



**HAL**  
open science

## Design of mechanisms with scissor linear joints for swept volume reduction

Damien Chablat, Luc Rolland

### ► To cite this version:

Damien Chablat, Luc Rolland. Design of mechanisms with scissor linear joints for swept volume reduction. The 4<sup>th</sup> Joint International Conference on Multibody System Dynamics, May 2016, Montréal, Canada. hal-01309180

**HAL Id: hal-01309180**

**<https://hal.science/hal-01309180v1>**

Submitted on 29 Apr 2016

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Design of mechanisms with scissor linear joints for swept volume reduction

Damien Chablat<sup>1</sup> and Luc Rolland<sup>2</sup>

<sup>1</sup>*Institut de Recherche en Communications et Cybernétique de Nantes, Damien.Chablat@cnr.fr*

<sup>2</sup>*High Performance Robotics Laboratory, Memorial University of Newfoundland, Luc.Rolland@gmail.com*

*ABSTRACT This paper addresses the optimal design of a new parallel robot, the Expteron, in which a networked scissor used as linear joint allows to reduce the swept volume of the kinematics chains. This robot is based on the Tripteron robot where the passive elbows constituted by two bars and three revolute joints are replaced by a set of networked scissors. By changing the architecture of the leg, only one working mode stands for each leg. The design is conducted on the basis of one prescribed cubic Cartesian workspace. The number of loops in the networked scissors are computed to avoid self-interference and to reduce the swept volume. A finite element analysis is made to define a preliminary stiffness model to predict the bending deformation when the mobile platform are subjected to unidirectional forces along the z-axis.*

## 1 Introduction

In order to increase the performance of manipulators, many researchers have studied the application of parallel architectures in mechanism designs to obtain better stiffness or higher accelerations, [1]. However, we can still identify many problems for such architectures as singular configurations, non-linear kinematic model, complex workspace shape and collision risks between linkages in the kinematics chains.

As any parallel robot is made by a set of legs connected from the fixed base to a mobile platform, the swept volume by each kinematics chain during the motion cannot be avoided and determines in encumbrance becoming also an important drawback. For example, the utilization of two arms or bars articulated with three revolute joints in many instances leading to problems of encumbrance during motion where proximal and distal body produce large sweeping volume, risk of collisions with the environment or self-collisions. This is probably because of this problem that the Stewart-Gough platforms have many applications [2].

The best architecture to produce translational motions is the Tripteron mechanism [3] and for Schönflies motion, the quadruperon mechanism [4]. The main property of the Tripteron robot is providing a mechanical structure with simple direct and inverse kinematic problems rendering them easier to model, simulate and control. The kinematics models can be just the identity matrix inside the workspace.

If these architectures are not suited to machining (buckling in the legs), applications of pick-and-place are asylum because of their simplicity and dynamic performance [5]. However, if these mechanisms have many advantages compared with other robots, either serial and parallel, the displacements of their legs can produce collisions with the environment. Furthermore, if one wishes to make robots with different workspace it is necessary to design new legs for each size.

The workspace limits are mainly defined by the lengths of the prismatic joints and the lengths of the legs. We have also to analyze the kinematic constraints to avoid serial singularities or self-collisions.

Existing technologies make use of scissors closed by slides which are utilized in scissor-lift platforms or even beforehand in Cuckoo clocks. Quite surprisingly, these were only recently examined in terms of their dynamics, [6, 7]. These are plagued by two slot or linear joints which can be subject to various problems such as misalignment, backlash and hard-spots reducing their dynamics capacity. Closing the scissor mechanism can be achieved through

two bars converging on a revolute joint [8]. These mechanism has been introduced and examined as linear actuator replacements, [9], where a DC motor provides for the motion.

In this paper, we will examine the application of scissors that are found in the foldable or deployable mechanisms, [10]. Thus, the swept volume is rendered smaller by using and multiplying series of small bars then that of two larges ones. This paper more specifically study how these scissor mechanisms, namely X-axes and their networking, can positively improve the design of the Tripteron manipulators. This approach enables a modular design of the robot’s legs that can be defined according to the dimensions of the desired workspace.

Understanding that the X-axes introduce higher dynamics than linear or prismatic joints at the expense of mechanical complexity, in depth geometric analysis can provide a compromise where the linkage collisions are completely prevented. The X-axis linkage is based on a parallel architecture allowing mass reduction leading to increased dynamics performance. Increased maximum acceleration is computed indicating larger bandwidth allowing the robot to proceed with oscillatory motions at higher frequencies.

The number of scissor networks is calculated in order to alleviate collisions by providing a breadth dimension as the design criteria. Kinematics analysis will provide the starting point to determine the extension and encumbrance of the scissors. Finally, the CATIA program allows to refine the study by introducing the scissors into the Tripteron. Finite element modeling allows comparing different design for a given external loading.

In the first section, the manipulator under study shall be reviewed. In the second section, the scissor legs shall be examined and sized. The third section will comprise an impact analysis of the combination of these utilizing the equivalent beam approach.

## 2 Mechanism under study: the Tripteron

### 2.1 Main properties

The Tripteron manipulator has been proposed as a 3-PRRR spatial parallel mechanism where three kinematics chains are comprised of one prismatic actuator displacing en elbow mechanism constituted by two bars and three revolute joints linking the mobile platform to the robot fixed frame. This architecture was presented and studied in several papers [11, 12, 13], more or less at the same time.

The main advantage of this robot is the simplicity of its implicit kinematics model which directly links the actuator positions with the Cartesian position of the end-effector. This leads to explicit linear forms for both the forward and inverse kinematics models comparable to a serial Cartesian manipulator. The expression of its Jacobian matrix is then the identity matrix, meaning that the kinetostatic behavior is constant in all the workspace. The kinematics simplicity renders robot control very simple. The Tripteron robot can avoid all the modeling problems which other parallel robots have in terms of non-linearities.

The Tripteron is the latest addition to the attempt to design translational parallel robots which was initiated with the advent of the Delta robot. It is considered the most efficient mechanism to achieve such displacements.

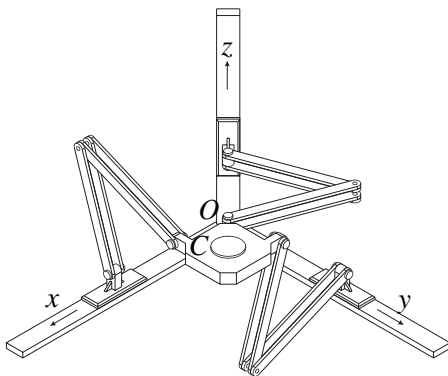


Fig. 1: Classic Tripteron with elbows

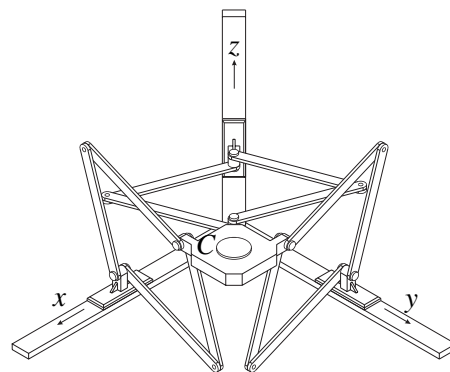


Fig. 2: Evolution of the Tripteron with rhombuses

However, as can be seen in Fig. 1, the elbow configuration will lead to two following problems:

- Collision risk between linkages,
- and elbows entering the workspace.

Moreover, for each kinematics chain, its chain elbow moves in a plane perpendicular to the actuator displacement providing significant bending in the linkages. Hence, as each leg is submitted to bending deformation, the elasto-static behavior varies according to the posture of the robot inside the Cartesian workspace. This will need to be examined in the third section.

## 2.2 Tripteron Kinematics modeling

From Figs. 1 and 2, if we defined that O and C are the origins of the fixed base and mobile platform reference frames, let the  $\rho_i$  be the  $i^{th}$  prismatic actuator length where  $i = 1 \dots 3$  and let  $OC$  be the vector of the end-effector position expressed in O. Then, the implicit geometric model then becomes:

$$\begin{aligned}\rho_1 &= OC_x \\ \rho_2 &= OC_y \\ \rho_3 &= OC_z.\end{aligned}\tag{1}$$

The Tripteron links the mobile platform to the prismatic actuators through three articulated legs which comprise two bars separated by one revolute joint providing motion in one plane perpendicular to the actuator displacement. In this case, the first, second and third prismatic actuators are parallel and aligned in the same direction as the fixed base reference frame X, Y and Z axes respectively. The mobile platform configuration ensures that the kinematics chains are connected so as to obtain the actuator to drive directly the center of the mobile platform.

In this case, the Tripteron robot yields only one explicit linear solution to the inverse kinematic model. The same can be stated about the forward kinematics problem. Furthermore, the Tripteron shares the same advantage as the Cartesian serial robots where each actuator directly and independently displaces the robot end-effector. This renders robot control straightforward for either position or velocity control.

The design problem is narrowed down to a problem of designing the passive linear mechanisms that join the linear motors to the mobile platform. The focus can then to improve the performance of the passive kinematics chains.

## 3 Design of the passive kinematics chains through scissor mechanisms

### 3.1 Introduction of the Rhombus

The first idea, as can be seen in Fig. 2, is to replace the serial elbow mechanism by a double-elbow version. We can also name those mechanisms, Rhombuses. This mechanism, in its simplest form, is actually defined as a four-bar mechanism where all four members are of equal lengths, [9, 14]. Introducing a parallel mechanism improves leg rigidity and even can provide for linkage mass reduction.

However, a single Rhombus does not improve the Tripteron's as compared with the elbow configuration and the listed problems are left unanswered.

As for the elbow leg, the main advantage of the Rhombus configuration is that their kinematics is not affecting the displacement of the Tripteron mobile platform. Its simple implicit kinematics is left unchanged and the actuator motion decoupling is maintained. The Rhombus is introducing a *telescopic* perpendicular motion to the platform at the kinematics chain level.

With the passivity of the perpendicular linkages, the resulting *Expteron* is a Tripteron featuring more constraints from the added kinematics chains. The new manipulator is performing tasks very similarly and it is not seeing a mobility reduction or a workspace reduction as it is often the case with over-constraint or parallel manipulators.

Since the Rhombus is a passive mechanism which reacts and adjusts its length to the position of its two extremities, one attached to the actuator which drives the kinematic chain and the other connected to the mobile platform which can apply an inertial force and/or task contact force, the singularities are found when the internal angle of the two adjacent bars are zero thereby blocking retraction. However, due to the physical limitations imposed by the body volumes themselves, the Rhombus cannot reach the blocking position, meaning that singularities cannot be reached in the real case.

Its mobility is provided in Fig. 3 and was introduced in [13].

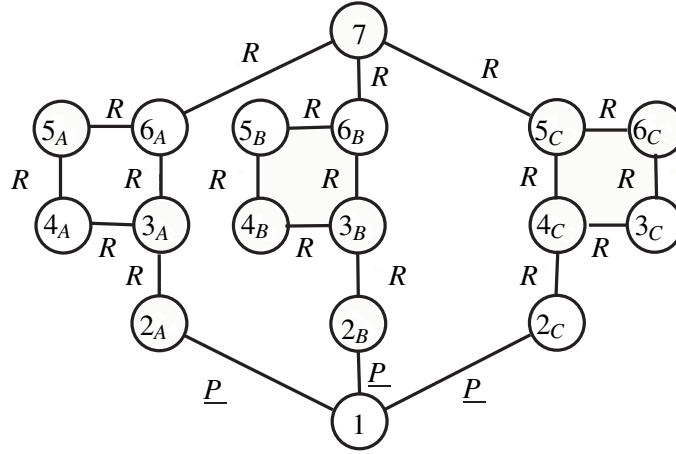


Fig. 3: The Mobility Graph of the Expteron with a single loop

### 3.2 Design specifications

To design a parallel robot whose end-effector displacements are limited to only translations and are preventing rotations, we can use the concept of regular dexterous workspace [15]:

- The shape of the workspace can be a cube or a rectangular parallelepiped
- The performance index used is a stiffness criterion. That is to say, under a given load, the maximum displacement of the end effector must be less than a given value.

Implicitly, we advocate that the robot's dimensions be set to their smallest values in order to keep their footprint minimal and increase complete manipulator compactness as one significant design criteria.

In our example, we will specify the Expteron manipulator from the size of the workspace is  $l_w$ ,  $720 \times 720 \times 720$  mm. The scissor linkage dimensions are set to the following  $50 \times 15$  mm respectively for width and thickness.

### 3.3 Rhombus networking

To reduce the volume swept by the manipulator Rhombus legs, the scissors-like mechanism may be multiplied and networked. In series, we can also name those mechanisms Rhombus networks. The rhombus networked mechanism keeps the simplest four-bar dimensions and actually extends two of the four-bar members. Their networking requires the extension of those two to extend beyond the middle point as to form an articulated X shape or scissor. Then, the second part of the scissor is closed by two single bars separated by one revolute joint.

The number of loops for each leg is defined according to the size of the workspace or the maximum encumbrance of the leg available. Taking into account the simplicity of the Tripteron kinematics, the design objective then becomes to determine the Rhombus bar sizes and their numbers to achieve the desired workspace. Hence, the first part of the design problem will become a geometric one at the level of the scissor-based linear legs.

Considering that each scissor can stretch and fold, the maximum length of each bar is fixed to prevent collisions between the legs. By placing the legs in three orthogonal planes as in Fig. 2, in top view, we can observe that the horizontal leg can meet in collision with the other two. This constraint sets a maximum length for the sides of the Rhombus mechanism. We must also consider the section of the beams used to define the minimum distance between two opposing revolute joints.

This is characterized by a minimum opening angle of Rhombus mechanism, called  $\alpha_{min}$ , as depicted in Fig. 4. To prevent the scissors from reaching a singularity, a maximum opening angle is also defined  $\alpha_{max}$ . The singularity is easily avoided by considering that the linkage width will prevent the scissors to reach the singularity position. These maximum and minimum values are actually determined by the linkage width.

The problem is to find the number of mechanisms in series to form a network by knowing:  $\alpha_{max}$ ,  $\alpha_{min}$  and  $L1_{max}$ . The distance  $l$  between  $A_x$  and  $P_x$  is written (respectively  $A_y$  and  $P_y$  or  $A_z$  and  $P_z$ )

$$l_{max} = 2nl(\sin(\alpha_{max}/2) - \sin(\alpha_{min}/2)) \quad (2)$$

To reach the corners of the planar sections of a cube, we have:

$$l_{max} = \sqrt{2} l_w \quad (3)$$

First, we must set the number of loop by taking the maximum size.

$$n = \frac{l_{max}}{2l(\sin(\alpha_{max}/2) - \sin(\alpha_{min}/2))} \quad (4)$$

Then we have to round up to the nearest larger integer.

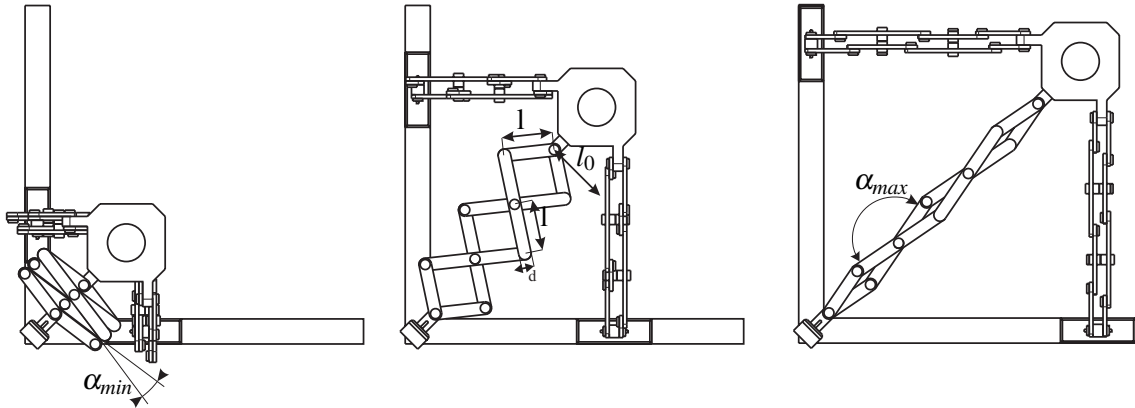


Fig. 4: Parameters to describe the Rhombus network with three loops

Another approach implements the notion of encumbrance calculations derived geometrically from the scissor extensions. The rhombus mechanism has to allow the whole workspace coverage, meaning that they will sustain a displacement along one of the workspace cube diagonal. From the workspace definition which we set to a cube with a dimension of 720, the resulting diagonal dimension is then  $r = 720\sqrt{2}$  where  $r$  stands for the rhombus range determined from the minimum extension to the maximum extension.

Let  $n$  be the number of rhombus networks,  $l$  the bar length and  $h$  the bar width, then taking  $C$  as the rhombus linear displacement or extension, the encumbrance is defined as function of the extension:

$$E_c = h + 2\sqrt{l^2 - \frac{C^2}{4n^2}} \quad (5)$$

Then, when the mechanism is folded on itself as shown on Fig. 2, we get the maximum encumbrance which is defined as:

$$e_{max} = 2\sqrt{l^2 - \left(\frac{h}{2}\right)^2} + h \quad (6)$$

When the mechanism reaches complete extension, the minimum encumbrance is encountered and it is restrained only to the width of two bars:  $e_{min} = 2h$ . The bar's width determines limits to the extensions which are now introduced. The maximum extension reaches:

$$\begin{aligned} C_{max} &= 2n \sqrt{l^2 - \left(\frac{h}{2}\right)^2} \\ C_{min} &= (n+1)h \end{aligned} \quad (7)$$

The rhombus mechanism reach or span is defined as:

$$C_{span} = C_{max} - C_{min} \quad (8)$$

At the design stage, we are trying to determine the rhombus networking which will limit the encumbrance to an arbitrary value sufficient to avoid collisions and scissors entering the workspace.

From the encumbrance Eq. 5, it is possible to derive a unit-less criteria allowing to compare scissor mechanisms without any dimensions concerns allowing to just comparing their performance in terms of encumbrance minimization. Let  $r = C_{span}$  be the rhombus mechanism range and  $c = \frac{C}{r}$  the extension ratio, then the encumbrance ratio is defined equal to:

$$e_c = \frac{h}{r} + 2 \sqrt{\frac{l^2}{r^2} - \frac{c^2}{4n^2}}; \quad (9)$$

It is then easy to plot the encumbrance functions with the standard tools available in Maple. Firstly, the Fig. 5

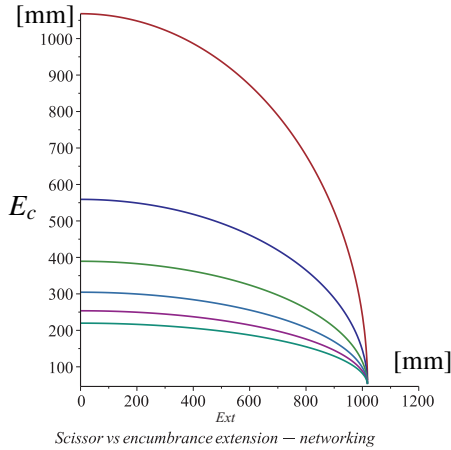


Fig. 5: Encumbrance vs extension of various rhombus networks

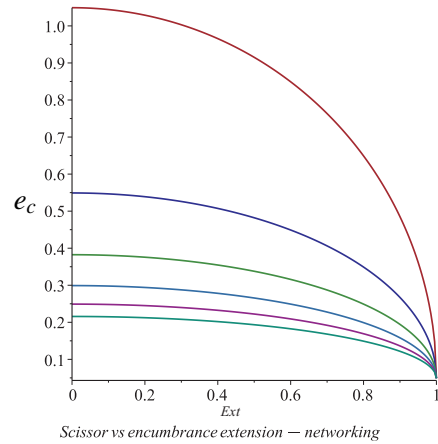


Fig. 6: Encumbrance ratio vs extension of various rhombus networks

presents the absolute  $E_C$  encumbrance in terms of the extension ranging from  $C_{min}$  to  $C_{max}$ . Then, Fig. 6 shows the  $e_c$ . From Figs. 5 and 6, we can observe the following:

- The first curve in red shows the encumbrance ratio for one Rhombus where the value exceeding one means that the maximum encumbrance exceeds the extension range. This case is thereby rejected for collision risks.
- The second (green) curve shows the results for the case with two Rhombuses where encumbrance drops to 55 % of extension range.
- The third (yellow) curve concerns the triple Rhombus networking where the encumbrance lowers even further to less than 40 %. This result is actually a good compromise between encumbrance and mechanical complexity.
- The fourth (violet) curve brings the results to 30 % but the mechanical complexity is becoming important. The following networking do yield only marginal improvements and do not seem to allow lower than 20 % encumbrance.

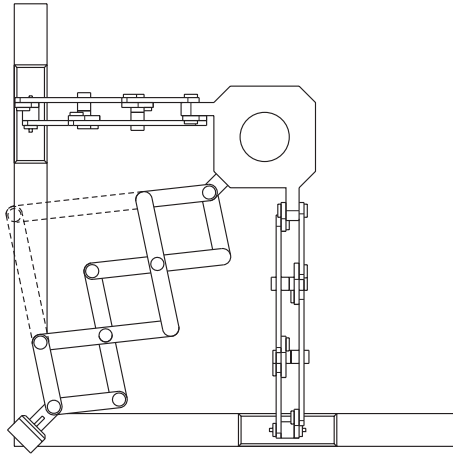


Fig. 7: Comparison between the legs of the Tripteron and Expteron robots

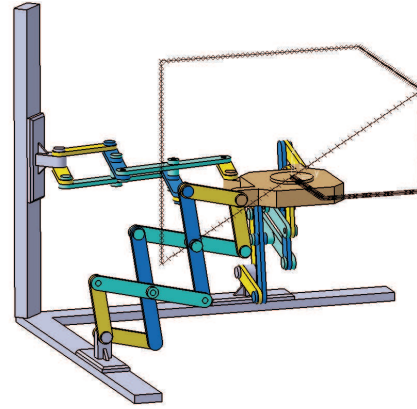


Fig. 8: Example of trajectory from the center of the workspace to the limits of the prescribed cubic workspace

In our case, the selected networking number is three. The Rhombus small segments are slightly larger than six times the value  $C_{span}$  to take into account the effect of bar widths reducing their extension range. Figure 7 depicts the new leg with respect to the previous one. The amplitude of motion is reduced by three.

In Fig. 7, the Tripteron elbow encumbrance is compared with the selected Expteron encumbrance in an almost mid position. It can be understood that the Expteron provides better than half of the encumbrance of the former in this specific position.

A test trajectory is introduced in Fig. 8. Figures 9 and 10 depict the swept volume of any parts of the robot along the same tested trajectory. We can easily conclude that the swept volume of the Expteron robot is more compact than the Tripteron one.

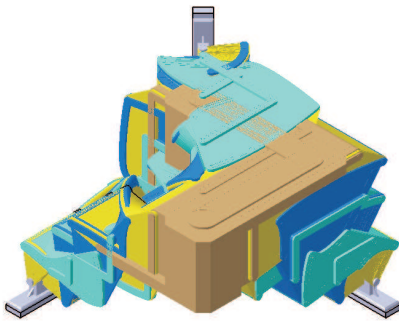


Fig. 9: Swept volume for the Expteron robot along the trajectory introduced in Fig. 8

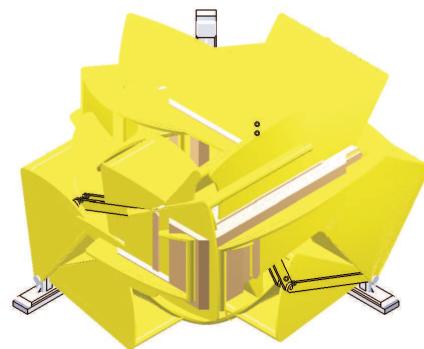


Fig. 10: Swept volume for the Tripteron robot along the trajectory introduced in Fig. 8

A movie of the Expteron robot in motion can be found in the following link: “<https://youtu.be/EdiXg4cOt9Y>”.

## 4 Equivalent beam

The Tripteron and Expteron robots are by definition over-constrained mechanisms. When a force is exerted on the moving platform along the Z axis, the leg of which the actuator is following this axis receives a large part of this force. However, this force will also create a torque in the other two legs. These forces and their transmissions



along the kinematics chains can be considered a complex problem which justifies the utilization of finite element analysis from CATIA.

#### 4.1 Simulation

The connections between the base and the prismatic joints is considered rigid. The revolute joints are considered perfect without clearances or friction. Rhombus mechanisms of the bars are made of steel and the effector is in aluminum. We use the moduli of elasticity conventionally used in the industry to make the pre-sizing.

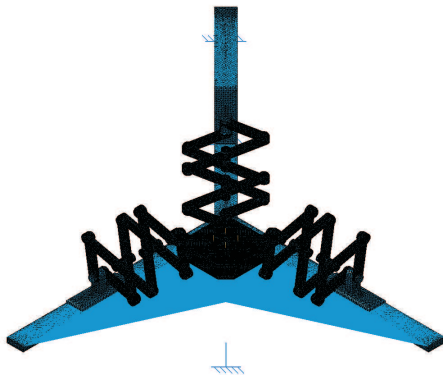


Fig. 11: Mesh of the Expteron

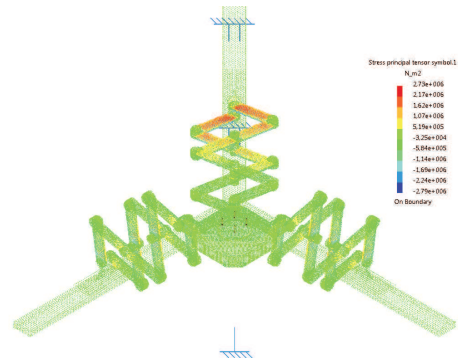


Fig. 12: Stress results in the Expteron

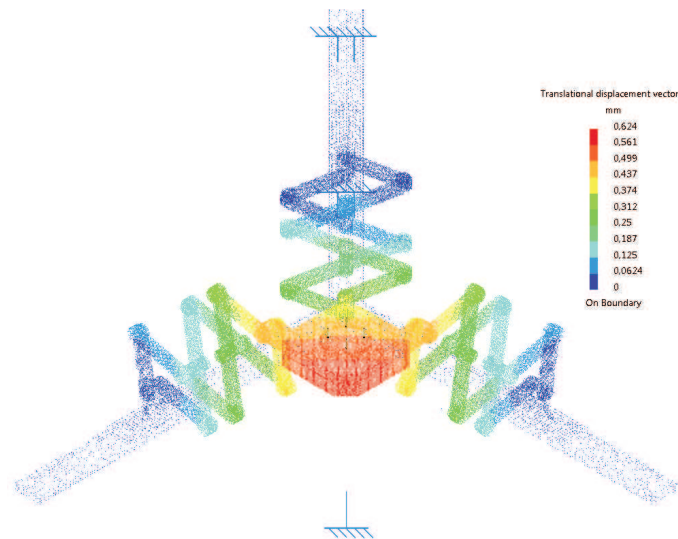


Fig. 13: Displacement field when 20 N are apply on the mobile platform of the Expteron

In Fig. 11, we can appreciate the mesh refinement which was selected for the finite element analysis. In Fig. 12, as expected, the results provide confirmation that the largest stresses are happening in the Rhombus network perpendicular to the Z axis prismatic actuator. The largest stresses appear the closest to the actuator indicating where the largest bending stress is occurring. Moreover, the results confirm some level of bending in the two other Rhombus networks. The levels are about 100 times less then for the vertical axis Rhombus network which comes from the fact that the other actuators are not moving. It will be therefore possible to determine bars' thicknesses in order to keep acceptable deflections at the end-effector. Figure 13 depicts the displacement field obtain when the end-effector is located at the center of the Cartesian workspace where an external loading of 20N is applied, corresponding to a weight of 2 kg.

## 4.2 Analysis of the results

To analyze the behavior of the robot inside his Cartesian workspace, we have applied the same forces over several postures. Figure 14 depicts the maximum deviation of the mobile platform by the color of the points.

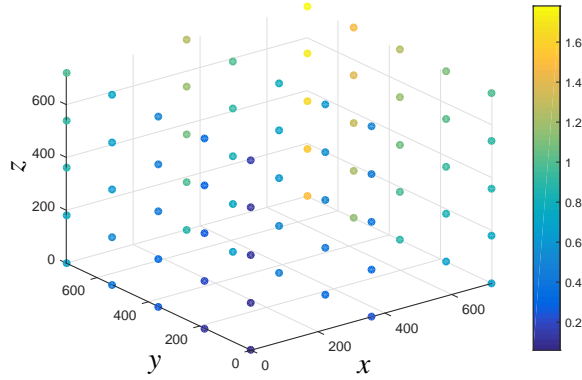


Fig. 14: Displacements of the mobile platform for a set of postures [mm]

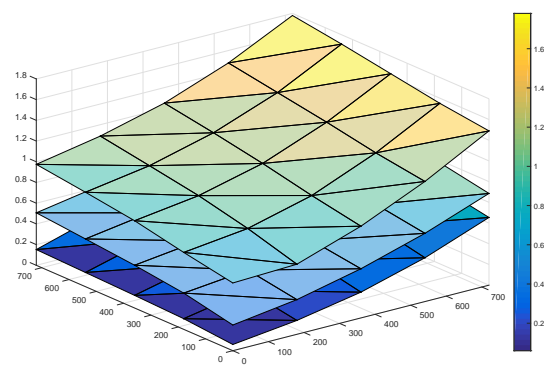


Fig. 15: Values of the displacements of the mobile platform [mm]

By adding the height of the bottom bar and the top bar, we can compare the bending stiffness of the bar with that of the Expteron. As a first approximation, we can consider that the scissor legs are 25 % less stiff than an equivalent section beam.

## 5 Conclusions

In this paper, we have presented an amendment which uses Rhombus mechanism networks to replace the legs typically performed by two arms with three revolute joints. Depending on the space between the legs and sections of the bars, it is easily possible to determine the number of loops to utilize so to reach the borders of a regular workspace in the shape of a cube or even a parallelepiped. The encumbrance serves as a design criteria for determining the Rhombus networking number and the bar lengths accordingly. From the results, three Rhombuses produce a good compromise between encumbrance reduction and mechanism complexity. It is also possible to estimate the deflection of the mobile platform when any leg is subjected to bending. The stiffness of the leg carried by a Rhombus mechanism networks is not equal to a beam whose height is the sum of the heights of the adjacent bars but reduced by 25%. The main advantage is the reduction in the volume swept by the legs. The reduction factor is directly related to the number of loops in the network. The only disadvantage is the proliferation of passive joints that require connections made with preloaded bearings to avoid backlash. Conversely, this approach allows to standardize the design of the legs and allows us to build families of Tripteron robots whose workspace size is adjusted to customer needs. Using a linear actuator catalog and a modular approach for the legs of the robot, a very large number of variants may be created.

## Acknowledgments

The authors would like to thank the French Embassy in Ottawa which supplied a Research Travel Grant allowing the authors to launch and prepare this research.

## References

- [1] C. M. Gosselin, M. T. Masouleh, V. Duchaine, P.-L. Richard, S. Foucault, and X. Kong, "Parallel mechanisms of the multipteron family: Kinematic architectures and benchmarking," in *Proceedings of the ICRA, IEEE International Conference on Robotics and Automation*, pp. 555–560, April 10 – 14, Roma, Italy 2007.
- [2] J.-P. Merlet, *Parallel robots*, vol. 74. Springer Science & Business Media, 2012.
- [3] C. Gosselin and X. Kong, "Cartesian parallel manipulators," May 4 2004. US Patent 6,729,202.
- [4] C. M. Gosselin, M. T. Masouleh, V. Duchaine, P.-L. Richard, S. Foucault, and X. Kong, "Parallel mechanisms of the multipteron family: kinematic architectures and benchmarking," in *Robotics and Automation, 2007 IEEE International Conference on*, pp. 555–560, IEEE, 2007.
- [5] C. Gosselin, "Compact dynamic models for the tripteron and quadrupteron parallel manipulators," *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, vol. 223, no. 1, pp. 1–12, 2009.
- [6] T. Islam, Y. Cheng, S. Jian, and L. Rolland, "Dynamic analysis of scissor lift mechanism through bond graph modeling," in *Proceedings of the IEEE/ASME AIM 2014, the IEEE/ASME International Conference on Advanced Intelligent Mechatronics*, pp. 1–10, July 7 – 10, Besancon, France 2014.
- [7] T. Islam and L. Rolland, "Parametric design optimization of scissor lift platform - a design of experiment (doe) approach," in *Proceedings of the 2015 CCToMM M3 Workshop*, pp. 1–10, May 29 – June 1, Ottawa, Canada 2015.
- [8] G. Gogu, *Structural Synthesis of Parallel Robots: Part 4: Other Topologies with Two and Three Degrees of Freedom*, vol. 183 of *SOLID MECHANICS AND ITS APPLICATIONS*. Dordrecht: Springer, 2012.
- [9] L. Rolland, *Kinematics Synthesis of a New Generation of Rapid Linear Actuators for High Velocity Robotics with Improved Performance Based on Parallel Architecture*, vol. In *Advanced Strategies for Robot Manipulators*. Dordrecht: Sciyo Publishers International, s. e. shafiei editor ed., 2010.
- [10] N. Hua, Z. Xie, L. Luo, X. Chen, D. J. Tang, and H. Zhang, "Design and analysis in multiple-scissor-linkage applied to the robotics arm," in *Proceedings of the 3rd ICMRA, International Conference on Mechatronics, Robotics and Automation*, pp. 482–485, Atlantis Press, May 14 – 15, Shenzhen, China 2015.
- [11] X. Kong and C. M. Gosselin, "Type synthesis of linear translational parallel manipulators," in *Advances in robot kinematics*, pp. 453–462, Springer, 2002.
- [12] M. Carricato and V. Parenti-Castelli, "Singularity-free fully-isotropic translational parallel manipulators," in *ASME 2002 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, pp. 1041–1050, American Society of Mechanical Engineers, 2002.
- [13] G. Gogu, *Structural Synthesis of Parallel Robots: Part 2: Translational Topologies with Two and Three Degrees of Freedom*, vol. 159 of *SOLID MECHANICS AND ITS APPLICATIONS*. Dordrecht: Springer, 2009.
- [14] T. Islam, "Design, development and control of a new generation high performance linear actuator for parallel robots and other applications," Master's thesis, Memorial University of Newfoundland, September 2015.
- [15] D. Chablat, P. Wenger, F. Majou, and J.-P. Merlet, "An interval analysis based study for the design and the comparison of three-degrees-of-freedom parallel kinematic machines," *The International Journal of Robotics Research*, vol. 23, no. 6, pp. 615–624, 2004.