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ABSTRACT

Optimal control techniques can actively maximize buildings efficiency by a smarter HVAC (Heating, Ventilation and Air-Conditioning) systems operation. All these techniques involve a model of the controlled system and an optimization process based on a cost function.

The main difficulty with these methods is to find an accurate model of a system with the right data for training and on-line operation. Our work introduces an adjoint based method aimed at efficiently computing an optimal predictive command law using a descriptive thermal model of a test case. Our approach has the advantage to enable fast analysis in order to identify the most influential inputs and parameters with respect to the final performance. In this paper, we present numerical results concerning a simple two-room test case.

INTRODUCTION

Recent evolutions in embedded informatics, sensor networks and wireless sensor networks, communication protocols dedicated to buildings applications (LonWorks, KNX, Zigbee...) have fostered the evolution of Building Management Systems (BMS). These techniques make buildings smarter, fully monitored and informatically controlled (Lalanda et al., 2010). All these improvements open new ways to control HVAC equipments in buildings, such as Model Predictive Control (MPC).

Model Predictive Control is a control technique initiated by (Richalet et al., 1978), and generalized by (Clarke et al., 1987). It is more powerful than a standard PID control (Maciejowski, 2002). It relies on a numerical model of the controlled system and a prediction of its future solicitations. The dynamical model of the controlled system is used to predict its evolution and compute the best command law *a priori*. Thus, MPC is formulated as an optimization problem over a given time period.

Some previous works intended to implement MPC techniques in modern buildings in order to take into account both thermal comfort and power consumption, while anticipating future energy gains. Numerical studies were carried out on several cases, such as water-based floor heating (Karlsson and Hagentoft, 2011), HVAC systems for commercial buildings (Henze et al., 2004), or real time control of subway-stations (Ansuini et al., 2013).

The main drawback of this method lies in the need of an accurate model and accurate predictions of its solicitations, otherwise important errors can be introduced in the computed command. A standard way to solve this problem is to use model identification techniques, which can sometimes be difficult to implement when the number of parameters is high.

Some sensitivity studies were performed on simulation and energetic models to identify what parameters are the most influential. In (Aude et al., 2000), local relative sensitivities of temperatures are based on standard differentiation. In (Corrado and Mechri, 2009), the Morris sensitivity method is applied on building heat balance terms and in (Tian and de Wilde, 2011), they are computed statistically for long term weather predictions. Nevertheless, from our point of view, it is also relevant to specifically identify important parameters for an optimal control in buildings.

For real time MPC, the efficiency of the optimization code is an important issue. Most implementations use gradient-descent techniques, and as the number of control parameters in modern buildings can be very high, standard differentiation techniques can be very time consuming.

In this work, we present an MPC algorithm to optimally control thermal comfort based on a criterion that includes the operative temperature. The algorithm computes an optimal command law for the electrical heating devices. We then propose a predictive control technique that uses a conjugate gradient algorithm for the optimization process, with an efficient gradient computation based on the use of an adjoint model. The computation cost proves very low and compliant with real-time application needs. We eventually present how this method can easily lead, with the same efficiency, to sensitivity studies of the optimal control performance. The adjoint theory widely used here is mainly based on the theory of optimal control of systems governed by partial differential equations (Lions, 1971).

A TWO-ROOM TEST CASE

The computation of a predictive command law for a particular system requires the use of a model accurately describing its dynamics. In this section, we present the test case selected for our studies and its mathematical description.

Test case description

The test case used for this study is a simple two-room building (see figure 1), used in a previous study of our team (Brouns et al., 2013). Both rooms are identical, each of them equipped with a heater, an individual indoor-outdoor air exchange system and a standard glazing.
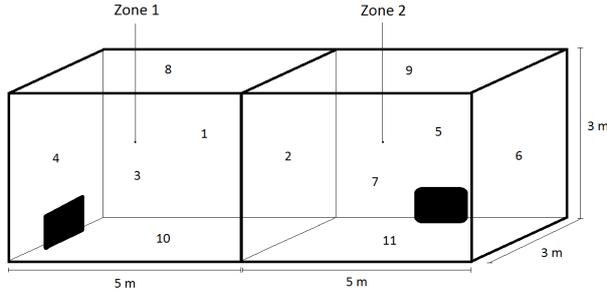

Figure 1 – Case study and wall numbering

Each heater is electric, and we assume that its power is fully controllable from of 0 to $3\,kW$. For this test case, we aim to control the electrical power of both heating devices in the most efficient way.

A physical model

In this work, we use the same mathematical model for both simulation and control. The model is built upon standard multizone assumptions for temperatures and heat flows (Clarke, 2001). These are homogenous temperature and pressure for each thermal zone, and one-directional thermal conduction through uniform walls. The temperature T_z in $°C$ of each zone $z \in \{1; 2\}$ is governed by the following equations:

$$\begin{cases} C_z \dfrac{dT_z}{dt} = \sum_{p \in \mathbb{P}_z} S_p h^0_{p;z} \left(\theta^s_p(t) - T_z\right) \\ \qquad\qquad + c_a q_z \left(T_a - T_z\right) + Q_z + W_z + \gamma_z \phi \\ T_z(t = 0) = T^0_z \end{cases}$$
(1)

where C_z is the heat capacity of room z in $J.K^{-1}$, S_p the area of the wall p in m^2, $h^0_{p;z}$ the convective exchange coefficient between zone z and the indoor surface of the adjacent wall p, c_a the air heat capacity, q_z the total air flow between the zone and the outdoors, T_a the outdoor temperature, Q_z the internal uncontrolled heat gains, γ_z the coefficient of solar heat gains due to the direct solar heat flow ϕ, and W_z the internal gains due to heaters in $J.s^{-1}$. $\mathbb{P}_z$ is the index set of walls adjacent to zone z, and θ^s_p their temperature on the adjacent surface. These equations are coupled with those used to describe wall temperature dynamics $\theta_p(x, t)$:

$$\begin{cases} S_p c_p \dfrac{\partial \theta_p}{\partial t} - S_p \dfrac{\partial}{\partial x}\left(k_p \dfrac{\partial \theta_p}{\partial x}\right) = 0 \\ -k_p S_p \dfrac{\partial \theta_p}{\partial x}(0;t) = \displaystyle\sum_{m \in \mathbb{P}_z} S_p \alpha_{p;m}\left(\theta_m(0;t) - \theta_p(0;t)\right) \\ \qquad\qquad\qquad + S_p h^0_{p;z}\left(T_z - \theta_p(0;t)\right) \\ k_p S_p \dfrac{\partial \theta_p}{\partial x}(L_p;t) = S_p h^L_p \left(T_a - \theta_p(L_p;t)\right) \\ \qquad\qquad\qquad + S_p \beta^L_p \left(T^\infty - \theta_p(L_p;t)\right) + S_p \varphi^L_p \\ \theta_p(x, t = 0) = \theta^0_p(x) \end{cases}$$
(2)

Each wall p has a thickness L_p, an equivalent heat capacity c_p in $J.m^{-3}.K^{-1}$, an equivalent thermal conductivity k_p in $J.s^{-1}.m^{-1}.K^{-1}$, and a radiative exchange coefficient $\alpha_{p;m}$ with wall m. φ^L_p is the radiative solar heat flux on outside surfaces, and β^L_p the radiative exchange coefficient with the sky temperature T^∞. To model each heater, we use a simple first order model, with an efficiency factor η_z and a time constant d_z.

$$\begin{cases} d_z \dfrac{dW_z}{dt} + W_z = \eta_z P_z \\ W_z(t = 0) = W^0_z \end{cases}$$
(3)

Equation (3) represents the dynamics between the electrical power P_z and the heat W_z in watts effectively delivered in the room z.

Note that for equation (2), $(x = 0)$ is the abscissa of the internal face and $(x = L_p)$ the external one. This equation is valid for each wall, except for walls 2, 10 and 11 where boundary conditions are different (2 is not facing outside, and 10 and 11 are lying on a ground of temperature T_s). For the sake of simplicity, we do not present here the complete equations for walls 2, 10 and 11.

MODEL PREDICTIVE CONTROL

In this section, we present a method to compute an optimal command law for the heaters of our system. This control method is formulated as an optimization problem and based on the model previously described.

Let $u = \{P_1, P_2\} \in U = \left(L^2\left([0, t_a]\right)\right)^2$ be our control vector, with $[0, t_a]$ the control period. We assume we know initial conditions $(T^0_z, \theta^0_p$ and $W^0_z)$ and all the static model parameters $(C_z, c_p, k_p, ...)$, and that a weather forecast is available, providing with estimates of dynamical parameters $(T_a(t), T^\infty(t), \varphi^L_p(t)$ and $T_s(t))$ for $t \in [0, t_a]$. We also assume that an estimation of $(Q_z(t), q_z(t))$ is available over the same period. With all this information, we are able to compute the model response $\{T_z, \theta_p, W_z\}$ over the control period for any given control vector u; and we aim to find it such that:

$$\begin{cases} J\left(u\right) = \inf\limits_{u \in U} J\left(u\right) \\[2mm] J = \dfrac{1}{2} \sum\limits_{z=1}^{N_z} \int_0^{t_a} a_z\left(t\right) P_z^2 dt \\[2mm] \qquad + \dfrac{1}{2} \sum\limits_{z=1}^{N_z} \int_0^{t_a} b_z\left(t\right) \left(T_{op_z}\left(u\right) - T_{op_z}^c\right)^2 dt \end{cases} \tag{4}$$

where $T_{op_z} = \dfrac{T_{mr_z} + T_z}{2}$ is the so-called operative temperature, with $T_{mr_z} = \overline{\theta_{p;z}^s}$ the mean of temperatures of surfaces adjacent to zone z. The use of the operative temperature is motivated by its better representativeness of the temperature felt by inhabitants; the formula used for T_{op_z} here is a simplified version, one can refer to (Nilsson, 2004) for further information.

$J\left(u\right)$ is a quadratic cost function that is a linear scalarization of the multi-objective problem that aims to minimize quadratic terms $\dfrac{1}{2} \sum\limits_{z=1}^{N_z} \int_0^{t_a} \left(t\right) P_z^2$ and $\dfrac{1}{2} \sum\limits_{z=1}^{N_z} \int_0^{t_a} \left(T_{op_z}\left(u\right) - T_{op_z}^c\right)^2 dt$ measuring respectively costs on power consumption and discomfort. Scalarization helps to simplify the multi-objective problem of control but weighting coefficients a_z and b_z managing the tradeoff between power consumption and comfort over the control time must be chosen wisely.

One can show (Chavent, 2010) that the optimization problem (4) is well posed and its solution is unique. As a matter of fact, the quadratic term $\dfrac{1}{2} \sum\limits_{z=1}^{N_z} \int_0^{t_a} \left(t\right) P_z^2$ acts like a regularization term (Engl et al., 1996). From this point of view, gradient based algorithms will perform much better than costly optimization algorithms such as genetic algorithms. To solve it we use a method based on adjoint models and the conjugate gradient algorithm, developed in the more general case of systems governed by partial differential equations (Lions, 1971).

This method aims to explicitly express gradients as a function of states of direct and adjoint models, so that we can compute them with the cost of only two simulations. In fact, this method is the most efficient to compute gradients and consequently the most promising for a real time implementation.

It is important to point out that such quadratic formulation has useful mathematical properties for optimization problem solving, but the quadratic norm on the electrical power doesn't represent the true consumed power. We use it there for the sake of simplicity, since it doesn't change our methodology, but a practical implementation should consider the true consumed power.

Numerical implementation

We show in the appendix of this paper how the adjoint method leads to an explicit gradient formulation for ∇J. To compute it, we just have to solve consecutively the direct and adjoint models. These simulations are done numerically using first order finite elements in space and with an Euler implicit scheme in time. Note that the adjoint model prescribes *final* conditions instead of initial conditions, and then must be solved backwards.

To solve the Euler's equation $\nabla \mathcal{L} = \nabla J = 0$, we implement a conjugate gradient algorithm that uses successive values of gradient computed with (18).

In our previous studies, the Levenberg-Marquardt approach coupled with the conjugate gradient has proven its efficiency on unconstrained and non-linear optimization problems of a similar kind (Bourquin and Nassiopoulos, 2011). In further studies, this method could be easily implemented to control non-linear parameters (like air ventilation q_z).

As standard heaters are considered, the power they can deliver is bounded, so that $0 \leq P_z \leq P_{z,max}$, $z \in \{1, 2\}$. We are in fact facing a constrained optimization problem:

$$\begin{cases} u \in U_c = \{v \in U \,;\, \varphi_i\left(v\right) \leq 0,\, 1 \leq i \leq m\} \\ J\left(u\right) = \inf\limits_{v \in U_c} J\left(v\right) \end{cases} \tag{5}$$

Where φ_i are linear constraint functions, positive if and only if the constraint is unsatisfied. To solve it, we use either an iterative projection version of the conjugate gradient algorithm or the Uzawa's method (Ciarlet, 1989), which is one of the simplest in the class of augmented Lagrangian methods (see the appendix for mathematical details).

First control results

For our test case described above, we perform an optimal command law computation and simulation over four days (from midnight to midnight). We use standard values for all model parameters, and the weather scenario is extracted from an EnergyPlus weather file. We choose to apply a room temperature setpoint T_{op}^c, for both rooms, only between 8 a.m. and 18 p.m., that implies $a_z(t) = 0$ for other times. The unique difference between rooms (excepted their weather exposition), is the presence of an uncontrolled thermal gain Q in room 2 of $2,5\,kW$ from 9 a.m. to 12 a.m. that models a human activity (several people plus the use of some equipments generating heat). For a better results analysis, we do not consider zone solar heat gains $\gamma_z \phi$.

Figure 2 gives the whole results for this computation : evolution of operative temperatures, heaters power and internal gains Q.
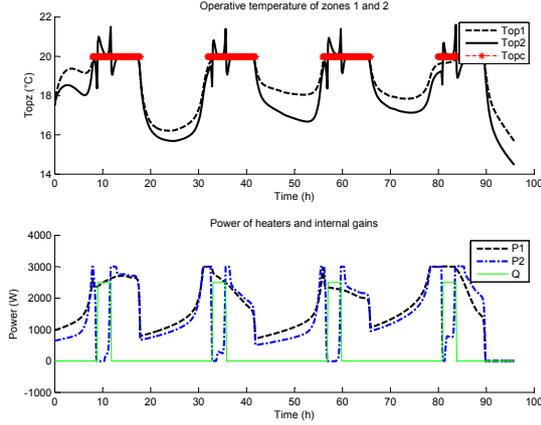
Figure 2 – Optimal command and temperature response

We clearly see that when the building is supposed to be unoccupied (i.e. the weights $b_z(t)$ are zero), the electrical power of heaters is not set to zero, but managed to give the required temperature at 9 a.m. . A standard regulator (like a PID controller) can't anticipate such setpoint changes.

This graph shows also some limits of our method. Firstly, boundary effects are quite important : even if all solicitations are supposed to be almost periodical, we observe an important break in periodicity at $t = 0$ and $t = t_a$. Moreover, we also observe some temperature oscillations on transitions of Q. These oscillations are due to the fact that u is sought in a L^2 space (Gibbs phenomenon) and that heaters have non-negligible dynamics.
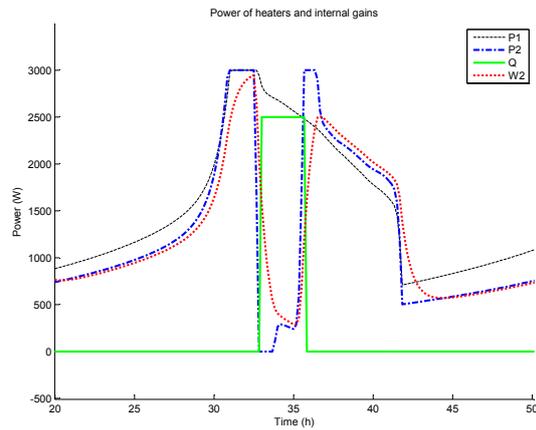

Figure 3 – Optimal command : Non controlled gains compensation

One can clearly see on figure 3 the anticipation of internal gains in room 2 by P_2, and the instantaneous power W_2 transmitted in room 2 by its heater. However, the power compensation is not perfect on transitions, which gives oscillations observed on T_2 at 9 a.m. and 12 a.m..

To solve these problems, some improvements can be done, like ignoring last instants of our solution or making direct improvements in the cost function : addition of a cost or a constraint associated to temperature derivatives, for example, to limit temperature oscillations (the maximum absolute value of derivatives is commonly called *slew rate*). A more rigorous technique is to use another functional space for control (Bourquin and Nassiopoulos, 2011).

In figure 4, we have a comparison of the zone 1 temperature with the corresponding operative temperature. In this case, walls temperature is lower than room temperature, which results in an operative temperature lower than the zone temperature.
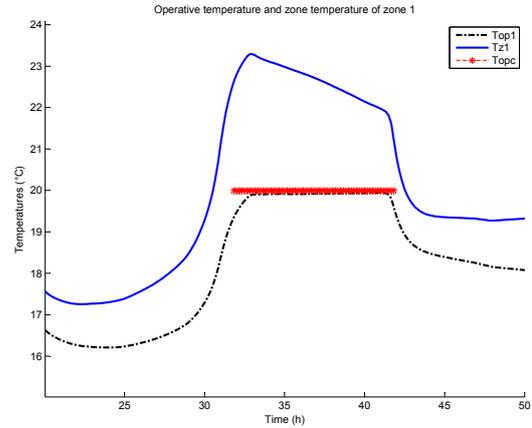

Figure 4 – Comparison between zone 1 temperature and corresponding operative temperature

We can also observe that zone temperature is not necessary constant while operative temperature is, because walls are progressively storing heat, reducing zone temperature contribution in operative temperature. This can lead to energy savings in comparison with a case where one would like to have a constant zone temperature.

SENSITIVITIES WITH RESPECT TO MODEL PARAMETERS

Uncertainties study

To compute an accurate command law, one has to provide accurate values for model parameters over the control period. If these values are false, the resulting command law will not be as efficient as expected. Unfortunately, obtaining these parameters with a good precision is a really difficult task, since theoretical values provided by construction data and meteorological forecasts often have a poor reliability. To deal with this problem, one can use on-site calibration techniques based on sensors measurements and inverse modeling (Hazyuk et al., 2012), to calibrate the model before computing a command law.

Figure 5 shows a typical workflow of the process and indicates the various steps at which uncertainties prop-

agate. In this figure, the model parameters p are divided in known parameters p_k and unknown ones p_u, $\hat{p}$ are estimates of parameters p after a model calibration, m are on-site measurements and J the cost function used for command computation (which is also our command performance indicator), while Δ represent an error.
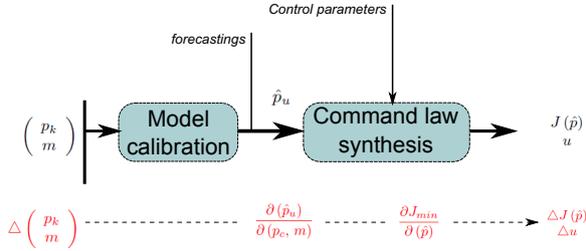

Figure 5 – Uncertainties propagation through calibration and command processes

In this section, we focus on the computation of $\dfrac{\partial J_{min}}{\partial (\hat{p})}$, which is essential to find the most influential parameters in the performance of the optimal control.

Sensitivity computations using the adjoint method

For the command synthesis process, we perform an adjoint based gradient computation. The computation of $\dfrac{\partial J}{\partial (p)}$ can be done in the same way for a given set of parameters p, by just rewriting the sensitivity model and the resulting adjoint model. If we compute an optimal control u^* for the constrained problem, derivatives of the Lagrangian $\mathcal{L}'(u^*)$ at this point give $\nabla_p J_{min}$ taking constraints into account (see appendix). Then we obtain a gradient of the minimum of the performance indicator for each parameter p in a very efficient way.

This gradient can be used to compute sensitivity indices.

We can build sensitivity indices, based on the assumption that every parameter as the same percent of uncertainty:

$$S_p = \frac{1}{J_{min}} |\langle \nabla_p J_{min}, \, p \rangle| \qquad (6)$$

With this expression, S_p measures the normalized variation of J_{min} for a relative variation of p. Consequently, if p is null, S_p is null too, but it does not mean that p is not influential.

Moreover, there is no unique way to build sensitivity indices. We can also compare influences of parameters using a L^2 norm on non-dimensional gradients:

$$\Upsilon_p = \left\| \frac{1}{J_{min}} \bar{p} . \nabla_p J_{min} \right\|_2 \qquad (7)$$

Because we assume here the architecture well known, parameters issued from geometrical characteristics (surfaces, radiative exchange coefficients...) are

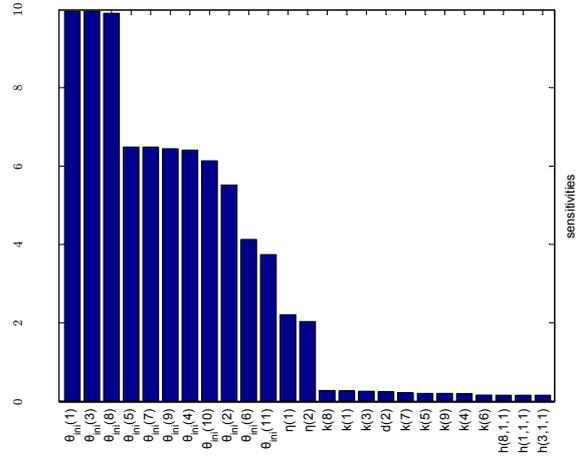known with a good accuracy and are not involved in our sensitivity studies.
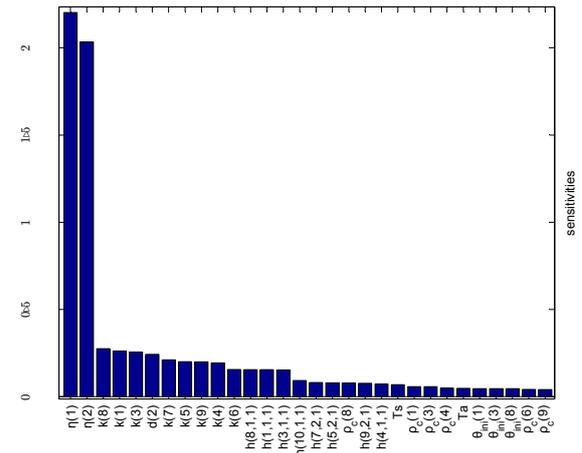

Figure 6 – Sensitivity indices S_p

Figure 7 – Sensitivity indices Υ_p using L^2 norm

For all the studied parameters, we obtain with formula (6) the results presented in figure 6.

The most influent parameters for this configuration seem to be initial wall temperatures, followed by heater efficiencies, wall conductivities (excepted for wall 2), air infiltration for room 2 and some convective exchange coefficients.

Equation (7), gives the results presented in figure 7. These results are quite similar, but not for wall temperatures that become less important. However , it's not surprising that parameters related to the amount of heat transfers with the outside environment have a such importance, since they are often chosen to characterize the insulation of a building.

These sensitivities are only local, because we use local derivatives at a particular point to compute them. It should be interesting to compute global sensitivity indices to give a better information on influential param-

eters. The DGSM technique (Sobol and Kucherenko, 2009; Kucherenko et al., 2009; Touzani and Busby, 2013), which involves local derivatives in different points, could be implemented in further studies.

Furthermore, they can change with the time horizon t_a. A precise study of the influence of this parameter could help to choose update times for our command law. For example, figure (8) shows that a bias on initial temperatures for the computation of our command naturally decreases in time, as it could be expected:
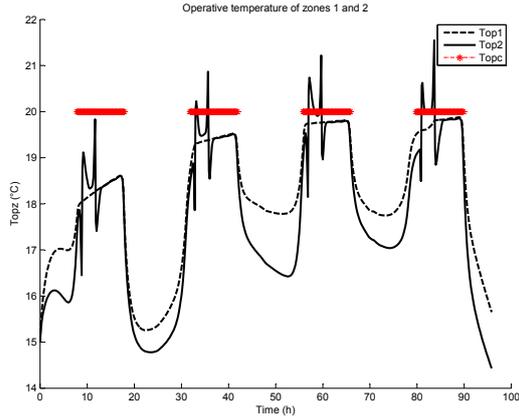

Figure 8 – Temperature response with a bias of $1.5\,^{\circ}C$ on initial temperatures

Consequently, the importance of initial temperatures can decrease as the terminal time t_a for predictions increases. Studies taking in account t_a can give some useful information on how and when the model used for control must be (re)calibrated.

CONCLUSION

This work presents a methodology to optimally control thermal comfort based on operative temperature: an MPC algorithm is used to compute the command law for heating devices. The adjoint method is put to work to provide a cost efficient implementation for gradient computations and sensitivity analysis needs. We present first numerical results on a simple test case, where the simulation model is the same as that used in command synthesis.

Future developments include a better modeling of thermal comfort and global sensitivity analysis. The cost function used for command computation should take into account temperature variations (that are involved in comfort perception), and local sensitivities should be extended to global sensitivities to obtain more general and reliable results on our test case. Despite slightly lower performances, automatic differentiation techniques are easier to implement and should be considered (Griesse and Walther, 2004). Nevertheless, our first results are encouraging for further studies in this way.

Our current test case is roughly close to the Predis MHI platform, located in G2ELab's quarters (Dang et al., 2013). This platform consists of two monitored office rooms, dedicated to the study of the monitoring of smart buildings. In future studies, we plan to adapt our test case to the more complex Predis MHI case for a real on-site implementation.

ACKNOWLEDGMENT

APPENDIX

The adjoint method

A standard way to solve the optimization problem (4) related to the linearized model is to use the Lagrangian formalism (Allaire, 2007). We define a Lagrangian $\mathcal{L}$ considering equations (1), (2) and (3) as constraints with associated Lagrange multipliers V_z, A_p and B_z for each zone z and wall p. We have :

$$\mathcal{L} = \mathcal{L}\left(u, T_z\left(u\right), \theta_p(u), W_z\left(u\right), V_z, A_p, B_z\right) \tag{8}$$

Solving (4) is equivalent to find the saddle point of $\mathcal{L}$ considering u and Lagrange multipliers as only one variables (and by definition verifying $\nabla\mathcal{L} = 0$).

Let $\delta u = \{\delta P_1, \delta P_2\} \in U$ be a small perturbation of the control vector. By definition, the gradient of $\mathcal{L}$ verifies :

$$D = \langle \nabla\mathcal{L}, \delta u\rangle_U + \mathcal{O}\left\|\delta u\right\|_U^2 \tag{9}$$

with

$$D = \mathcal{L}\left(u + \delta u\right) - \mathcal{L}\left(u\right) \tag{10}$$

One can shows that D also depends on $\delta\theta_p$, δT_z and δW_z such that :

$$\begin{aligned}
\theta_p(u + \delta u) &\simeq \theta_p(u) + \delta\theta_p(\delta u)\\
T_z(u + \delta u) &\simeq T_z(u) + \delta T_z(\delta u)\\
T_{op_z}(u + \delta u) &\simeq T_{op_z}(u) + \delta T_{op_z}(\delta u)
\end{aligned} \tag{11}$$

Where δW_z, δT_z and $\delta\theta_p$ are solutions of the sensitivity model around u :

$$\begin{cases}
C_z \dfrac{d\delta T_z}{dt} = \displaystyle\sum_{p\in\mathbb{P}_z} S_p h_{p;z}^0 \left(\delta\theta_p(0;t) - \delta T_z\right)\\
\qquad\qquad - c_a q_z \delta T_z + \delta W_z\\
\delta T_z(t = 0) = 0
\end{cases} \tag{12}$$

$$\begin{cases} S_p\rho c_p \dfrac{\partial \delta\theta_p}{\partial t} - S_p \dfrac{\partial}{\partial x}\left(k_p \dfrac{\partial \delta\theta_p}{\partial x}\right) = 0 \\[2mm] -k_p S_p \dfrac{\partial \delta\theta_p}{\partial x}(0;t) = \displaystyle\sum_{m\in\mathbb{P}_z} S_p\alpha_{p;m}\left(\delta\theta_m(0;t) - \delta\theta_p(0;t)\right) \\[2mm] \qquad\qquad\qquad + S_p h^0_{p;z}\left(\delta T_z - \delta\theta_p(0;t)\right) \\[2mm] k_p S_p \dfrac{\partial \delta\theta_p}{\partial x}(L_p;t) = -S_p\left(h^L_p + \beta^L_p\right)\delta\theta_p(L_p;t) \\[2mm] \delta\theta_p(x,t=0) = 0 \end{cases}$$
(13)

$$\begin{cases} d_z \dfrac{d\delta W_z}{dt} + \delta W_z = \eta_z \delta P_z \\[2mm] \delta W_z(t=0) = 0 \end{cases}$$
(14)

By performing some integrations by parts in the full expression of $\mathcal{L}$, one can show that if V_z, A_p and B_z are the states of of the so-called adjoint model of the sensitivity model, we ensure that $\forall (z,p),\ \nabla_{(V_z,\,A_p,\,B_z)}\mathcal{L} = 0$ and consequently that $\nabla J = \nabla \mathcal{L}$.

The adjoint model consists of the following equations (15), (16) and (17), where V_z, A_p and B_z are respectively adjoint states of T_z, θ_p and W_z.

$$\begin{cases} -C_z \dfrac{dV_z}{dt} = \displaystyle\sum_{p\in\mathbb{P}_z} S_p h^0_{p;z}\left(A_p(0;t) - V_z\right) \\[2mm] \qquad\qquad - c_a q_z V_z + \dfrac{b_z}{2}\left(T_{op_z} + \delta T_{op_z} - T^d_{op_z}\right) \\[2mm] V_z(t=t_a) = 0 \end{cases}$$
(15)

$$\begin{cases} -S_p\rho c_p \dfrac{\partial A_p}{\partial t} - S_p \dfrac{\partial}{\partial x}\left(k_p \dfrac{\partial A_p}{\partial x}\right) = \\[2mm] \qquad\qquad \dfrac{b_z}{12}\displaystyle\sum_{z=1}^{N_z}\left(T_{op_z} + \delta T_{op_z} - T^d_{op_z}\right) \\[2mm] -k_p S_p \dfrac{\partial A_p}{\partial x}(0;t) = \displaystyle\sum_{p\in\mathbb{P}_z} S_p\alpha_{p;m}\left(A_m(0;t) - A_p(0;t)\right) \\[2mm] \qquad\qquad\qquad + S_p h^0_{p;z}\left(V_z - A_p(0;t)\right) \\[2mm] k_p S_p \dfrac{\partial A_p}{\partial x}(L_p;t) = S_p h^L_{p;z}\left(V_z - A_p(L_p;t)\right) \\[2mm] \qquad\qquad\qquad - S_p\left(h^L_p + \beta^L_p\right)A_p(L_p;t) \\[2mm] A_p(x,t=t_a) = 0 \end{cases}$$
(16)

$$\begin{cases} -d_z \dfrac{dB_z}{dt} + B_z = V_z \\[2mm] B_z(t=t_a) = 0 \end{cases}$$
(17)

As a consequence, we obtain :

$$\langle \nabla J(u); \delta P_z\rangle_U = \int_0^{t_a}\left[\eta_z B_z + a_z P_z\right]\delta P_z\, dt$$
(18)

Constrained optimization problem

Uzawa's method consists in defining a sequence $\left(u^k, \lambda^k\right)_{k\geq 0}$, where λ^0 is arbitrarily chosen and for each $k \geq 0$. For our problem, we have :

$$\begin{cases} J\left(u^k\right) + \displaystyle\sum_{i=1}^{m}\int \lambda_i^k(t)\varphi_i\left(u^k(t)\right)dt = \\[2mm] \displaystyle\inf_{v\in U}\left\{J(v) + \sum_{i=1}^{m}\int \lambda_i^k(t)\varphi_i(v(t))dt\right\} \\[2mm] \lambda_i^{k+1} = \max\left\{\lambda_i^k + \rho\varphi_i\left(u^k\right),\,0\right\},\ 1\leq i\leq m \end{cases}$$
(19)

With a particular $\rho_c \in \mathbb{R}^{+*}$ such that convergence of $\left(u^k, \lambda^k\right)_{k\geq 0}$ towards the solution of (5) is guaranteed if $0 \leq \rho \leq \rho_c$. With this method, the resolution becomes a sequence of unconstrained problems. Eventually, for a k where the limit of $\left(u^k, \lambda^k\right)_{k\geq 0}$ is almost reached, the solution of the constrained problem (5) is almost the same for the unconstrained problem :

$$\begin{cases} u \in U \\[2mm] J(u) + \displaystyle\sum_{i=1}^{m}\int \lambda_i^k(t)\varphi_i(u(t))dt = \\[2mm] \displaystyle\inf_{v\in U}\left\{J\left(v^k\right) + \sum_{i=1}^{m}\int \lambda_i^k(t)\varphi_i(v(t))dt\right\} \end{cases}$$
(20)

In our case, we have four constraint functions : $\left\{(P_z - P_{z,max}),\ -P_z\right\};\ z \in \{1,2\}$. For each unconstrained problem of our sequence, we have to compute gradients of $J'_k = J\left(u^k\right) + \displaystyle\sum_{i=1}^{m}\int \lambda_i^k(t)\varphi_i\left(u^k(t)\right)dt$ using the augmented Lagrangian $\mathcal{L}'_k = \mathcal{L}(u) + \displaystyle\sum_{i=1}^{m}\int \lambda_i^k(t)\varphi_i(u(t))dt$, that gives :

$$\langle \nabla J'_k; \delta P_z\rangle_U = \int_0^{t_a}\left[\eta_z B_z + a_z P_z \right. \\ \left. + \lambda_{z,max}^k - \lambda_{z,min}^k\right]\delta P_z\, dt$$
(21)

Note that this method easily extends to cases with more complex constraints.

REFERENCES

Allaire, G., May 2007. Numerical Analysis and Optimization: An Introduction to Mathematical Modelling and Numerical Simulation. OUP Oxford.

Ansuini, R., Vaccarini, M., Giretti, A., Ruffini, S., 2013. Models for the real-time control of subway stations. In: Proceedings of BS 2013: 13th Conference of the International Building Performance Simulation Association. pp. 1794–1801.

Aude, P., Tabary, L., Depecker, P., 2000. Sensitivity analysis and validation of buildings' thermal models using adjoint-code method. Energy and Buildings 31 (3), 267–283.

Bourquin, F., Nassiopoulos, A., 2011. Inverse reconstruction of initial and boundary conditions of a heat transfer problem with accurate final state.

International Journal of Heat and Mass Transfer 54 (15–16), 3749–3760.

Brouns, J., Nassiopoulos, A., Bourquin, F., Limam, K., 2013. State-parameter identification problems for accurate building energy audits.

Chavent, G., Mar. 2010. Nonlinear Least Squares for Inverse Problems: Theoretical Foundations and Step-by-Step Guide for Applications. Springer.

Ciarlet, P. G., Aug. 1989. Introduction to Numerical Linear Algebra and Optimisation. Cambridge University Press.

Clarke, D. W., Mohtadi, C., Tuffs, P. S., 1987. Generalized predictive control—Part i. the basic algorithm. Automatica 23 (2), 137–148.

Clarke, J., 2001. Energy Simulation in Building Design. Taylor & Francis.

Corrado, V., Mechri, H. E., Jun. 2009. Uncertainty and sensitivity analysis for building energy rating. Journal of Building Physics 33 (2), 125–156.

Dang, H. A., Gaaloul, S., Delinchant, B., Wurtz, F., 2013. Building simulation of energy consumption and ambient temperature: application to the PRE-DIS platform. In: Building simulation 2013.

Engl, H. W., Hanke, M., Neubauer, A., Jul. 1996. Regularization of Inverse Problems. Springer.

Griesse, R., Walther, A., Jul. 2004. Evaluating gradients in optimal control: Continuous adjoints versus automatic differentiation. Journal of Optimization Theory and Applications 122 (1), 63–86.

Hazyuk, I., Ghiaus, C., Penhouet, D., 2012. Optimal temperature control of intermittently heated buildings using model predictive control: Part i – building modeling. Building and Environment 51, 379–387.

Henze, G. P., Felsmann, C., Knabe, G., 2004. Evaluation of optimal control for active and passive building thermal storage. International Journal of Thermal Sciences 43 (2), 173–183.

Karlsson, H., Hagentoft, C.-E., Mar. 2011. Application of model based predictive control for water-based floor heating in low energy residential buildings. Building and Environment 46 (3), 556–569.

Kucherenko, S., Rodriguez-Fernandez, M., Pantelides, C., Shah, N., 2009. Monte carlo evaluation of derivative-based global sensitivity measures. Reliability Engineering & System Safety 94 (7), 1135–1148.

Lalanda, P., Bourcier, J., Bardin, J., Chollet, S., Feb. 2010. Smart home systems. In: A., M. (Ed.), Smart Home Systems. InTech.

Lions, J. L., 1971. Optimal control of systems governed by partial differential equations. Springer-Verlag.

Maciejowski, J. M., Jan. 2002. Predictive Control: With Constraints. Pearson Education.

Nilsson, H. O., 2004. Comfort climate evaluation with thermal manikin methods and computer simulation models. Arbetslivsinstitutet, Stockholm.

Richalet, J., Rault, A., Testud, J. L., Papon, J., Sep. 1978. Model predictive heuristic control: Applications to industrial processes. Automatica 14 (5), 413–428.

Sobol, I., Kucherenko, S., Jun. 2009. Derivative based global sensitivity measures and their link with global sensitivity indices. Mathematics and Computers in Simulation 79 (10), 3009–3017.

Tian, W., de Wilde, P., 2011. Uncertainty and sensitivity analysis of building performance using probabilistic climate projections: A UK case study. Automation in Construction 20 (8), 1096–1109.

Touzani, S., Busby, D., Feb. 2013. Screening method using the derivative-based global sensitivity indices with application to reservoir simulator.