Comparison of different FFT-based methods for computing the mechanical response of heteregoneous materials

H. Moulinec^{*}, F. Silva¹, P. Suquet²

CNRS LMA, 31 chemin Joseph Aiguier 13402 Marseille cedex 20 FRANCE * moulinec@lma.cnrs-mrs.fr ¹ silva@lma.cnrs-mrs.fr ² suquet@lma.cnrs-mrs.fr

The last decade has witnessed a growing interest for the so-called "FFT-based methods" for computing the overall and local properties of heterogeneous materials submitted to mechanical solicitations. Since the original method was introduced by Moulinec and Suquet [1], several authors have proposed different algorithms to better deal with non-linear materials or with materials with highly contrasted mechanical properties between their constituents.

The study concerns a linear elastic material - although the methods involved can be extended into the case of non-linear behavior - submitted to a prescribed overall strain *E*. The stiffness tensor c(x)of the material varies with the position *x*. The numerical method proposed by Moulinec & Suquet lies on the iterative resolution of the Lippmann-Schwinger equation and can be summarized by the following relation between two successive iterates ε^{i} and ε^{i+1} of the strain field:

$$\boldsymbol{\varepsilon}^{\mathbf{i+1}}(x) = -\Gamma^0 * \left((c(x) - c^0) : \boldsymbol{\varepsilon}^{\mathbf{i}}(x) \right) + E \quad ,$$

where c^0 is the stiffness tensor of a reference medium supposed to be linear elastic, where Γ^0 is a Green operator associated to c^0 and where * denotes the convolution operator.

Eyre & Milton [2], Michel et al. [3] and Monchiet & Bonnet [4] proposed different schemes to accelerate the convergence of the initial scheme. It has been recently demonstrated in [5] that the two first schemes are particular cases of the last one. On the other hand, Zeman et al. [6] proposed to use a conjugate gradient method for solving the Lippmann-Schwinger equation.

The present paper aims to compare these different methods with a special attention paid to their relative efficiency and their rates of convergence.

References

- [1] H. Moulinec and P. Suquet. A fast numerical method for computing the linear and nonlinear properties of composites. C. R. Acad. Sc. Paris II, **318**, 1417–1423, 1994.
- [2] D.J. Eyre and G.W. Milton. A fast numerical scheme for computing the response of composites using grid refinement. J. Physique III, 6, 41–47, 1999.
- [3] J.C. Michel, H. Moulinec, P Suquet. A computational method based on augmented Lagrangians and Fast Fourier Transforms for composites with high contrast. *Comput. Modelling Engng. Sc.*, 1 (2), 79–88, 2000.
- [4] V. Monchiet and G. Bonnet. A polarization-based FFT iterative scheme for computing the effective properties of elastic composites with arbitrary contrast. *Int. J. Numer. Meth. Engng*, 89, 1419–1436, 2012.
- [5] Moulinec H, Silva F. Comparison of three accelerated FFT-based schemes for computing the mechanical response of composite materials *Int. J. Numer. Meth. Engng* **97** 13, 960–985, 2014.
- [6] J. Zeman, J. Vondřejc, J. Novák, I. Marek. Accelerating a FFT-based solver for numerical homogenisation of periodic media by conjugate gradients. *Journal of Computational Physics* 229 (21) 8065–8071, 2010.