} + \dots + R_{2,1,\dots,n_N,\dots,n_{X-2}}) \right. \right. \\
& \left. \left. + (2^{\binom{n}{N+1} + \dots + \binom{n}{x}} - \dots - 2^{\binom{n}{x}} - 1) R_{2,1,\dots,n_N} \right) \right] + \dots + 2(-2^{\binom{n}{x} + 1} R_{2,1,\dots,n_N,\dots,n_{X-2},1} + \dots + R_{2,1,\dots,n_N,\dots,n_{X-1}-1}) \\
& + (2^{\binom{n}{x} - 1} R_{2,1,\dots,n_N,\dots,n_{X-1}}) \Big] \\
& - 2R_{2,1,\dots,n_N,\dots,n_{X-1}} (R_{2,1,\dots,n_N,\dots,n_{X-1},1} + \dots + R_{2,1,\dots,n_N,\dots,n_{X-2}})
\end{aligned}$$

Exceptions de la formule de $R_{2,1,\dots,n_N,n_2,\dots,n_X}$:

Soit n_b , le premier indice de valeur ≥ 2 dans la notation de $A_{2,1,\dots,n_N,\dots,n_X}$, avec $N+1 \leq b \leq x$

Pour $n_{N+1}, \dots, n_X = 1$, on considère que $b=x$

Exemple: pour $R_{2,1,1,3,1,1,5,2,3}$, $b=7$ (septième indice) de valeur $n_b = 5$

F1:

*Pour $n_N = 2$, on a la relation

$$\frac{3 \cdot 5^{2N} - 31(-3)^N}{28} 2^{2(b-N+2)} + 31(-3)^{N-1} 2^{b-N+2} 2^{\binom{n}{b} + \dots + \binom{n}{x} - 2}$$

*Pour $n_N = 2$, $n_{N+1}, \dots, n_X = 1$, on a la relation

$$\frac{9.5 \cdot 2^N - 31(-3)^{N+1} 2^{2(x-N+1)} + 31(-3)^{N-1} 2^{x-N+1}}{28}$$

F2:

*Pour $n_N = 2$, $-(21 \cdot 2^{2(b-N+2)} + 49 \cdot 2^{b-N+4}) 2^{\binom{n_b + \dots + n_x}{b}} - 2$

*Pour $n_N = 2$, $n_{N+1}, \dots, n_X = 1$, $-(3 \cdot 21 \cdot 2^{2(b-N+1)} + 49 \cdot 2^{b-N+3})$

F3:

* $[(35 \cdot 2^N - 21) 2^{2(b-N)+3} - (35 \cdot 2^N - 21) 2^{b-N+3}] 2^{\binom{n_b + \dots + n_x}{b}} - 2$

*Pour $n_{N+1}, \dots, n_X = 1$, $3(35 \cdot 2^N - 21) 2^{2(b-N)+1} - (35 \cdot 2^N - 21) 2^{b-N+2}$

F4:

* $-(2^{2(b-N)+3} - 2^{b-N+3}) 2^{\binom{n_b + \dots + n_x}{b}} - 2$

*Pour $n_{N+1}, \dots, n_X = 1$, $-(3 \cdot 2^{2(b-N)+1} - 2^{b-N+2})$

E3:

Soit n_k , le dernier indice de valeur ≥ 2 dans la notation de $A_2, 1, \dots, n_N, \dots, n_X$. Le coefficient placé devant $R_2, 1, \dots, n_N, \dots, n_{k-1}$ est une exception qui obéit à la règle E3

Exemple: pour $R_2, 1, 1, 3, 5, 2, 1, 3, 1, 1$, $k=8$ (huitième indice) de valeur $n_k = 3$, ainsi, le coefficient placé devant $R_2, 1, 1, 3, 5, 2, 1, 2$ est une exception

$$[4((3/2)5 \cdot 2^{\binom{n_N + \dots + n_k}{N} + 2(x-k-1)} - 3(2^{\binom{n_{N+1} + \dots + n_k}{N+1}} + 2^{\binom{n_{N+2} + \dots + n_k}{N+2}} + \dots + 2^{\binom{n_k}{k}}))((3/2)2^{2(x-k)+1} - 2^{x-k}) - 3((3/2)2^{2(x-k+1)} - 2^{x-k+2}) - 5 \cdot 2^{\binom{n_N + \dots + n_k}{N} + x - k - 3}) - 2(R_2, 1, \dots, n_N, \dots, n_{k-1}, 1 + \dots + R_2, 1, \dots, n_N, \dots, n_{k-2})((3/2)2^{2(x-k+1)} - 2^{x-k+2})] R_2, 1, \dots, n_N, \dots, n_{k-1}$$

E4:

$$[(2^{\binom{n_{N+1} + \dots + n_k}{N+1}} + 2^{\binom{n_{N+2} + \dots + n_k}{N+2}} + \dots + 2^{\binom{n_k}{k}})((3/2)2^{2(x-k)+1} - 2^{x-k}) + ((3/2)2^{2(x-k+1)} - 2^{x-k+2})] R_2, 1, \dots, n_N, \dots, n_{k-1}(R_2, 1, \dots, 1_N + \dots + R_2, 1, \dots, n_{N-1})$$

E5:

[termes identiques à ceux du crochet précédent, sauf que les + deviennent des -] $R_{n_1, \dots, n_{k-1}} \times R_n$

E6:

$$[(2^{n_k}) ((3/2)^{2(x-k)+1} - 2^{x-k}) + ((3/2)^{2(x-k+1)} - 2^{x-k+2})] R_{n_1, \dots, n_{k-1}} (R_{2,1, \dots, n_N, \dots, n_{k-2}, 1} + \dots + R_{2,1, \dots, n_N, \dots, n_{k-1}})$$

E7

[termes identiques à ceux du crochet précédent, sauf que les + deviennent des -]

$$R_{2,1, \dots, n_N, \dots, n_{k-1}} \times R_{2,1, \dots, n_N, \dots, n_{k-1}}$$

Surface, pour une zone $N \geq 2$: on pose $(x-N)=d$, x représente l'ordre du dernier indice= nombre d'indices, tend vers l'infini, ainsi que tous les n .

$$8 \left[\frac{(3.5^{2N} - 25.31(-3))^{N-1} [(n_{N-1}) + (n_{N-2})d] + 6.5^{2N} - 15.31(-3)^{N-1} (2+d)}{7.5^{2N} \cdot 2^3} + \frac{3.5^{2N} - 31(-3)^N (d^2 + 3d) + 31(-3)^{N-1} d}{7.5^{2N} \cdot 2^4} \right]$$

$$+ \left(\frac{35}{5} \cdot 2^{2N} \cdot 2^2 \right) S(R_{211 \dots 1N}, (1/2, n_{N-1})) - \left(\frac{35}{5} \cdot 2^{2N} \cdot 2^{n_{N+1}} \right) S(R_{211 \dots 1N}, (1, n_{N-1})) +$$

$$d \left[\left(\frac{35}{5} \cdot 2^{2N} \cdot 2^3 \right) S(R_{211 \dots 1N}, (1/2, n_{N-2})) - \left(\frac{35}{5} \cdot 2^{2N} \cdot 2^{n_{N+1}} \right) S(R_{211 \dots 1N}, (1, n_{N-2})) \right]$$

$$+ \left(\frac{35}{5} \cdot 2^{2N} \cdot 2^5 \right) (4d^2 + 14d + 6) S(R_{211 \dots 1N}, (1/2, n_{N-1})) - \left(\frac{21}{5} \cdot 2^{2N} \cdot 2^4 \right) (2d^2 + 2d) S(R_{211 \dots 1N}, (1/4, n_{N-1})) +$$

$$d \left[\left(\frac{35}{5} \cdot 2^{2N} \cdot 2^3 \right) S(R_{211 \dots 1N}, (1/2, n_{N-2})) - \left(\frac{35}{5} \cdot 2^{2N} \cdot 2^{n_{N+1}} \right) S(R_{211 \dots 1N}, (1, n_{N-2})) \right] \left(1 + \frac{2}{7} \right) (R_{21} + R_{211} + \dots + R_{211 \dots 1n_{N-1}})$$

$$- \left(\frac{175}{5} \cdot 2^{2N} \cdot 2 \right) ((n_{N-1}) + d(n_{N-2})) - \left(\frac{105}{5} \cdot 2^{2N} \cdot 2 \right) (2+d) + \left(\frac{21}{5} \cdot 2^{2N} \cdot 2^3 \right) (d^2 + 3d) + d \left(\frac{49}{5} \cdot 2^{2N} \cdot 2 \right) \left(R_{211 \dots 1n_{N-1}} + (-3) R_{211 \dots 1n_{N-2}} + (-3)^{N-3} R_{21} \right) + \left(\frac{2}{7} \right) (R_{211 \dots 1n_{N-1}} (R_{21} + R_{211} + \dots + R_{211 \dots 1n_{N-2}}) + (-3) R_{211 \dots 1n_{N-2}} (R_{21} + R_{211} + \dots + R_{211 \dots 1n_{N-3}}) + (-3)^{N-4} R_{211} R_{21})$$

$$- d \left(\frac{35}{5} \cdot 2^{2N} \cdot 2^3 \right) S(R_{211 \dots 1N}, (1/2, n_N)) - \left(\frac{21}{5} \cdot 2^{2N} \cdot 2^4 \right) S(R_{211 \dots 1N}, (1/4, n_N)) \left(1 + \frac{2}{7} \right) (R_{21} + R_{211} + \dots + R_{211 \dots 1n_{N-1}})$$

$$+ d \left(\frac{5}{5} \cdot 2^{2N} \cdot 2^2 \right) S(R_{211 \dots 1N}, (1/2, n_{N-2})) - \left(\frac{5}{5} \cdot 2^{2N} \cdot 2^{n_N} \right) S(R_{211 \dots 1N}, (1, n_{N-2})) \left(R_{21} + R_{211} + \dots + R_{211 \dots 1n_{N-1}} \right)$$

$$+ d \left(\frac{15}{5} \cdot 2^{2N} \cdot 2^2 \right) S(R_{211 \dots 1N}, (1/2, n_{N-1})) - \left(\frac{3}{5} \cdot 2^{2N} \cdot 2^2 \right) S(R_{211 \dots 1N}, (1/4, n_{N-1})) \left(R_{21} + R_{211} + \dots + R_{211 \dots 1n_{N-1}} \right)$$

$$+ \left(- \frac{(d^2 + d)}{5} \cdot 2^{2N} \cdot 2^5 \right) S(R_{211 \dots 1N} R_{211 \dots 2N}, (1, n_{N-2}) (1/4, n_{N-1})) + d \left(\frac{21}{5} \cdot 2^{2N} \cdot 2^4 \right) S(R_{211 \dots 1N} R_{211 \dots 2N}, (1/4, n_{N-1}))$$

[à partir d'ici, les termes sont similaires à ceux relatifs à la zone $N=1$, il convient toutefois d'apporter les modifications suivantes:

*on remplace chacun des termes x suivants par $(d+1)$

*on remplace chaque terme (n_1) par (n_N) , (n_2) par (n_{N+1}) et ainsi de suite.

*on remplace chaque terme de la forme $(R_{311 \dots 1x-1})$ par $(R_{211 \dots 1N11 \dots 1x-1})$ et $(R_{n_111 \dots 1x-1})$ par $(R_{211 \dots n_N11 \dots 1x-1})$

$$\begin{aligned}
& + \left(\left[(7/2) \left((x-2/5)^{2N-1} \cdot 2^4 \right) S(R31, (1/2, n2-2), (1/2, n1)) - (x-2/5)^{2N-1} \cdot 2^{n2+2} S(R31, (1, n2-2), (1/2, n1)) \right] - (7/4)3 \left((x-2/5)^{2N} \cdot 2^4 \right) S(R31, (1/2, n2-2), (1/4, n1)) - (x-2/5)^{2N} \cdot 2^{n2+2} S(R31, (1, n2-2), (1/4, n1)) \right) + \dots + \left[(7/2) \left((1/5)^{2N-1} \cdot 2^{x+1} \right) S(R311\dots 1_{x-1}, (1/2, n_{x-1}-2), (1/2, n_{x-2}), \dots, (1/2, n1)) - (x-2/5)^{2N-1} \cdot 2^{n_{x-1}+x+2} S(R311\dots 1_{x-1}, (1, n_{x-1}-2), (1/2, n_{x-2}), \dots, (1/2, n1)) \right] - (7/4)3 \left((1/5)^{2N} \cdot 2^{x+1} \right) S(R311\dots 1_{x-1}, (1/2, n_{x-1}-2), (1/2, n_{x-2}), \dots, (1/4, n1)) - (1/5)^{2N} \cdot 2^{n_{x-1}+x-1} S(R311\dots 1_{x-1}, (1, n_{x-1}-2), (1/2, n_{x-2}), \dots, (1/4, n1)) + \left((1/5)^{2N} \cdot 2^{x+2} \right) S(R311\dots 1_{x-1}, (1/2, n_{x-1}-2), (1/2, n_{x-2}), \dots, (1/4, n2), (1/4, n1)) - (1/5)^{2N} \cdot 2^{n_{x-1}+x} S(R311\dots 1_{x-1}, (1, n_{x-1}-2), (1/2, n_{x-2}), \dots, (1/4, n2), (1/4, n1)) + \dots + \left((1/5)^{2N} \cdot 2^{2x-2} \right) S(R311\dots 1_{x-1}, (1/2, n_{x-1}-2), (1/4, n_{x-2}), \dots, (1/4, n1)) - (1/5)^{2N} \cdot 2^{n_{x-1}+2x-4} S(R311\dots 1_{x-1}, (1, n_{x-1}-2), (1/4, n_{x-2}), \dots, (1/4, n1)) \right) \left[(1+(2/7)(R21+R211+\dots+R211\dots 1_{nN-1})) \right. \\
& + \left. \left(\left[(7/2)(1/5)^{2N-1} \cdot 2^5 \right] (2(x-1)^2-2) S(R31, (1/2, n2-1), (1/2, n1)) (1+(2/7)(R21+R211+\dots+R211\dots 1_{nN-1})) - (7/4)3 \left((1/5)^{2N} \cdot 2^5 \right) (2(x-1)^2-2) S(R31, (1/2, n2-1), (1/4, n1)) + (1/5)^{2N} \cdot 2^5 \left((x-1)^2 - (x-1) \right) S(R31, (1/4, n2-1), (1/4, n1)) \right) \left[(1+(2/7)(R21+R211+\dots+R211\dots 1_{nN-1})) - (1/5)^{2N} \cdot 2^7 \left((x-1)^2 - (x-1) \right) S(R32R31, (1/4, n2-1), (1, n2-2), (1/4, n1)) \right] + \dots + \left[(7/2)(1/5)^{2N-1} \cdot 2^{x+2} \right] (2(2)^2-2) S(R311\dots 1_{x-1}, (1/2, n_{x-1}-1), (1/2, n_{x-2}), \dots, (1/2, n1)) (1+(2/7)(R21+R211+\dots+R211\dots 1_{nN-1})) - (7/4)3 \left((1/5)^{2N} \cdot 2^{x+2} \right) (2(2)^2-2) \left(S(R311\dots 1_{x-1}, (1/2, n_{x-1}-1), (1/2, n_{x-2}), \dots, (1/2, n2), (1/4, n1)) + (1/2) S(R311\dots 1_{x-1}, (1/2, n_{x-1}-1), (1/2, n_{x-2}), \dots, (1/2, n3), (1/4, n2), (1/4, n1)) + \dots + (1/2)^{x-3} S(R311\dots 1_{x-1}, (1/2, n_{x-1}-1), (1/4, n_{x-2}), \dots, (1/4, n1)) \right) + (1/5)^{2N} \cdot 2^{2x-1} \left((2)^2-2 \right) S(R311\dots 1_{x-1}, (1/4, n_{x-1}-1), (1/4, n_{x-2}), \dots, (1/4, n1)) \right) \left[(1+(2/7)(R21+R211+\dots+R211\dots 1_{nN-1})) - (1/5)^{2N} \cdot 2^{2x+1} \left((2)^2-2 \right) \right] S(R311\dots 2_{x-1} R311\dots 1_{x-1}, (1/4, n_{x-1}-1), (1, n_{x-1}-2), (1/4, n_{x-2}), \dots, (1/4, n1)) \left. \right] \\
& - \left(\left[(7/2) \left((n3-1)/5^{2N-1} \cdot 2^3 \right) S(R31, (1/2, n2), (1/2, n1)) (1+(2/7)(R21+R211+\dots+R211\dots 1_{nN-1})) - (7/4)3 \left((n3-1)/5^{2N} \cdot 2^3 \right) S(R31, (1/2, n2), (1/4, n1)) + (n3/5)^{2N} \cdot 2^4 S(R31, (1/4, n2), (1/4, n1)) \right] \left[(1+(2/7)(R21+R211+\dots+R211\dots 1_{nN-1})) - (n3/5)^{2N} \cdot 2^6 S(R32R31, (1/4, n2), (1, n2-1), (1/4, n1)) \right] (x-2) + \dots + \left[(7/2) \left((n_{x-1})/5^{2N-1} \cdot 2^x \right) S(R311\dots 1_{x-1}, (1/2, n_{x-1}), (1/2, n_{x-2}), \dots, (1/2, n1)) (1+(2/7)(R21+R211+\dots+R211\dots 1_{nN-1})) - (7/4)3 \left((n_{x-1})/5^{2N} \cdot 2^x \right) \left(S(R311\dots 1_{x-1}, (1/2, n_{x-1}), (1/2, n_{x-2}), \dots, (1/2, n2), (1/4, n1)) + (1/2) S(R311\dots 1_{x-1}, (1/2, n_{x-1}), (1/2, n_{x-2}), \dots, (1/2, n3), (1/4, n2), (1/4, n1)) + \dots + (1/2)^{x-3} S(R311\dots 1_{x-1}, (1/2, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n1)) \right) + (n_{x-1}/5)^{2N} \cdot 2^{2x-2} S(R311\dots 1_{x-1}, (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n1)) \right) \left[(1+(2/7)(R21+R211+\dots+R211\dots 1_{nN-1})) - (n_{x-1}/5)^{2N} \cdot 2^{2x} S(R311\dots 2_{x-1} R311\dots 1_{x-1}, (1/4, n_{x-1}), (1, n_{x-1}-1), (1/4, n_{x-2}), \dots, (1/4, n1)) \right] (1) \left. \right] \\
& + \left(\left[- \left(\left[(1/5)^{2N} \cdot 2^6 \right] S(R3R41, (1/2, n2-2), (1, n1-1)(1/4, n1)) - (1/5)^{2N} \cdot 2^{n2+4} S(R3R41, (1, n2-2), (1, n1-1)(1/4, n1)) - \left((n3-1)/5^{2N} \cdot 2^5 \right) S(R3R41, (1/2, n2), (1, n1-1)(1/4, n1)) + (n3/5)^{2N} \cdot 2^6 S(R3R41, (1/4, n2), (1, n1-1)(1/4, n1)) \right) \right] (x-2) + \dots + \left[(1/5)^{2N} \cdot 2^{x+3} \right] \left(S(R3R411\dots 1_{x-1}, (1/2, n_{x-1}-2), (1/2, n_{x-2}), \dots, (1/2, n2), (1, n1-1)(1/4, n1)) + (1/2) S(R3R411\dots 1_{x-1}, (1/2, n_{x-1}-2), (1/2, n_{x-2}), \dots, \right.
\end{aligned}$$

$$\begin{aligned}
& (1/4, n_2), (1, n_1-1)(1/4, n_1)) + \dots + (1/2^{x-3})S(R3R411\dots 1_{x-1}, (1/2, n_{x-1-2}), (1/4, n_{x-2}), \dots, (1/4, n_2), \\
& (1, n_1-1)(1/4, n_1)) - (1/5^{2N} \cdot 2^{n_{x-1}+x+1}) (S(R3R411\dots 1_{x-1}, (1, n_{x-1-2}), (1/2, n_{x-2}), \dots, (1/2, n_2), (1, \\
& n_1-1)(1/4, n_1)) + (1/2)S(R3R411\dots 1_{x-1}, (1, n_{x-1-2}), (1/2, n_{x-2}), \dots, (1/4, n_2), (1, n_1-1)(1/4, n_1)) + \dots \\
& + (1/2^{x-3})S(R3R411\dots 1_{x-1}, (1, n_{x-1-2}), (1/4, n_{x-2}), \dots, (1/4, n_2), (1, n_1-1)(1/4, n_1)) - (((nx- \\
& 1)/5^{2N} \cdot 2^{x+2}) (S(R3R411\dots 1_{x-1}, (1/2, n_{x-1}), (1/2, n_{x-2}), \dots, (1/2, n_2), (1, n_1-1)(1/4, n_1)) + \\
& (1/2)S(R3R411\dots 1_{x-1}, (1/2, n_{x-1}), (1/2, n_{x-2}), \dots, (1/4, n_2), (1, n_1-1)(1/4, n_1)) + \dots + (1/2^{x- \\
& 3})S(R3R411\dots 1_{x-1}, (1/2, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_2), (1, n_1-1)(1/4, n_1)) + (nx/5^{2N} \cdot 2^{2x}) \\
& S(R3R411\dots 1_{x-1}, (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_2), (1, n_1-1)(1/4, n_1))) \rfloor (1)) \\
& + ((1/5^{2N} \cdot 2^7)(2(x-1)^2-2) S(R3R41, (1/2, n_2-1), (1, n_1-1)(1/4, n_1)) + (1/5^{2N} \cdot 2^7)((x-1)^2-(x-1)) \\
& S(R3R41, (1/4, n_2-1), (1, n_1-1)(1/4, n_1)) + \dots + \rfloor (1/5^{2N} \cdot 2^{x+4})(2(2)^2-2) (S(R3R411\dots 1_{x-1}, (1/2, \\
& n_{x-1-1}), (1/2, n_{x-2}), \dots, (1/2, n_2), (1, n_1-1)(1/4, n_1)) + (1/2)S(R3R411\dots 1_{x-1}, (1/2, n_{x-1-1}), (1/2, n_{x-2}) \\
& \dots, (1/4, n_2), (1, n_1-1)(1/4, n_1)) + \dots + (1/2^{x-3})S(R3R411\dots 1_{x-1}, (1/2, n_{x-1-1}), (1/4, n_{x-2}), \dots, (1/4, \\
& n_2), (1, n_1-1)(1/4, n_1)) + (1/5^{2N} \cdot 2^{2x+1})((2)^2-2) S(R3R411\dots 1_{x-1}, (1/4, n_{x-1-1}), (1/4, n_{x-2}), \dots, \\
& (1/4, n_2), (1, n_1-1)(1/4, n_1)) \rfloor) \\
& + (1/4) ((\rfloor (1/5^{2N} \cdot 2^4) S(R3R31, (1/2, n_2-2), (1/4, n_1)) - (1/5^{2N} \cdot 2^{n_2+2})S(R3R31, (1, n_2-2), (1/4, \\
& n_1)) - (((n_3-1)/5^{2N} \cdot 2^3) S(R3R31, (1/2, n_2), (1/4, n_1)) - (n_3/5^{2N} \cdot 2^4) S(R3R31, (1/4, n_2), (1/4, \\
& n_1))) \rfloor (x-2) + \dots + \rfloor (1/5^{2N} \cdot 2^{x+1}) (S(R3R311\dots 1_{x-1}, (1/2, n_{x-1-2}), (1/2, n_{x-2}), \dots, (1/2, n_2), (1/4, \\
& n_1)) - (1/2)S(R3R311\dots 1_{x-1}, (1/2, n_{x-1-2}), (1/2, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1)) - \dots - (1/2^{x- \\
& 3})S(R3R311\dots 1_{x-1}, (1/2, n_{x-1-2}), (1/4, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1)) - (1/5^{2N} \cdot 2^{n_{x-1}+x-1}) \\
& (S(R3R311\dots 1_{x-1}, (1, n_{x-1-2}), (1/2, n_{x-2}), \dots, (1/2, n_2), (1/4, n_1)) - (1/2)S(R3R311\dots 1_{x-1}, (1, n_{x-1- \\
& 2), (1/2, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1)) - \dots - (1/2^{x-3})S(R3R311\dots 1_{x-1}, (1, n_{x-1-2}), (1/4, n_{x-2}), \dots, \\
& (1/4, n_2), (1/4, n_1)) - (((nx-1)/25 \cdot 2^x) (S(R3R311\dots 1_{x-1}, (1/2, n_{x-1}), (1/2, n_{x-2}), \dots, (1/2, n_2), \\
& (1/4, n_1)) - (1/2)S(R3R311\dots 1_{x-1}, (1/2, n_{x-1}), (1/2, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1)) - \dots - (1/2^{x- \\
& 3})S(R3R311\dots 1_{x-1}, (1/2, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1)) - (nx/5^{2N} \cdot 2^{2x-2}) \\
& S(R3R311\dots 1_{x-1}, (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1))) \rfloor (1)) \\
& + ((1/5^{2N} \cdot 2^5)(2(x-1)^2-2) S(R3R31, (1/2, n_2-1), (1/4, n_1)) - (1/5^{2N} \cdot 2^5)((x-1)^2-(x-1)) S(R3R31, \\
& (1/4, n_2-1), (1/4, n_1)) - \dots - \rfloor (1/5^{2N} \cdot 2^{x+2})(2(2)^2-2) (S(R3R311\dots 1_{x-1}, (1/2, n_{x-1-1}), (1/2, n_{x- \\
& 2}), \dots, (1/2, n_2), (1/4, n_1)) - (1/2)S(R3R311\dots 1_{x-1}, (1/2, n_{x-1-1}), (1/2, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1)) \\
& - \dots - (1/2^{x-3})S(R3R311\dots 1_{x-1}, (1/2, n_{x-1-1}), (1/4, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1)) - (1/5^{2N} \cdot 2^{2x-1}) \\
& ((2)^2-2) S(R3R311\dots 1_{x-1}, (1/4, n_{x-1-1}), (1/4, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1)) \rfloor) \rfloor (crochets roses ci- \\
& contres= terme pour x=3, on a R311\dots 2_{x-2} 1_{x-1} = R4 pour x=3)
\end{aligned}$$

+ +

$$\begin{aligned}
& \left[- \left(\left(\left(\left(\left(\frac{1}{5^{2N}} \cdot 2^{2x} \right) S(R311...1_{x-2} R311...2_{x-2} 1_{x-1}, (1/2, n_{x-1-2}), (1, n_{x-2-1})(1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) - \left(\frac{1}{5^{2N}} \cdot 2^{n_{x-1}+2x-2} \right) S(R311...1_{x-2} R311...2_{x-2} 1_{x-1}, (1, n_{x-1-2}), (1, n_{x-2-1})(1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) - \left(\frac{(n_x-1)}{5^{2N}} \cdot 2^{2x-1} \right) S(R311...1_{x-2} R311...2_{x-2} 1_{x-1}, (1/2, n_{x-1}), (1, n_{x-2-1})(1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) + \left(\frac{n_x}{5^{2N}} \cdot 2^{2x} \right) S(R311...1_{x-2} R311...2_{x-2} 1_{x-1}, (1/4, n_{x-1}), (1, n_{x-2-1})(1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) \right) \right] (1) \right) \\
& + \left(\left(\frac{1}{5^{2N}} \cdot 2^{2x+1} \right) (2(2)^2-2) S(R311...1_{x-2} R311...2_{x-2} 1_{x-1}, (1/2, n_{x-1-1}), (1, n_{x-2-1})(1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) + \left(\frac{1}{5^{2N}} \cdot 2^{2x+1} \right) ((2)^2-(2)) S(R311...1_{x-2} R311...2_{x-2} 1_{x-1}, (1/4, n_{x-1-1}), (1, n_{x-2-1})(1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) \right) \right) \\
& + \left(\frac{1}{4} \right) \left(\left(\left(\left(\left(\frac{1}{5^{2N}} \cdot 2^{2x-2} \right) S(R311...1_{x-2} R311...1_{x-2} 1_{x-1}, (1/2, n_{x-1-2}), (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) - \left(\frac{1}{5^{2N}} \cdot 2^{n_{x-1}+2x-4} \right) S(R311...1_{x-2} R311...1_{x-2} 1_{x-1}, (1, n_{x-1-2}), (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) - \left(\frac{(n_x-1)}{5^{2N}} \cdot 2^{2x-3} \right) S(R311...1_{x-2} R311...1_{x-2} 1_{x-1}, (1/2, n_{x-1}), (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) - \left(\frac{n_x}{5^{2N}} \cdot 2^{2x-2} \right) S(R311...1_{x-2} R311...1_{x-2} 1_{x-1}, (1/4, n_{x-1}), (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) \right) \right] (1) \right) \\
& + \left(\left(\frac{1}{5^{2N}} \cdot 2^{2x-1} \right) (2(2)^2-2) S(R311...1_{x-2} R311...1_{x-2} 1_{x-1}, (1/2, n_{x-1-1}), (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) - \left(\frac{1}{5^{2N}} \cdot 2^{2x-1} \right) ((2)^2-(2)) S(R311...1_{x-2} R311...1_{x-2} 1_{x-1}, (1/4, n_{x-1-1}), (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) \right) \right) \left] \right) \\
& + \left(\left[\frac{7}{2} \right] \left(\frac{1}{5^{2N-1}} \cdot 2^3 \right) S(R31, (1/2, n_{2-1}), (1/2, n_1)) - \left(\frac{1}{5^{2N-1}} \cdot 2^{n_2+2} \right) S(R31, (1, n_{2-1}), (1/2, n_1)) \right) - \left(\frac{7}{4} \right) \left[\frac{3}{3} \right] \left(\frac{1}{5^{2N}} \cdot 2^3 \right) S(R31, (1/2, n_{2-1}), (1/4, n_1)) - \left(\frac{1}{5^{2N}} \cdot 2^{n_2+2} \right) S(R31, (1, n_{2-1}), (1/4, n_1)) \right) \right] + \dots + \left[\frac{7}{2} \right] \left(\frac{1}{5^{2N-1}} \cdot 2^{x+1} \right) S(R311...1_x, (1/2, n_{x-1}), (1/2, n_{x-1}), \dots, (1/2, n_1)) - \left(\frac{1}{5^{2N-1}} \cdot 2^{n_x+x} \right) S(R311...1_x, (1, n_x), (1/2, n_{x-1}), \dots, (1/2, n_1)) \right) - \left(\frac{7}{4} \right) \left[\frac{3}{3} \right] \left(\frac{1}{5^{2N}} \cdot 2^{x+1} \right) S(R311...1_x, (1/2, n_{x-1}), (1/2, n_{x-1}), \dots, (1/4, n_1)) - \left(\frac{1}{5^{2N}} \cdot 2^{n_x+x} \right) S(R311...1_x, (1, n_{x-1}), (1/2, n_{x-1}), \dots, (1/4, n_1)) \right) + \left(\frac{1}{5^{2N}} \cdot 2^{x+2} \right) S(R311...1_x, (1/2, n_{x-1}), (1/2, n_{x-1}), \dots, (1/4, n_2), (1/4, n_1)) - \left(\frac{1}{5^{2N}} \cdot 2^{n_x+x+1} \right) S(R311...1_x, (1, n_{x-1}), (1/2, n_{x-1}), \dots, (1/4, n_2), (1/4, n_1)) \right) + \dots + \left(\frac{1}{5^{2N}} \cdot 2^{2x-1} \right) S(R311...1_x, (1/2, n_{x-1}), (1/4, n_{x-1}), \dots, (1/4, n_1)) - \left(\frac{1}{5^{2N}} \cdot 2^{n_{x-1}+2x-2} \right) S(R311...1_x, (1, n_{x-1}), (1/4, n_{x-1}), \dots, (1/4, n_1)) \right) \right] (1+(2/7)(R_{21}+R_{211}+\dots+R_{211\dots 1n_N})) \\
& + \left(\left[- \left[\frac{1}{5^{2N}} \cdot 2^5 \right) S(R3R41, (1/2, n_{2-1}), (1, n_{1-1})(1/4, n_1)) - \left(\frac{1}{5^{2N}} \cdot 2^{n_2+4} \right) S(R3R41, (1, n_{2-1}), (1, n_{1-1})(1/4, n_1)) \right] + \left(\frac{1}{4} \right) \left[\left[\frac{1}{5^{2N}} \cdot 2^3 \right) S(R3R31, (1/2, n_{2-1}), (1/4, n_1)) - \left(\frac{1}{5^{2N}} \cdot 2^{n_2+2} \right) S(R3R31, (1, n_{2-1}), (1/4, n_1)) \right] \right] \right) \\
& + \left[\left(- \left[\frac{1}{5^{2N}} \cdot 2^6 \right) S(R3R411, (1/2, n_{3-1}), (1/2, n_2), (1, n_{1-1})(1/4, n_1)) - \left(\frac{1}{5^{2N}} \cdot 2^{n_3+5} \right) S(R3R411, (1, n_{3-1}), (1/2, n_2), (1, n_{1-1})(1/4, n_1)) \right] + \left[\frac{1}{5^{2N}} \cdot 2^7 \right) S(R3R411, (1/2, \right. \right.
\end{aligned}$$

$$\begin{aligned}
& n3-1), (1/4, n2), (1, n1-1)(1/4, n1)) - (1/5^{2N} \cdot 2^{n3+6}) S(R3R411, (1, n3-1), (1/4, n2), (1, n1-1)(1/4, \\
& n1)) \Big] + (1/4) \Big(\Big[(1/5^{2N} \cdot 2^4) S(R3R311, (1/2, n3-1), (1/2, n2), (1/4, n1)) - \\
& (1/5^{2N} \cdot 2^{n3+3}) S(R3R311, (1, n3-1), (1/2, n2), (1/4, n1)) \Big] - \Big[(1/5^{2N} \cdot 2^5) S(R3R311, (1/2, n3-1), \\
& (1/4, n2), (1/4, n1)) - (1/5^{2N} \cdot 2^{n3+4}) S(R3R311, (1, n3-1), (1/4, n2), (1/4, n1)) \Big] \Big) + \Big(- \\
& \Big[(1/5^{2N} \cdot 2^7) S(R31R321, (1/2, n3-1), (1, n2-1)(1/4, n2), (1/4, n1)) - (1/5^{2N} \cdot 2^{n3+6}) S(R31R321, (1, \\
& n3-1), (1, n2-1)(1/4, n2), (1/4, n1)) \Big] + (1/4) \Big[(1/5^{2N} \cdot 2^5) S(R31R311, (1/2, n3-1), (1/4, n2), (1/4, \\
& n1)) - (1/5^{2N} \cdot 2^{n3+4}) S(R31R311, (1, n3-1), (1/4, n2), (1/4, n1)) \Big] \Big) \Big] \\
& + \dots + \\
& \Big[\Big(- \Big(\Big[(1/5^{2N} \cdot 2^{x+3}) S(R3R411\dots 1x, (1/2, nx-1), (1/2, nx-1), \dots, (1, n1-1)(1/4, n1)) - \\
& (1/5^{2N} \cdot 2^{nx+x+2}) S(R3R411\dots 1x, (1, nx-1), (1/2, nx-1), \dots, (1, n1-1)(1/4, n1)) \Big] + \Big[(1/5^{2N} \cdot 2^{x+4}) \\
& S(R3R411\dots 1x, (1/2, nx-1), (1/2, nx-1), \dots, (1/4, n2), (1, n1-1)(1/4, n1)) - (1/5^{2N} \cdot 2^{nx+x+3}) \\
& S(R3R411\dots 1x, (1, nx-1), (1/2, nx-1), \dots, (1/4, n2), (1, n1-1)(1/4, n1)) \Big] + \dots + \Big[(1/5^{2N} \cdot 2^{2x+1}) \\
& S(R3R411\dots 1x, (1/2, nx-1), (1/4, nx-1), \dots, (1/4, n2), (1, n1-1)(1/4, n1)) - (1/5^{2N} \cdot 2^{nx+2x}) \\
& S(R3R411\dots 1x, (1, nx-1), (1/4, nx-1), \dots, (1/4, n2), (1, n1-1)(1/4, n1)) \Big] \Big) + (1/4) \Big(\Big[(1/5^{2N} \cdot 2^{x+1}) \\
& S(R3R311\dots 1x, (1/2, nx-1), (1/2, nx-1), \dots, (1/4, n1)) - (1/5^{2N} \cdot 2^{nx+x}) S(R3R311\dots 1x, (1, nx-1), \\
& (1/2, nx-1), \dots, (1/4, n1)) \Big] - \Big[(1/5^{2N} \cdot 2^{x+2}) S(R3R311\dots 1x, (1/2, nx-1), (1/2, nx-1), \dots, (1/4, n2), \\
& (1/4, n1)) - (1/5^{2N} \cdot 2^{nx+x+1}) S(R3R311\dots 1x, (1, nx-1), (1/2, nx-1), \dots, (1/4, n2), (1/4, n1)) \Big] - \dots - \\
& \Big[(1/5^{2N} \cdot 2^{2x-1}) S(R3R311\dots 1x, (1/2, nx-1), (1/4, nx-1), \dots, (1/4, n2), (1/4, n1)) - (1/5^{2N} \cdot 2^{nx+2x-2}) \\
& S(R3R311\dots 1x, (1, nx-1), (1/4, nx-1), \dots, (1/4, n2), (1/4, n1)) \Big] \Big) + \Big(- \Big(\Big[(1/5^{2N} \cdot 2^{x+4}) \\
& S(R31R321\dots 1x, (1/2, nx-1), (1/2, nx-1), \dots, (1, n2-1)(1/4, n2), (1/4, n1)) - (1/5^{2N} \cdot 2^{nx+x+3}) \\
& S(R31R321\dots 1x, (1, nx-1), (1/2, nx-1), \dots, (1, n2-1)(1/4, n2), (1/4, n1)) \Big] + \Big[(1/5^{2N} \cdot 2^{x+5}) \\
& S(R31R321\dots 1x, (1/2, nx-1), (1/2, nx-1), \dots, (1/4, n3), (1, n2-1)(1/4, n2), (1/4, n1)) - \\
& (1/5^{2N} \cdot 2^{nx+x+4}) S(R31R321\dots 1x, (1, nx-1), (1/2, nx-1), \dots, (1/4, n3), (1, n2-1)(1/4, n2), (1/4, n1)) \\
& \Big] + \dots + \Big[(1/5^{2N} \cdot 2^{2x+2}) S(R31R321\dots 1x, (1/2, nx-1), (1/4, nx-1), \dots, (1/4, n3), (1, n2-1)(1/4, n2), \\
& (1/4, n1)) - (1/5^{2N} \cdot 2^{nx+2x+1}) S(R31R321\dots 1x, (1, nx-1), (1/4, nx-1), \dots, (1/4, n3), (1, n2-1)(1/4, \\
& n2), (1/4, n1)) \Big] \Big) + (1/4) \Big(\Big[(1/5^{2N} \cdot 2^{x+2}) S(R31R311\dots 1x, (1/2, nx-1), (1/2, nx-1), \dots, (1/4, n2), \\
& (1/4, n1)) - (1/5^{2N} \cdot 2^{nx+x+1}) S(R31R311\dots 1x, (1, nx-1), (1/2, nx-1), \dots, (1/4, n2), (1/4, n1)) \Big] - \\
& \Big[(1/5^{2N} \cdot 2^{x+3}) S(R31R311\dots 1x, (1/2, nx-1), (1/2, nx-1), \dots, (1/4, n3), (1/4, n2), (1/4, n1)) - \\
& (1/5^{2N} \cdot 2^{nx+x+2}) S(R31R311\dots 1x, (1, nx-1), (1/2, nx-1), \dots, (1/4, n3), (1/4, n2), (1/4, n1)) \Big] - \dots - \\
& \Big[(1/5^{2N} \cdot 2^{2x}) S(R31R311\dots 1x, (1/2, nx-1), (1/4, nx-1), \dots, (1/4, n3), (1/4, n2), (1/4, n1)) - \\
& (1/5^{2N} \cdot 2^{nx+2x-1}) S(R31R311\dots 1x, (1, nx-1), (1/4, nx-1), \dots, (1/4, n3), (1/4, n2), (1/4, n1)) \Big] \Big) + \\
& \dots + \Big(- \Big(\Big[(1/5^{2N} \cdot 2^{2x+1}) S(R311\dots 1x-1R31\dots 2x-1 1x, (1/2, nx-1), (1, nx-1-1)(1/4, nx-1), ((1/4, nx-
\end{aligned}$$

$$\begin{aligned}
& 2), \dots, (1/4, n1)) - (1/5^{2N} \cdot 2^{n_x+2x}) S(R311\dots 1_{x-1}R31\dots 2_{x-1}1_x, (1, n_x-1), (1, n_{x-1}-1)(1/4, n_x-1), ((1/4, \\
& n_x-2), \dots, (1/4, n1)) \Big] + (1/4) \Big[(1/5^{2N} \cdot 2^{2x-1}) S(R311\dots 1_{x-1}R31\dots 1_{x-1}1_x, (1/2, n_x-1), (1/4, n_x-1), \\
& ((1/4, n_x-2), \dots, (1/4, n1)) - (1/5^{2N} \cdot 2^{n_x+2x-2}) S(R311\dots 1_{x-1}R31\dots 1_{x-1}1_x, (1, n_x-1), (1/4, n_x-1), \\
& ((1/4, n_x-2), \dots, (1/4, n1)) \Big] \Big] \\
& - \Big[(7/4)(3/5^{2N} \cdot 2^2) S(R3, (1/4, n1))(1+(2/7)(R21+R211+\dots+R211\dots 1_{n_{N-1}})) + (1/5^{2N} \cdot 2^4) S(R3R4, \\
& (1, n1-1)(1/4, n1)) \Big] + \Big[(7/4)(3/5^{2N} \cdot 2^4) S(R31, (1/4, n2), (1/4, n1))(1+(2/7)(R21+R211+\dots \\
& +R211\dots 1_{n_{N-1}})) + (1/4)(1/5^{2N} \cdot 2^4) (S(R3R31, (1/4, n2), (1, n1)(1/4, n1)) \Big] + (1/4) \\
& (1/5^{2N} \cdot 2^6) S(R31R32, (1, n2-1)(1/4, n2), (1/4, n1)) \Big] + \dots + \Big[(7/4)(3/5^{2N} \cdot 2^{2x}) S(R311\dots 1_x, (1/4, \\
& n_x), \dots, (1/4, n1))(1+(2/7)(R21+R211+\dots+R211\dots 1_{n_{N-1}})) + (1/4)(1/5^{2N} \cdot 2^{2x}) (S(R3R311\dots 1_x, (1/4, \\
& n_x), \dots, (1, n1)(1/4, n1)) + S(R31R311\dots 1_x, (1/4, n_x), \dots, (1, n2)(1/4, n2), (1/4, n1)) R311\dots 1_x, \\
& (1/4, n_x), \dots, (1/4, n1)) + \dots + S(R311\dots 1_{x-1} R311\dots 1_x, (1/4, n_x), (1, n_x-1)(1/4, n_x-1), \dots, (1/4, n2), \\
& (1/4, n1)) \Big] + (1/4)(1/5^{2N} \cdot 2^{2x+2}) S(R311\dots 1_x R311\dots 2_x, (1, n_x-1)(1/4, n_x), (1/4, n_x-1), \dots, (1/4, n2), \\
& (1/4, n1)) \Big] \Big]
\end{aligned}$$

Calcul de la surface totale du cercle et expression de Pi

La sommation, jusqu'à l'infini, de la surface des différentes zones: zone C, zone N=1 et zones $N \geq 2$ nous donne la surface globale du cercle. La relation $\pi=4S/D^2$ nous donne la valeur de pi
On a x, n, d, valant l'infini

$$\begin{aligned}
& \pi = \\
& 4 \Big[-(3/4)x^3 + (831/392)x^2 + (26217/9800)x + (2059/980) - \frac{26\sqrt{2}}{125} - \frac{52\sqrt{402}}{875} - \frac{2\sqrt{201}}{25} \\
& - \frac{4\sqrt{2}}{175} (X211+(1/5)X2111+(1/5)^2X21111+\dots+(1/5)^{N-3} X21,1,\dots,1N) \\
& - (4/4375) \Big[((7/2) + X21)X211 + (1/25)((7/2) + X21+X211)X2111+\dots+(1/25)^{N-3} ((7/2) \\
& +X21+X211+ \dots+ X211,\dots,1_{N-1}) X211,\dots,1_N \Big] \\
& + 8 \Big[\Big[(x-1/5 \cdot 2^3) S(R3, (1/2, n1-2)) - (x-1/5 \cdot 2^{n1-1}) S(R3, (1, n1-2)) \Big] + \Big[(x-2/5 \cdot 2^4) S(R31, \\
& (1/2, n2-2), (1/2, n1)) - (x-2/5 \cdot 2^{n2+2}) S(R31, (1, n2-2), (1/2, n1)) \Big] - 3 \Big[(x-2/25 \cdot 2^4) S(R31, (1/2, \\
& n2-2), (1/4, n1)) - (x-2/25 \cdot 2^{n2+2}) S(R31, (1, n2-2), (1/4, n1)) \Big] + \dots + \Big[(1/5 \cdot 2^{x+1}) S(R311\dots 1_x \\
& 1, (1/2, n_{x-1}-2), (1/2, n_x-2), \dots, (1/2, n1)) - (x-2/5 \cdot 2^{n_{x-1}+x+2}) S(R311\dots 1_{x-1}, (1, n_{x-1}-2), (1/2, n_x-2), \dots, \\
& (1/2, n1)) \Big] - 3 \Big[(1/25 \cdot 2^{x+1}) S(R311\dots 1_{x-1}, (1/2, n_{x-1}-2), (1/2, n_x-2), \dots, (1/4, n1)) - (1/25 \cdot 2^{n_{x-1}+x-}
\end{aligned}$$

$$\begin{aligned}
& \left[\left(\frac{1}{25.2^{x+2}} \right) S(R311\dots 1_{x-1}, (1, n_{x-1-2}), (1/2, n_{x-2}), \dots, (1/4, n_1)) \right] + \left(\frac{1}{25.2^{x+2}} \right) S(R311\dots 1_{x-1}, (1/2, n_{x-1-2}), \\
& (1/2, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1)) - \left(\frac{1}{25.2^{n_{x-1} + x}} \right) S(R311\dots 1_{x-1}, (1, n_{x-1-2}), (1/2, n_{x-2}), \dots, (1/4, n_2), \\
& (1/4, n_1)) \right] + \dots + \left(\frac{1}{25.2^{2x-2}} \right) S(R311\dots 1_{x-1}, (1/2, n_{x-1-2}), (1/4, n_{x-2}), \dots, (1/4, n_1)) - \left(\frac{1}{25.2^{n_{x-1} + 2x-4}} \right) \\
& S(R311\dots 1_{x-1}, (1, n_{x-1-2}), (1/4, n_{x-2}), \dots, (1/4, n_1)) \right] \\
& + \left(\left[\left(\frac{1}{5.2^4} \right) (2x^2-2) S(R3, (1/2, n_1-1)) - 3 \left(\frac{1}{25.2^3} \right) (x^2-x) S(R3, (1/4, n_1-1)) - \left(\frac{1}{25.2^5} \right) (x^2-x) \right. \right. \\
& S(R4R3, (1/4, n_1-1)(1, n_1-2)) \right] + \left[\left(\frac{1}{5.2^5} \right) (2(x-1)^2-2) S(R31, (1/2, n_2-1), (1/2, n_1)) - 3 \left(\frac{1}{25.2^5} \right) \right. \\
& (2(x-1)^2-2) S(R31, (1/2, n_2-1), (1/4, n_1)) + \left. \left(\frac{1}{25.2^5} \right) ((x-1)^2-(x-1)) S(R31, (1/4, n_2-1), (1/4, n_1)) \right] \\
& - \left(\frac{1}{25.2^7} \right) ((x-1)^2-(x-1)) S(R32R31, (1/4, n_2-1)(1, n_2-2), (1/4, n_1)) \right] + \dots + \left[\left(\frac{1}{5.2^{x+2}} \right) (2(2)^2-2) \right. \\
& S(R311\dots 1_{x-1}, (1/2, n_{x-1-1}), (1/2, n_{x-2}), \dots, (1/2, n_1)) - 3 \left(\frac{1}{25.2^{x+2}} \right) (2(2)^2-2) \left(S(R311\dots 1_{x-1}, (1/2, \right. \\
& n_{x-1-1}), (1/2, n_{x-2}), \dots, (1/2, n_2), (1/4, n_1)) + \left. \left(\frac{1}{2} \right) S(R311\dots 1_{x-1}, (1/2, n_{x-1-1}), (1/2, n_{x-2}), \dots, (1/2, \right. \\
& n_3), (1/4, n_2), (1/4, n_1)) + \dots + \left. \left(\frac{1}{2} \right)^{x-3} S(R311\dots 1_{x-1}, (1/2, n_{x-1-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) \right] + \\
& \left(\frac{1}{25.2^{2x-1}} \right) ((2)^2-2) S(R311\dots 1_{x-1}, (1/4, n_{x-1-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) \right] - \left(\frac{1}{25.2^{2x+1}} \right) ((2)^2- \\
& (2)) S(R311\dots 2_{x-1} R311\dots 1_{x-1}, (1/4, n_{x-1-1})(1, n_{x-1-2}), (1/4, n_{x-2}), \dots, (1/4, n_1)) \right] \\
& - \left(\left[\left(\frac{(n_2-1)}{5.2^2} \right) S(R3, (1/2, n_1)) - 3 \left(\frac{n_2}{25.2^2} \right) S(R3, (1/4, n_1)) - \left(\frac{n_2}{25.2^4} \right) S(R4R3, (1/4, n_1) \right. \right. \\
& (1, n_1-1)) \right] (x-1) + \left[\left(\frac{(n_3-1)}{5.2^3} \right) S(R31, (1/2, n_2), (1/2, n_1)) - 3 \left(\frac{(n_3-1)}{25.2^3} \right) S(R31, (1/2, n_2), \right. \\
& (1/4, n_1)) + \left. \left(\frac{n_3}{25.2^4} \right) S(R31, (1/4, n_2), (1/4, n_1)) \right] - \left. \left(\frac{n_3}{25.2^6} \right) S(R32R31, (1/4, n_2)(1, n_2-1), \right. \\
& (1/4, n_1)) \right] (x-2) + \dots + \left[\left(\frac{(n_x-1)}{5.2^x} \right) S(R311\dots 1_{x-1}, (1/2, n_{x-1}), (1/2, n_{x-2}), \dots, (1/2, n_1)) - \right. \\
& \left. 3 \left(\frac{(n_x-1)}{25.2^x} \right) \left(S(R311\dots 1_{x-1}, (1/2, n_{x-1}), (1/2, n_{x-2}), \dots, (1/2, n_2), (1/4, n_1)) + \left(\frac{1}{2} \right) \right. \right. \\
& S(R311\dots 1_{x-1}, (1/2, n_{x-1}), (1/2, n_{x-2}), \dots, (1/2, n_3), (1/4, n_2), (1/4, n_1)) + \dots + \left. \left(\frac{1}{2} \right)^{x-3} S(R311\dots 1_{x-1}, \right. \\
& (1/2, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) \right] + \left. \left(\frac{n_x}{25.2^{2x-2}} \right) S(R311\dots 1_{x-1}, (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, \right. \\
& (1/4, n_1)) \right] - \left(\frac{n_x}{25.2^{2x}} \right) S(R311\dots 2_{x-1} R311\dots 1_{x-1}, (1/4, n_{x-1})(1, n_{x-1-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) \right] \\
& \left. \right] (1) \\
& + \left(\left[- \left(\left[\left(\frac{1}{25.2^6} \right) S(R3R41, (1/2, n_2-2), (1, n_1-1)(1/4, n_1)) - \left(\frac{1}{25.2^{n_2+4}} \right) S(R3R41, (1, n_2-2), \right. \right. \right. \right. \right. \\
& (1, n_1-1)(1/4, n_1)) - \left. \left(\frac{(n_3-1)}{25.2^5} \right) S(R3R41, (1/2, n_2), (1, n_1-1)(1/4, n_1)) + \left. \left(\frac{n_3}{25.2^6} \right) \right. \right. \\
& S(R3R41, (1/4, n_2), (1, n_1-1)(1/4, n_1)) \right] (x-2) + \dots + \left[\left(\frac{1}{25.2^{x+3}} \right) \left(S(R3R41\dots 1_{x-1}, (1/2, n_{x-1-2}), \right. \right. \\
& (1/2, n_{x-2}), \dots, (1/2, n_2), (1, n_1-1)(1/4, n_1)) + \left. \left(\frac{1}{2} \right) S(R3R41\dots 1_{x-1}, (1/2, n_{x-1-2}), (1/2, n_{x-2}), \dots, \right. \\
& (1/4, n_2), (1, n_1-1)(1/4, n_1)) + \dots + \left. \left(\frac{1}{2} \right)^{x-3} S(R3R41\dots 1_{x-1}, (1/2, n_{x-1-2}), (1/4, n_{x-2}), \dots, (1/4, n_2), \right. \\
& (1, n_1-1)(1/4, n_1)) \right] - \left(\frac{1}{25.2^{n_{x-1} + x + 1}} \right) \left(S(R3R41\dots 1_{x-1}, (1, n_{x-1-2}), (1/2, n_{x-2}), \dots, (1/2, n_2), (1, \right. \\
& n_1-1)(1/4, n_1)) + \left. \left(\frac{1}{2} \right) S(R3R41\dots 1_{x-1}, (1, n_{x-1-2}), (1/2, n_{x-2}), \dots, (1/4, n_2), (1, n_1-1)(1/4, n_1)) + \dots \right. \\
& \left. + \left(\frac{1}{2} \right)^{x-3} S(R3R41\dots 1_{x-1}, (1, n_{x-1-2}), (1/4, n_{x-2}), \dots, (1/4, n_2), (1, n_1-1)(1/4, n_1)) \right] - \left(\frac{(n_x- \right. \\
& 1)}{25.2^{x+2}} \right) \left(S(R3R41\dots 1_{x-1}, (1/2, n_{x-1}), (1/2, n_{x-2}), \dots, (1/2, n_2), (1, n_1-1)(1/4, n_1)) + \right. \\
& \left. \left(\frac{1}{2} \right) S(R3R41\dots 1_{x-1}, (1/2, n_{x-1}), (1/2, n_{x-2}), \dots, (1/4, n_2), (1, n_1-1)(1/4, n_1)) + \dots + \left(\frac{1}{2} \right)^{x-} \right)
\end{aligned}$$

$$\begin{aligned}
&^3)S(\text{R3R411...1}_{x-1}, (1/2, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_2), (1, n_{1-1})(1/4, n_1)) \Big) + (n_x/25.2^{2x}) \\
&S(\text{R3R411...1}_{x-1}, (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_2), (1, n_{1-1})(1/4, n_1)) \Big) \Big| (1) \Big) \\
&+ \Big((1/25.2^7)(2(x-1)^2-2) S(\text{R3R41}, (1/2, n_2-1), (1, n_{1-1})(1/4, n_1)) + (1/25.2^7)((x-1)^2-(x-1)) \\
&S(\text{R3R41}, (1/4, n_2-1), (1, n_{1-1})(1/4, n_1)) + \dots + \Big| (1/25.2^{x+4})(2(2)^2-2) \Big(S(\text{R3R411...1}_{x-1}, (1/2, \\
&n_{x-1}-1), (1/2, n_{x-2}), \dots, (1/2, n_2), (1, n_{1-1})(1/4, n_1)) + (1/2)S(\text{R3R411...1}_{x-1}, (1/2, n_{x-1}-1), (1/2, n_{x-2}), \\
&\dots, (1/4, n_2), (1, n_{1-1})(1/4, n_1)) + \dots + (1/2^{x-3})S(\text{R3R411...1}_{x-1}, (1/2, n_{x-1}-1), (1/4, n_{x-2}), \dots, (1/4, \\
&n_2), (1, n_{1-1})(1/4, n_1)) \Big) + (1/25.2^{2x+1})((2)^2-2) S(\text{R3R411...1}_{x-1}, (1/4, n_{x-1}-1), (1/4, n_{x-2}), \dots, (1/4, \\
&n_2), (1, n_{1-1})(1/4, n_1)) \Big) \Big) \\
&+ \Big(\Big(\Big| (1/25.2^4) S(\text{R3R31}, (1/2, n_2-2), (1/4, n_1)) - (1/25.2^{n_2+2})S(\text{R3R31}, (1, n_2-2), (1/4, n_1)) - \\
&((n_3-1)/25.2^3) S(\text{R3R31}, (1/2, n_2), (1/4, n_1)) - (n_3/25.2^4) S(\text{R3R31}, (1/4, n_2), (1/4, n_1)) \Big) \Big| (x-2) \\
&+ \dots + \Big| (1/25.2^{x+1}) \Big(S(\text{R3R311...1}_{x-1}, (1/2, n_{x-1}-2), (1/2, n_{x-2}), \dots, (1/2, n_2), (1/4, n_1)) - \\
&(1/2)S(\text{R3R311...1}_{x-1}, (1/2, n_{x-1}-2), (1/2, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1)) - \dots - (1/2^{x-3})S(\text{R3R311...1}_{x-1}, \\
&(1/2, n_{x-1}-2), (1/4, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1)) \Big) - (1/25.2^{n_{x-1}+x-1}) \Big(S(\text{R3R311...1}_{x-1}, (1, n_{x-1}- \\
&2), (1/2, n_{x-2}), \dots, (1/2, n_2), (1/4, n_1)) - (1/2)S(\text{R3R311...1}_{x-1}, (1, n_{x-1}-2), (1/2, n_{x-2}), \dots, (1/4, n_2), \\
&(1/4, n_1)) - \dots - (1/2^{x-3})S(\text{R3R311...1}_{x-1}, (1, n_{x-1}-2), (1/4, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1)) \Big) - ((n_x- \\
&1)/25.2^x) \Big(S(\text{R3R311...1}_{x-1}, (1/2, n_{x-1}), (1/2, n_{x-2}), \dots, (1/2, n_2), (1/4, n_1)) - (1/2)S(\text{R3R311...1}_{x-1}, \\
&(1/2, n_{x-1}), (1/2, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1)) - \dots - (1/2^{x-3})S(\text{R3R311...1}_{x-1}, (1/2, n_{x-1}), (1/4, n_{x- \\
&2}), \dots, (1/4, n_2), (1/4, n_1)) \Big) - (n_x/25.2^{2x-2}) S(\text{R3R311...1}_{x-1}, (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_2), \\
&(1/4, n_1)) \Big) \Big| (1) \Big) \\
&+ \Big((1/25.2^5)(2(x-1)^2-2) S(\text{R3R31}, (1/2, n_2-1), (1/4, n_1)) - (1/25.2^5)((x-1)^2-(x-1)) S(\text{R3R31}, (1/4, \\
&n_2-1), (1/4, n_1)) - \dots - \Big| (1/25.2^{x+2})(2(2)^2-2) \Big(S(\text{R3R311...1}_{x-1}, (1/2, n_{x-1}-1), (1/2, n_{x-2}), \dots, (1/2, \\
&n_2), (1/4, n_1)) - (1/2)S(\text{R3R311...1}_{x-1}, (1/2, n_{x-1}-1), (1/2, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1)) - \dots - (1/2^{x- \\
&3})S(\text{R3R311...1}_{x-1}, (1/2, n_{x-1}-1), (1/4, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1)) \Big) - (1/25.2^{2x-1})((2)^2-2) \\
&S(\text{R3R311...1}_{x-1}, (1/4, n_{x-1}-1), (1/4, n_{x-2}), \dots, (1/4, n_2), (1/4, n_1)) \Big) \Big) \Big| (crochets roses ci-contre= \\
&\text{terme pour } x=3, \text{ on a } \text{R311...2}_{x-2} \text{1}_{x-1} = \text{R4 pour } x=3)
\end{aligned}$$

$$\begin{aligned}
&+ \dots + \\
&\Big[- \Big(\Big| (1/25.2^{2x}) S(\text{R311...1}_{x-2} \text{R311...2}_{x-2} \text{1}_{x-1}, (1/2, n_{x-1}-2), (1, n_{x-2}-1)(1/4, n_{x-1}), (1/4, n_{x-2}), \dots, \\
&(1/4, n_1)) - (1/25.2^{n_{x-1}+2x-2}) S(\text{R311...1}_{x-2} \text{R311...2}_{x-2} \text{1}_{x-1}, (1, n_{x-1}-2), (1, n_{x-2}-1)(1/4, n_{x-1}), (1/4, \\
&n_{x-2}), \dots, (1/4, n_1)) - ((n_x-1)/25.2^{2x-1}) S(\text{R311...1}_{x-2} \text{R311...2}_{x-2} \text{1}_{x-1}, (1/2, n_{x-1}), (1, n_{x-2}-1)(1/4, \\
&n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) + (n_x/25.2^{2x}) S(\text{R311...1}_{x-2} \text{R311...2}_{x-2} \text{1}_{x-1}, (1/4, n_{x-1}), (1, n_{x-2}-1) \\
&(1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) \Big) \Big| (1) \Big)
\end{aligned}$$

$$\begin{aligned}
& + \left((1/25.2^{2x+1})(2(2)^2-2) S(R311...1_{x-2} R311...2_{x-2} 1_{x-1}, (1/2, n_{x-1-1}), (1, n_{x-2-1})(1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) + (1/25.2^{2x+1})((2)^2-(2)) S(R311...1_{x-2} R311...2_{x-2} 1_{x-1}, (1/4, n_{x-1-1}), (1, n_{x-2-1})(1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) \right) \\
& + \left(\left((1/25.2^{2x-2}) S(R311...1_{x-2} R311...1_{x-2} 1_{x-1}, (1/2, n_{x-1-2}), (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) - (1/25.2^{n_{x-1}+2x-4}) S(R311...1_{x-2} R311...1_{x-2} 1_{x-1}, (1, n_{x-1-2}), (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) - ((n_{x-1})/25.2^{2x-3}) S(R311...1_{x-2} R311...1_{x-2} 1_{x-1}, (1/2, n_{x-1}), (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) - (n_x/25.2^{2x-2}) S(R311...1_{x-2} R311...1_{x-2} 1_{x-1}, (1/4, n_{x-1}), (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) \right) \right) (1) \\
& + \left((1/25.2^{2x-1})(2(2)^2-2) S(R311...1_{x-2} R311...1_{x-2} 1_{x-1}, (1/2, n_{x-1-1}), (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) - (1/25.2^{2x-1})((2)^2-(2)) S(R311...1_{x-2} R311...1_{x-2} 1_{x-1}, (1/4, n_{x-1-1}), (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& + \left(\left((1/5.2^2) S(R3, (1/2, n_1-1)) - (1/5.2^{n_1-1}) S(R3, (1, n_1-1)) \right) + \left((1/5.2^3) S(R31, (1/2, n_2-1), (1/2, n_1)) - (1/5.2^{n_2+2}) S(R31, (1, n_2-1), (1/2, n_1)) \right) - 3 \left((1/25.2^3) S(R31, (1/2, n_2-1), (1/4, n_1)) - (1/25.2^{n_2+2}) S(R31, (1, n_2-1), (1/4, n_1)) \right) \right) + \dots + \left((1/5.2^{x+1}) S(R311...1_x, (1/2, n_{x-1}), (1/2, n_{x-1}), \dots, (1/2, n_1)) - (1/5.2^{n_x+x}) S(R311...1_x, (1, n_x), (1/2, n_{x-1}), \dots, (1/2, n_1)) \right) - 3 \left((1/25.2^{x+1}) S(R311...1_x, (1/2, n_{x-1}), (1/2, n_{x-1}), \dots, (1/4, n_1)) - (1/25.2^{n_x+x}) S(R311...1_x, (1, n_{x-1}), (1/2, n_{x-1}), \dots, (1/4, n_1)) \right) + \left((1/25.2^{x+2}) S(R311...1_x, (1/2, n_{x-1}), (1/2, n_{x-1}), \dots, (1/4, n_2), (1/4, n_1)) - (1/25.2^{n_x+x+1}) S(R311...1_x, (1, n_{x-1}), (1/2, n_{x-1}), \dots, (1/4, n_2), (1/4, n_1)) \right) + \dots + \left((1/25.2^{2x-1}) S(R311...1_x, (1/2, n_{x-1}), (1/4, n_{x-1}), \dots, (1/4, n_1)) - (1/25.2^{n_{x-1}+2x-2}) S(R311...1_x, (1, n_{x-1}), (1/4, n_{x-1}), \dots, (1/4, n_1)) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& + \left(\left[- \left((1/25.2^5) S(R3R41, (1/2, n_2-1), (1, n_1-1)(1/4, n_1)) - (1/25.2^{n_2+4}) S(R3R41, (1, n_2-1), (1, n_1-1)(1/4, n_1)) \right) + \left((1/25.2^3) S(R3R31, (1/2, n_2-1), (1/4, n_1)) - (1/25.2^{n_2+2}) S(R3R31, (1, n_2-1), (1/4, n_1)) \right) \right] \right) \\
& + \left[\left(- \left((1/25.2^6) S(R3R411, (1/2, n_3-1), (1/2, n_2), (1, n_1-1)(1/4, n_1)) - (1/25.2^{n_3+5}) S(R3R411, (1, n_3-1), (1/2, n_2), (1, n_1-1)(1/4, n_1)) \right) + \left((1/25.2^7) S(R3R411, (1/2, n_3-1), (1/4, n_2), (1, n_1-1)(1/4, n_1)) - (1/25.2^{n_3+6}) S(R3R411, (1, n_3-1), (1/4, n_2), (1, n_1-1)(1/4, n_1)) \right) \right) + \left((1/25.2^4) S(R3R311, (1/2, n_3-1), (1/2, n_2), (1/4, n_1)) - (1/25.2^{n_3+3}) S(R3R311, (1, n_3-1), (1/2, n_2), (1/4, n_1)) \right) - \left((1/25.2^5) S(R3R311, (1/2, n_3-1), (1/4, n_2), (1/4, n_1)) - (1/25.2^{n_3+4}) S(R3R311, (1, n_3-1), (1/4, n_2), (1/4, n_1)) \right) \right) + \left(- \left((1/25.2^7) S(R31R321, (1/2, n_3-1), (1, n_2-1)(1/4, n_2), (1/4, n_1)) - \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{1}{25.2^{n^3+6}} \right) S(R31R321, (1, n3-1), (1, n2-1)(1/4, n2), (1/4, n1)) \Big| + \left[\frac{1}{25.2^5} \right] S(R31R311, (1/2, \\
& n3-1), (1/4, n2), (1/4, n1)) - \left(\frac{1}{25.2^{n^3+4}} \right) S(R31R311, (1, n3-1), (1/4, n2), (1/4, n1)) \Big| \Big] \\
& + \dots + \\
& \left[\left(- \left[\frac{1}{25.2^{x+3}} \right] S(R3R411...1x, (1/2, nx-1), (1/2, nx-1), \dots, (1, n1-1)(1/4, n1)) - \left(\frac{1}{25.2^{n_x+x+2}} \right) \right. \right. \\
& S(R3R411...1x, (1, nx-1), (1/2, nx-1), \dots, (1, n1-1)(1/4, n1)) \Big| + \left[\frac{1}{25.2^{x+4}} \right] S(R3R411...1x, (1/2, \\
& nx-1), (1/2, nx-1), \dots, (1/4, n2), (1, n1-1)(1/4, n1)) - \left(\frac{1}{25.2^{n_x+x+3}} \right) S(R3R411...1x, (1, nx-1), (1/2, \\
& nx-1), \dots, (1/4, n2), (1, n1-1)(1/4, n1)) \Big| + \dots + \left[\frac{1}{25.2^{2x+1}} \right] S(R3R411...1x, (1/2, nx-1), (1/4, nx- \\
& 1), \dots, (1/4, n2), (1, n1-1)(1/4, n1)) - \left(\frac{1}{25.2^{n_x+2x}} \right) S(R3R411...1x, (1, nx-1), (1/4, nx-1), \dots, (1/4, \\
& n2), (1, n1-1)(1/4, n1)) \Big| \Big] + \left(\left[\frac{1}{25.2^{x+1}} \right] S(R3R311...1x, (1/2, nx-1), (1/2, nx-1), \dots, (1/4, n1)) - \right. \\
& \left. \left(\frac{1}{25.2^{n_x+x}} \right) S(R3R311...1x, (1, nx-1), (1/2, nx-1), \dots, (1/4, n1)) \Big| - \left[\frac{1}{25.2^{x+2}} \right] S(R3R311...1x, \right. \\
& \left. (1/2, nx-1), (1/2, nx-1), \dots, (1/4, n2), (1/4, n1)) - \left(\frac{1}{25.2^{n_x+x+1}} \right) S(R3R311...1x, (1, nx-1), (1/2, nx- \\
& 1), \dots, (1/4, n2), (1/4, n1)) \Big| - \dots - \left[\frac{1}{25.2^{2x-1}} \right] S(R3R311...1x, (1/2, nx-1), (1/4, nx-1), \dots, (1/4, n2), \\
& (1/4, n1)) - \left(\frac{1}{25.2^{n_x+2x-2}} \right) S(R3R311...1x, (1, nx-1), (1/4, nx-1), \dots, (1/4, n2), (1/4, n1)) \Big| \Big) \Big] + \left(- \right. \\
& \left. \left[\frac{1}{25.2^{x+4}} \right] S(R31R321...1x, (1/2, nx-1), (1/2, nx-1), \dots, (1, n2-1)(1/4, n2), (1/4, n1)) - \right. \\
& \left. \left(\frac{1}{25.2^{n_x+x+3}} \right) S(R31R321...1x, (1, nx-1), (1/2, nx-1), \dots, (1, n2-1)(1/4, n2), (1/4, n1)) \Big| + \right. \\
& \left[\frac{1}{25.2^{x+5}} \right] S(R31R321...1x, (1/2, nx-1), (1/2, nx-1), \dots, (1/4, n3), (1, n2-1)(1/4, n2), (1/4, n1)) - \\
& \left. \left(\frac{1}{25.2^{n_x+x+4}} \right) S(R31R321...1x, (1, nx-1), (1/2, nx-1), \dots, (1/4, n3), (1, n2-1)(1/4, n2), (1/4, n1)) \Big| \right. \\
& \left. + \dots + \left[\frac{1}{25.2^{2x+2}} \right] S(R31R321...1x, (1/2, nx-1), (1/4, nx-1), \dots, (1/4, n3), (1, n2-1)(1/4, n2), (1/4, \\
& n1)) - \left(\frac{1}{25.2^{n_x+2x+1}} \right) S(R31R321...1x, (1, nx-1), (1/4, nx-1), \dots, (1/4, n3), (1, n2-1)(1/4, n2), (1/4, \\
& n1)) \Big| \Big] + \left(\left[\frac{1}{25.2^{x+2}} \right] S(R31R311...1x, (1/2, nx-1), (1/2, nx-1), \dots, (1/4, n2), (1/4, n1)) - \right. \\
& \left. \left(\frac{1}{25.2^{n_x+x+1}} \right) S(R31R311...1x, (1, nx-1), (1/2, nx-1), \dots, (1/4, n2), (1/4, n1)) \Big| - \left[\frac{1}{25.2^{x+3}} \right] \right. \\
& S(R31R311...1x, (1/2, nx-1), (1/2, nx-1), \dots, (1/4, n3), (1/4, n2), (1/4, n1)) - \left(\frac{1}{25.2^{n_x+x+2}} \right) \\
& S(R31R311...1x, (1, nx-1), (1/2, nx-1), \dots, (1/4, n3), (1/4, n2), (1/4, n1)) \Big| - \dots - \left[\frac{1}{25.2^{2x}} \right] \\
& S(R31R311...1x, (1/2, nx-1), (1/4, nx-1), \dots, (1/4, n3), (1/4, n2), (1/4, n1)) - \left(\frac{1}{25.2^{n_x+2x-1}} \right) \\
& S(R31R311...1x, (1, nx-1), (1/4, nx-1), \dots, (1/4, n3), (1/4, n2), (1/4, n1)) \Big| \Big) \Big] + \dots + \left(- \left(\right. \right. \\
& \left. \left[\frac{1}{25.2^{2x+1}} \right] S(R311...1x-1R31...2x-11x, (1/2, nx-1), (1, nx-1-1)(1/4, nx-1), ((1/4, nx-2), \dots, (1/4, \\
& n1)) - \left(\frac{1}{25.2^{n_x+2x}} \right) S(R311...1x-1R31...2x-11x, (1, nx-1), (1, nx-1-1)(1/4, nx-1), ((1/4, nx-2), \dots, \\
& (1/4, n1)) \Big| \Big) + \left(\left[\frac{1}{25.2^{2x-1}} \right] S(R311...1x-1R31...1x-11x, (1/2, nx-1), (1/4, nx-1), ((1/4, nx-2), \dots, \\
& (1/4, n1)) - \left(\frac{1}{25.2^{n_x+2x-2}} \right) S(R311...1x-1R31...1x-11x, (1, nx-1), (1/4, nx-1), ((1/4, nx-2), \dots, (1/4, \\
& n1)) \Big| \Big) \Big) \Big] \Big]
\end{aligned}$$

$$- \left(\left[(3/25.2^2) S(R3, (1/4, n1)) + (1/25.2^4) S(R3R4, (1, n1-1)(1/4, n1)) \right] + \left[(3/25.2^4) S(R31, (1/4, n2), (1/4, n1)) + (1/25.2^4) S(R3R31, (1/4, n2), (1, n1)(1/4, n1)) \right] + (1/25.2^6) S(R31R32, (1, n2-1)(1/4, n2), (1/4, n1)) \right] + \dots + \left[(3/25.2^{2x}) S(R311\dots 1x, (1/4, nx), \dots, (1/4, n1)) + (1/25.2^{2x}) S(R3R311\dots 1x, (1/4, nx), \dots, (1, n1)(1/4, n1)) + S(R31R311\dots 1x, (1/4, nx), \dots, (1, n2)(1/4, n2), (1/4, n1)) \right] R311\dots 1x, (1/4, nx), \dots, (1/4, n1)) + \dots + S(R311\dots 1x-1 R311\dots 1x, (1/4, nx), (1, nx-1)(1/4, nx-1), \dots, (1/4, n2), (1/4, n1)) \right] + (1/25.2^{2x+2}) S(R311\dots 1x R311\dots 2x, (1, nx-1)(1/4, nx), (1/4, nx-1), \dots, (1/4, n2), (1/4, n1)) \right] \right]$$

$$+ 8 \sum_{N=2}^{N=\infty}$$

$$\left[\left((35/5^{2N}.2^2) S(R211\dots 1N, (1/2, nN-1)) - (35/5^{2N}.2^{nN+1}) S(R211\dots 1N, (1, nN-1)) + d[(35/5^{2N}.2^3) S(R211\dots 1N, (1/2, nN-2)) - (35/5^{2N}.2^{nN+1}) S(R211\dots 1N, (1, nN-2))] \right) + \left((35/5^{2N}.2^5) (4d^2+14d+6) S(R211\dots 1N, (1/2, nN-1)) - (21/5^{2N}.2^4) (2d^2+2d) S(R211\dots 1N, (1/4, nN-1)) + d[(35/5^{2N}.2^3) S(R211\dots 1N, (1/2, nN-2)) - (35/5^{2N}.2^{nN+1}) S(R211\dots 1N, (1, nN-2))] \right) \right] \left(1 + (2/7)(R21+R211+\dots+R211\dots 1nN-1) \right) - \left((175/5^{2N}.2) ((nN-1)+d(nN-2)) - (105/5^{2N}.2) (2+d) + (21/5^{2N}.2^3) (d^2+3d) + d(49/5^{2N}.2) \right) \left(R211\dots 1nN-1 + (-3)R211\dots 1nN-2 + (-3)^{N-3} R21 \right) + (2/7)(R211\dots 1nN-1(R21+R211+\dots+R211\dots 1nN-2) + (-3)R211\dots 1nN-2(R21+R211+\dots+R211\dots 1nN-3) + (-3)^{N-4} R211R21) - d \left((35(nN+1)/5^{2N}.2^3) S(R211\dots 1N, (1/2, nN)) - (21nN+1/5^{2N}.2^4) S(R211\dots 1N, (1/4, nN)) \right) \left(1 + (2/7)(R21+R211+\dots+R211\dots 1nN-1) \right) + d \left((5/5^{2N}.2^2) S(R211\dots 1N, (1/2, nN-2)) - (5/5^{2N}.2^{nN}) S(R211\dots 1N, (1, nN-2)) \right) (R21+R211+\dots+R211\dots 1nN-1) + d \left((15/5^{2N}.2^2) S(R211\dots 1N, (1/2, nN-1)) - (3/5^{2N}.2^2) S(R211\dots 1N, (1/4, nN-1)) \right) (R21+R211+\dots+R211\dots 1nN-1) + \left(-(d^2+d)/5^{2N}.2^5) S(R211\dots 1N R211\dots 2N, (1, nN-2)(1/4, nN-1)) + d(nN+1/5^{2N}.2^4) S(R211\dots 1N R211\dots 2N, (1/4, nN-1)) \right) \right]$$

[à partir d'ici, les termes sont similaires à ceux relatifs à la zone N=1, il convient toutefois d'apporter les modifications suivantes:

- *on remplace chacun des termes x suivants par (d+1)
 - *on remplace chaque terme (n1) par (nN), (n2) par (nN+1) et ainsi de suite.
 - *on remplace chaque terme de la forme (R311\dots 1x-1) par (R211\dots 1N11\dots 1x-1) et (Rn111\dots 1x-1) par (R211\dots 1nN11\dots 1x-1)
- $$+ \left((7/2) \left((x-2/5^{2N-1}.2^4) S(R31, (1/2, n2-2), (1/2, n1)) - (x-2/5^{2N-1}.2^{n2+2}) S(R31, (1, n2-2), (1/2, n1)) \right) - (7/4) \left((x-2/5^{2N}.2^4) S(R31, (1/2, n2-2), (1/4, n1)) - (x-2/5^{2N}.2^{n2+2}) S(R31, (1, n2-$$

$$\begin{aligned}
& 2), (1/4, n1)) \Big] + \dots + \Big[(7/2) \left((1/5^{2N-1} \cdot 2^{x+1}) S(R311\dots 1_{x-1}, (1/2, n_{x-1-2}), (1/2, n_{x-2}), \dots, (1/2, \right. \\
& n1) \Big) - (x-2/5^{2N-1} \cdot 2^{n_{x-1}+x+2}) S(R311\dots 1_{x-1}, (1, n_{x-1-2}), (1/2, n_{x-2}), \dots, (1/2, n1) \Big) - (7/4)3 \Big(\\
& \left((1/5^{2N} \cdot 2^{x+1}) S(R311\dots 1_{x-1}, (1/2, n_{x-1-2}), (1/2, n_{x-2}), \dots, (1/4, n1) \Big) - (1/5^{2N} \cdot 2^{n_{x-1}+x-1}) \right. \\
& S(R311\dots 1_{x-1}, (1, n_{x-1-2}), (1/2, n_{x-2}), \dots, (1/4, n1) \Big) + \left((1/5^{2N} \cdot 2^{x+2}) S(R311\dots 1_{x-1}, (1/2, n_{x-1-2}), \right. \\
& (1/2, n_{x-2}), \dots, (1/4, n2), (1/4, n1) \Big) - (1/5^{2N} \cdot 2^{n_{x-1}+x}) S(R311\dots 1_{x-1}, (1, n_{x-1-2}), (1/2, n_{x-2}), \dots, (1/4, \\
& n2), (1/4, n1) \Big) + \dots + \left((1/5^{2N} \cdot 2^{2x-2}) S(R311\dots 1_{x-1}, (1/2, n_{x-1-2}), (1/4, n_{x-2}), \dots, (1/4, n1) \Big) - \right. \\
& \left. (1/5^{2N} \cdot 2^{n_{x-1}+2x-4}) S(R311\dots 1_{x-1}, (1, n_{x-1-2}), (1/4, n_{x-2}), \dots, (1/4, n1) \Big) \Big] \Big((1+(2/7)(R21+R211+\dots \\
& +R211\dots 1_{nN-1})) \right. \\
& + \left(\Big[(7/2)(1/5^{2N-1} \cdot 2^5)(2(x-1)^2-2) S(R31, (1/2, n2-1), (1/2, n1))(1+(2/7)(R21+R211+\dots \\
& +R211\dots 1_{nN-1})) - (7/4)3 \left((1/5^{2N} \cdot 2^5)(2(x-1)^2-2) S(R31, (1/2, n2-1), (1/4, n1)) + (1/5^{2N} \cdot 2^5)((x-1)^2- \right. \right. \\
& (x-1)) S(R31, (1/4, n2-1), (1/4, n1)) \Big] + (2/7)(R21+R211+\dots+R211\dots 1_{nN-1}) - (1/5^{2N} \cdot 2^7)((x-1)^2- \\
& (x-1)) S(R32R31, (1/4, n2-1)(1,n2-2), (1/4, n1) \Big] + \dots + \Big[(7/2)(1/5^{2N-1} \cdot 2^{x+2})(2(2)^2-2) \\
& S(R311\dots 1_{x-1}, (1/2, n_{x-1-1}), (1/2, n_{x-2}), \dots, (1/2, n1))(1+(2/7)(R21+R211+\dots+R211\dots 1_{nN-1})) - \\
& (7/4)3 \left((1/5^{2N} \cdot 2^{x+2})(2(2)^2-2) \left(S(R311\dots 1_{x-1}, (1/2, n_{x-1-1}), (1/2, n_{x-2}), \dots, (1/2, n2), (1/4, n1)) + \right. \right. \\
& (1/2) S(R311\dots 1_{x-1}, (1/2, n_{x-1-1}), (1/2, n_{x-2}), \dots, (1/2, n3), (1/4, n2), (1/4, n1)) + \dots + (1/2^{x-} \\
& ^3) S(R311\dots 1_{x-1}, (1/2, n_{x-1-1}), (1/4, n_{x-2}), \dots, (1/4, n1) \Big) + (1/5^{2N} \cdot 2^{2x-1})((2)^2-2) S(R311\dots 1_{x-1}, \\
& (1/4, n_{x-1-1}), (1/4, n_{x-2}), \dots, (1/4, n1) \Big) \Big] + (2/7)(R21+R211+\dots+R211\dots 1_{nN-1}) - (1/5^{2N} \cdot 2^{2x+1}) \\
& ((2)^2-(2)) S(R311\dots 2_{x-1} R311\dots 1_{x-1}, (1/4, n_{x-1-1})(1, n_{x-1-2}), (1/4, n_{x-2}), \dots, (1/4, n1) \Big] \Big) \\
& - \left(\Big[(7/2)((n3-1)/5^{2N-1} \cdot 2^3) S(R31, (1/2, n2), (1/2, n1))(1+(2/7)(R21+R211+\dots+R211\dots 1_{nN-1})) - \right. \\
& (7/4)3 \left(((n3-1)/5^{2N} \cdot 2^3) S(R31, (1/2, n2), (1/4, n1)) + (n3/5^{2N} \cdot 2^4) S(R31, (1/4, n2), (1/4, n1)) \right) \Big] \\
& (1+(2/7)(R21+R211+\dots+R211\dots 1_{nN-1})) - (n3/5^{2N} \cdot 2^6) S(R32R31, (1/4, n2)(1,n2-1), (1/4, n1) \Big] \Big((x- \\
& 2) + \dots + \Big[(7/2)((n_{x-1})/5^{2N-1} \cdot 2^x) S(R311\dots 1_{x-1}, (1/2, n_{x-1}), (1/2, n_{x-2}), \dots, (1/2, n1))(1+(2/7) \\
& (R21+R211+\dots+R211\dots 1_{nN-1})) - (7/4)3 \left(((n_{x-1})/5^{2N} \cdot 2^x) \left(S(R311\dots 1_{x-1}, (1/2, n_{x-1}), (1/2, n_{x-2}), \dots, \right. \right. \\
& (1/2, n2), (1/4, n1)) + (1/2) S(R311\dots 1_{x-1}, (1/2, n_{x-1}), (1/2, n_{x-2}), \dots, (1/2, n3), (1/4, n2), (1/4, n1)) \\
& + \dots + (1/2^{x-3}) S(R311\dots 1_{x-1}, (1/2, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n1) \Big) + (n_{x-1}/5^{2N} \cdot 2^{2x-2}) S(R311\dots 1_{x-1}, \\
& (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n1) \Big) \Big] + (2/7)(R21+R211+\dots+R211\dots 1_{nN-1}) - \\
& (n_{x-1}/5^{2N} \cdot 2^{2x}) S(R311\dots 2_{x-1} R311\dots 1_{x-1}, (1/4, n_{x-1})(1, n_{x-1-1}), (1/4, n_{x-2}), \dots, (1/4, n1) \Big] \Big(1 \Big) \Big) \\
& + \left(\Big[- \left(\Big[(1/5^{2N} \cdot 2^6) S(R3R41, (1/2, n2-2), (1, n1-1)(1/4, n1)) - (1/5^{2N} \cdot 2^{n2+4}) S(R3R41, (1, n2- \right. \right. \\
& 2), (1, n1-1)(1/4, n1)) - \left(((n3-1)/5^{2N} \cdot 2^5) S(R3R41, (1/2, n2), (1, n1-1)(1/4, n1)) + (n3/5^{2N} \cdot 2^6) \right. \\
& S(R3R41, (1/4, n2), (1, n1-1)(1/4, n1)) \Big] \Big((x-2) + \dots + \Big[(1/5^{2N} \cdot 2^{x+3}) \left(S(R3R411\dots 1_{x-1}, (1/2, n_{x-1-} \right. \\
& 2), (1/2, n_{x-2}), \dots, (1/2, n2), (1, n1-1)(1/4, n1)) + (1/2) S(R3R411\dots 1_{x-1}, (1/2, n_{x-1-2}), (1/2, n_{x-2}), \dots, \\
& (1/4, n2), (1, n1-1)(1/4, n1)) + \dots + (1/2^{x-3}) S(R3R411\dots 1_{x-1}, (1/2, n_{x-1-2}), (1/4, n_{x-2}), \dots, (1/4, n2), \\
& (1, n1-1)(1/4, n1)) \Big] - (1/5^{2N} \cdot 2^{n_{x-1}+x+1}) \left(S(R3R411\dots 1_{x-1}, (1, n_{x-1-2}), (1/2, n_{x-2}), \dots, (1/2, n2), (1, \right.
\end{aligned}$$

$$\begin{aligned}
& n1-1)(1/4, n1)) + (1/2)S(R3R411...1_{x-1}, (1, n_{x-1}-2), (1/2, n_{x-2}), \dots, (1/4, n_2), (1, n1-1)(1/4, n1)) + \dots \\
& + (1/2^{x-3})S(R3R411...1_{x-1}, (1, n_{x-1}-2), (1/4, n_{x-2}), \dots, (1/4, n_2), (1, n1-1)(1/4, n1)) - (((n_{x-1}-1)/5^{2N} \cdot 2^{x+2}) (S(R3R411...1_{x-1}, (1/2, n_{x-1}), (1/2, n_{x-2}), \dots, (1/2, n_2), (1, n1-1)(1/4, n1)) + \\
& (1/2)S(R3R411...1_{x-1}, (1/2, n_{x-1}), (1/2, n_{x-2}), \dots, (1/4, n_2), (1, n1-1)(1/4, n1)) + \dots + (1/2^{x-3}) \\
& ^3)S(R3R411...1_{x-1}, (1/2, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_2), (1, n1-1)(1/4, n1))) + (n_x/5^{2N} \cdot 2^{2x}) \\
& S(R3R411...1_{x-1}, (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_2), (1, n1-1)(1/4, n1))) \mathbb{I}(1) \\
& + ((1/5^{2N} \cdot 2^7)(2(x-1)^2-2) S(R3R41, (1/2, n2-1), (1, n1-1)(1/4, n1)) + (1/5^{2N} \cdot 2^7)((x-1)^2-(x-1)) \\
& S(R3R41, (1/4, n2-1), (1, n1-1)(1/4, n1)) + \dots + \mathbb{I} (1/5^{2N} \cdot 2^{x+4})(2(2)^2-2) (S(R3R411...1_{x-1}, (1/2, \\
& n_{x-1}-1), (1/2, n_{x-2}), \dots, (1/2, n_2), (1, n1-1)(1/4, n1)) + (1/2)S(R3R411...1_{x-1}, (1/2, n_{x-1}-1), (1/2, n_{x-2}) \\
& \dots, (1/4, n_2), (1, n1-1)(1/4, n1)) + \dots + (1/2^{x-3})S(R3R411...1_{x-1}, (1/2, n_{x-1}-1), (1/4, n_{x-2}), \dots, (1/4, \\
& n_2), (1, n1-1)(1/4, n1))) + (1/5^{2N} \cdot 2^{2x+1})(2)^2-2) S(R3R411...1_{x-1}, (1/4, n_{x-1}-1), (1/4, n_{x-2}), \dots, \\
& (1/4, n_2), (1, n1-1)(1/4, n1)) \mathbb{I}) \\
& + (1/4) ((\mathbb{I} (1/5^{2N} \cdot 2^4) S(R3R31, (1/2, n2-2), (1/4, n1))- (1/5^{2N} \cdot 2^{n2+2})S(R3R31, (1, n2-2), (1/4, \\
& n1)) - (((n3-1)/5^{2N} \cdot 2^3) S(R3R31, (1/2, n2), (1/4, n1)) - (n3/5^{2N} \cdot 2^4) S(R3R31, (1/4, n2), (1/4, \\
& n1))) \mathbb{I} (x-2) + \dots + \mathbb{I} (1/5^{2N} \cdot 2^{x+1}) (S(R3R311...1_{x-1}, (1/2, n_{x-1}-2), (1/2, n_{x-2}), \dots, (1/2, n_2), (1/4, \\
& n1)) - (1/2)S(R3R311...1_{x-1}, (1/2, n_{x-1}-2), (1/2, n_{x-2}), \dots, (1/4, n_2), (1/4, n1)) - \dots - (1/2^{x-3}) \\
& ^3)S(R3R311...1_{x-1}, (1/2, n_{x-1}-2), (1/4, n_{x-2}), \dots, (1/4, n_2), (1/4, n1))) - (1/5^{2N} \cdot 2^{n_{x-1}+x-1}) \\
& (S(R3R311...1_{x-1}, (1, n_{x-1}-2), (1/2, n_{x-2}), \dots, (1/2, n_2), (1/4, n1)) - (1/2)S(R3R311...1_{x-1}, (1, n_{x-1}-2), \\
& (1/2, n_{x-2}), \dots, (1/4, n_2), (1/4, n1)) - \dots - (1/2^{x-3})S(R3R311...1_{x-1}, (1, n_{x-1}-2), (1/4, n_{x-2}), \dots, \\
& (1/4, n_2), (1/4, n1))) - (((n_{x-1})/25 \cdot 2^x) (S(R3R311...1_{x-1}, (1/2, n_{x-1}), (1/2, n_{x-2}), \dots, (1/2, n_2), \\
& (1/4, n1)) - (1/2)S(R3R311...1_{x-1}, (1/2, n_{x-1}), (1/2, n_{x-2}), \dots, (1/4, n_2), (1/4, n1)) - \dots - (1/2^{x-3}) \\
& ^3)S(R3R311...1_{x-1}, (1/2, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_2), (1/4, n1))) - (n_x/5^{2N} \cdot 2^{2x-2}) \\
& S(R3R311...1_{x-1}, (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_2), (1/4, n1))) \mathbb{I}(1) \\
& + ((1/5^{2N} \cdot 2^5)(2(x-1)^2-2) S(R3R31, (1/2, n2-1), (1/4, n1)) - (1/5^{2N} \cdot 2^5)((x-1)^2-(x-1)) S(R3R31, \\
& (1/4, n2-1), (1/4, n1)) - \dots - \mathbb{I} (1/5^{2N} \cdot 2^{x+2})(2(2)^2-2) (S(R3R311...1_{x-1}, (1/2, n_{x-1}-1), (1/2, n_{x-2}) \\
& \dots, (1/2, n_2), (1/4, n1)) - (1/2)S(R3R311...1_{x-1}, (1/2, n_{x-1}-1), (1/2, n_{x-2}), \dots, (1/4, n_2), (1/4, n1)) \\
& - \dots - (1/2^{x-3})S(R3R311...1_{x-1}, (1/2, n_{x-1}-1), (1/4, n_{x-2}), \dots, (1/4, n_2), (1/4, n1))) - (1/5^{2N} \cdot 2^{2x-1}) \\
& ((2)^2-2) S(R3R311...1_{x-1}, (1/4, n_{x-1}-1), (1/4, n_{x-2}), \dots, (1/4, n_2), (1/4, n1)) \mathbb{I}) \mathbb{I}) \text{ (crochets roses ci-} \\
& \text{contres= terme pour } x=3, \text{ on a } R311...2_{x-2} 1_{x-1} = R4 \text{ pour } x=3)
\end{aligned}$$

$$\begin{aligned}
& + \dots + \\
& \mathbb{I} - ((\mathbb{I} (1/5^{2N} \cdot 2^{2x}) S(R311...1_{x-2} R311...2_{x-2} 1_{x-1}, (1/2, n_{x-1}-2), (1, n_{x-2}-1)(1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n1))- (1/5^{2N} \cdot 2^{n_{x-1}+2x-2}) S(R311...1_{x-2} R311...2_{x-2} 1_{x-1}, (1, n_{x-1}-2), (1, n_{x-2}-1)(1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n1)) - (((n_{x-1})/5^{2N} \cdot 2^{2x-1}) S(R311...1_{x-2} R311...2_{x-2} 1_{x-1}, (1/2, n_{x-1}), (1, n_{x-2}-1)
\end{aligned}$$

$$\begin{aligned}
& (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1) + (n_x/5^{2N} \cdot 2^{2x}) S(R311\dots 1_{x-2} R311\dots 2_{x-2} 1_{x-1}, (1/4, n_{x-1}), (1, n_{x-2-1})(1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) \mathbb{I}(1) \\
& + ((1/5^{2N} \cdot 2^{2x+1}) (2(2)^2-2) S(R311\dots 1_{x-2} R311\dots 2_{x-2} 1_{x-1}, (1/2, n_{x-1-1}), (1, n_{x-2-1})(1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) + (1/5^{2N} \cdot 2^{2x+1}) ((2)^2-(2)) S(R311\dots 1_{x-2} R311\dots 2_{x-2} 1_{x-1}, (1/4, n_{x-1-1}), (1, n_{x-2-1})(1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1))) \\
& + (1/4) (((1/5^{2N} \cdot 2^{2x-2}) S(R311\dots 1_{x-2} R311\dots 1_{x-2} 1_{x-1}, (1/2, n_{x-1-2}), (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) - (1/5^{2N} \cdot 2^{n_{x-1}+2x-4}) S(R311\dots 1_{x-2} R311\dots 1_{x-2} 1_{x-1}, (1, n_{x-1-2}), (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) - ((n_{x-1})/5^{2N} \cdot 2^{2x-3}) S(R311\dots 1_{x-2} R311\dots 1_{x-2} 1_{x-1}, (1/2, n_{x-1}), (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) - (n_x/5^{2N} \cdot 2^{2x-2}) S(R311\dots 1_{x-2} R311\dots 1_{x-2} 1_{x-1}, (1/4, n_{x-1}), (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1))) \mathbb{I}(1) \\
& + ((1/5^{2N} \cdot 2^{2x-1}) (2(2)^2-2) S(R311\dots 1_{x-2} R311\dots 1_{x-2} 1_{x-1}, (1/2, n_{x-1-1}), (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1)) - (1/5^{2N} \cdot 2^{2x-1}) ((2)^2-(2)) S(R311\dots 1_{x-2} R311\dots 1_{x-2} 1_{x-1}, (1/4, n_{x-1-1}), (1/4, n_{x-1}), (1/4, n_{x-2}), \dots, (1/4, n_1))) \mathbb{I})
\end{aligned}$$

$$\begin{aligned}
& + (\mathbb{I} (7/2) ((1/5^{2N-1} \cdot 2^3) S(R31, (1/2, n_2-1), (1/2, n_1)) - (1/5^{2N-1} \cdot 2^{n_2+2}) S(R31, (1, n_2-1), (1/2, n_1))) - (7/4) 3 ((1/5^{2N} \cdot 2^3) S(R31, (1/2, n_2-1), (1/4, n_1)) - (1/5^{2N} \cdot 2^{n_2+2}) S(R31, (1, n_2-1), (1/4, n_1)))) \mathbb{I} + \dots + \mathbb{I} (7/2) ((1/5^{2N-1} \cdot 2^{x+1}) S(R311\dots 1_x, (1/2, n_{x-1}), (1/2, n_{x-1}), \dots, (1/2, n_1)) - (1/5^{2N-1} \cdot 2^{n_x+x}) S(R311\dots 1_x, (1, n_x), (1/2, n_{x-1}), \dots, (1/2, n_1))) - (7/4) 3 ((1/5^{2N} \cdot 2^{x+1}) S(R311\dots 1_x, (1/2, n_{x-1}), (1/2, n_{x-1}), \dots, (1/4, n_1)) - (1/5^{2N} \cdot 2^{n_x+x}) S(R311\dots 1_x, (1, n_x-1), (1/2, n_{x-1}), \dots, (1/4, n_1))) + ((1/5^{2N} \cdot 2^{x+2}) S(R311\dots 1_x, (1/2, n_{x-1}), (1/2, n_{x-1}), \dots, (1/4, n_2), (1/4, n_1)) - (1/5^{2N} \cdot 2^{n_x+x+1}) S(R311\dots 1_x, (1, n_{x-1}), (1/2, n_{x-1}), \dots, (1/4, n_2), (1/4, n_1))) + \dots + ((1/5^{2N} \cdot 2^{2x-1}) S(R311\dots 1_x, (1/2, n_{x-1}), (1/4, n_{x-1}), \dots, (1/4, n_1)) - (1/5^{2N} \cdot 2^{n_{x-1}+2x-2}) S(R311\dots 1_x, (1, n_{x-1}), (1/4, n_{x-1}), \dots, (1/4, n_1)))) \mathbb{I}) (1+(2/7)(R21+R211+\dots+R211\dots 1_{n_{N-1}}))
\end{aligned}$$

$$\begin{aligned}
& + (\mathbb{I} - \mathbb{I} (1/5^{2N} \cdot 2^5) S(R3R41, (1/2, n_2-1), (1, n_1-1)(1/4, n_1)) - (1/5^{2N} \cdot 2^{n_2+4}) S(R3R41, (1, n_2-1), (1, n_1-1)(1/4, n_1)) \mathbb{I} + (1/4) \mathbb{I} (1/5^{2N} \cdot 2^3) S(R3R31, (1/2, n_2-1), (1/4, n_1)) - (1/5^{2N} \cdot 2^{n_2+2}) S(R3R31, (1, n_2-1), (1/4, n_1)) \mathbb{I} \\
& + \mathbb{I} ((- \mathbb{I} (1/5^{2N} \cdot 2^6) S(R3R411, (1/2, n_3-1), (1/2, n_2), (1, n_1-1)(1/4, n_1)) - (1/5^{2N} \cdot 2^{n_3+5}) S(R3R411, (1, n_3-1), (1/2, n_2), (1, n_1-1)(1/4, n_1)) \mathbb{I} + \mathbb{I} (1/5^{2N} \cdot 2^7) S(R3R411, (1/2, n_3-1), (1/4, n_2), (1, n_1-1)(1/4, n_1)) - (1/5^{2N} \cdot 2^{n_3+6}) S(R3R411, (1, n_3-1), (1/4, n_2), (1, n_1-1)(1/4, n_1)) \mathbb{I}) + (1/4) (\mathbb{I} (1/5^{2N} \cdot 2^4) S(R3R311, (1/2, n_3-1), (1/2, n_2), (1/4, n_1)) - (1/5^{2N} \cdot 2^{n_3+3}) S(R3R311, (1, n_3-1), (1/2, n_2), (1/4, n_1)) \mathbb{I} - \mathbb{I} (1/5^{2N} \cdot 2^5) S(R3R311, (1/2, n_3-1),
\end{aligned}$$

$$\begin{aligned}
& \left((1/4, n_2), (1/4, n_1) \right) - (1/5^{2N} \cdot 2^{n_3+4}) S(R_3R_3R_{311}, (1, n_3-1), (1/4, n_2), (1/4, n_1)) \Big) + \left(- \right. \\
& \left. \left[(1/5^{2N} \cdot 2^7) S(R_3R_1R_3R_{321}, (1/2, n_3-1), (1, n_2-1)(1/4, n_2), (1/4, n_1)) - (1/5^{2N} \cdot 2^{n_3+6}) S(R_3R_1R_3R_{321}, (1, \right. \right. \\
& n_3-1), (1, n_2-1)(1/4, n_2), (1/4, n_1)) \Big] + (1/4) \left[(1/5^{2N} \cdot 2^5) S(R_3R_1R_3R_{311}, (1/2, n_3-1), (1/4, n_2), (1/4, \right. \\
& n_1)) - (1/5^{2N} \cdot 2^{n_3+4}) S(R_3R_1R_3R_{311}, (1, n_3-1), (1/4, n_2), (1/4, n_1)) \Big] \Big) \\
& + \dots + \\
& \left[\left(- \left[(1/5^{2N} \cdot 2^{x+3}) S(R_3R_4R_{11} \dots 1x, (1/2, n_x-1), (1/2, n_x-i), \dots, (1, n_1-1)(1/4, n_1)) - \right. \right. \right. \\
& (1/5^{2N} \cdot 2^{n_x+x+2}) S(R_3R_4R_{11} \dots 1x, (1, n_x-1), (1/2, n_x-i), \dots, (1, n_1-1)(1/4, n_1)) \Big] + \left[(1/5^{2N} \cdot 2^{x+4}) \right. \\
& S(R_3R_4R_{11} \dots 1x, (1/2, n_x-1), (1/2, n_x-i), \dots, (1/4, n_2), (1, n_1-1)(1/4, n_1)) - (1/5^{2N} \cdot 2^{n_x+x+3}) \\
& S(R_3R_4R_{11} \dots 1x, (1, n_x-1), (1/2, n_x-i), \dots, (1/4, n_2), (1, n_1-1)(1/4, n_1)) \Big] + \dots + \left[(1/5^{2N} \cdot 2^{2x+1}) \right. \\
& S(R_3R_4R_{11} \dots 1x, (1/2, n_x-1), (1/4, n_x-i), \dots, (1/4, n_2), (1, n_1-1)(1/4, n_1)) - (1/5^{2N} \cdot 2^{n_x+2x}) \\
& S(R_3R_4R_{11} \dots 1x, (1, n_x-1), (1/4, n_x-i), \dots, (1/4, n_2), (1, n_1-1)(1/4, n_1)) \Big] + (1/4) \left(\left[(1/5^{2N} \cdot 2^{x+1}) \right. \right. \\
& S(R_3R_3R_{311} \dots 1x, (1/2, n_x-1), (1/2, n_x-i), \dots, (1/4, n_1)) - (1/5^{2N} \cdot 2^{n_x+x}) S(R_3R_3R_{311} \dots 1x, (1, n_x-1), \\
& (1/2, n_x-i), \dots, (1/4, n_1)) \Big] - \left[(1/5^{2N} \cdot 2^{x+2}) S(R_3R_3R_{311} \dots 1x, (1/2, n_x-1), (1/2, n_x-i), \dots, (1/4, n_2), \right. \\
& (1/4, n_1)) - (1/5^{2N} \cdot 2^{n_x+x+1}) S(R_3R_3R_{311} \dots 1x, (1, n_x-1), (1/2, n_x-i), \dots, (1/4, n_2), (1/4, n_1)) \Big] - \dots - \\
& \left. \left[(1/5^{2N} \cdot 2^{2x-1}) S(R_3R_3R_{311} \dots 1x, (1/2, n_x-1), (1/4, n_x-i), \dots, (1/4, n_2), (1/4, n_1)) - (1/5^{2N} \cdot 2^{n_x+2x-2}) \right. \right. \\
& S(R_3R_3R_{311} \dots 1x, (1, n_x-1), (1/4, n_x-i), \dots, (1/4, n_2), (1/4, n_1)) \Big] \Big) + \left(- \left[(1/5^{2N} \cdot 2^{x+4}) \right. \right. \\
& S(R_3R_1R_3R_{321} \dots 1x, (1/2, n_x-1), (1/2, n_x-i), \dots, (1, n_2-1)(1/4, n_2), (1/4, n_1)) - (1/5^{2N} \cdot 2^{n_x+x+3}) \\
& S(R_3R_1R_3R_{321} \dots 1x, (1, n_x-1), (1/2, n_x-i), \dots, (1, n_2-1)(1/4, n_2), (1/4, n_1)) \Big] + \left[(1/5^{2N} \cdot 2^{x+5}) \right. \\
& S(R_3R_1R_3R_{321} \dots 1x, (1/2, n_x-1), (1/2, n_x-i), \dots, (1/4, n_3), (1, n_2-1)(1/4, n_2), (1/4, n_1)) - \\
& (1/5^{2N} \cdot 2^{n_x+x+4}) S(R_3R_1R_3R_{321} \dots 1x, (1, n_x-1), (1/2, n_x-i), \dots, (1/4, n_3), (1, n_2-1)(1/4, n_2), (1/4, n_1)) \Big] \\
& \left. + \dots + \left[(1/5^{2N} \cdot 2^{2x+2}) S(R_3R_1R_3R_{321} \dots 1x, (1/2, n_x-1), (1/4, n_x-i), \dots, (1/4, n_3), (1, n_2-1)(1/4, n_2), \right. \right. \\
& (1/4, n_1)) - (1/5^{2N} \cdot 2^{n_x+2x+1}) S(R_3R_1R_3R_{321} \dots 1x, (1, n_x-1), (1/4, n_x-i), \dots, (1/4, n_3), (1, n_2-1)(1/4, \\
& n_2), (1/4, n_1)) \Big] + (1/4) \left(\left[(1/5^{2N} \cdot 2^{x+2}) S(R_3R_1R_3R_{311} \dots 1x, (1/2, n_x-1), (1/2, n_x-i), \dots, (1/4, n_2), \right. \right. \\
& (1/4, n_1)) - (1/5^{2N} \cdot 2^{n_x+x+1}) S(R_3R_1R_3R_{311} \dots 1x, (1, n_x-1), (1/2, n_x-i), \dots, (1/4, n_2), (1/4, n_1)) \Big] - \\
& \left[(1/5^{2N} \cdot 2^{x+3}) S(R_3R_1R_3R_{311} \dots 1x, (1/2, n_x-1), (1/2, n_x-i), \dots, (1/4, n_3), (1/4, n_2), (1/4, n_1)) - \right. \\
& (1/5^{2N} \cdot 2^{n_x+x+2}) S(R_3R_1R_3R_{311} \dots 1x, (1, n_x-1), (1/2, n_x-i), \dots, (1/4, n_3), (1/4, n_2), (1/4, n_1)) \Big] - \dots - \\
& \left. \left[(1/5^{2N} \cdot 2^{2x}) S(R_3R_1R_3R_{311} \dots 1x, (1/2, n_x-1), (1/4, n_x-i), \dots, (1/4, n_3), (1/4, n_2), (1/4, n_1)) - \right. \right. \\
& (1/5^{2N} \cdot 2^{n_x+2x-1}) S(R_3R_1R_3R_{311} \dots 1x, (1, n_x-1), (1/4, n_x-i), \dots, (1/4, n_3), (1/4, n_2), (1/4, n_1)) \Big] \Big) + \\
& \dots + \left(- \left[(1/5^{2N} \cdot 2^{2x+1}) S(R_3R_{11} \dots 1x - 1R_3R_{11} \dots 2x-1 1x, (1/2, n_x-1), (1, n_x-1-1)(1/4, n_x-1), ((1/4, n_x- \right. \right. \\
& 2), \dots, (1/4, n_1)) - (1/5^{2N} \cdot 2^{n_x+2x}) S(R_3R_{11} \dots 1x - 1R_3R_{11} \dots 2x-1 1x, (1, n_x-1), (1, n_x-1-1)(1/4, n_x-1), ((1/4, \\
& n_x-2), \dots, (1/4, n_1)) \Big] + (1/4) \left(\left[(1/5^{2N} \cdot 2^{2x-1}) S(R_3R_{11} \dots 1x - 1R_3R_{11} \dots 1x-1 1x, (1/2, n_x-1), (1/4, n_x-1), \right. \right.
\end{aligned}$$

$$\begin{aligned}
& ((1/4, n_x-2), \dots, (1/4, n_1)) - (1/5^{2N} \cdot 2^{n_x+2x-2}) S(R311\dots 1_{x-1}R31\dots 1_{x-1}1_x, (1, n_x-1), (1/4, n_x-1), \\
& ((1/4, n_x-2), \dots, (1/4, n_1))) \\
& - \left(\left[(7/4)(3/5^{2N} \cdot 2^2) S(R3, (1/4, n_1))(1+(2/7)(R21+R211+\dots+R211\dots 1_{n-1})) + (1/5^{2N} \cdot 2^4) S(R3R4, \right. \right. \\
& (1, n_1-1)(1/4, n_1)) \left. \right] + \left[(7/4)(3/5^{2N} \cdot 2^4) S(R31, (1/4, n_2), (1/4, n_1))(1+(2/7)(R21+R211+\dots \right. \\
& +R211\dots 1_{n-1})) + (1/4)(1/5^{2N} \cdot 2^4) (S(R3R31, (1/4, n_2), (1, n_1)(1/4, n_1))) + (1/4) \\
& (1/5^{2N} \cdot 2^6) S(R31R32, (1, n_2-1)(1/4, n_2), (1/4, n_1)) \left. \right] + \dots + \left[(7/4)(3/5^{2N} \cdot 2^{2x}) S(R311\dots 1_x, (1/4, \right. \\
& n_x), \dots, (1/4, n_1))(1+(2/7)(R21+R211+\dots+R211\dots 1_{n-1})) + (1/4)(1/5^{2N} \cdot 2^{2x}) (S(R3R311\dots 1_x, (1/4, \\
& n_x), \dots, (1, n_1)(1/4, n_1)) + S(R31R311\dots 1_x, (1/4, n_x), \dots, (1, n_2)(1/4, n_2), (1/4, n_1)) R311\dots 1_x, \\
& (1/4, n_x), \dots, (1/4, n_1)) + \dots + S(R311\dots 1_{x-1} R311\dots 1_x, (1/4, n_x), (1, n_x-1)(1/4, n_x-1), \dots, (1/4, n_2), \\
& (1/4, n_1)) \left. \right] + (1/4)(1/5^{2N} \cdot 2^{2x+2}) S(R311\dots 1_x R311\dots 2_x, (1, n_x-i)(1/4, n_x), (1/4, n_x-1), \dots, (1/4, n_2), \\
& (1/4, n_1)) \left. \right] \left. \right]
\end{aligned}$$