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# Color texture classification method based on a statistical multi-model and geodesic distance

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## Abstract

In this letter, we propose a novel color texture classification method based on statistical characterization. The approach consists in modeling complex wavelet coefficients of both luminance and chrominance components separately leading to a multi-modeling approach. The copula theory allows to take into account the spatial dependencies which exist within the intra-luminance sub-bands via the luminance model  $M_L$ , and also between the inter-chrominance subband coefficients via the chrominance model  $M_{C_r}$ . The multi-model, i.e  $M_L$  and  $M_{C_r}$ , is used to develop a Bayesian classifier based on the softmax principal. To derive the classifier, we propose a closed-form expression for the Rao geodesic distance between two copulas. Experiments on two sub-families of luminance-chrominance color spaces namely Lab and HSV have been carried out for a wide range of color texture databases. The combination of different statistical sub-models show that the multi-modeling performs better than some existing methods in term of classification rates.

*Keywords:* Color texture, wavelet representation, Rao geodesic distance, Bayesian classification

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## 1. Introduction

For color texture analysis, the question of the color space selection to characterize the visual content may be raised. RGB space is the most natural since

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14 it represents the acquisition-reference space. However, R, G and B channels are  
15 known to exhibit simultaneously high statistical dependencies relatively to: 1)  
16 the intra-channel or spatial one, 2) the inter-band for scale-space representation  
17 one and 3) the inter-channel or multicomponent one. In recent works, some au-  
18 thors have proposed various stochastic models to characterize the inter-channel  
19 dependencies alone. From scale-space representation such as 2-D wavelet or  
20 others, previous approaches consist in fitting the histograms of the wavelet sub-  
21 bands with a given parametric probability density function (pdf) of the trivariate  
22 color vector samples for each level of the decomposition. Recent works are based  
23 on multivariate generalized Gaussian distribution in [Verdoolaege et al. 2012]  
24 or copulas based multivariate Weibull distribution in [Kwitt et al. 2011]. The  
25 parameters of the pdf are the covariance matrix and different auxiliary param-  
26 eters such as the shape parameter characterizing the non-Gaussianness. Ex-  
27 tending these models to take into account jointly intra-band, inter-band and  
28 inter-channel dependencies, leads to a high-dimensional model in terms of num-  
29 ber of parameters. This is a main drawback of these proposals since, in practice,  
30 for a fixed data size, dealing with high-dimensional model may be a critical issue  
31 in terms of parameter estimation performance. Considering color space, another  
32 alternative to the RGB space is to consider the family of luminance-chrominance  
33 (LC) color spaces [DeYoe et al. 1996, Mojsilovic et al. 2000]. This family in-  
34 cludes perceptual (HSV, HSI and HSB) and perceptually uniform (Lab, Yuv,  
35 YCbCr) color spaces [Sarifuddin et al. 2005]. An extended comparison of those  
36 spaces is given by [Qazi et al. 2011]. These authors note that the luminance  
37 channel is uncorrelated with the chrominance channels. From this independ-  
38 ence in LC spaces, they proposed a model taking into account separately intra  
39 and inter dependencies, i.e. multi-model, with few parameters compared to the  
40 required number by using the RGB space. Qazi et al. proposed to consider a  
41 multi-model based on a 2-D complex quarter plan autoregressive (2-D QP AR)  
42 model for luminance as well as for chrominance information. This may lead to a  
43 lack of flexibility because the luminance and chrominance are decorrelated and  
44 thus, have distributions of different natures.

45 In this letter, considering the LC color spaces, we propose a multi-modeling ap-  
46 proach based on multivariate stochastic modeling. The contribution is twofold:

- 47 • First, we propose a multi-model based on two non-Gaussian sub-models,  
48 i.e.  $M_L$  and  $M_{C_r}$ , devoted to describe respectively 1) the intra-channel  
49 dependency by considering the spatial structure and inter-band depen-  
50 dence of the luminance channel, and 2) the inter-channel dependency by  
51 considering a bivariate pdf for the two chrominance channels. Both  $M_L$   
52 and  $M_{C_r}$  are based on copula as a powerful tool to construct non-Gaussian  
53 joint models that fit data with even different natures of margins. Linear  
54 and/or circular models are derived depending on the component natures  
55 (ex: the Hue in HSV color space is a circular channel).
- 56 • Second, for the similarity measurement used to compare two texture sam-  
57 ples, we derive a closed form expression for the Rao geodesic distance for  
58 copula based multivariate models. Contrary to the conventional Kullback-  
59 leibler (KL) divergence, the Rao geodesic metric has the advantage to  
60 satisfy distance properties such as symmetry and triangular inequality  
61 [Atkinson et al. 1981].

62 The use of different models for characterizing the wavelet coefficients has been  
63 recently investigated by [Rakvongthai et al. 2013], for texture retrieval in a  
64 noisy environment. Authors proposed different univariate statistical models for  
65 the real/imaginary, magnitude and phase parts of complex subband coefficients.  
66 The parameter vector of the studied texture is then formed by the concatena-  
67 tion of parameters estimated from each of these models. Also, the similarity  
68 measure has been expressed by a formula combining both magnitude and phase  
69 KL divergences using a mixing parameter. While the cited work did not con-  
70 sider the color information at all, it should be emphasized that the present work  
71 considers the color and texture information jointly for image characterization.  
72 Concerning the similarity measure, the originality of our method is the use of  
73 Rao geodesic distance which represents a good alternative to the KL divergence  
74 since it satisfies the distance properties. For this, a closed-form expression of

75 Rao geodesic distance between copula based distributions is developed.  
76 Note also that this work stands out from our previous work [Maliani et al. 2012]  
77 which presents a preliminary study of the multi-modeling concept that consid-  
78 ered only the special case of color representation, the HSV color space. More-  
79 over, the paper did not discuss the dimensionality reduction issue when consider-  
80 ing the three dependencies (intra-band, inter-band and inter-channel). Also, in  
81 the classification step, we proposed the K-Nearest Neighborhood classifier tak-  
82 ing the weighted sum of the two similarity measures between the luminance and  
83 chrominance sub-models. The weights have been tuned experimentally which is  
84 the same as for [Rakvongthai et al. 2013], which led us to test exhaustively on  
85 several scales of weights to get the best classification rate. The multi-modeling  
86 proposed in this letter, helps in dimensionality reduction of the extracted fea-  
87 tures to characterize the textural content. According to several dependence  
88 experiments presented in section 2, the spatial structure of the texture and the  
89 inter-band dependence is only considered for the luminance component and the  
90 inter color-channel dependency consists of a bivariate model. The performance  
91 of the multi-model is tested using a Bayesian classifier based on the softmax  
92 principal [Bishop 2006] thanks to its suitability to combine several similarity  
93 measures through a posteriori probability fusion. The experimental protocol  
94 adopted here, aims to find the optimal configuration of our multi-model. This  
95 starts with the choice of statistical distribution related to color components in  
96 the field of dual-tree complex wavelet CT-DWT [Kingsbury 1998]. We proposed  
97 to build joint models by testing different combinations of Gamma , Weibull and  
98 VonMises distributions for both luminance and chrominance components. First  
99 comparisons were considered to show the relevance of each model for the color  
100 spaces. Then, we compared our best selected models with the state of the art  
101 methods in terms of classification rate, the dimensionality reduction and the  
102 computation time.

103 This paper is organized as follows. Section 2 deals with the proposed multi-  
104 modeling approach for textures in LC color spaces. In Section 3, we present the  
105 classification framework, and the proposed similarity measurement. Then, we

106 show experimental results in Section 4. Finally, we give concluding remarks in  
107 Section 5.

## 108 **2. Color texture modeling in LC color spaces**

### 109 *2.1. Dependence characterization*

110 Let us consider a stationary color texture  $x_T$  represented in a given LC color  
111 space, and let  $l$  be the luminance component,  $cr_1/cr_2$  respectively the first and  
112 the second chrominance component of  $x_T$ . Each color component is first de-  
113 composed into a set of wavelet subbands. In wavelet domain texture character-  
114 ization, it is common to model the statistical distribution of wavelet coefficients  
115 in different subbands, which corresponds to the detail subbands. The low-  
116 pass wavelet coefficients are excluded ([Mallat 1999], [Boubchir et al. 2010]).  
117 For 2-D wavelet transforms, each scale usually has several subbands corre-  
118 sponding to the different orientations. Let us consider  $K$  subbands, where  
119  $K = S \times O$  with  $S$  the number of scales and  $O$  the number of orientations.  
120 The subbands are respectively  $l^{(k)}, cr_1^{(k)}, cr_2^{(k)}$ ,  $k = 1, \dots, K$ . The statistical  
121 distribution of wavelet detail coefficients of natural textures has been exten-  
122 sively studied ([Simoncelli 1997], [Boubchir et al. 2010], [Lasmar et al. 2010],  
123 [Verdoolaege et al. 2012], [Kwitt et al. 2011]). It has been shown that the em-  
124 pirical distribution is generally heavy-tailed with a sharp peak at zero. This  
125 leptokurtic behavior leads to consider non-Gaussian marginal or joint density.  
126 Contrary to the marginal approach, the joint modeling needs to define the de-  
127 pendence which exists between subbands coefficients. Intra-band, inter-band  
128 and inter-channel dependencies have to be considered. Let us detail each of  
129 these dependencies *for the three color components*.

#### 130 *2.1.1. Intra-band dependency*

131 Intra-band dependence refers to the spatial neighborhood dependence be-  
132 tween coefficients of the same wavelet subband. It corresponds to the spatial  
133 structure of the studied subband. Let  $s$  be the location of a reference coefficient

134 from the  $k^{th}$  subband of a given color channel. If we consider the luminance  
 135 channel for example, let  $m$  be the size of the chosen neighborhood. The collec-  
 136 tion of the reference coefficient neighbors is specified by  $l_s^{(k)} = (l_{1,s}^{(k)}, \dots, l_{m,s}^{(k)})$ .  
 137 Then, considering all the subbands coefficient neighbors, the dataset to be ob-  
 138 served is:

$$l_{\text{intra}} = [l_1^{(k)}, \dots, l_s^{(k)}, \dots, l_P^{(k)}]^T \quad (1)$$

139  $l_{\text{intra}}$  is a  $P$  by  $m$  matrix, where  $P$  is the size of the wavelet subband. If we  
 140 consider, the first chrominance channels, intra-band dependency for a given  
 141 subband  $k$  is studied by observing the following dataset:

$$cr_{1\text{intra}} = [cr_{1_1}^{(k)}, \dots, cr_{1_s}^{(k)}, \dots, cr_{1_P}^{(k)}]^T \quad (2)$$

142 For the second chrominance channel, we have:

$$cr_{2\text{intra}} = [cr_{2_1}^{(k)}, \dots, cr_{2_s}^{(k)}, \dots, cr_{2_P}^{(k)}]^T \quad (3)$$

### 143 2.1.2. Inter-band dependency

144 To model the inter-band dependence, we consider subbands in different ori-  
 145 entations for the same color channel. Thus for luminance inter-band we observe  
 146 a dataset represented by a  $P$  by  $K$  matrix, where  $K$  is the total number of  
 147 subbands:

$$l_{\text{interbd}} = [l^{(1)} \dots l^{(K)}] \quad (4)$$

148 for chrominance inter-band:

$$cr_{1\text{interbd}} = [cr_1^{(1)} \dots cr_1^{(K)}] \quad (5)$$

$$cr_{2\text{interbd}} = [cr_2^{(1)} \dots cr_2^{(K)}] \quad (6)$$

149 we note that  $l^{(k)}$ ,  $cr_1^{(k)}$  and  $cr_2^{(k)}$  are respectively the column vectors containing  
 150 all coefficients of the  $k^{th}$  subband of luminance, first and second chrominance  
 151 channels.

152 *2.1.3. Inter-channel dependency*

153 It refers to the dependence between coefficients of subbands from different  
 154 color channel, considering a fixed orientation  $k$ . The observation matrix is as  
 155 follows:

$$D_{\text{interch}} = \begin{bmatrix} l^{(k)} & cr_1^{(k)} & cr_2^{(k)} \end{bmatrix} \quad (7)$$

156 *2.1.4. Full dependence*

157 Considering all the aforementioned dependencies leads to a full dataset that  
 158 considers intra-band, inter-band and inter-channel dependencies jointly. It is  
 159 represented by the following matrix of size  $P$  by  $(3 \times K \times m)$ :

$$D = \begin{bmatrix} l_1^{(1)}, \dots, l_P^{(1)}, \dots, l_1^{(K)}, \dots, l_P^{(K)}, cr_{11}^{(1)}, \dots, cr_{1P}^{(1)}, \\ \dots, cr_{11}^{(K)}, \dots, cr_{1P}^{(K)}, cr_{21}^{(1)}, \dots, cr_{2P}^{(1)}, \dots, cr_{21}^{(K)}, \\ \dots, cr_{2P}^{(K)} \end{bmatrix}^T \quad (8)$$

160 Consideration of this matrix means, first, that all the three components carry  
 161 the spatial structure information (intra-band). Second, that dependence exists  
 162 between subbands of all orientations for each channel (inter-band), and third,  
 163 that the representation exhibits dependence among the three color channels  
 164 (inter-channel).

165 *2.2. Multi-model approach*

166 *2.2.1. Experimental analysis of dependence*

167 Contrary to the RGB representation, LC color spaces offer separability be-  
 168 tween luminance and chrominance. At first glance, this indicates that the inter-  
 169 channel dependence have not to be considered between luminance and chromi-  
 170 nance. A more precise study of dependence is done according to the correlation  
 171 coefficients. Table 1 shows degrees of dependence when textures are represented  
 172 in Lab color space. This is obtained by averaging the Pearson, Kendall, Spear-  
 173 man and the mutual information coefficients on all subbands of 40 color textures  
 174 from the Vistex database [Vistex]. We recall that mutual information is a mea-  
 175 sure of the statistical dependence between two variables. If we suppose that



176  $X$  and  $Y$  are two random variables,  $f(x)$  and  $f(y)$  the marginal probability  
177 distribution functions of  $X$  and  $Y$  respectively, and  $f(x, y)$  the joint probability  
178 distribution function of  $X$  and  $Y$ . The mutual information between  $X$  and  $Y$   
179 is then:

$$I(X, Y) = \sum_{x,y} f(x, y) \log \frac{f(x, y)}{f(x)f(y)} \quad (9)$$

We deduce from these results that intra-band dependence have to be consid-

Table 1: Averaging measure of dependence on the Vistex database, using Pearson, Kendall, Spearman and mutual information for 40 color textures from the Vistex database in the Lab color space.

	Intra-band			Inter-band			Inter-channel		
	$l$	$cr_1$	$cr_2$	$l$	$cr_1$	$cr_2$	$\{l, cr_1\}$	$\{l, cr_2\}$	$\{cr_1, cr_2\}$
Pearson	0.47	0.041	0.0372	0.33	0.037	0.041	0.003	0.003	1
Kendall	0.45	0.038	0.037	0.28	0.037	0.038	0.002	0.002	1
Spearman	0.32	0.026	0.025	0.25	0.025	0.026	0.001	0.001	1
MI	4.46	1.56	1.56	4.20	1.03	1.04	0.74	0.75	5.88

180

181 ered only for luminance, and that inter-channel dependence exists only between  
182 chrominance components. They also show that we can restrict the study of  
183 inter-band dependence to the luminance component.

### 184 2.2.2. Multi-model formulation

185 Our contribution here is to exploit the independence between luminance and  
186 chrominance. So we propose a multi-model based characterization that consid-  
187 ers the different natures of these information ([Qazi et al. 2011]). Two joint  
188 and independent models  $M_L$  and  $M_{C_r}$  are used to respectively characterize  
189 the luminance information with spatial and inter-band dependence considering  
190 observations of a vector  $\vec{l}_s$  (see subsection 2.4.1) and the chrominance one de-  
191 voted to inter-channel dependence by observing realizations of a vector  $cr_s$  (see

192 subsection 2.4.2):

$$p(x_T, \theta) = p(x_T | \{M_L, M_{C_r}\}) \propto p(\bar{l}_s, \theta_L) \prod_{k=1}^K p(cr_s^{(k)} | \theta_{c_r}) \quad (10)$$

193 where  $\theta_L$  and  $\theta_{c_r}$  are the vectors of parameters of luminance and chrominance  
 194 models, respectively. The multi-modeling permits to reduce the dimension of  
 195 the observed dependence dataset  $D$  (subsection 2.1).

### 196 2.3. Copula based multivariate models

197 Different multivariate models can be used for defining  $M_L$  and  $M_{C_r}$ , such  
 198 as the multivariate generalized Gaussian (MGGD) [Verdoolaege et al. 2012], the  
 199 multivariate Bessel K forms [Boubchir et al. 2010], the spherically invariant ran-  
 200 dom vector (SIRV) [Lasmar et al. 2010]. Those models have been proposed to  
 201 characterize different kinds of dependencies such as the intra-band in the case  
 202 of SIRV and the inter-channel in the case of MGGD. Each of them is dedicated  
 203 to a specific dependency rule, either for intra-band or inter-channel but never  
 204 taking account of both together. Obviously, extending each model simultane-  
 205 ously to the both dependencies is possible but the equivalent model exhibits  
 206 the main drawback the common definition of the leptokurtic and platykurtic for  
 207 the whole set components of the multivariate model. If we suppose to have a  
 208 set of dependent variables  $T = [t_1, \dots, t_n]$ , with the MGGD or SIRV multivari-  
 209 ate modeling, each marginal  $t_i$  has finally a similar shape value. This behavior  
 210 is confirmed in the study of [Rangaswamy et al. 2002], the characterization of  
 211 one marginal is sufficient to determine the shape of the whole distribution of  
 212 a SIRV. Table 2 show averages of scale and shape parameters supposing that  
 213 subbands of the three components are described by the same marginal, Weibull  
 214 or Gamma for example. It is clear from the table that the marginal modeling of  
 215 luminance and chrominance exhibits different shape and scale parameters in the  
 216 case of Lab color space, contrary to RGB. In the HSV color space, we observe  
 217 that parameters are different even between the chrominance components H and  
 218 S.

Table 2: Average of parameters (considering wavelet subbands for 40 color textures from Vistex database), supposing that subbands of the three color components are described by the same marginals (Weibull or Gamma) in the case of Lab, HSV and RGB color spaces

	Weibull		Gamma	
	scale	shape	scale	shape
L	11.47	1.54	4.97	2.18
a	3.21	1.63	1.26	2.41
b	2.20	1.62	0.92	2.39
H	26.83	1.08	27.15	1.29
S	0.12	1.53	0.05	2.16
V	0.0081	1.52	0.03	2.21
R	31.51	1.53	13.83	2.16
G	29.91	1.52	13.33	2.13
B	31.25	1.52	13.85	2.14

219 The copula theory allows the joint modeling by separating the dependence  
220 structure from the marginal modeling. It is thus easy to merge different marginal  
221 laws into the same multivariate pdf. [Nelsen 2006] defined the copula as a  
222 multivariate uniform distribution used to construct multivariate models. Given  
223 that  $F$  is a  $d$ -dimensional cumulative distribution function (cdf) with continuous  
224 marginals  $F_1, \dots, F_d$ , the Sklar's theorem [Sklar et al. 1959] shows that there  
225 exist a unique copula  $C$  such that:

$$F(x_1, \dots, x_d) = C(F_1(x_1), \dots, F_d(x_d)) \quad (11)$$

226 Further, if  $C$  is continuous and differentiable, the copula density ( $c$ ) is given by:

$$c(u_1, \dots, u_d) = \frac{\partial^d C(u_1, \dots, u_d)}{\partial u_1 \dots \partial u_d} \quad (12)$$

227 The joint pdf is uniquely deduced from the marginals and the copula density as  
228 follows:

$$f(x_1, \dots, x_d) = c(F_1(x_1), \dots, F_d(x_d)) \prod_{i=1}^d f_i(x_i) \quad (13)$$

229 where  $f_i$ ,  $i = 1, \dots, d$ , represent the marginal pdfs that can follow different  
 230 laws.

231 Estimation of the copula parameters is done using the canonical maximum like-  
 232 lihood (CML) [Durrleman et al. 1959] method. This is a pseudo parametric es-  
 233 timation method contrary to the fully parametric method, termed the inference  
 234 from marginals (IFM) [Joe. 1997]. The CML method consists in transforming  
 235 the data  $(x_1, \dots, x_d)$  into uniform data  $(\hat{u}_1, \dots, \hat{u}_d)$  using the empirical distribu-  
 236 tions (empirical cdfs) and then estimates the copula parameters considering all  
 237 the observations as:

$$\hat{\Sigma} = \operatorname{argmax}_{\Sigma} \sum_{i=1}^n \log c(\hat{u}_{i1}, \dots, \hat{u}_{id}; \Sigma) \quad (14)$$

238 where  $n$  denotes the number of observations, and  $\Sigma$  represents the correlation  
 239 matrix.

240 For marginal parameters, the estimation is done using the conventional max-  
 241 imum likelihood method. Experiments reveal that the CML estimator is the  
 242 best since the method does not rely on parametric assumption about marginals.  
 243 This means that less pertinent choice of marginals do not affect estimation of  
 244 the copula parameters.

245 For texture modeling, we repose on copulas to construct the models  $M_L$  and  
 246  $M_{C_r}$ . We select the Gaussian copula for which the density is defined by:

$$c_{\phi}(u_1, \dots, u_d) = \frac{1}{|\Sigma|^{1/2}} \exp \left[ -\frac{1}{2} \vartheta^T (\Sigma^{-1} - I) \vartheta \right] \quad (15)$$

247 with  $\vartheta^T = (\phi^{-1}(u_1), \dots, \phi^{-1}(u_d))$ ,  $\phi$  denotes the cdf of the normalized Gaus-  
 248 sian density,  $\Sigma$  the correlation matrix, and  $I$  the  $d$ -dimensional identity matrix.

249 The choice of the Gaussian copula is justified by:

- 250 • the dependence representation, which is based on the well known correla-  
 251 tion coefficient.

252 • the existence of closed form of Rao distance for the Gaussian copula which  
 253 allow us to derive a closed form of Rao distance for the joint model as a  
 254 similarity measure (subsection 3.1).

#### 255 2.4. $M_L$ and $M_{C_r}$ models

256 Considering the full dependence dataset  $D$  (see sub-section 2.1.4) leads to  
 257 high parametrization and thus to very cumbersome estimation of parameters.  
 258 To alleviate this complexity, we use the proposition made by [Qazi et al. 2011]  
 259 about dependencies between coefficient subbands when color textures are rep-  
 260 resented in LC color spaces. The proposition consists in a multi-modeling ap-  
 261 proach to reduce the dimensionality of the observed dataset.

##### 262 2.4.1. Multivariate spatial model for luminance ( $M_L$ )

263 As a first step for reducing dimension of the full dataset  $D$ , the spatial  
 264 structure along intra- and inter-band dependence are studied only for luminance  
 265 via the model  $M_L$ . For a subband coefficient at the position  $s$ , intra inter-  
 266 band dependence is represented by the vector  $\bar{l}_s = (l_{1,s}, \dots, l_{m,s}, \dots, l_{d,s})$ , where  
 267  $d = m \times K$ ,  $m$  is the neighborhood size and  $K$  is the number of wavelet subbands.  
 268 Considering all positions at the luminance subbands, the dataset to be modeled  
 269 is a  $P$  by  $d$  matrix of realizations of the vector  $\bar{l}$ :

$$L = \begin{bmatrix} \bar{l}_1 \\ \bar{l}_2 \\ \vdots \\ \bar{l}_s \\ \vdots \\ \bar{l}_P \end{bmatrix}$$

270 where  $P$  is the size of a wavelet subband. The observation matrix is:

$$L = \begin{bmatrix} l_{1,1} & \cdots & l_{m,1} & \cdots & l_{d,1} \\ \vdots & & \vdots & & \vdots \\ l_{1,s} & \cdots & l_{m,s} & \cdots & l_{d,s} \\ \vdots & & \vdots & & \vdots \\ l_{1,P} & \cdots & l_{m,P} & \cdots & l_{d,P} \end{bmatrix}$$

271 Based on the Gaussian copula, the pdf of the model  $M_L$  is defined by:

$$f_{M_L}(\bar{l}_s; \theta) = \frac{1}{|\Sigma_L|^{1/2}} \exp \left[ -\frac{1}{2} \vartheta_s^T (\Sigma_L^{-1} - I) \vartheta_s \right] \prod_{i=1}^d f_s(\bar{l}_{i,s}) \quad (16)$$

272 where  $\theta = (w_1, w_2, \dots, w_d; \Sigma_L)$  is the vector of hyperparameters,  $w_s = \{w_{sj}\}_{j \in \mathbb{N}}$ ,  
 273  $s = 1, \dots, d$  is the vector of parameters for the marginal  $f_s$  and  $\Sigma_L$  is the  
 274 covariance matrix of the Gaussian copula.

275 In the case of Gaussian copula, the CML estimator of covariance matrix is the  
 276 sample correlation matrix of Gaussian observations  $\vartheta_1, \dots, \vartheta_P$  :

$$\hat{\Sigma}_L = \frac{1}{P} \sum_{s=1}^P \vartheta_s \vartheta_s^T \quad (17)$$

277 with  $\vartheta_s = (\vartheta_{1,s}, \dots, \vartheta_{d,s})$  is obtained by transforming the observation  $\bar{l}_s =$   
 278  $(l_{1,s}, \dots, l_{d,s})$  from the observation matrix  $L$  by  $\vartheta_{i,s} = \phi^{-1}(F_i(l_{i,s}))$ ,  $i = 1, \dots, d$   
 279 and  $F_s$  is the empirical marginal cdf.

#### 280 2.4.2. Circular/Linear bivariate model for chrominance ( $M_{C_r}$ )

281 We consider the inter-channel dependence only between chrominance com-  
 282 ponent subband represented by the vector  $cr = (cr_1^{(k)}, cr_2^{(k)})$ . Thus, considering  
 283 all coefficients of the chrominance subbands, the dataset to be modeled by  $M_{C_r}$   
 284 is a  $P$  by 2 matrix:

$$cr = \begin{bmatrix} cr_{1,1} & cr_{2,1} \\ \vdots & \vdots \\ \vdots & \vdots \\ cr_{1,P} & cr_{2,P} \end{bmatrix}$$

285  $M_{C_r}$  is a bivariate model defined reposing on the Gaussian copula:

$$f_{M_{C_r}}(cr_s^{(k)}; \theta) = \frac{1}{|\Sigma_{C_r}|^{1/2}} \exp \left[ -\frac{1}{2} \vartheta_s^T (\Sigma_{C_r}^{-1} - I) \vartheta_s \right] \prod_{i=1}^2 f_i(cr_i) \quad (18)$$

286 where  $\theta = (w_1, w_2; \Sigma_{C_r})$ ,  $w_i = \{w_{ij}\}_{j \in \mathbb{N}}$ . The index  $i \in \{1, 2\}$  is the vector  
 287 of parameters for the marginal  $f_i$ , and  $\Sigma_{C_r}$  the covariance matrix. The CML  
 288 estimator of the covariance matrix is as follows:

$$\hat{\Sigma}_{C_r} = \frac{1}{P} \sum_{s=1}^P \vartheta_s \vartheta_s^T \quad (19)$$

289 with  $\vartheta_s = (\vartheta_{1,s}, \vartheta_{2,s})$  is obtained by transforming the observation  $cr_s = (cr_{1,s}, cr_{2,s})$   
 290 from the observation matrix  $cr$  by  $\vartheta_{i,s} = \phi^{-1}(F_i(cr_{i,s}))$ ,  $i = 1, 2$  and  $F_s$  is the  
 291 empirical marginal cdf.

292 The model is called circular/linear because it offers the possibility of using dif-  
 293 ferent marginals  $f_i$  to come up with a bivariate circular/linear, circular/circular  
 294 or linear/linear model for chrominance channels depending on the nature of the  
 295 used color space.

### 296 2.4.3. Texture joint modeling

297 Taking into account the previous sub-models, respectively  $M_L$  and  $M_{C_r}$   
 298 termed probabilistic because they describe probability distributions over sub-  
 299 band coefficients of the texture, let us consider a stationary observed texture,  
 300 we have:

$$p(x_T | \{M_L, M_{C_r}\}) \propto p(\bar{l}_s | w_s, \Sigma_L) \prod_{k=1}^K p(cr_s^{(k)} | w_{cr_1}^{(k)}, w_{cr_2}^{(k)}, \Sigma_{C_r}^{(k)}) \quad (20)$$

301 which can be written in a more compact parametric form as:

$$p(x_T | \{M_L, M_{C_r}\}) \propto p(x_T | \theta_s) \quad (21)$$

302 with  $\theta_s = \left( w_s, \Sigma_L, \{w_{cr_1}^{(k)}, w_{cr_2}^{(k)}, \Sigma_{C_r}^{(k)}\}_{k=1, \dots, K} \right)$

303 **3. Classification**

304 In the framework of stochastic parametric classifier, we consider supervised  
 305 Bayes classifier to assign the most probable class to a textured image. If we  
 306 consider the equiprobability of each class, the Bayes classifier consists in finding  
 307 the maximum of the likelihood such as:

$$\hat{b} = \operatorname{argmax}_{b \in \{1, 2, \dots, B\}} [p(x_T, \theta_b)] \quad (22)$$

308 where  $\theta_b$  is the vector of parameters associated to the class  $b$ . The vector  $\theta_b$  is  
 309 estimated during the learning step from the set of texture samples representing  
 310 the class  $b$ . Considering equation 22, the decision of the classifier is provided  
 311 by the numerical evaluation of the likelihood which is time consuming because  
 312 of evaluating equation 21 from the three color channel data. In order to avoid  
 313 this expensive step, a direct comparison between parametric vectors, i.e. one  
 314 representing the class and the other the texture query  $x_T$ , is expected. Thus,  
 315 the class assignment is obtained from a similarity measure. To do this, the  
 316 Softmax approach or normalized exponential function [Bishop 2006] is selected.

$$\operatorname{argmax}_{b \in \{1, 2, \dots, B\}} \frac{\exp(a_b)}{\sum_{j=1}^B \exp(a_j)} \quad (23)$$

317 with  $a_b = \ln \left[ \frac{1}{1 + L_i(x_T, x_b)} \right]$ .

318 The advantage of using the softmax principal is the smoothed version of the  
 319 maximum, since if  $a_b \gg a_j$  for all  $j \neq b$ , then  $p(C_b|x_T) \simeq 1$  and  $p(C_j|x_T) \simeq 0$ .  
 320 The proposed approach is thus based on the selection of a similarity measure,  
 321 i.e  $L_i(x_T, x_b)$  exhibiting a closed form in terms of model parameters. Various  
 322 probabilistic measures of similarity propose this kind of properties such as the  
 323 Kullback-Leibler or Hellinger measures. All these possibilities have been unified  
 324 in the seminal theory of the information geometry. This consists in considering  
 325 that the parametric model of finite dimension forms a sub-manifold character-  
 326 ized by a smooth Riemannian space. Some of these measures are divergences  
 327 such as Kullback-Leibler. A divergence is a non-symmetric measure and can



328 be not respecting the triangle inequality. Some others such as Bhattacharya,  
 329 Hellinger or others are really some distances. Considering the properties of the  
 330 Riemannian manifold, the well-founded distance associated to this type of man-  
 331 ifold is the geodesic or Rao distance [Atkinson et al. 1981]. The main drawback  
 332 of the geodesic distance is that a closed form for probabilistic model is rarely  
 333 available. This is the reason for which the Kullback-Leibler divergence is usualy  
 334 used in place to the geodesic distance. However, the true distance according to  
 335 the Riemannian space is the geodesic distance.

336 In the next subsection, we derive for our context a closed form for the geodesic  
 337 distance for the Copula modeling.

### 338 3.1. Proposed Rao distance as similarity measure

339 For a probability density  $f(x; \theta)$  with  $\theta = (\theta^1, \dots, \theta^N)$  a vector of parame-  
 340 ters, the Rao distance is a Riemannian metric defined by the fisher information  
 341 matrix as:

$$ds^2 = \sum_{p,q=1}^N g_{pq}(\theta) d\theta^i d\theta^j \quad (24)$$

342 where  $g_{pq}$  represent the Fisher matrix elements:

$$g_{pq}(\theta) = -E \left[ \frac{\partial^2}{\partial \theta^p \partial \theta^q} \log f(x; \theta) \right] \quad (25)$$

343 Let  $f$  be a Gaussian copula based joint pdf ( $M_L$  or  $M_{C_r}$ ),  $\theta = \{w_i, \Sigma\}$  is the  
 344 vector of hyper-parameters of  $f$ ,  $w_i = \{w_{ij}\}_{j \in \mathbb{N}}$  is the vector of parameters  
 345 for the marginal  $f_i$ ,  $i = 1, \dots, d$  (for example if the marginal is a Weibull we  
 346 have  $w_i = \{\alpha_i, \beta_i\}$  representing shape and scale parameters),  $d$  is the length  
 347 of the vector of one observation and  $\Sigma$  represents the covariance matrix of the  
 348 Gaussian copula. By applying equations 13 and 15 in equation 25 we have:

$$g_{pq}(\theta) = -E \left[ \frac{\partial^2}{\partial \theta^p \partial \theta^q} \left( \log c_{\Phi}(u; \Sigma) + \log \prod_{i=1}^d f_i(x_i; w_i) \right) \right] \quad (26)$$

349 where  $u = (u_1, \dots, u_d)$  represents the vector of uniform coefficients that are  
 350 obtained by transforming the wavelet coefficients  $(x_1, \dots, x_d)$  using their cdfs

351 ( $u_i = F^{-1}(x_i, w_i)$ ) or using their empirical cdfs (see **MATLAB**'s routine "ecdf"  
352 ) ( $u_i = eF^{-1}(x_i)$ ). In this latter case, no assumptions are made on the para-  
353 metric form of the marginal distributions. The copula parameters are then  
354 independent from any marginal parameter  $w_{ij}$ :

$$\frac{\partial}{\partial w_{ij}} c_{\Phi}(u; \Sigma) = 0 \quad (27)$$

355 so then,  $g_{w_{ij}\Sigma}(\theta) = g_{\Sigma w_{ij}}(\theta) = 0$ ,  $i = 1 \cdots d$ ,  $j \in \aleph$

356 Thus,

$$g_{\Sigma\Sigma}(\theta) = -E \left[ \frac{\partial^2}{\partial \Sigma \partial \Sigma} \log c_{\Phi}(u; \Sigma) \right] \quad (28)$$

357 and

$$g_{\mu\nu}(\theta) = -E \left[ \frac{\partial^2}{\partial \mu \partial \nu} \log \prod_{i=1}^d f_i(x_i; w_i) \right] \quad (29)$$

358 with  $\mu, \nu \in \{w_{ij}\}$ ,  $i = 1 \cdots d$ ,  $j \in \aleph$

$$g_{\mu\nu}(\theta) = -E \left[ \frac{\partial^2}{\partial \mu \partial \nu} \sum_{i=1}^d \log f_i(x_i; w_i) \right] \quad (30)$$

359

$$= -E \left[ \sum_{i=1}^d \frac{\partial^2}{\partial \mu \partial \nu} \log f_i(x_i; w_i) \right] \quad (31)$$

360

$$= \sum_{i=1}^d -E \left[ \frac{\partial^2}{\partial \mu \partial \nu} \log f_i(x_i; w_i) \right] \quad (32)$$

361 Thus, from (12):

$$ds^2 = g_{\Sigma\Sigma} d\Sigma d\Sigma + \sum_{i=1}^d \sum_{\mu, \nu} g_{\mu\nu} \mu \nu dt \quad (33)$$

$$= ds_{\text{Gauss}}^2 + \sum_{i=1}^d ds_{\text{Margins}}^2 \quad (34)$$

362 Hence, given two copula based probability distributions  $f^{(1)}(x; \theta_1)$  and  $f^{(2)}(x; \theta_2)$   
363 with  $\theta_1 = (w_1^{(1)}, \dots, w_d^{(1)}, \Sigma_1)$  and  $\theta_2 = (w_1^{(2)}, \dots, w_d^{(2)}, \Sigma_2)$ , we can compute the

364 Rao geodesic distance as the sum of the Rao distances of the Gaussian distri-  
 365 bution and the Rao distances between marginals [El Maliani et al. 2011]:

$$L^2 = \int_{\theta_1}^{\theta_2} ds^2 = \int_{\theta_1}^{\theta_2} ds_{\text{Gauss}}^2 + \sum_{i=1}^d \int_{\theta_1}^{\theta_2} ds_{\text{Margins}}^2 \quad (35)$$

366

$$L^2 = L_{\text{Gauss}}^2(f^{(1)}(x; \Sigma_1) || f^{(2)}(x; \Sigma_2)) +$$

367

$$\sum_{i=1}^d L_{\text{Margins}}^2(f_i^{(1)}(x; w_i^{(1)}) || f_i^{(2)}(x; w_i^{(2)})) \quad (36)$$

368 that is:

$$L^2 = \frac{1}{2} \sum_{i=1}^d (\ln \lambda^i)^2 +$$

369

$$\sum_{i=1}^d L_{\text{Margins}}^2(f_i^{(1)}(x; w_i^{(1)}) || f_i^{(2)}(x; w_i^{(2)})) \quad (37)$$

370 where  $\lambda^i, i = 1, \dots, d$  represents the eigenvalues of  $\Sigma_1^{-1} \Sigma_2$ .

#### 371 4. EXPERIMENTAL RESULTS

372 The experimental section is concerned with five major issues:

- 373 1. Choice of the multi-model for color texture: for this, we investigate a  
 374 variety of models for different color spaces.
- 375 2. Choice of the LC color space: we compare performances between two sub-  
 376 families of LC color spaces, namely the perceptually uniform color spaces  
 377 (represented by Lab), and the perceptual color spaces (represented by  
 378 HSV).
- 379 3. Comparison with existing approaches: we evaluate performances of our  
 380 approach in comparison with existing uni-modeling approaches in LC  
 381 color spaces [Qazi et al. 2010, Qazi et al. 2011] and in RGB color space  
 382 [Verdoolaege et al. 2012, Kwitt et al. 2011].

- 383 4. Runtime: we address the runtime issue by comparing the proposed Rao  
384 distance based similarity measure with the Monte-carlo based Kullback-  
385 leibler divergence between copula based pdfs.
- 386 5. Curse of dimensionality: we compare our approach with the full feature  
387 vector based approach (subsection 2.1).

#### 388 4.1. Experiments versus data diversity

389 In order to conduct representative experiments, we use different databases  
390 and various configurations of the experimental protocol leading to a large view  
391 of the proposed multi-model performance. The conventional Vistex database  
392 [Verdoolaege et al. 2012, Kwitt et al. 2011, Do et al. 2002] is used considering  
393 two sizes of samples respectively  $32 \times 32$  and  $128 \times 128$ . Experiments are also  
394 conducted on the Outex, ALOT and Stex databases which are more challenging  
395 color texture database, since the color and texture information are not easily  
396 distinguishable. We consider the following scenarios:

- 397 • First scenario, i.e. DB1, addressing a set of 24 textured images of size  
398  $512 \times 512$  from the Vistex database [Vistex]. The protocol follows the work  
399 of [Permuter et al. 2006] and [Qazi et al. 2010]. Each image was divided  
400 into subimages of size  $32 \times 32$  pixels resulting of a large database of 6144  
401 small textures. We consider 96 image from the resulting 256 subimages as  
402 the training set, while the remaining 160 subimages are considered as the  
403 test set. This dataset is used in order to evaluate robustness of our model  
404 even if very local spatial structures are considered.
- 405 • Second scenario, i.e. DB2, addressing a set of 54 textured images of size  
406  $512 \times 512$  from the Vistex database. The protocol follows the work of  
407 [Do et al. 2002]. Each image was divided into subimages of size  $128 \times 128$   
408 pixels. This dataset is available on the Outex web site [Ojala et al., Outex]  
409 as test suit Contrib\_TC\_00006. For each texture, subimages are considered  
410 to form a checkerboard. The white half of subimages is then considered

411 as the training set and the black half is used as the testing set. Hence, the  
412 training procedure will account for non uniformity of the original images.

413 • Third scenario, i.e. DB3, addressing a set of 68 textured images of size  
414  $746 \times 538$  from the Outex database. This dataset is also available on  
415 the Outex web site as Outex\_TC\_00013. Each image was divided into 20  
416 subimages of size  $128 \times 128$  pixels. Training and test sets are obtained as  
417 in DB2.

418 • Fourth scenario, i.e. DB4, addressing large set of classes from ALOT  
419 [ALOT] database. 250 images of size  $384 \times 256$  from ALOT database are  
420 considered, then each image is split into 16 non-overlapping subimages  
421 resulting on two large databases of 4000 images of size  $96 \times 64$ . These  
422 images have been captured under different viewing angles and illumination.  
423 Here again training and test sets are obtained as in DB2.

424 • Fifth scenario, i.e. DB5, addressing large set of classes from Stex [STex]  
425 database. 476 textures from STex of size  $512 \times 512$  are considered. Then,  
426 each image is split into 16 non-overlapping subimages resulting on two  
427 large databases of 7616 images of size  $128 \times 128$ . Here again training and  
428 test sets are obtained as in DB2.

429 Every color component of each subimage was normalized by subtracting its  
430 mean and dividing by its standard deviation, and then decomposed using a 2-  
431 scale dual-tree complex wavelet transform (DTCWT) [Kingsbury 1998] with a  
432 Q-shift(14,14) tap filter. In addition to its rich directional analysis, shift invari-  
433 ance and low redundancy properties, the DTCWT was chosen for its reduced  
434 computational time.

#### 435 *4.2. Quantitative evaluation of performance*

436 Evaluation of the classification performance of each approach consists on two  
437 criteria, namely the percentage of classification and the precision. We recall that  
438 for each class of textures:

- 439 • True positives (TP): refers to the number of subimages that have been  
440 successfully returned by the classifier as belonging to a given class.
- 441 • False positives (FP): is the number of subimages that have been wrongly  
442 classified as belonging to a given class.
- 443 • False negatives (FN): is the number of subimages that are classified as  
444 not belonging to a given class while, in fact, they belong to this class of  
445 textures.
- 446 • True negatives (TN): is the number of subimages that are correctly clas-  
447 sified as not belonging to a given class.

448 The percentage of classification (or Sensitivity) is the proportion of textures  
449 which were well labeled by the classifier. It represents the ability of the classifier  
450 in correct positive assignment of a texture to its real class. A 100% percent-  
451 age means that the classifier makes no false negative labeling (FN = 0). The  
452 Percentage classification is given as follows:

$$\text{Sn} = \frac{\text{TP}}{\text{TP} + \text{FN}} \times 100\% \quad (38)$$

453 However, theoretically, a system that always returns positive assignment (even  
454 wrongly) will achieve 100% percentage classification. The problem with per-  
455 centage classification is that it tells nothing about textures that were wrongly  
456 labeled as belonging to a given class (FP assignments). For this, we take into  
457 account also the precision to evaluate the classifier performance.

458 The precision criterion indicates the ability of the classifier to detect true class  
459 membership. The precision is given by:

$$\text{Sp} = \frac{\text{TP}}{\text{TP} + \text{FP}} \quad (39)$$

460 A classifier with very high precision makes no mistakes in assigning a texture  
461 to its class (FP = 0).

#### 462 *4.3. Results and discussions*

463 One of the advantages of the proposed multi-modeling approach is the flex-  
464 ibility in term of modeling textures from different LC color spaces. The joint

Table 3: Notations, descriptions and pdf expressions for the used models

Model	Description	pdf
CopMGam	Gaussian copula based multivariate Gamma (all marginals are Gamma)	$f(x, \theta) = \frac{1}{ \Sigma ^{1/2}} \exp[-\frac{1}{2}\vartheta^T(\Sigma^{-1} - I)\vartheta] \times (\frac{\beta^{-\alpha}}{\Gamma(\alpha)})^d \prod_{i=1}^d x_i^{\alpha-1} \exp - \sum_{i=1}^d (\frac{x_i}{\beta}), \theta = (\alpha, \beta, \Sigma)$
CopMWbl	Gaussian copula based multivariate Weibull (all marginals are Weibull)	$f(x, \theta) = \frac{1}{ \Sigma ^{1/2}} \exp[-\frac{1}{2}\vartheta^T(\Sigma^{-1} - I)\vartheta] \times (\frac{\tau}{\lambda})^d \prod_{i=1}^d x_i^{\tau-1} \exp - \sum_{i=1}^d (\frac{x_i}{\lambda})^\tau, \theta = (\tau, \lambda, \Sigma)$
{Gamma,Weibull}	Gaussian copula based bivariate model with Gamma as the first marginal and Weibull as the second marginal	$f(x, \theta) = \frac{1}{ \Sigma ^{1/2}} \exp[-\frac{1}{2}\vartheta^T(\Sigma^{-1} - I)\vartheta] \times \frac{\tau\beta^{-\alpha}x_2^{\alpha-1}}{\lambda\Gamma(\alpha)} (\frac{x_1}{\lambda})^{\tau-1} \times \exp \left[ -(\frac{x_1}{\lambda})^\tau - \frac{x_2}{\beta} \right], \theta = (\alpha, \beta, \tau, \lambda, \Sigma)$
{vonMises,Weibull}	Gaussian copula based circular/linear bivariate model with vonMises as the first marginal and Weibull as the second marginal	$f(x, \theta) = \frac{1}{ \Sigma ^{1/2}} \exp[-\frac{1}{2}\vartheta^T(\Sigma^{-1} - I)\vartheta] \times \frac{\tau}{2\pi\lambda I_0(\nu)} (\frac{x_2}{\lambda})^{\tau-1} \exp[\nu \cos(x_1 - \mu) - (\frac{x_2}{\lambda})^\tau], \theta = (\mu, \nu, \tau, \lambda, \Sigma)$
{vonMises,Gamma}	Gaussian copula based circular/linear bivariate model with vonMises as the first marginal and Gamma as the second marginal	$f(x, \theta) = \frac{1}{ \Sigma ^{1/2}} \exp[-\frac{1}{2}\vartheta^T(\Sigma^{-1} - I)\vartheta] \times \frac{\beta^{-\alpha}}{2\pi\Gamma(\alpha)I_0(\nu)} (\frac{x_2}{\beta})^{\alpha-1} \exp[\nu \cos(x_1 - \mu) - (\frac{x_2}{\beta})^\alpha], \theta = (\mu, \nu, \alpha, \beta, \Sigma)$

Table 4: Performances of different versions of the multi-model in Lab and HSV color spaces.

	DB1		DB2		DB3		DB4		DB5	
	Sn	Sp	Sn	Sp	Sn	Sp	Sn	Sp	Sn	Sn
Lab1	<b>97.5</b>	<b>0.96</b>	<b>98.3</b>	<b>0.98</b>	<b>89.7</b>	<b>0.88</b>	<b>58.2</b>	<b>0.55</b>	<b>77.6</b>	<b>0.75</b>
Lab2	94.5	0.92	95	0.93	84.6	0.84	54.3	0.52	74.7	0.73
Lab3	93.3	0.92	93.8	0.90	82	0.82	53.6	0.52	73.4	0.71
HSV1	<b>95.6</b>	<b>0.94</b>	<b>96.7</b>	<b>0.94</b>	<b>86</b>	<b>0.86</b>	<b>55.5</b>	<b>0.54</b>	<b>76.9</b>	<b>0.75</b>
HSV2	93.1	0.93	93.4	0.93	83.1	0.83	53.6	0.53	73.9	0.73
HSV3	90.4	0.88	90.8	0.88	80.5	0.82	51.3	0.49	70.4	0.68

models  $M_L$  and  $M_{C_r}$  enable us to consider different parametric modeling for marginals between luminance and chrominance channels. As already mentioned, when chrominance components are defined in a cylindrical space, the *hue* represents a circular component and the *saturation* is the axe of the cylinder. We can then benefit from the advantage of copulas in merging different marginals in the same joint model to construct a circular/linear bivariate model for chrominance such as {vonMises,Weibull} or {vonMises,Gamma} (see Table 3).

Regarding the first part of our experiments, Table 4 presents color texture classification performance in Lab and HSV color spaces respectively, using different multi-models on DB1, DB2, DB3, DB4 and DB5. We note that:

- Lab1: refers to a multi-model that considers CopMGam (Table 3) for the luminance model  $M_L$  and CopMWbl for the chrominance model  $M_{C_r}$  in Lab color space.
- Lab2: refers to CopMWbl for  $M_L$  and CopMGam for  $M_{C_r}$  in Lab color space.
- Lab3: refers to CopMGam model for  $M_L$  and the bivariate model {Gamma,Weibull} for  $M_{C_r}$  in Lab color space.
- HSV1: refers to CopMGam for  $M_L$  and the bivariate model {vonMises,Weibull}



- 483 for  $M_{C_r}$  in HSV color space. (vonMises for the Hue marginal, and Weibull  
484 for the Saturation marginal)
- 485 • HSV2: refers to CopMWbl for  $M_L$  and the bivariate model {vonMises, Gamma}  
486 for  $M_{C_r}$  in HSV color space.
  - 487 • HSV3: refers to CopMGam for  $M_L$  and and the bivariate model {Weibull, Gamma}  
488 for  $M_{C_r}$  in HSV color space.

489 We remark that in Lab, and for all the five databases, best performance is  
490 achieved using the multi-model Lab1. This indicates that the spatial struc-  
491 ture along with the inter-band dependency are better characterized using the  
492 Gaussian copula in conjunction with Gamma marginals (CopMGam). This also  
493 indicates that the dependency between "a" and "b" chrominance components  
494 is better characterized using the Gaussian copula in conjunction with Weibull  
495 marginals (bivariate CopMWbl). For HSV, the multi-model HSV1 leads to the  
496 best performance in all databases. This is due to the ability of CopMGam  
497 to model spatial structure and inter-band dependence for luminance, and the  
498 circular/linear bivariate {vonMises, Weibull} to characterize the chrominance  
499 dependency.

500 We next would like to compare between the perceptually uniform and percep-  
501 tual subfamilies of LC color spaces, via their respective representatives Lab and  
502 HSV spaces. Considering the best multi-models in each case (Lab1 for Lab  
503 and HSV1 for HSV), we clearly observe from Table 4 that Lab representation  
504 outperforms the HSV one. This is a fair comparison even the models  $M_L$  and  
505  $M_{C_r}$  are not the same in multi-models Lab1 and HSV1, since these latters are  
506 the best relatively the nature of the color space. Then, if Lab1 outperforms  
507 HSV1, we can conclude that in best modeling case the Lab representation is  
508 better than the HSV one. These results are explained by the better luminance-  
509 chrominance representation of Lab. In other words, we can say that Lab offers  
510 more decorrelation property between luminance and chrominance, which is the  
511 central interest of our approach. We do not ignore that percentage classification  
512 and precision rates are lower for large databases DB3, DB4 and DB5.

Table 5: Average percentage classification (Sn) using the best multi-models in Lab and HSV in comparison with existing approaches.

	DB1	DB2	DB3	DB4	DB5
Lab1 (proposed method)	97.5	98.3	89.7	58.2	77.6
[Qazi et al. 2011], in Lab	97.2	96.5	88	52.1	78.5
HSV1 (proposed method)	95.6	97.8	86	55.5	76.9
[Qazi et al. 2010], in IHLS	95.4	97.4	84.1	49.3	77.1
[Verdoolaege et al. 2012], in RGB	92.2	94.7	78.5	59.6	75.1
[Kwitt et al. 2011], in RGB	91.5	94.1	79.8	55.4	74.9

513 Regarding the effectiveness of our approach in comparison with existing ones,  
 514 i.e uni-modeling in LC and RGB color spaces, Table 5 shows average percentage  
 515 classification Sn of textures of DB1, DB2, DB3, DB4 and DB5 using our best  
 516 multi-models (Lab1 and HSV1) against the four following approaches:

- 517 • [Qazi et al. 2010], in Lab: Linear prediction models based characteriza-  
 518 tion in the Lab color space,
- 519 • [Qazi et al. 2011], in IHLS: Linear prediction models based characteriza-  
 520 tion in the improved hue luminance saturation (IHLS ) color space,
- 521 • [Verdoolaege et al. 2012], in RGB: Dependence between R, G and B color  
 522 components using the MGGD model,
- 523 • [Kwitt et al. 2011], in RGB: Dependence between R, G and B using the  
 524 t-copula based multivariate Weibull model,

525 We observe from Table 5 that our Lab1 proposed model leads to better per-  
 526 formance when compared with the linear prediction modeling [Qazi et al. 2010]  
 527 and [Qazi et al. 2011]. Results presented in the latter studies are the best of our  
 528 knowledge, but it should be highlighted that the characterization of both lumi-  
 529 nance and chrominance channels by using the 2-D auto regressive (AR) models

Table 6: Average percentage classification Sn and runtime (in minutes) on DB2, using Rao distance, ML based divergence and Monte-carlo based KL divergence as similarity measure for Lab1 and HSV1 multi-models.

	Rao		KL <sub>ML</sub>		KL <sub>MC</sub>	
	Sn	Runtime	Sn	Runtime	Sn	Runtime
Lab1	98.3	4.9 min	98	40.5 min	97.2	60 min
HSV1	96.7	6.5 min	95.9	44.3 min	96.2	63 min

Table 7: Classifier performances for full feature vector (spatial structure for luminance and chrominance)

	DB1		DB2		DB3	
	Sn	Sp	Sn	Sp	Sn	Sp
Lab1	86.9	0.83	90	0.88	75.8	0.78
HSV1	84.5	0.82	89.6	0.88	72.2	0.70

530 with the same order, seems a limitation because the two channels are indepen-  
531 dent. However, it is noteworthy to precise that the results shown in Table 5 for  
532 the two studies consider only the case when authors use the spatial structure and  
533 chrominance information without adding the color information (3-D color his-  
534 tograms) in the characterization. We note that there is a slight performance for  
535 this later for DB5 database and for [Verdoolaege et al. 2012] for DB4 database.  
536 We also remark from the same table the superiority of the characterization in LC  
537 color spaces compared to the RGB one except for DB4. This is theoretically con-  
538 firmed since considering the image as two separable luminance and chrominance  
539 information is more intuitive and correlated with the human perception than the  
540 RGB space which considers the color as an addition of the primary colors red,  
541 green and blue. Here, we also note that [Verdoolaege et al. 2012] considered  
542 uniquely the dependency between R, G and B components (inter-channel only),  
543 and that [Kwitt et al. 2011] considered the R, G and B dependency besides the  
544 inter-band one (for the three components). As we stressed in the introduction,

545 extending these models to take into account jointly intra-band, inter-band and  
546 inter-channel dependencies leads to high-dimensionality, due to the huge depen-  
547 dence between R, G and B components, and to the fact that RGB representation  
548 offers no separation between spatial structure and chrominance.

549 Another contribution of our approach is the definition of a closed form ex-  
550 pression for the Rao distance in the variety of copula based probability densi-  
551 ties [El Maliani et al. 2011]. As it is known, deriving a similarity measure in  
552 the case of copula based multivariate models is a challenging task. For this,  
553 in [Kwitt et al. 2009] and [Sakji-Nsibi et al. 2010], authors proposed a Monte-  
554 carlo approximation of the KL divergence. But, this approach is computation-  
555 ally expensive and is not deterministic, since the KL divergence differs depending  
556 on the random number generation. An alternative approach was proposed in  
557 [Kwitt et al. 2011], using the maximum likelihood (ML) selection rule as a simi-  
558 larity measure. This significantly reduces the execution time in comparison with  
559 the Monte-carlo based similarity measurement. A more attractive expression of  
560 the KL divergence is provided in [Lasmar et al. 2012], where the separability  
561 between the copula space and the marginal space is used to derive a closed-  
562 form and fast KL based similarity measurement. However, KL divergence does  
563 not satisfy properties of symmetry and triangular inequality, and thus is not a  
564 distance in the right sense of the parametric manifold. Table 6 shows average  
565 percentage classification along with the execution time using both similarity  
566 measures (Rao and KL) for the textures of DB2. We recall that according to  
567 equation 37, we need expression of the Rao distance for marginals and the used  
568 copula. Expressions of the Rao distance for Weibull, Gamma and vonMises  
569 can be found respectively in [El Maliani et al. 2011], [Reverter et al. 2003] and  
570 [Ceolini et al. 2010]. We can see that the proposed similarity measure slightly  
571 outperforms the ML based and Monte-Carlo based approach in term of per-  
572 centage classification. However, the improvement is more significant in term of  
573 runtime which is 9 times and 12 times less than the ML based and Monte-carlo  
574 based approaches respectively. We note that the experiments were done using  
575 Matlab environment on an HP Compaq equipped with an Intel Core 2 Duo

576 CPU at 3GHZ workstation

577 Finally, we want to validate experimentally the hypothesis made in introduction  
578 about dimensionality reduction when we consider multi-modeling in LC color  
579 spaces. We compare our approach to the full dependence based approach of sub-  
580 section 2.1. Table 7 shows performance of the classifier for multi-models Lab1  
581 and HSV1 with full feature vectors length. It can be seen from these results  
582 that the high parametrisation does not improve performances, but on contrary  
583 it makes them decrease by about 10%. Separability between luminance and  
584 chrominance helps then to avoid redundancy in characterization which is the  
585 case in RGB where spatial structure resides in the three color components.

## 586 **5. Conclusion**

587 We proposed in this letter a multi-model based approach for color textures  
588 characterization in the LC color spaces. This approach takes into account the  
589 separability between luminance and chrominance. We especially took care of  
590 the circular/linear nature of chrominance components. We also addressed the  
591 dimensionality reduction issue for the study of the full dependence in term of  
592 intra-band, inter-band and inter-channel relation between coefficients color com-  
593 ponent subbands. Results on five wide range of databases show the superiority of  
594 the proposed approach in comparison with the existing unimodeling approaches.  
595 We know that our method as any compared methods, presents some limitations  
596 when using large databases and also against rotation and illumination effects.  
597 Furthermore, considering the runtime issue, results show the effectiveness of the  
598 Rao geodesic distance in comparison with the Kullback-leibler divergence.

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