Didactic contract and secondary-tertiary transition: a focus on resources and their use

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Abstract: In this article we claim that the concept of didactic contract can help to develop a deeper understanding of the secondary-tertiary transition, in particular by showing changes at three different levels: at the general, institutional level; at the level of the discipline concerning mathematical practices; and at the level of a given mathematical content. In fact, we argue that the didactic contract is linked to the use of and interaction with different resources, by teachers and students, in the sense that their use is shaped by the contract; and at the same time the available resources shape the mathematics taught. We draw here on two studies, one in the UK and one in France, to illustrate how a focus on resources can inform us about contract rules at the different levels.

Didactic contract and interaction with resources: framework and research questions

The study presented here is a contribution to research on the transition from secondary school to university mathematics (Gueudet 2008; Pepin 2014). Whilst different theoretical perspectives can enlighten what happens during this transition (Nardi et al. 2014), we retain a socio-cultural approach in this paper. We consider secondary school and university as two different institutions (Chevallard 2006), with different mathematical practices. In particular, the didactic contract (Brousseau 1997) is different in these two institutions. The didactic contract is defined as a set of rules, some explicit but most of them implicit, framing the mathematical practices of both teacher and student/s, which can be presented as a sharing of responsibilities between teacher and student/s. Moreover, we have a specific interest in the links between the didactic contract and the resources intervening in the students' mathematical work. In previous works (e.g. Gueudet, Pepin & Trouche 2012) we have shown that the use of resources – we consider here curriculum resources, like textbooks, websites, but also lecture notes, for example - contributes to shaping mathematics instruction and learning; and it is likely to shape in particular the didactic contract.

The didactic contract can be considered at different levels (Chevallard 2006, Winsløw et al. 2014); we distinguish here between three such levels (De Vleeschouwer & Gueudet 2011):

- a general, institutional contract: its rules apply for all subjects taught in the given institution. For example, at secondary school the teacher writes on the blackboard all that the students need to write down; at University, at least a part of the content is only provided orally by the teachers and the student him/herself is responsible for taking written notes. We interpret this as a change of contract rules during the transition, likely to raise difficulties for many students.

 - a contract at the level of the subject (here, mathematics): its rules apply for all mathematical contents. For example, new expectations at university in terms of rigor are changes in the didactic contract at this level.

- a didactic contract for particular mathematical contents: here the rule concerns a particular mathematical content, like Linear Algebra, Calculus and so on. Such rules are likely to change during transition for contents taught both at secondary school and at university (e.g. different teaching of calculus: "calculational" at school (how to calculate an integral); and more mathematical at university (precise definitions and assumptions, etc.)).

Our central research questions are the following:

What are the resources and what is their "expected" (by the institution) use by students in higher education mathematics (first year), as compared to the resources and their uses at secondary school? How do these resources and their uses inform us about the didactic contract at each level?

We draw on two different data sets: one in the UK concerning the student transition from upper secondary school to university mathematics[1]; and one in France concerning the teaching of number theory at first year university. To emphasize, we do not intend to compare these two cases; we consider them as complementary, since the first one informs us on the contract at the general and at the subject level, while the second informs us at the subject and the content level.

Resources and institutional contract: the Transmath project

In the UK, our data are anchored in the TransMaths project, where we investigated how students experienced different mathematics teaching-and-learning practices at both sides of the transition point, and they developed different strategies to make the transition successful (or not). As this project used a mixed-method approach, we could identify and analyse particular resources and their use, both from interviews with students (and lecturers), and from the accompanying student survey.

From interviews with Sunny (and his friends), who studied at City University, we could identify the main resources used in their first year of study (see appendix): lecture and lecture notes; the lecturer him/herself (during office hours); the coursework, tutorial and tutor; their friends/study group. It was clear that these resources were quite different, in nature, from what students were used to at school: at school students had a textbook (which was portraying mathematics as something that one can learn by solving "tons of exercises"); and the teacher who was available for individual questions and explanations during lesson time (and even out of school time for special revision lessons). Friendship group did not seem to be important, as students could discuss their problems with the teacher, who was seen as the authority in terms of correctness and learning of the subject.

Indeed, at university one mathematics lecturer said that "students [have to] learn 'from day one' that they are not in school but in a university mathematics department". This included a clear distance between students and lecturers, which was also mentioned by Sunny:

"I think it's, it's more like the learning here is more general like in a way, like in sixth form it was more personalised kind of. You kind of, you was closer to the teacher, you was, you had constant like, you was talking to them- you was after school you was chatting to them. You

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[1] Transmaths project at the University of Manchester: http://www.transmaths.org

saw them around, like here it's so funny cos when we see the lecturers walking around it's like they're like celebrities. Cos we haven't got, we haven't quite got that personalised you know, thing with them so they're from a distance you know. 'That's Professor ..., that's Professor…wow!' You're like wow, they're about. So I suppose it's less personal in a way." (DP5, Sunny)

Returning to resources used and their nature, at university the main resources were clearly the lecture and the lecture notes provided by the lecturer/professor (sometimes supported by a textbook). However, these were not always "understandable" and students would have needed individual help from the lecturer (during particular office hours), but most students did not dare to go (as they were afraid of asking "stupid questions"). In addition, the coursework (provided once a week) was to support student understanding of the lecture, through exercises. Sunny and his friends/learning group emphasized that unless the coursework was well aligned with the lectures, it did not help their understanding of the topic (see calculus as compared to geometry lectures/coursework). Indeed, in some cases students did not know what to ask in tutorial time, or in lectures, so little had they understood the topic area. Other resources included textbooks (suggested/approved by the lecturers); and particular self-support schemes (where higher year students help their 'younger' peers) - these were seen as less helpful than the notes and support provided by lecturers and tutors, in particular as students were often "learning to the test". However, the same resources (e.g. lecture notes) were often evaluated very differently by students, in terms of support for their learning, so much so that Sunny (as student representative) had asked for a change in form and practice concerning lecture notes: as students did not want to be presented with "one slide after another", they asked for hand-written lectures during the lecture (so that they would have time to think and process the notes, and perhaps ask questions).

At the same time institutional practices and accompanying resources played a crucial role in the ways that mathematics, and what it meant to "do mathematics", was portrayed, which often hindered students developing a mathematical disposition that supported their engagement with demanding mathematics. From the student surveys at entry to university, as compared to a year later, we could see that students adjusted to particular practices and routines, and socio-mathematical norms (see Pepin 2014). In particular, "whole-group/class teaching" (and listening/writing in lectures) and "working in groups" (either with friends, or in the tutorial) was seen as essential to pass the examinations. "Taking notes in lectures" and "studying from your own notes" depended on what the lecturer provided as learning resources. For example, one lecturer (of geometry) apparently provided "perfect notes", that is lecture notes that suited students' level of understanding and learning pace, and that were aligned with the coursework (exercises) and the examinations. So, students felt well-prepared by the lectures, the lecturer's explanations during the lecture, and the coursework, to pass the examinations.

On the basis of video footage of selected lectures and pre- and post-video stimulated recall discussions with lecturers, one could identify meanings that were attached to particular practices. Particular lectures reflected the kinds of things that a "rigorous mathematician" may need to learn:

- 'reasoning and proof' based thinking and practices were expected to be developed through Geometry and Linear Algebra;

- 'procedural fluency' (methods) was seen to be developed through Calculus;

- practical and context relatedness was regarded to be developed through Statistics.

In terms of Didactic Contract for mathematics, it can be argued that there was a clear *institutional didactic contract* at City University, made explicit in discussion with lecturers and students, and mediated by particular practices. This contract was about helping students to become a "rigorous mathematician" and attain the "very highest academic standards". Becoming a "rigorous mathematician" included making sense of the mathematics in lectures, and different lectures (different mathematical topic areas) appeared to provide the key to particular competencies (e.g. reasoning and proof was developed through Algebra). However, how students were expected to learn and develop these was not clear. It can be argued that this change of Didactic Contract from school to university appeared to necessitate students becoming more independent learners. In terms of resources (and their use) we retain that the change of Didactic Contract at transition from school to university mathematics education has implications for students, (1) in terms of the change in nature of the resources: teachers 'change into' lecturers; lessons into lectures; homework into coursework; textbooks into course materials and lecture notes; tests into examinations; and school mathematics into university mathematics; and (2) in terms of the expectations of their use: the teacher could be accesses (nearly) all the time, whereas the (individual) lecturer is 'only' available for a limited number of minutes/hours; textbooks in school are seen as a support of teachers' teaching, mainly in terms of provision of exercises, whereas at university lecture notes are to be "understood" and studied, with the support of the lecture and the coursework/tutorials.

Resources, didactic contract and mathematical content: the case of number theory

In this section we focus on the Didactical Contract at the level of a particular mathematical content. Number theory is known as a difficult topic for students of different levels (Zazkis & Campbell 2006). In France, where our study took place, the scientific students taking the "mathematics specialty" in grade 12 (last year before university) learned advanced topics like prime numbers, Bézout's and Gauss' theorems and congruencies, with the aim to develop reasoning and proof skills. However, Battie (2010) who examined exercises proposed at the Baccalauréat (end of secondary school examination) argues that the expectations for grade 12 students concerning number theory was mostly limited to computation, and the application of methods they learned. We interpret this in terms of the Didactic Contract (Brousseau 1997): an important rule of the Didactic Contract for number theory in grade 12 was that developing an original solution method was not part of the students' responsibilities.

At university level we investigated a teaching unit on number theory taught at the first semester of the first year in a university in France. This teaching unit addressed topics such as: Euclidean division and Euclidean algorithm; prime numbers; and congruencies. The main resources available for the students in this teaching were actually the mathematical "texts" (e.g. exercises from textbooks; etc.). We firstly consider the level of mathematics as a discipline.

We asked the students about their use of mathematical texts (considered as resources) for this teaching unit; we also asked the responsible lecturer of this course about the uses she would expect (from students). We proposed an online questionnaire to the 140 students enrolled and obtained 85 answers. The resources offered by the institution were: a "polycopie" (lecture notes, comprising of all the definitions, the theorems and their proofs- instead of a textbook); exercise sheets; previous examination papers, all on paper and available online as pdf files. Moreover, the students had their own course notes (five classes followed this course, with five different teachers). According to the lecturer responsible for the course, the students should work on the polycopie in order to learn the theorems and to work on the proofs. Considering the answers to our questionnaire, the actual situation seemed quite different: only 52% of the students declared that they found the polycopie helpful. They considered that the lecture was enough, and that they used the polycopie only for the final examinations (83%). Moreover 90% would like to find worked examples in the polycopie; and 44% looked for additional resources on the Internet, in particular worked examples.

We interpret these answers as follows: whilst the teachers at university expected that the polycopie would be used to work on the text of the lecture (i.e. in terms of definitions, theorems, proofs etc.), the students considered that their responsibility was to work on the exercises, and that this was an efficient way to prepare the test. In secondary school they were used to find methods presented in the textbook, in particular many worked examples (Rezat 2013). In this teaching unit, the polycopie did not incorporate the presentation of methods how to solve problems, or worked examples. In fact, most of the exercises proposed did not correspond to the application of a given method.

Let us now consider the level of a particular mathematical content. Differences between the school and university Didactic Contracts could be observed by analyzing the questions/exercises proposed (e.g. in examinations, or in tutorials). For example, at the final examination the following exercise was proposed (figure 1):

Exercice 4   *3 points*

On définit une application de $\mathbb{N}$ dans $\mathbb{N}$ par $f(n) = \text{pgcd}(42, n)$ pour tout entier naturel n.
1. Calculer $f(0)$, $f(2)$, $f(10)$ et $f(5)$. L'application f est-elle injective ?
2. L'application f est-elle surjective ? Déterminer $f(\mathbb{N})$.

Figure 1. Exercise 4. An application from IN to IN is defined by f(n)=gcd(n,42) for all integer n. 1. Compute f(0), f(2), f(10) and f(5). Is f an injective application? 2. Is f surjective? Determine f(IN). The exam lasts two hours, and comprises of 6 exercises. The exam is marked on 20 points, exercise 4 is marked on 3 points.

In this exercise, for the first questions the students just needed to apply the definition of a gcd to compute f(0), f(2), f(10) and f(5). Then they found that f(2)=2=f(10), and concluded by applying the definition of injectivity that f is not injective. The second question required more personal initiative. The students had to determine the range of f; the question is "open", thus they firstly need to decide if they will try to prove that f is surjective or not. Then they searched for *f(IN)*. This required to choose the relevant property of the gcd: gcd(n, 42) divides 42, so it belongs to {1, 2, 3, 6, 7, 14, 21, 42}; and then to justify that each of these values was reached by f.

No similar exercise had been solved in the tutorials, or given as example in the polycopie. Surjective functions was indeed a topic studied at the beginning of the teaching unit, during the second or third week, while gcd was studied during the weeks 8 and 9. In the tutorials, no exercises associated gcd and functions.

Analyzing the mathematical problems/exercises proposed (in tutorials, examinations or in tutorials) showed that students were not given "recipe" solutions, but that they were expected to use their understandings of the course lectures to find a solution method. The example given above with gcd(42,n) was typical: the students had to go back to the definitions and properties presented in the course (here: the gcd of two numbers is in particular a divisor of these two numbers) to build their own solution method. Thus, unlike secondary school, in the Didactic Contract at the level of mathematics at university, building the method was the students' responsibility, and this would direct their use of resources for their individual work. However, in the first year at university the students had not yet entered into this contract, they still looked for worked examples in order to observe and reproduce solution methods.

Discussion

Our investigations led us to observe changes in the rules of the Didactic Contract between school and university, at the institutional level and at the level of mathematics in UK; at the level of mathematics and at the level of a particular content in France. These rules were associated with the use of particular resources, which subsequently became indicators for these changes.

At the institutional level, we retain from our study in UK the increasing responsibility of the students towards their own developing understanding, and the "replacement" of the teacher by other students as a central resource. At the level of mathematics, both in UK and in France we observed that the lecturer expected that the text of the lecture would be used by the students, not only to learn and understand the concepts, but also as a model for his/her own mathematical practices, mathematical proof in particular. In France we observed that the novice students did not adhere to this rule, and searched for worked examples as model. At the level of a particular content, number theory, we observed expected changes in the students' responsibilities through and by analyzing the mathematical texts. At university this also included developing or identifying/choosing a method to solve an exercise.

We claim that the available and expected usages of resources contributed to the shaping of the Didactic Contract. At the same time the resources were shaped by the teachers' and students' expectations. Hence, we contend that investigating the changes in resources and their actual or expected usages can inform about changes in the contract, at different levels.

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