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Color texture classification method based on a statistical multi-model and geodesic distance

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Abstract

In this letter, we propose a novel color texture classification method based on statistical characterization. The approach consists in modeling complex wavelet coefficients of both luminance and chrominance components separately leading to a multi-modeling approach. The copula theory allows to take into account the spatial dependencies which exist within the intra-luminance sub-bands via the luminance model M_L , and also between the inter-chrominance subband coefficients via the chrominance model M_{C_r} . The multi-model, i.e M_L and M_{C_r} , is used to develop a Bayesian classifier based on the softmax principal. To derive the classifier, we propose a closed-form expression for the Rao geodesic distance between two copulas. Experiments on two sub-families of luminance-chrominance color spaces namely Lab and HSV have been carried out for a wide range of color texture databases. The combination of different statistical sub-models show that the multi-modeling performs better than some existing methods in term of classification rates.

Keywords: Color texture, wavelet representation, Rao geodesic distance, Bayesian classification

1. Introduction

For color texture analysis, the question of the color space selection to characterize the visual content may be raised. RGB space is the most natural since

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14 it represents the acquisition-reference space. However, R, G and B channels are
15 known to exhibit simultaneously high statistical dependencies relatively to: 1)
16 the intra-channel or spatial one, 2) the inter-band for scale-space representation
17 one and 3) the inter-channel or multicomponent one. In recent works, some au-
18 thors have proposed various stochastic models to characterize the inter-channel
19 dependencies alone. From scale-space representation such as 2-D wavelet or
20 others, previous approaches consist in fitting the histograms of the wavelet sub-
21 bands with a given parametric probability density function (pdf) of the trivariate
22 color vector samples for each level of the decomposition. Recent works are based
23 on multivariate generalized Gaussian distribution in [Verdoolaege et al. 2012]
24 or copulas based multivariate Weibull distribution in [Kwitt et al. 2011]. The
25 parameters of the pdf are the covariance matrix and different auxiliary param-
26 eters such as the shape parameter characterizing the non-Gaussianness. Ex-
27 tending these models to take into account jointly intra-band, inter-band and
28 inter-channel dependencies, leads to a high-dimensional model in terms of num-
29 ber of parameters. This is a main drawback of these proposals since, in practice,
30 for a fixed data size, dealing with high-dimensional model may be a critical issue
31 in terms of parameter estimation performance. Considering color space, another
32 alternative to the RGB space is to consider the family of luminance-chrominance
33 (LC) color spaces [DeYoe et al. 1996, Mojsilovic et al. 2000]. This family in-
34 cludes perceptual (HSV, HSI and HSB) and perceptually uniform (Lab, Yuv,
35 YCbCr) color spaces [Sarifuddin et al. 2005]. An extended comparison of those
36 spaces is given by [Qazi et al. 2011]. These authors note that the luminance
37 channel is uncorrelated with the chrominance channels. From this independ-
38 ence in LC spaces, they proposed a model taking into account separately intra
39 and inter dependencies, i.e. multi-model, with few parameters compared to the
40 required number by using the RGB space. Qazi et al. proposed to consider a
41 multi-model based on a 2-D complex quarter plan autoregressive (2-D QP AR)
42 model for luminance as well as for chrominance information. This may lead to a
43 lack of flexibility because the luminance and chrominance are decorrelated and
44 thus, have distributions of different natures.

45 In this letter, considering the LC color spaces, we propose a multi-modeling ap-
46 proach based on multivariate stochastic modeling. The contribution is twofold:

- 47 • First, we propose a multi-model based on two non-Gaussian sub-models,
48 i.e. M_L and M_{C_r} , devoted to describe respectively 1) the intra-channel
49 dependency by considering the spatial structure and inter-band depen-
50 dence of the luminance channel, and 2) the inter-channel dependency by
51 considering a bivariate pdf for the two chrominance channels. Both M_L
52 and M_{C_r} are based on copula as a powerful tool to construct non-Gaussian
53 joint models that fit data with even different natures of margins. Linear
54 and/or circular models are derived depending on the component natures
55 (ex: the Hue in HSV color space is a circular channel).

- 56 • Second, for the similarity measurement used to compare two texture sam-
57 ples, we derive a closed form expression for the Rao geodesic distance for
58 copula based multivariate models. Contrary to the conventional Kullback-
59 leibler (KL) divergence, the Rao geodesic metric has the advantage to
60 satisfy distance properties such as symmetry and triangular inequality
61 [Atkinson et al. 1981].

62 The use of different models for characterizing the wavelet coefficients has been
63 recently investigated by [Rakvongthai et al. 2013], for texture retrieval in a
64 noisy environment. Authors proposed different univariate statistical models for
65 the real/imaginary, magnitude and phase parts of complex subband coefficients.
66 The parameter vector of the studied texture is then formed by the concatena-
67 tion of parameters estimated from each of these models. Also, the similarity
68 measure has been expressed by a formula combining both magnitude and phase
69 KL divergences using a mixing parameter. While the cited work did not con-
70 sider the color information at all, it should be emphasized that the present work
71 considers the color and texture information jointly for image characterization.
72 Concerning the similarity measure, the originality of our method is the use of
73 Rao geodesic distance which represents a good alternative to the KL divergence
74 since it satisfies the distance properties. For this, a closed-form expression of

75 Rao geodesic distance between copula based distributions is developed.
76 Note also that this work stands out from our previous work [Maliani et al. 2012]
77 which presents a preliminary study of the multi-modeling concept that consid-
78 ered only the special case of color representation, the HSV color space. More-
79 over, the paper did not discuss the dimensionality reduction issue when consider-
80 ing the three dependencies (intra-band, inter-band and inter-channel). Also, in
81 the classification step, we proposed the K-Nearest Neighborhood classifier tak-
82 ing the weighted sum of the two similarity measures between the luminance and
83 chrominance sub-models. The weights have been tuned experimentally which is
84 the same as for [Rakvongthai et al. 2013], which led us to test exhaustively on
85 several scales of weights to get the best classification rate. The multi-modeling
86 proposed in this letter, helps in dimensionality reduction of the extracted fea-
87 tures to characterize the textural content. According to several dependence
88 experiments presented in section 2, the spatial structure of the texture and the
89 inter-band dependence is only considered for the luminance component and the
90 inter color-channel dependency consists of a bivariate model. The performance
91 of the multi-model is tested using a Bayesian classifier based on the softmax
92 principal [Bishop 2006] thanks to its suitability to combine several similarity
93 measures through a posteriori probability fusion. The experimental protocol
94 adopted here, aims to find the optimal configuration of our multi-model. This
95 starts with the choice of statistical distribution related to color components in
96 the field of dual-tree complex wavelet CT-DWT [Kingsbury 1998]. We proposed
97 to build joint models by testing different combinations of Gamma , Weibull and
98 VonMises distributions for both luminance and chrominance components. First
99 comparisons were considered to show the relevance of each model for the color
100 spaces. Then, we compared our best selected models with the state of the art
101 methods in terms of classification rate, the dimensionality reduction and the
102 computation time.

103 This paper is organized as follows. Section 2 deals with the proposed multi-
104 modeling approach for textures in LC color spaces. In Section 3, we present the
105 classification framework, and the proposed similarity measurement. Then, we

106 show experimental results in Section 4. Finally, we give concluding remarks in
107 Section 5.

108 **2. Color texture modeling in LC color spaces**

109 *2.1. Dependence characterization*

110 Let us consider a stationary color texture x_T represented in a given LC color
111 space, and let l be the luminance component, cr_1/cr_2 respectively the first and
112 the second chrominance component of x_T . Each color component is first de-
113 composed into a set of wavelet subbands. In wavelet domain texture character-
114 ization, it is common to model the statistical distribution of wavelet coefficients
115 in different subbands, which corresponds to the detail subbands. The low-
116 pass wavelet coefficients are excluded ([Mallat 1999], [Boubchir et al. 2010]).
117 For 2-D wavelet transforms, each scale usually has several subbands corre-
118 sponding to the different orientations. Let us consider K subbands, where
119 $K = S \times O$ with S the number of scales and O the number of orientations.
120 The subbands are respectively $l^{(k)}, cr_1^{(k)}, cr_2^{(k)}$, $k = 1, \dots, K$. The statistical
121 distribution of wavelet detail coefficients of natural textures has been exten-
122 sively studied ([Simoncelli 1997], [Boubchir et al. 2010], [Lasmar et al. 2010],
123 [Verdoolaege et al. 2012], [Kwitt et al. 2011]). It has been shown that the em-
124 pirical distribution is generally heavy-tailed with a sharp peak at zero. This
125 leptokurtic behavior leads to consider non-Gaussian marginal or joint density.
126 Contrary to the marginal approach, the joint modeling needs to define the de-
127 pendence which exists between subbands coefficients. Intra-band, inter-band
128 and inter-channel dependencies have to be considered. Let us detail each of
129 these dependencies *for the three color components*.

130 *2.1.1. Intra-band dependency*

131 Intra-band dependence refers to the spatial neighborhood dependence be-
132 tween coefficients of the same wavelet subband. It corresponds to the spatial
133 structure of the studied subband. Let s be the location of a reference coefficient

134 from the k^{th} subband of a given color channel. If we consider the luminance
 135 channel for example, let m be the size of the chosen neighborhood. The collec-
 136 tion of the reference coefficient neighbors is specified by $l_s^{(k)} = (l_{1,s}^{(k)}, \dots, l_{m,s}^{(k)})$.
 137 Then, considering all the subbands coefficient neighbors, the dataset to be ob-
 138 served is:

$$l_{\text{intra}} = [l_1^{(k)}, \dots, l_s^{(k)}, \dots, l_P^{(k)}]^T \quad (1)$$

139 l_{intra} is a P by m matrix, where P is the size of the wavelet subband. If we
 140 consider, the first chrominance channels, intra-band dependency for a given
 141 subband k is studied by observing the following dataset:

$$cr_{1\text{intra}} = [cr_{1_1}^{(k)}, \dots, cr_{1_s}^{(k)}, \dots, cr_{1_P}^{(k)}]^T \quad (2)$$

142 For the second chrominance channel, we have:

$$cr_{2\text{intra}} = [cr_{2_1}^{(k)}, \dots, cr_{2_s}^{(k)}, \dots, cr_{2_P}^{(k)}]^T \quad (3)$$

143 2.1.2. Inter-band dependency

144 To model the inter-band dependence, we consider subbands in different ori-
 145 entations for the same color channel. Thus for luminance inter-band we observe
 146 a dataset represented by a P by K matrix, where K is the total number of
 147 subbands:

$$l_{\text{interbd}} = [l^{(1)} \dots l^{(K)}] \quad (4)$$

148 for chrominance inter-band:

$$cr_{1\text{interbd}} = [cr_1^{(1)} \dots cr_1^{(K)}] \quad (5)$$

$$cr_{2\text{interbd}} = [cr_2^{(1)} \dots cr_2^{(K)}] \quad (6)$$

149 we note that $l^{(k)}$, $cr_1^{(k)}$ and $cr_2^{(k)}$ are respectively the column vectors containing
 150 all coefficients of the k^{th} subband of luminance, first and second chrominance
 151 channels.

152 *2.1.3. Inter-channel dependency*

153 It refers to the dependence between coefficients of subbands from different
 154 color channel, considering a fixed orientation k . The observation matrix is as
 155 follows:

$$D_{\text{interch}} = \begin{bmatrix} l^{(k)} & cr_1^{(k)} & cr_2^{(k)} \end{bmatrix} \quad (7)$$

156 *2.1.4. Full dependence*

157 Considering all the aforementioned dependencies leads to a full dataset that
 158 considers intra-band, inter-band and inter-channel dependencies jointly. It is
 159 represented by the following matrix of size P by $(3 \times K \times m)$:

$$D = \begin{bmatrix} l_1^{(1)}, \dots, l_P^{(1)}, \dots, l_1^{(K)}, \dots, l_P^{(K)}, cr_{11}^{(1)}, \dots, cr_{1P}^{(1)}, \\ \dots, cr_{11}^{(K)}, \dots, cr_{1P}^{(K)}, cr_{21}^{(1)}, \dots, cr_{2P}^{(1)}, \dots, cr_{21}^{(K)}, \\ \dots, cr_{2P}^{(K)} \end{bmatrix}^T \quad (8)$$

160 Consideration of this matrix means, first, that all the three components carry
 161 the spatial structure information (intra-band). Second, that dependence exists
 162 between subbands of all orientations for each channel (inter-band), and third,
 163 that the representation exhibits dependence among the three color channels
 164 (inter-channel).

165 *2.2. Multi-model approach*

166 *2.2.1. Experimental analysis of dependence*

167 Contrary to the RGB representation, LC color spaces offer separability be-
 168 tween luminance and chrominance. At first glance, this indicates that the inter-
 169 channel dependence have not to be considered between luminance and chromi-
 170 nance. A more precise study of dependence is done according to the correlation
 171 coefficients. Table 1 shows degrees of dependence when textures are represented
 172 in Lab color space. This is obtained by averaging the Pearson, Kendall, Spear-
 173 man and the mutual information coefficients on all subbands of 40 color textures
 174 from the Vistex database [Vistex]. We recall that mutual information is a mea-
 175 sure of the statistical dependence between two variables. If we suppose that

176 X and Y are two random variables, $f(x)$ and $f(y)$ the marginal probability
177 distribution functions of X and Y respectively, and $f(x, y)$ the joint probability
178 distribution function of X and Y . The mutual information between X and Y
179 is then:

$$I(X, Y) = \sum_{x,y} f(x, y) \log \frac{f(x, y)}{f(x)f(y)} \quad (9)$$

We deduce from these results that intra-band dependence have to be consid-

Table 1: Averaging measure of dependence on the Vistex database, using Pearson, Kendall, Spearman and mutual information for 40 color textures from the Vistex database in the Lab color space.

	Intra-band			Inter-band			Inter-channel		
	l	cr_1	cr_2	l	cr_1	cr_2	$\{l, cr_1\}$	$\{l, cr_2\}$	$\{cr_1, cr_2\}$
Pearson	0.47	0.041	0.0372	0.33	0.037	0.041	0.003	0.003	1
Kendall	0.45	0.038	0.037	0.28	0.037	0.038	0.002	0.002	1
Spearman	0.32	0.026	0.025	0.25	0.025	0.026	0.001	0.001	1
MI	4.46	1.56	1.56	4.20	1.03	1.04	0.74	0.75	5.88

180

181 ered only for luminance, and that inter-channel dependence exists only between
182 chrominance components. They also show that we can restrict the study of
183 inter-band dependence to the luminance component.

184 2.2.2. Multi-model formulation

185 Our contribution here is to exploit the independence between luminance and
186 chrominance. So we propose a multi-model based characterization that consid-
187 ers the different natures of these information ([Qazi et al. 2011]). Two joint
188 and independent models M_L and M_{C_r} are used to respectively characterize
189 the luminance information with spatial and inter-band dependence considering
190 observations of a vector \bar{l}_s (see subsection 2.4.1) and the chrominance one de-
191 voted to inter-channel dependence by observing realizations of a vector cr_s (see

192 subsection 2.4.2):

$$p(x_T, \theta) = p(x_T | \{M_L, M_{C_r}\}) \propto p(\bar{l}_s, \theta_L) \prod_{k=1}^K p(cr_s^{(k)} | \theta_{c_r}) \quad (10)$$

193 where θ_L and θ_{c_r} are the vectors of parameters of luminance and chrominance
 194 models, respectively. The multi-modeling permits to reduce the dimension of
 195 the observed dependence dataset D (subsection 2.1).

196 *2.3. Copula based multivariate models*

197 Different multivariate models can be used for defining M_L and M_{C_r} , such
 198 as the multivariate generalized Gaussian (MGGD) [Verdoolaege et al. 2012], the
 199 multivariate Bessel K forms [Boubchir et al. 2010], the spherically invariant ran-
 200 dom vector (SIRV) [Lasmar et al. 2010]. Those models have been proposed to
 201 characterize different kinds of dependencies such as the intra-band in the case
 202 of SIRV and the inter-channel in the case of MGGD. Each of them is dedicated
 203 to a specific dependency rule, either for intra-band or inter-channel but never
 204 taking account of both together. Obviously, extending each model simultane-
 205 ously to the both dependencies is possible but the equivalent model exhibits
 206 the main drawback the common definition of the leptokurtic and platykurtic for
 207 the whole set components of the multivariate model. If we suppose to have a
 208 set of dependent variables $T = [t_1, \dots, t_n]$, with the MGGD or SIRV multivari-
 209 ate modeling, each marginal t_i has finally a similar shape value. This behavior
 210 is confirmed in the study of [Rangaswamy et al. 2002], the characterization of
 211 one marginal is sufficient to determine the shape of the whole distribution of
 212 a SIRV. Table 2 show averages of scale and shape parameters supposing that
 213 subbands of the three components are described by the same marginal, Weibull
 214 or Gamma for example. It is clear from the table that the marginal modeling of
 215 luminance and chrominance exhibits different shape and scale parameters in the
 216 case of Lab color space, contrary to RGB. In the HSV color space, we observe
 217 that parameters are different even between the chrominance components H and
 218 S.

Table 2: Average of parameters (considering wavelet subbands for 40 color textures from Vistex database), supposing that subbands of the three color components are described by the same marginals (Weibull or Gamma) in the case of Lab, HSV and RGB color spaces

	Weibull		Gamma	
	scale	shape	scale	shape
L	11.47	1.54	4.97	2.18
a	3.21	1.63	1.26	2.41
b	2.20	1.62	0.92	2.39
H	26.83	1.08	27.15	1.29
S	0.12	1.53	0.05	2.16
V	0.0081	1.52	0.03	2.21
R	31.51	1.53	13.83	2.16
G	29.91	1.52	13.33	2.13
B	31.25	1.52	13.85	2.14

219 The copula theory allows the joint modeling by separating the dependence
 220 structure from the marginal modeling. It is thus easy to merge different marginal
 221 laws into the same multivariate pdf. [Nelsen 2006] defined the copula as a
 222 multivariate uniform distribution used to construct multivariate models. Given
 223 that F is a d -dimensional cumulative distribution function (cdf) with continuous
 224 marginals F_1, \dots, F_d , the Sklar's theorem [Sklar et al. 1959] shows that there
 225 exist a unique copula C such that:

$$F(x_1, \dots, x_d) = C(F_1(x_1), \dots, F_d(x_d)) \quad (11)$$

226 Further, if C is continuous and differentiable, the copula density (c) is given by:

$$c(u_1, \dots, u_d) = \frac{\partial^d C(u_1, \dots, u_d)}{\partial u_1 \dots \partial u_d} \quad (12)$$

227 The joint pdf is uniquely deduced from the marginals and the copula density as
 228 follows:

$$f(x_1, \dots, x_d) = c(F_1(x_1), \dots, F_d(x_d)) \prod_{i=1}^d f_i(x_i) \quad (13)$$

229 where f_i , $i = 1, \dots, d$, represent the marginal pdfs that can follow different
 230 laws.

231 Estimation of the copula parameters is done using the canonical maximum like-
 232 lihood (CML) [Durrleman et al. 1959] method. This is a pseudo parametric es-
 233 timation method contrary to the fully parametric method, termed the inference
 234 from marginals (IFM) [Joe. 1997]. The CML method consists in transforming
 235 the data (x_1, \dots, x_d) into uniform data $(\hat{u}_1, \dots, \hat{u}_d)$ using the empirical distribu-
 236 tions (empirical cdfs) and then estimates the copula parameters considering all
 237 the observations as:

$$\hat{\Sigma} = \underset{\Sigma}{\operatorname{argmax}} \sum_{i=1}^n \log c(\hat{u}_{i1}, \dots, \hat{u}_{id}; \Sigma) \quad (14)$$

238 where n denotes the number of observations, and Σ represents the correlation
 239 matrix.

240 For marginal parameters, the estimation is done using the conventional max-
 241 imum likelihood method. Experiments reveal that the CML estimator is the
 242 best since the method does not rely on parametric assumption about marginals.
 243 This means that less pertinent choice of marginals do not affect estimation of
 244 the copula parameters.

245 For texture modeling, we repose on copulas to construct the models M_L and
 246 M_{C_r} . We select the Gaussian copula for which the density is defined by:

$$c_\phi(u_1, \dots, u_d) = \frac{1}{|\Sigma|^{1/2}} \exp \left[-\frac{1}{2} \vartheta^T (\Sigma^{-1} - I) \vartheta \right] \quad (15)$$

247 with $\vartheta^T = (\phi^{-1}(u_1), \dots, \phi^{-1}(u_d))$, ϕ denotes the cdf of the normalized Gaus-
 248 sian density, Σ the correlation matrix, and I the d -dimensional identity matrix.

249 The choice of the Gaussian copula is justified by:

- 250 • the dependence representation, which is based on the well known correla-
 251 tion coefficient.

252 • the existence of closed form of Rao distance for the Gaussian copula which
 253 allow us to derive a closed form of Rao distance for the joint model as a
 254 similarity measure (subsection 3.1).

255 *2.4. M_L and M_{C_r} models*

256 Considering the full dependence dataset D (see sub-section 2.1.4) leads to
 257 high parametrization and thus to very cumbersome estimation of parameters.
 258 To alleviate this complexity, we use the proposition made by [Qazi et al. 2011]
 259 about dependencies between coefficient subbands when color textures are rep-
 260 resented in LC color spaces. The proposition consists in a multi-modeling ap-
 261 proach to reduce the dimensionality of the observed dataset.

262 *2.4.1. Multivariate spatial model for luminance (M_L)*

263 As a first step for reducing dimension of the full dataset D , the spatial
 264 structure along intra- and inter-band dependence are studied only for luminance
 265 via the model M_L . For a subband coefficient at the position s , intra inter-
 266 band dependence is represented by the vector $\bar{l}_s = (l_{1,s}, \dots, l_{m,s}, \dots, l_{d,s})$, where
 267 $d = m \times K$, m is the neighborhood size and K is the number of wavelet subbands.
 268 Considering all positions at the luminance subbands, the dataset to be modeled
 269 is a P by d matrix of realizations of the vector \bar{l} :

$$L = \begin{bmatrix} \bar{l}_1 \\ \bar{l}_2 \\ \vdots \\ \bar{l}_s \\ \vdots \\ \bar{l}_P \end{bmatrix}$$

270 where P is the size of a wavelet subband. The observation matrix is:

$$L = \begin{bmatrix} l_{1,1} & \cdots & l_{m,1} & \cdots & l_{d,1} \\ \vdots & & \vdots & & \vdots \\ l_{1,s} & \cdots & l_{m,s} & \cdots & l_{d,s} \\ \vdots & & \vdots & & \vdots \\ l_{1,P} & \cdots & l_{m,P} & \cdots & l_{d,P} \end{bmatrix}$$

271 Based on the Gaussian copula, the pdf of the model M_L is defined by:

$$f_{M_L}(\bar{l}_s; \theta) = \frac{1}{|\Sigma_L|^{1/2}} \exp \left[-\frac{1}{2} \vartheta_s^T (\Sigma_L^{-1} - I) \vartheta_s \right] \prod_{i=1}^d f_s(\bar{l}_{i,s}) \quad (16)$$

272 where $\theta = (w_1, w_2, \dots, w_d; \Sigma_L)$ is the vector of hyperparameters, $w_s = \{w_{sj}\}_{j \in \mathbb{N}}$,
 273 $s = 1, \dots, d$ is the vector of parameters for the marginal f_s and Σ_L is the
 274 covariance matrix of the Gaussian copula.

275 In the case of Gaussian copula, the CML estimator of covariance matrix is the
 276 sample correlation matrix of Gaussian observations $\vartheta_1, \dots, \vartheta_P$:

$$\hat{\Sigma}_L = \frac{1}{P} \sum_{s=1}^P \vartheta_s \vartheta_s^T \quad (17)$$

277 with $\vartheta_s = (\vartheta_{1,s}, \dots, \vartheta_{d,s})$ is obtained by transforming the observation $\bar{l}_s =$
 278 $(l_{1,s}, \dots, l_{d,s})$ from the observation matrix L by $\vartheta_{i,s} = \phi^{-1}(F_i(l_{i,s}))$, $i = 1, \dots, d$
 279 and F_s is the empirical marginal cdf.

280 2.4.2. Circular/Linear bivariate model for chrominance (M_{C_r})

281 We consider the inter-channel dependence only between chrominance com-
 282 ponent subband represented by the vector $cr = (cr_1^{(k)}, cr_2^{(k)})$. Thus, considering
 283 all coefficients of the chrominance subbands, the dataset to be modeled by M_{C_r}
 284 is a P by 2 matrix:

$$cr = \begin{bmatrix} cr_{1,1} & cr_{2,1} \\ \vdots & \vdots \\ \vdots & \vdots \\ cr_{1,P} & cr_{2,P} \end{bmatrix}$$

285 M_{C_r} is a bivariate model defined reposing on the Gaussian copula:

$$f_{M_{C_r}}(cr_s^{(k)}; \theta) = \frac{1}{|\Sigma_{C_r}|^{1/2}} \exp \left[-\frac{1}{2} \vartheta_s^T (\Sigma_{C_r}^{-1} - I) \vartheta_s \right] \prod_{i=1}^2 f_i(cr_i) \quad (18)$$

286 where $\theta = (w_1, w_2; \Sigma_{C_r})$, $w_i = \{w_{ij}\}_{j \in \mathbb{N}}$. The index $i \in \{1, 2\}$ is the vector
 287 of parameters for the marginal f_i , and Σ_{C_r} the covariance matrix. The CML
 288 estimator of the covariance matrix is as follows:

$$\hat{\Sigma}_{C_r} = \frac{1}{P} \sum_{s=1}^P \vartheta_s \vartheta_s^T \quad (19)$$

289 with $\vartheta_s = (\vartheta_{1,s}, \vartheta_{2,s})$ is obtained by transforming the observation $cr_s = (cr_{1,s}, cr_{2,s})$
 290 from the observation matrix cr by $\vartheta_{i,s} = \phi^{-1}(F_i(cr_{i,s}))$, $i = 1, 2$ and F_s is the
 291 empirical marginal cdf.

292 The model is called circular/linear because it offers the possibility of using dif-
 293 ferent marginals f_i to come up with a bivariate circular/linear, circular/circular
 294 or linear/linear model for chrominance channels depending on the nature of the
 295 used color space.

296 2.4.3. Texture joint modeling

297 Taking into account the previous sub-models, respectively M_L and M_{C_r}
 298 termed probabilistic because they describe probability distributions over sub-
 299 band coefficients of the texture, let us consider a stationary observed texture,
 300 we have:

$$p(x_T | \{M_L, M_{C_r}\}) \propto p(\bar{l}_s | w_s, \Sigma_L) \prod_{k=1}^K p(cr_s^{(k)} | w_{cr_1}^{(k)}, w_{cr_2}^{(k)}, \Sigma_{C_r}^{(k)}) \quad (20)$$

301 which can be written in a more compact parametric form as:

$$p(x_T | \{M_L, M_{C_r}\}) \propto p(x_T | \theta_s) \quad (21)$$

302 with $\theta_s = (w_s, \Sigma_L, \{w_{cr_1}^{(k)}, w_{cr_2}^{(k)}, \Sigma_{C_r}^{(k)}\}_{k=1, \dots, K})$

303 **3. Classification**

304 In the framework of stochastic parametric classifier, we consider supervised
 305 Bayes classifier to assign the most probable class to a textured image. If we
 306 consider the equiprobability of each class, the Bayes classifier consists in finding
 307 the maximum of the likelihood such as:

$$\hat{b} = \operatorname{argmax}_{b \in \{1, 2, \dots, B\}} [p(x_T, \theta_b)] \quad (22)$$

308 where θ_b is the vector of parameters associated to the class b . The vector θ_b is
 309 estimated during the learning step from the set of texture samples representing
 310 the class b . Considering equation 22, the decision of the classifier is provided
 311 by the numerical evaluation of the likelihood which is time consuming because
 312 of evaluating equation 21 from the three color channel data. In order to avoid
 313 this expensive step, a direct comparison between parametric vectors, i.e. one
 314 representing the class and the other the texture query x_T , is expected. Thus,
 315 the class assignment is obtained from a similarity measure. To do this, the
 316 Softmax approach or normalized exponential function [Bishop 2006] is selected.

$$\operatorname{argmax}_{b \in \{1, 2, \dots, B\}} \frac{\exp(a_b)}{\sum_{j=1}^B \exp(a_j)} \quad (23)$$

317 with $a_b = \ln \left[\frac{1}{1 + L_i(x_T, x_b)} \right]$.

318 The advantage of using the softmax principal is the smoothed version of the
 319 maximum, since if $a_b \gg a_j$ for all $j \neq b$, then $p(C_b|x_T) \simeq 1$ and $p(C_j|x_T) \simeq 0$.
 320 The proposed approach is thus based on the selection of a similarity measure,
 321 i.e $L_i(x_T, x_b)$ exhibiting a closed form in terms of model parameters. Various
 322 probabilistic measures of similarity propose this kind of properties such as the
 323 Kullback-Leibler or Hellinger measures. All these possibilities have been unified
 324 in the seminal theory of the information geometry. This consists in considering
 325 that the parametric model of finite dimension forms a sub-manifold character-
 326 ized by a smooth Riemannian space. Some of these measures are divergences
 327 such as Kullback-Leibler. A divergence is a non-symmetric measure and can

328 be not respecting the triangle inequality. Some others such as Bhattacharya,
 329 Hellinger or others are really some distances. Considering the properties of the
 330 Riemannian manifold, the well-founded distance associated to this type of man-
 331 ifold is the geodesic or Rao distance [Atkinson et al. 1981]. The main drawback
 332 of the geodesic distance is that a closed form for probabilistic model is rarely
 333 available. This is the reason for which the Kullback-Leibler divergence is usualy
 334 used in place to the geodesic distance. However, the true distance according to
 335 the Riemannian space is the geodesic distance.

336 In the next subsection, we derive for our context a closed form for the geodesic
 337 distance for the Copula modeling.

338 3.1. Proposed Rao distance as similarity measure

339 For a probability density $f(x; \theta)$ with $\theta = (\theta^1, \dots, \theta^N)$ a vector of parame-
 340 ters, the Rao distance is a Riemannian metric defined by the fisher information
 341 matrix as:

$$ds^2 = \sum_{p,q=1}^N g_{pq}(\theta) d\theta^i d\theta^j \quad (24)$$

342 where g_{pq} represent the Fisher matrix elements:

$$g_{pq}(\theta) = -E \left[\frac{\partial^2}{\partial \theta^p \partial \theta^q} \log f(x; \theta) \right] \quad (25)$$

343 Let f be a Gaussian copula based joint pdf (M_L or M_{C_r}), $\theta = \{w_i, \Sigma\}$ is the
 344 vector of hyper-parameters of f , $w_i = \{w_{ij}\}_{j \in \mathbb{N}}$ is the vector of parameters
 345 for the marginal f_i , $i = 1, \dots, d$ (for example if the marginal is a Weibull we
 346 have $w_i = \{\alpha_i, \beta_i\}$ representing shape and scale parameters), d is the length
 347 of the vector of one observation and Σ represents the covariance matrix of the
 348 Gaussian copula. By applying equations 13 and 15 in equation 25 we have:

$$g_{pq}(\theta) = -E \left[\frac{\partial^2}{\partial \theta^p \partial \theta^q} \left(\log c_{\Phi}(u; \Sigma) + \log \prod_{i=1}^d f_i(x_i; w_i) \right) \right] \quad (26)$$

349 where $u = (u_1, \dots, u_d)$ represents the vector of uniform coefficients that are
 350 obtained by transforming the wavelet coefficients (x_1, \dots, x_d) using their cdfs

351 ($u_i = F^{-1}(x_i, w_i)$) or using their empirical cdfs (see **MATLAB**'s routine "ecdf"
352) ($u_i = eF^{-1}(x_i)$). In this latter case, no assumptions are made on the para-
353 metric form of the marginal distributions. The copula parameters are then
354 independent from any marginal parameter w_{ij} :

$$\frac{\partial}{\partial w_{ij}} c_{\Phi}(u; \Sigma) = 0 \quad (27)$$

355 so then, $g_{w_{ij}\Sigma}(\theta) = g_{\Sigma w_{ij}}(\theta) = 0$, $i = 1 \cdots d$, $j \in \aleph$

356 Thus,

$$g_{\Sigma\Sigma}(\theta) = -E \left[\frac{\partial^2}{\partial \Sigma \partial \Sigma} \log c_{\Phi}(u; \Sigma) \right] \quad (28)$$

357 and

$$g_{\mu\nu}(\theta) = -E \left[\frac{\partial^2}{\partial \mu \partial \nu} \log \prod_{i=1}^d f_i(x_i; w_i) \right] \quad (29)$$

358 with $\mu, \nu \in \{w_{ij}\}$, $i = 1 \cdots d$, $j \in \aleph$

$$g_{\mu\nu}(\theta) = -E \left[\frac{\partial^2}{\partial \mu \partial \nu} \sum_{i=1}^d \log f_i(x_i; w_i) \right] \quad (30)$$

359

$$= -E \left[\sum_{i=1}^d \frac{\partial^2}{\partial \mu \partial \nu} \log f_i(x_i; w_i) \right] \quad (31)$$

360

$$= \sum_{i=1}^d -E \left[\frac{\partial^2}{\partial \mu \partial \nu} \log f_i(x_i; w_i) \right] \quad (32)$$

361 Thus, from (12):

$$ds^2 = g_{\Sigma\Sigma} d\Sigma d\Sigma + \sum_{i=1}^d \sum_{\mu, \nu} g_{\mu\nu} \mu \nu dt \quad (33)$$

$$= ds_{\text{Gauss}}^2 + \sum_{i=1}^d ds_{\text{Margins}}^2 \quad (34)$$

362 Hence, given two copula based probability distributions $f^{(1)}(x; \theta_1)$ and $f^{(2)}(x; \theta_2)$
363 with $\theta_1 = (w_1^{(1)}, \dots, w_d^{(1)}, \Sigma_1)$ and $\theta_2 = (w_1^{(2)}, \dots, w_d^{(2)}, \Sigma_2)$, we can compute the

364 Rao geodesic distance as the sum of the Rao distances of the Gaussian distri-
 365 bution and the Rao distances between marginals [El Maliani et al. 2011]:

$$L^2 = \int_{\theta_1}^{\theta_2} ds^2 = \int_{\theta_1}^{\theta_2} ds_{\text{Gauss}}^2 + \sum_{i=1}^d \int_{\theta_1}^{\theta_2} ds_{\text{Margins}}^2 \quad (35)$$

366

$$L^2 = L_{\text{Gauss}}^2(f^{(1)}(x; \Sigma_1) || f^{(2)}(x; \Sigma_2)) +$$

367

$$\sum_{i=1}^d L_{\text{Margins}}^2(f_i^{(1)}(x; w_i^{(1)}) || f_i^{(2)}(x; w_i^{(2)})) \quad (36)$$

368 that is:

$$L^2 = \frac{1}{2} \sum_{i=1}^d (\ln \lambda^i)^2 +$$

369

$$\sum_{i=1}^d L_{\text{Margins}}^2(f_i^{(1)}(x; w_i^{(1)}) || f_i^{(2)}(x; w_i^{(2)})) \quad (37)$$

370 where $\lambda^i, i = 1, \dots, d$ represents the eigenvalues of $\Sigma_1^{-1}\Sigma_2$.

371 4. EXPERIMENTAL RESULTS

372 The experimental section is concerned with five major issues:

- 373 1. Choice of the multi-model for color texture: for this, we investigate a
 374 variety of models for different color spaces.
- 375 2. Choice of the LC color space: we compare performances between two sub-
 376 families of LC color spaces, namely the perceptually uniform color spaces
 377 (represented by Lab), and the perceptual color spaces (represented by
 378 HSV).
- 379 3. Comparison with existing approaches: we evaluate performances of our
 380 approach in comparison with existing uni-modeling approaches in LC
 381 color spaces [Qazi et al. 2010, Qazi et al. 2011] and in RGB color space
 382 [Verdoolaege et al. 2012, Kwitt et al. 2011].

- 383 4. Runtime: we address the runtime issue by comparing the proposed Rao
384 distance based similarity measure with the Monte-carlo based Kullback-
385 leibler divergence between copula based pdfs.
- 386 5. Curse of dimensionality: we compare our approach with the full feature
387 vector based approach (subsection 2.1).

388 4.1. Experiments versus data diversity

389 In order to conduct representative experiments, we use different databases
390 and various configurations of the experimental protocol leading to a large view
391 of the proposed multi-model performance. The conventional Vistex database
392 [Verdoolaege et al. 2012, Kwitt et al. 2011, Do et al. 2002] is used considering
393 two sizes of samples respectively 32×32 and 128×128 . Experiments are also
394 conducted on the Outex, ALOT and Stex databases which are more challenging
395 color texture database, since the color and texture information are not easily
396 distinguishable. We consider the following scenarios:

- 397 • First scenario, i.e. DB1, addressing a set of 24 textured images of size
398 512×512 from the Vistex database [Vistex]. The protocol follows the work
399 of [Permuter et al. 2006] and [Qazi et al. 2010]. Each image was divided
400 into subimages of size 32×32 pixels resulting of a large database of 6144
401 small textures. We consider 96 image from the resulting 256 subimages as
402 the training set, while the remaining 160 subimages are considered as the
403 test set. This dataset is used in order to evaluate robustness of our model
404 even if very local spatial structures are considered.
- 405 • Second scenario, i.e. DB2, addressing a set of 54 textured images of size
406 512×512 from the Vistex database. The protocol follows the work of
407 [Do et al. 2002]. Each image was divided into subimages of size 128×128
408 pixels. This dataset is available on the Outex web site [Ojala et al., Outex]
409 as test suit Contrib_TC_00006. For each texture, subimages are considered
410 to form a checkerboard. The white half of subimages is then considered

411 as the training set and the black half is used as the testing set. Hence, the
412 training procedure will account for non uniformity of the original images.

413 • Third scenario, i.e. DB3, addressing a set of 68 textured images of size
414 746×538 from the Outex database. This dataset is also available on
415 the Outex web site as Outex_TC_00013. Each image was divided into 20
416 subimages of size 128×128 pixels. Training and test sets are obtained as
417 in DB2.

418 • Fourth scenario, i.e. DB4, addressing large set of classes from ALOT
419 [ALOT] database. 250 images of size 384×256 from ALOT databse are
420 considered, then each image is split into 16 non-overlapping subimages
421 resulting on two large databases of 4000 images of size 96×64 . These
422 images have been captured uder different viewing angles and illumination.
423 Here again training and test sets are obtained as in DB2.

424 • Fifth scenario, i.e. DB5, addressing large set of classes from Stex [STex]
425 database. 476 textures from STex of size 512×512 are considered. Then,
426 each image is split into 16 non-overlapping subimages resulting on two
427 large databases of 7616 images of size 128×128 . Here again training and
428 test sets are obtained as in DB2.

429 Every color component of each subimage was normalized by subtracting its
430 mean and dividing by its standard deviation, and then decomposed using a 2-
431 scale dual-tree complex wavelet transform (DTCWT) [Kingsbury 1998] with a
432 Q-shift(14,14) tap filter. In addition to its rich directional analysis, shift invari-
433 ance and low redundancy properties, the DTCWT was chosen for its reduced
434 computational time.

435 *4.2. Quantitative evaluation of performance*

436 Evaluation of the classification performance of each approach consists on two
437 criteria, namely the percentage of classification and the precision. We recall that
438 for each class of textures:

- 439 • True positives (TP): refers to the number of subimages that have been
440 successfully returned by the classifier as belonging to a given class.
- 441 • False positives (FP): is the number of subimages that have been wrongly
442 classified as belonging to a given class.
- 443 • False negatives (FN): is the number of subimages that are classified as
444 not belonging to a given class while, in fact, they belong to this class of
445 textures.
- 446 • True negatives (TN): is the number of subimages that are correctly clas-
447 sified as not belonging to a given class.

448 The percentage of classification (or Sensitivity) is the proportion of textures
449 which were well labeled by the classifier. It represents the ability of the classifier
450 in correct positive assignment of a texture to its real class. A 100% percent-
451 age means that the classifier makes no false negative labeling (FN = 0). The
452 Percentage classification is given as follows:

$$\text{Sn} = \frac{\text{TP}}{\text{TP} + \text{FN}} \times 100\% \quad (38)$$

453 However, theoretically, a system that always returns positive assignment (even
454 wrongly) will achieve 100% percentage classification. The problem with per-
455 centage classification is that it tells nothing about textures that were wrongly
456 labeled as belonging to a given class (FP assignments). For this, we take into
457 account also the precision to evaluate the classifier performance.

458 The precision criterion indicates the ability of the classifier to detect true class
459 membership. The precision is given by:

$$\text{Sp} = \frac{\text{TP}}{\text{TP} + \text{FP}} \quad (39)$$

460 A classifier with very high precision makes no mistakes in assigning a texture
461 to its class (FP = 0).

462 *4.3. Results and discussions*

463 One of the advantages of the proposed multi-modeling approach is the flex-
464 ibility in term of modeling textures from different LC color spaces. The joint

Table 3: Notations, descriptions and pdf expressions for the used models

Model	Description	pdf
CopMGam	Gaussian copula based multivariate Gamma (all marginals are Gamma)	$f(x, \theta) = \frac{1}{ \Sigma ^{1/2}} \exp[-\frac{1}{2}\vartheta^T(\Sigma^{-1} - I)\vartheta] \times (\frac{\beta^{-\alpha}}{\Gamma(\alpha)})^d \prod_{i=1}^d x_i^{\alpha-1} \exp - \sum_{i=1}^d (\frac{x_i}{\beta}), \theta = (\alpha, \beta, \Sigma)$
CopMWbl	Gaussian copula based multivariate Weibull (all marginals are Weibull)	$f(x, \theta) = \frac{1}{ \Sigma ^{1/2}} \exp[-\frac{1}{2}\vartheta^T(\Sigma^{-1} - I)\vartheta] \times (\frac{\tau}{\lambda})^d \prod_{i=1}^d x_i^{\tau-1} \exp - \sum_{i=1}^d (\frac{x_i}{\lambda})^\tau, \theta = (\tau, \lambda, \Sigma)$
{Gamma,Weibull}	Gaussian copula based bivariate model with Gamma as the first marginal and Weibull as the second marginal	$f(x, \theta) = \frac{1}{ \Sigma ^{1/2}} \exp[-\frac{1}{2}\vartheta^T(\Sigma^{-1} - I)\vartheta] \times \frac{\tau\beta^{-\alpha}x_2^{\alpha-1}}{\lambda\Gamma(\alpha)} (\frac{x_1}{\lambda})^{\tau-1} \times \exp \left[-(\frac{x_1}{\lambda})^\tau - \frac{x_2}{\beta} \right], \theta = (\alpha, \beta, \tau, \lambda, \Sigma)$
{vonMises,Weibull}	Gaussian copula based circular/linear bivariate model with vonMises as the first marginal and Weibull as the second marginal	$f(x, \theta) = \frac{1}{ \Sigma ^{1/2}} \exp[-\frac{1}{2}\vartheta^T(\Sigma^{-1} - I)\vartheta] \times \frac{\tau}{2\pi\lambda I_0(\nu)} (\frac{x_2}{\lambda})^{\tau-1} \exp[\nu \cos(x_1 - \mu) - (\frac{x_2}{\lambda})^\tau], \theta = (\mu, \nu, \tau, \lambda, \Sigma)$
{vonMises,Gamma}	Gaussian copula based circular/linear bivariate model with vonMises as the first marginal and Gamma as the second marginal	$f(x, \theta) = \frac{1}{ \Sigma ^{1/2}} \exp[-\frac{1}{2}\vartheta^T(\Sigma^{-1} - I)\vartheta] \times \frac{\beta^{-\alpha}}{2\pi\Gamma(\alpha)I_0(\nu)} (\frac{x_2}{\beta})^{\alpha-1} \exp[\nu \cos(x_1 - \mu) - (\frac{x_2}{\beta})^\alpha], \theta = (\mu, \nu, \alpha, \beta, \Sigma)$

Table 4: Performances of different versions of the multi-model in Lab and HSV color spaces.

	DB1		DB2		DB3		DB4		DB5	
	Sn	Sp	Sn	Sp	Sn	Sp	Sn	Sp	Sn	Sn
Lab1	97.5	0.96	98.3	0.98	89.7	0.88	58.2	0.55	77.6	0.75
Lab2	94.5	0.92	95	0.93	84.6	0.84	54.3	0.52	74.7	0.73
Lab3	93.3	0.92	93.8	0.90	82	0.82	53.6	0.52	73.4	0.71
HSV1	95.6	0.94	96.7	0.94	86	0.86	55.5	0.54	76.9	0.75
HSV2	93.1	0.93	93.4	0.93	83.1	0.83	53.6	0.53	73.9	0.73
HSV3	90.4	0.88	90.8	0.88	80.5	0.82	51.3	0.49	70.4	0.68

465 models M_L and M_{C_r} enable us to consider different parametric modeling for
 466 marginals between luminance and chrominance channels. As already mentioned,
 467 when chrominance components are defined in a cylindrical space, the *hue* repre-
 468 sents a circular component and the *saturation* is the axe of the cylinder. We can
 469 then benefit from the advantage of copulas in merging different marginals in the
 470 same joint model to construct a circular/linear bivariate model for chrominance
 471 such as {vonMises,Weibull} or {vonMises,Gamma} (see Table 3).

472 Regarding the first part of our experiments, Table 4 presents color texture clas-
 473 sification performance in Lab and HSV color spaces respectively, using different
 474 multi-models on DB1, DB2, DB3, DB4 and DB5. We note that:

- 475 • Lab1: refers to a multi-model that considers CopMGam (Table 3) for the
 476 luminance model M_L and CopMWbl for the chrominance model M_{C_r} in
 477 Lab color space.
- 478 • Lab2: refers to CopMWbl for M_L and CopMGam for M_{C_r} in Lab color
 479 space.
- 480 • Lab3: refers to CopMGam model for M_L and the bivariate model {Gamma,Weibull}
 481 for M_{C_r} in Lab color space.
- 482 • HSV1: refers to CopMGam for M_L and the bivariate model {vonMises,Weibull}

- 483 for M_{C_r} in HSV color space. (vonMises for the Hue marginal, and Weibull
484 for the Saturation marginal)
- 485 • HSV2: refers to CopMWbl for M_L and the bivariate model {vonMises, Gamma}
486 for M_{C_r} in HSV color space.
 - 487 • HSV3: refers to CopMGam for M_L and and the bivariate model {Weibull, Gamma}
488 for M_{C_r} in HSV color space.

489 We remark that in Lab, and for all the five databases, best performance is
490 achieved using the multi-model Lab1. This indicates that the spatial struc-
491 ture along with the inter-band dependency are better characterized using the
492 Gaussian copula in conjunction with Gamma marginals (CopMGam). This also
493 indicates that the dependency between "a" and "b" chrominance components
494 is better characterized using the Gaussian copula in conjunction with Weibull
495 marginals (bivariate CopMWbl). For HSV, the multi-model HSV1 leads to the
496 best performance in all databases. This is due to the ability of CopMGam
497 to model spatial structure and inter-band dependence for luminance, and the
498 circular/linear bivariate {vonMises, Weibull} to characterize the chrominance
499 dependency.

500 We next would like to compare between the perceptually uniform and percep-
501 tual subfamilies of LC color spaces, via their respective representatives Lab and
502 HSV spaces. Considering the best multi-models in each case (Lab1 for Lab
503 and HSV1 for HSV), we clearly observe from Table 4 that Lab representation
504 outperforms the HSV one. This is a fair comparison even the models M_L and
505 M_{C_r} are not the same in multi-models Lab1 and HSV1, since these latters are
506 the best relatively the nature of the color space. Then, if Lab1 outperforms
507 HSV1, we can conclude that in best modeling case the Lab representation is
508 better than the HSV one. These results are explained by the better luminance-
509 chrominance representation of Lab. In other words, we can say that Lab offers
510 more decorrelation property between luminance and chrominance, which is the
511 central interest of our approach. We do not ignore that percentage classification
512 and precision rates are lower for large databases DB3, DB4 and DB5.

Table 5: Average percentage classification (Sn) using the best multi-models in Lab and HSV in comparison with existing approaches.

	DB1	DB2	DB3	DB4	DB5
Lab1 (proposed method)	97.5	98.3	89.7	58.2	77.6
[Qazi et al. 2011], in Lab	97.2	96.5	88	52.1	78.5
HSV1 (proposed method)	95.6	97.8	86	55.5	76.9
[Qazi et al. 2010], in IHLS	95.4	97.4	84.1	49.3	77.1
[Verdoolaege et al. 2012], in RGB	92.2	94.7	78.5	59.6	75.1
[Kwitt et al. 2011], in RGB	91.5	94.1	79.8	55.4	74.9

513 Regarding the effectiveness of our approach in comparison with existing ones,
 514 i.e uni-modeling in LC and RGB color spaces, Table 5 shows average percentage
 515 classification Sn of textures of DB1, DB2, DB3, DB4 and DB5 using our best
 516 multi-models (Lab1 and HSV1) against the four following approaches:

- 517 • [Qazi et al. 2010], in Lab: Linear prediction models based characteriza-
 518 tion in the Lab color space,
- 519 • [Qazi et al. 2011], in IHLS: Linear prediction models based characteriza-
 520 tion in the improved hue luminance saturation (IHLS) color space,
- 521 • [Verdoolaege et al. 2012], in RGB: Dependence between R, G and B color
 522 components using the MGGD model,
- 523 • [Kwitt et al. 2011], in RGB: Dependence between R, G and B using the
 524 t-copula based multivariate Weibull model,

525 We observe from Table 5 that our Lab1 proposed model leads to better per-
 526 formance when compared with the linear prediction modeling [Qazi et al. 2010]
 527 and [Qazi et al. 2011]. Results presented in the latter studies are the best of our
 528 knowledge, but it should be highlighted that the characterization of both lumi-
 529 nance and chrominance channels by using the 2-D auto regressive (AR) models

Table 6: Average percentage classification Sn and runtime (in minutes) on DB2, using Rao distance, ML based divergence and Monte-carlo based KL divergence as similarity measure for Lab1 and HSV1 multi-models.

	Rao		KL _{ML}		KL _{MC}	
	Sn	Runtime	Sn	Runtime	Sn	Runtime
Lab1	98.3	4.9 min	98	40.5 min	97.2	60 min
HSV1	96.7	6.5 min	95.9	44.3 min	96.2	63 min

Table 7: Classifier performances for full feature vector (spatial structure for luminance and chrominance)

	DB1		DB2		DB3	
	Sn	Sp	Sn	Sp	Sn	Sp
Lab1	86.9	0.83	90	0.88	75.8	0.78
HSV1	84.5	0.82	89.6	0.88	72.2	0.70

530 with the same order, seems a limitation because the two channels are indepen-
531 dent. However, it is noteworthy to precise that the results shown in Table 5 for
532 the two studies consider only the case when authors use the spatial structure and
533 chrominance information without adding the color information (3-D color his-
534 tograms) in the characterization. We note that there is a slight performance for
535 this later for DB5 database and for [Verdoolaege et al. 2012] for DB4 database.
536 We also remark from the same table the superiority of the characterization in LC
537 color spaces compared to the RGB one except for DB4. This is theoretically con-
538 firmed since considering the image as two separable luminance and chrominance
539 information is more intuitive and correlated with the human perception than the
540 RGB space which considers the color as an addition of the primary colors red,
541 green and blue. Here, we also note that [Verdoolaege et al. 2012] considered
542 uniquely the dependency between R, G and B components (inter-channel only),
543 and that [Kwitt et al. 2011] considered the R, G and B dependency besides the
544 inter-band one (for the three components). As we stressed in the introduction,

545 extending these models to take into account jointly intra-band, inter-band and
 546 inter-channel dependencies leads to high-dimensionality, due to the huge depen-
 547 dence between R, G and B components, and to the fact that RGB representation
 548 offers no separation between spatial structure and chrominance.

549 Another contribution of our approach is the definition of a closed form ex-
 550 pression for the Rao distance in the variety of copula based probability densi-
 551 ties [El Maliani et al. 2011]. As it is known, deriving a similarity measure in
 552 the case of copula based multivariate models is a challenging task. For this,
 553 in [Kwitt et al. 2009] and [Sakji-Nsibi et al. 2010], authors proposed a Monte-
 554 carlo approximation of the KL divergence. But, this approach is computation-
 555 ally expensive and is not deterministic, since the KL divergence differs depending
 556 on the random number generation. An alternative approach was proposed in
 557 [Kwitt et al. 2011], using the maximum likelihood (ML) selection rule as a simi-
 558 larity measure. This significantly reduces the execution time in comparison with
 559 the Monte-carlo based similarity measurement. A more attractive expression of
 560 the KL divergence is provided in [Lasmar et al. 2012], where the separability
 561 between the copula space and the marginal space is used to derive a closed-
 562 form and fast KL based similarity measurement. However, KL divergence does
 563 not satisfy properties of symmetry and triangular inequality, and thus is not a
 564 distance in the right sense of the parametric manifold. Table 6 shows average
 565 percentage classification along with the execution time using both similarity
 566 measures (Rao and KL) for the textures of DB2. We recall that according to
 567 equation 37, we need expression of the Rao distance for marginals and the used
 568 copula. Expressions of the Rao distance for Weibull, Gamma and vonMises
 569 can be found respectively in [El Maliani et al. 2011], [Reverter et al. 2003] and
 570 [Ceolini et al. 2010]. We can see that the proposed similarity measure slightly
 571 outperforms the ML based and Monte-Carlo based approach in term of per-
 572 centage classification. However, the improvement is more significant in term of
 573 runtime which is 9 times and 12 times less than the ML based and Monte-carlo
 574 based approaches respectively. We note that the experiments were done using
 575 Matlab environment on an HP Compaq equipped with an Intel Core 2 Duo

576 CPU at 3GHZ workstation

577 Finally, we want to validate experimentally the hypothesis made in introduction
578 about dimensionality reduction when we consider multi-modeling in LC color
579 spaces. We compare our approach to the full dependence based approach of sub-
580 section 2.1. Table 7 shows performance of the classifier for multi-models Lab1
581 and HSV1 with full feature vectors length. It can be seen from these results
582 that the high parametrisation does not improve performances, but on contrary
583 it makes them decrease by about 10%. Separability between luminance and
584 chrominance helps then to avoid redundancy in characterization which is the
585 case in RGB where spatial structure resides in the three color components.

586 **5. Conclusion**

587 We proposed in this letter a multi-model based approach for color textures
588 characterization in the LC color spaces. This approach takes into account the
589 separability between luminance and chrominance. We especially took care of
590 the circular/linear nature of chrominance components. We also addressed the
591 dimensionality reduction issue for the study of the full dependence in term of
592 intra-band, inter-band and inter-channel relation between coefficients color com-
593 ponent subbands. Results on five wide range of databases show the superiority of
594 the proposed approach in comparison with the existing unimodeling approaches.
595 We know that our method as any compared methods, presents some limitations
596 when using large databases and also against rotation and illumination effects.
597 Furthermore, considering the runtime issue, results show the effectiveness of the
598 Rao geodesic distance in comparison with the Kullback-leibler divergence.

599 **References**

600 [Verdoolaege et al. 2012] G. Verdoolaege and P.Scheunders, “On the Geometry
601 of Multivariate Generalized Gaussian Models,” *Journal of Mathematical*
602 *Imaging and Vision*, vol. 43, pp. 180-193, 2012.

- 603 [Kwitt et al. 2011] R. Kwitt, P. Meerwald and A. Uhl, "Efficient Texture Image
604 Retrieval Using Copulas in a Bayesian Framework," *IEEE transactions on*
605 *image processing*, vol. 20, pp. 2063-2077, 2011.
- 606 [Maliani et al. 2012] A.D. El Maliani, M. El Hassouni, Y. Berthoumieu, D.
607 Aboutajdine, "Multi-model Approach for Multicomponent Texture Classi-
608 fication," *Image and Signal Processing Lecture Notes in Computer Science*,
609 vol. 7340, pp. 36-44, 2011.
- 610 [Qazi et al. 2011] I. Qazi, O. Alata, J.C. Burie, A. Moussa and C.F. Maloigne,
611 "Choice of a pertinent color space for color texture characterization using
612 parametric spectral analysis," *Pattern Recognition*, vol. 44, pp. 16-33, 2011.
- 613 [Qazi et al. 2010] I. Qazi, O. Alata, J.C. Burie and C.F. Maloigne "Color spec-
614 tral analysis for spatial structure characterization of textures in IHLS color
615 space," *Pattern Recognition*, vol. 43, pp. 663-675, 2010.
- 616 [Rakvongthai et al. 2013] Y. Rakvongthai, S. Oraintara "Statistical texture re-
617 trieval in noise using complex wavelets," *Signal Processing: Image Com-*
618 *munication*, vol. 28, pp. 14941505, 2013.
- 619 [DeYoe et al. 1996] E. DeYoe and D. van Essen, "Concurrent processing
620 streams in monkey visual cortex". *Trends. Neurosci.*, 11:219226, 1996.
- 621 [Mojsilovic et al. 2000] A. Mojsilovic, J. Kovacevic, D. Kall, R. Safranek, and
622 S. Ganapathy, "Matching and retrieval based on the vocabulary and gram-
623 mar of color patterns". *IEEE Transactions on Image Processing*, 9(1):3854,
624 2000.
- 625 [Sarifuddin et al. 2005] M. Sarifuddin and R. Missaoui, "A New Perceptually
626 Uniform Color Space with Associated Color Similarity Measure for Content
627 Based Image and Video Retrieval", *Proceedings of Multimedia Information*
628 *Retrieval Workshop*, 28th annual ACM SIGIR Conference, pp. 1-8, 2005.
- 629 [Simoncelli 1997] E. P. Simoncelli, "Statistical models for images: Compression,
630 restoration and synthesis", *IEEE Conference on Signals, Systems & Com-*

- 631 *puters, 1997.*, Conference Record of the Thirty-First Asilomar, pp. 673-678,
632 1997.
- 633 [Kwitt et al. 2009] R. Kwitt and A. Uhl, “A joint model of complex wavelet
634 coefficients for texture retrieval,” in *Image Processing, 2009. ICIP 2009.*
635 *16th IEEE International Conference*, 2009, pp. 1877-1880.
- 636 [Boubchir et al. 2010] L. Boubchir, A. Nait-Ali and E. Petit, “Multivariate sta-
637 tistical modeling of images in sparse multiscale transforms domain,” in
638 *Proceeding of the 17th IEEE International Conference on Image Process-*
639 *ing (ICIP’10)*, 2010, pp. 1877-1880.
- 640 [Lasmar et al. 2010] N. Lasmar and Y. Berthoumieu, “Multivariate statistical
641 modeling for texture analysis using wavelet transforms,” in *Proceeding of*
642 *the IEEE International Conference on Acoustics Speech and Signal Pro-*
643 *cessing (ICASSP’10)*, 2010 pp. 790-793.
- 644 [Sakji-Nsibi et al. 2010] S. Sakji-Nsibi and A. Benazza-Benyahia, “Fast scal-
645 able retrieval of multispectral images with kullback-leibler diver-
646 gence,” in *Image Processing, 2010. ICIP 2010. 17th IEEE*
647 *International Conference*, 2010, pp. 2333-2336.
- 648 [Nelsen 2006] R. B. Nelsen, *An Introduction to Copulas*. Springer Series in
649 Statistics. Springer, second edition, 2006.
- 650 [Sklar et al. 1959] M. Sklar, “Fonctions de répartition à n dimensions et leurs
651 marges,”. *Publications de l’institut de Statistique de l’Université de Paris*.
652 vol. 8, pp. 229-231, 1959.
- 653 [Joe. 1997] H. Joe, *Multivariate Models and Dependence Concepts*. Monographs
654 on Statistics and Applied Probability. Chapman & Hall, 1997.
- 655 [Durrleman et al. 1959] V. Durrleman, A. Nikeghbali, and T. Roncalli, “Which
656 copula is the right one,” *Groupe de Recherche Operationnelle, Credit Lyon-*
657 *nais*, 2000.

- 658 [Lasmar et al. 2012] N. Lasmar, Y. Berthoumieu, "Gaussian Copula Multivari-
659 ate Modeling for Image Texture Retrieval Using Wavelet Transforms," *sub-*
660 *mitted to IEEE transactions on image processing.*
- 661 [Bishop 2006] C. M. Bishop, "Pattern recognition and machine learning",
662 *Springer*, 2006
- 663 [Mallat 1999] S. G. Mallat, "A wavelet tour of signal processing", *2nd edition*,
664 *San Diego: Academic press*, 1999.
- 665 [El Maliani et al. 2011] A. D. EL Maliani, M. EL Hassouni, N. Lasmar and
666 Y. Berthoumieu, "Color texture classification using rao distance between
667 multivariate copula based models," in *Proceedings of the 14th international*
668 *conference on Computer Analysis of Images and Patterns*, CAIP11, pp.
669 498505, 2011.
- 670 [Reverter et al. 2003] F. Reverter and J.M. Oller, "Computing the Rao distance
671 for Gamma distributions," *Journal of Computational and Applied Mathe-*
672 *matics*, vol. 157, pp. 155-167, 2003.
- 673 [Ceolini et al. 2010] S. Ceolini and E. R. Hancock, "Characterising facial gender
674 difference using fisher-rao metric," in *Proceedings of the 2010 20th Inter-*
675 *national Conference on Pattern Recognition*, ICPR10, pp. 43084311, 2010.
- 676 [Do et al. 2002] M. Do and M. Vetterli, "Wavelet-based texture retrieval using
677 generalized Gaussian density and Kullback-Leibler distance," *IEEE trans-*
678 *actions on image processing*, vol. 11, pp. 146–158, 2002.
- 679 [Rangaswamy et al. 2002] M. Rangaswamy, and D. Weiner, Donald and A. Oz-
680 turk, "Non-Gaussian random vector identification using spherically invari-
681 ant random processes," *IEEE Transactions on Aerospace and Electronic*
682 *Systems*, vol. 29, pp. 111–124, 1993.
- 683 [Vistex] "MIT vision and modeling group," [Online], Available from:
684 <http://vismod.media.mit.edu>.

- 685 [Outex] “Available from: <http://www.outex.oulu.fi/>,”
- 686 [ALOT] Amsterdam Library of Textures. [Online]. Available from:
687 http://staff.science.uva.nl/aloi/public_alot
- 688 [STex] Salzburg Textures [Online]. Available from:
689 <http://wavelab.at/sources/STex>
- 690 [Ojala et al.] T. Ojala, M. Pietikinen, T. Menp, J. Viertola, J. Kyllnen, and S.
691 Huovinen. ”Outex - new framework for empirical evaluation of texture anal-
692 ysis algorithms”. In *16th International Conference on Pattern Recognition*,
693 Quebec, Canada, August 2002.
- 694 [Permuter et al. 2006] H. Permuter, J. Francos, and I. Jermyn, “A study of
695 gaussian mixture models of color and texture features for image classifica-
696 tion and segmentation,” *Pattern Recognition*, vol. 39(4), pp. 695-706, 2006.
- 697 [Atkinson et al. 1981] C. Atkinson, A.F. Mitchell, ”Rao’s distance measure”.
698 *Sankhy: The Indian Journal of Statistics*, Series A, 345-365. 1981.
- 699 [Kingsbury 1998] N. Kingsbury, ”*The Dual-Tree Complex Wavelet Transform:*
700 *A new Technique for Shift-Invariance and Directional Filters*,” in Proceed-
701 ings of the 8th IEEE DSP Workshop, Aug. 1998, pp. 9-12.