



HAL
open science

There are infinitely many twin primes $30n+11$ and $30n+13$, $30n+17$ and $30n+19$, $30n+29$ and $30n+31$

Sibiri Christian Bandre

► **To cite this version:**

Sibiri Christian Bandre. There are infinitely many twin primes $30n+11$ and $30n+13$, $30n+17$ and $30n+19$, $30n+29$ and $30n+31$. 2016. hal-01307789

HAL Id: hal-01307789

<https://hal.science/hal-01307789>

Preprint submitted on 3 May 2016

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution - NonCommercial - NoDerivatives 4.0 International License

There are infinitely many twin primes $30n+11$ and $30n+13$, $30n+17$ and $30n+19$, $30n+29$ and $30n+31$

By CHRISTIAN BANDRÉ

Abstract

We proved that $\liminf_{n \rightarrow +\infty} (p_{n+1} - p_n) = 2$ where p_n is the n -th prime number. We showed the conditions on which an integer $10X + 1$, $10X + 3$, $10X + 7$ or $10X + 9$ can be a prime number. We studied the conditions required to get some twin primes and proved the twin primes conjecture. First we showed that some sets of formulas are governing each of the eight sets of composite integers $30n+11$, $30n+31$, $30n+13$, $30n+23$, $30n+7$, $30n+17$, $30n+19$ and $30n+29$. Then we showed that there are infinitely many couple of integers $30n+11$ and $30n+13$, $30n+17$ and $30n+19$, $30n+29$ and $30n+31$ which are not generated by the formulas of the corresponding sets of composite integers respectively. As a side result, it will open a new era for the study of gaps between primes and leads to the proofs of many open problems linked to prime numbers.

Contents

1. Introduction	2
2. Notation	2
3. The composite numbers and prime numbers $10X+1$	3
4. The composite numbers and prime numbers $10X+3$	5
5. The twin primes $30n+11$ and $30n+13$	7
6. The composite numbers and prime numbers $10X+7$	8
7. The composite numbers and prime numbers $10X+9$	9
8. The twin primes $30n+17$ and $30n+19$	10
9. The twin primes $30n+29$ and $30n+31$	11
References	12

1. Introduction

The issue of distribution of primes and the twin primes conjecture is a puzzle for mathematicians since the time of Euclid of Alexandria [1, 2]. The sequence of prime number seems to be random. This messy look of the sequence of prime numbers requires an explanation. The twin primes conjecture is one of the oldest mathematics problems [3]. It asserts that "there exist infinitely many primes p such that $p+2$ is a prime". Recently Yitang Zhang has solved the Bounded Gap Conjecture [4]. The aim of this work is to give a strong and elegant proof of the twin primes conjecture and the distribution of prime numbers. Our method consisted in discovering the formulas of the eight groups of composite numbers, unveiling the deterministic structure which organizes the distribution of prime numbers.

THEOREM 1. $\liminf_{n \rightarrow +\infty} (p_{n+1} - p_n) = 2$ where p_n is the n -th prime number.

In the following sections we give the proof of the theorem above. First we showed that some set of formulas are governing each of the eight set of composite integers $30n+31$, $30n+11$, $30n+13$, $30n+23$, $30n+7$, $30n+17$, $30n+19$, $30n+29$. Then we showed that there are infinitely many integers which are not generated by the formulas of the sets of composite $30n+11$ and $30n+13$, $30n+17$ and $30n+19$, $30n+29$ and $30n+31$.

2. Notation

$X, Y, n, p, q, x, y, i, j, k, a, b, c$ are all natural integers

PROPOSITION 1. X is an integer > 0 .

$10X + 1$ can be a prime if only $X \equiv 0 \pmod{3}$ or $X \equiv 1 \pmod{3}$.

$10X + 3$ can be a prime if only $X \equiv 1 \pmod{3}$ or $X \equiv 2 \pmod{3}$.

$10X + 7$ can be a prime if only $X \equiv 0 \pmod{3}$ or $X \equiv 1 \pmod{3}$.

$10X + 9$ can be a prime if only $X \equiv 1 \pmod{3}$ or $X \equiv 2 \pmod{3}$.

$10X + 1$ and $10X + 3$ can be twin primes if only $X \equiv 1 \pmod{3}$.

$10X + 7$ and $10X + 9$ can be twin primes if only $X \equiv 1 \pmod{3}$.

$10X + 9$ and $10X + 11$ can be twin primes if only $X \equiv 2 \pmod{3}$.

Table 1. Values of $10X+1$, $10X+3$, $10X+7$, $10X+9$ and $10X+11$ given X

X	$10X+1$	$10X+3$	$10X+7$	$10X+9$	$10X+11$
$3n$	$30n+1$	$3(10n+1)$	$30n+7$	$3(10n+3)$	$30n+11$
$3n+1$	$30n+11$	$30n+13$	$30n+17$	$30n+19$	$3(10n+7)$
$3n+2$	$3(10n+7)$	$30n+23$	$3(10n+9)$	$30n+29$	$30n+31$

Proof. The table 1 shows that if $n \geq 0$ the twin primes can be only :
 $10(3n + 1) + 1$ and $10(3n + 1) + 3$
 $10(3n + 1) + 7$ and $10(3n + 1) + 9$
 $10(3n + 2) + 9$ and $10(3n + 2) + 11$.

□

3. The composite numbers and prime numbers $10X+1$

PROPOSITION 2. *If $X \equiv 0 \pmod{3}$ then $10X + 1 = 30n + 1$ is a prime number if only X is not a term of at least one of the following sequences (1) to (6) :*

$$(1) \quad (A_{x,y}^1) = 3(30xy + x + y) \text{ with } x > 0 \text{ and } y > 0$$

$$(2) \quad (B_{x,y}^1) = 3(30xy + 11x + 11y + 4)$$

$$(3) \quad (C_{x,y}^1) = 3(30xy + 7x + 13y + 3)$$

$$(4) \quad (D_{x,y}^1) = 3(30xy + 17x + 23y + 13)$$

$$(5) \quad (E_{x,y}^1) = 3(30xy + 19x + 19y + 12)$$

$$(6) \quad (F_{x,y}^1) = 3(30xy + 29x + 29y + 28)$$

The sequences (1) to (6) constitute the set C^{30n+1} .

The sequences (7) to (10) constitute the set C^{30n+11} .

If $X = 3n + 1$ then $10X + 1 = 30n + 11$ is a prime number if only X is not a term of at least one of the following sequences (7) to (10):

$$(7) \quad (U_{x,y}^1) = 3(30xy + x + 11y) + 1 \text{ with } y > 0$$

$$(8) \quad (V_{x,y}^1) = 3(30xy + 7x + 23y + 5) + 1$$

$$(9) \quad (W_{x,y}^1) = 3(30xy + 17x + 13y + 7) + 1$$

$$(10) \quad (Z_{x,y}^1) = 3(30xy + 19x + 29y + 18) + 1$$

Proof. Consider the integers $10X + 1, p, q, x$ and y . If $10X + 1$ is not a prime number then $10X + 1$ has at least two factors such that :

$$10X + 1 = (10p + 1)(10q + 1) \text{ with } p > 0 \text{ and } q > 0$$

or

$$10X + 1 = (10p + 3)(10q + 7)$$

or

$$10X + 1 = (10p + 9)(10q + 9)$$

.

It is easy to check that by setting $p=3x$ or $3x+1$ or $3x+2$ and $q=3y$ or $3y+1$ or $3y+2$ then we get the previous sequences.

Example : For $10X + 1 = (10p + 1)(10q + 1)$ with $p = 3x + 1$ and $q = 3y$ then

$$\begin{aligned} 10X + 1 &= (30x + 11)(30y + 1) \\ &= 10(90xy + 3x + 33y + 1) + 1 \end{aligned}$$

or also

$$\begin{aligned} X &= 90xy + 3x + 33y + 1 \\ &= 3(30xy + x + 11y) + 1 \end{aligned}$$

$p > 0$ and $q > 0$ means that $y > 0$

For $10X + 1 = (10p + 3)(10q + 7)$ with $p = 3x + 1$ and $q = 3y + 1$ then

$$\begin{aligned} 10X + 1 &= (30x + 13) \times (30y + 17) \\ &= 900xy + 510x + 390y + 221 \\ &= 10(90xy + 51x + 39y + 22) + 1 \end{aligned}$$

and

$$\begin{aligned} X &= 90xy + 51x + 39y + 22 \\ &= 3(30xy + 17x + 13y + 7) + 1 \end{aligned}$$

For $10X + 1 = (10p + 3)(10q + 7)$ with $p = 3x + 2$ and $q = 3y$ then

$$\begin{aligned} X &= 90xy + 21x + 69y + 16 \\ &= 3(30xy + 7x + 23y + 5) + 1 \end{aligned}$$

For $10X + 1 = (10p + 9)(10q + 9)$ with $p = 3x + 1$ and $q = 3y + 2$ then

$$\begin{aligned} 10X + 1 &= (30x + 19)(30y + 29) \\ &= 900xy + 870x + 570y + 551 \\ &= 10(90xy + 87x + 57y + 55) + 1 \end{aligned}$$

and

$$\begin{aligned} X &= 90xy + 87x + 57y + 55 \\ &= 3(30xy + 29x + 19y + 18) + 1 \end{aligned}$$

It follows that if $X = 3n + 1$ is a term of at least one of the sequences (7) to (10) then $10X + 1 = 30n + 11$ is a composite number. Thus, we conclude that $10X + 1 = 30n + 11$ is a prime number if X is not a term of at least one of the sequences (7) to (10). QED

11, 41, 71, 101, 131 are primes because 1, 4, 7, 10 and 13 are not some terms of at least one of the sequences (7) to (10)

Similarly, if $X = 3n$ is not a term of at least one of the sequences (1) to (6) then $30n+1$ is prime. QED

31, 61, 151, 181, 211 are primes because 3, 6, 15, 18 and 21 are not some terms of at least one of the sequences (1) to (6)

□

4. The composite numbers and prime numbers $10X+3$

PROPOSITION 3. *If $X = 3n + 1$ then $10X+3=30n+13$ is a prime number if only X is not a term of at least one of the following sequences (11) to (14):*

$$(11) \quad (U_{x,y}^3) = 3(30xy + 13x + y) + 1 \text{ with } x > 0$$

$$(12) \quad (V_{x,y}^3) = 3(30xy + 23x + 11y + 8) + 1$$

$$(13) \quad (W_{x,y}^3) = 3(30xy + 19x + 7y + 4) + 1$$

$$(14) \quad (Z_{x,y}^3) = 3(30xy + 29x + 17y + 16) + 1$$

The sequences (11) to (14) constitute the set C^{30n+13}

If $X = 3n + 2$ then $10X+3=30n+23$ is a prime number if only X is not a term of at least one of the following sequences (15) to (18):

$$(15) \quad (A_{x,y}^3) = 3(30xy + 13x + 11y + 3) + 2$$

$$(16) \quad (B_{x,y}^3) = 3(30xy + 23x + y) + 2 \quad x > 0$$

$$(17) \quad (C_{x,y}^3) = 3(30xy + 19x + 17y + 10) + 2$$

$$(18) \quad (D_{x,y}^3) = 3(30xy + 29x + 7y + 6) + 2$$

The sequences (15) to (18) constitute the set C^{30n+23}

Proof. Consider the integers $10X+3, p, q, x$ and y

If $10X+3$ is not a prime number then it has at least two factors such that :

$$10X + 3 = (10p + 1)(10q + 3) \text{ with } p > 0$$

or

$$10X + 3 = (10p + 7)(10q + 9)$$

It is easy to check that by setting $p=3x$ or $3x+1$ or $3x+2$ and $q=3y$ or $3y+1$ or $3y+2$ and using the same method of transformations in 3 then we shall get the sequences (11) to (14) and (15) to (18)

It follows that if $X = 3n + 1$ then $10X+3=30n+13$ is a prime number if only X is not a term of at least one of the sequences (11) to (14) QED.

13, 43, 73, 103, 163 are primes because 1, 4, 7, 10 and 16 are not some terms of at least one of the sequences (11) to (14)

If $X = 3n + 2$ then $10X+3=30n+23$ is a prime number if only X is not a term of at least one of the sequences (15) to (18) QED.

23, 53, 83, 113, 173 are primes because 2, 5, 8, 11 and 17 are not some terms of at least one of the sequences (15) to (18)

□

5. The twin primes $30n+11$ and $30n+13$

From (1), (2) and (3) it follows that $10X+1$ and $10X+3$ are twin primes if only $X=3n+1$ and is not a term of at least one of the sequences (7) to (14).

To prove that the twin primes $30n+11$ and $30n+13$ are infinite we must prove that the positive integers $X=3n+1$ which are not terms of at least one of the sequences (7) to (14) are infinite.

PROPOSITION 4.

Consider the numerical sequence $U_n = 3(an + b) + \lambda$

We must have at least a set of (a) numerical sequences

$U_{n,i} = 3(an+b+i)+\lambda$ with i taking the values 0 to $(a-1)$ to get all the consecutive

terms of the numerical sequence $P_n = 3(n + b) + \lambda$ infinitely

Example The 2 sequences $6n+1$ and $6n+4$ generates all the integers $3n+1 : 1, 4, 7, 10, 13, 16, \dots$. The 3 sequences $9n+1, 9n+4$ and $9n+7$ generates all the integers $3n+1 : 1, 4, 7, 10, 13, 16, \dots$. The 4 sequences $12n+1, 12n+4$ and $12n+7, 12n+10$ generates all the integers $3n+1 : 1, 4, 7, 10, 13, 16, 19, 22, \dots$

Proof. Consider the numerical sequence $P_n = 3(n + b) + \lambda$ It follows that $P_{n+1} = P_n + 3$ and the common difference of P_n is 3

Consider the numerical sequence $U_n = 3(an + b) + \lambda$ It follows that $U_{n+1} = U_n + 3a$

The common difference of U_n is $3a$.

Thus between U_n and $U_n + 3a$ we have the terms $U_n + 3i$ of P_n with i taking the values 1 to $(a - 1)$.

Thus if we have the set of numerical sequences $U_n, U_n + 3, \dots, U_n + 3(a - 1)$ we will get all the terms of $P_n = 3(n + b) + \lambda$ infinitely.

if we have the set of (a) sequences $U_n, U_n + 3, \dots, U_n + 3(a - 1)$

we will get all the terms of $P_n = 3(n + b) + \lambda$ infinitely.

It follows that if we have a set of q numerical sequences where $q < a$ we will not get all the terms of $P_n = 3n + b + \lambda$ between U_n and U_{n+1} . \square

Given that the set of the sequences (7) to (14) does not satisfy the proved proposition 4 which is a condition both necessary and sufficient, it follows that we can find infinitely some terms of the sequence $3n + 1$ that are not generated by the sequences (7) to (14). We conclude that the twin primes $30n+11$ and

$30n+13$ are infinite. We also conclude that the number of twin primes p and $p + 2$ is infinite. example : $19 = 3 * 6 + 1$ but 19 is not generated by at least one of the sequences (7) to (14). this is why 191 and 193 are twin primes. $64 = 3 * 21 + 1$ but 64 is not generated by at least one of the the sequences (7) to (14). This is why 641 and 643 are twin primes. $67 = 90(0)(2) + 33(2) + 1$ and 671 is not a prime number, this is why 671 and 673 are not twin primes $70 = 90(1)(0) + 57(1) + 21(0) + 13$ so 703 is not a prime number, this is why 701 and 703 are not twin primes

6. The composite numbers and prime numbers $10X+7$

PROPOSITION 5. *If $X \equiv 0 \pmod{3}$ then $10X+7=30n+7$ is a prime number if only X is not generated by at least one of the following formulas (19) to (22) :*

$$(19) \quad (A_{x,y}^7) = 3(30xy + 7x + y) \text{ with } x > 0$$

$$(20) \quad (B_{x,y}^7) = 3(30xy + 17x + 11y + 6)$$

$$(21) \quad (C_{x,y}^7) = 3(30xy + 19x + 13y + 8)$$

$$(22) \quad (D_{x,y}^7) = 3(30xy + 29x + 23y + 22)$$

If $X \equiv 1 \pmod{3}$ then $10X + 7 = 30n + 17$ is a prime number if only X is not generated by at least one of the following sequences (23) to (26):

$$(23) \quad (U_{x,y}^7) = 3(30xy + 7x + 11y + 2) + 1$$

$$(24) \quad (V_{x,y}^7) = 3(30xy + 17x + y) + 1, x > 0$$

$$(25) \quad (W_{x,y}^7) = 3(30xy + 19x + 23y + 14) + 1$$

$$(26) \quad (Z_{x,y}^7) = 3(30xy + 29x + 13y + 12) + 1$$

The sequences (19) to (22) constitute the set C^{30n+7}

The sequences (23) to (26) constitute the set C^{30n+17}

Proof. Consider the integers $10X + 7, p, q, x$ and y

If $10X + 7$ is not a prime number then it has at least two factors such that

:

$$10X + 7 = (10p + 1)(10q + 7) \text{ with } p > 0$$

or

$$10X + 7 = (10p + 3)(10q + 9)$$

It is easy to check that by setting $p = 3x$ or $p = 3x + 1$ or $p = 3x + 2$ and $q = 3y$ or $q = 3y + 1$ or $q = 3y + 2$ and using the same method of transformations in 3 then we shall get the sequences (19) to (22):
and (23) to (26):

7, 37, 67, 97, 127, 157 are primes because 0, 3, 6, 9, 12 and 15 are not some terms of at least one of the sequences (19) to (22)
17, 47, 107, 137, 167, 197, 227, 257 are primes because 1, 4, 10, 13, 16, 19, 22 and 25 are not some terms of at least one of the sequences (23) to (26)

□

7. The composite numbers and prime numbers $10X+9$

PROPOSITION 6. *If $X \equiv 1 \pmod{3}$ then $10X+9=30n+19$ is a prime number if only X is not generated by at least one of the following sequences (27) to (32):*

$$(27) \quad (U_{x,y}^9) = 3(30xy + 19x + y) + 1 \quad x > 0$$

$$(28) \quad (V_{x,y}^9) = 3(30xy + 29x + 11y + 10) + 1$$

$$(29) \quad (W_{x,y}^9) = 3(30xy + 13x + 13y + 5) + 1$$

$$(30) \quad (X_{x,y}^9) = 3(30xy + 23x + 23y + 17) + 1$$

$$(31) \quad (Y_{x,y}^9) = 3(30xy + 7x + 7y + 1) + 1$$

$$(32) \quad (Z_{x,y}^9) = 3(30xy + 17x + 17y + 9) + 1$$

If $X \equiv 2 \pmod{3}$ then $10X+9=30n+29$ is a prime number if only X is not a generated by at least one of the following formulas (37) to (40):

$$(33) \quad (A_{x,y}^9) = 3(30xy + 19x + 11y + 6) + 2$$

$$(34) \quad (B_{x,y}^9) = 3(30xy + 29x + y) + 2 \quad x > 0$$

$$(35) \quad (C_{x,y}^9) = 3(30xy + 13x + 23y + 9) + 2$$

$$(36) \quad (D_{x,y}^9) = 3(30xy + 7x + 17y + 3) + 2$$

The sequences (27) to (32) constitute the set C^{30n+19}

The sequences (37) to (40) constitute the set C^{30n+29}

Proof. Consider the integers $10X + 9, p, q, x$ and y . If $10X + 9$ is not a prime number then it has at least two factors such that :

$$10X + 9 = (10p + 1)(10q + 9) \text{ with } p > 0$$

or

$$10X + 9 = (10p + 3)(10q + 3)$$

or

$$10X + 9 = (10p + 7)(10q + 7)$$

It is easy to check that by setting $p = 3x$ or $p = 3x + 1$ or $p = 3x + 2$ and $q = 3y$ or $q = 3y + 1$ or $q = 3y + 2$ and using the same method of transformations in 3 then we shall get the sequences (27) to (32) and (37) to (40):

19, 79, 109, 139, 199, 229 are primes because 1, 7, 10, 13, 19 and 22 are not some terms of at least one of the sequences (27) to (32)

29, 59, 89, 149, 179, 239 are primes because 2, 5, 8, 14, 17 and 23 are not some terms of at least one of the sequences (37) to (40) \square

8. The twin primes $30n+17$ and $30n+19$

From theorem (1), propositions (5) and (6) it follows that if $X=3n+1$ is not a generated by the set the set $C^{30n+17} C^{30n+19}$ then $30n+17$ and $30n+19$ are twin primes.

To prove that the twin primes $30n+17$ and $30n+19$ are infinite we must prove that the numbers $X=3n+1$ which are not a generated by at least one of the following formulas (23) to (32) are infinite.

It is easy to check that the set of sequences (23) to (32) cannot be converted to a set of sequences

$$U_{n,i} = 3(an + b + i) + \lambda$$

with i taking the values 0 to $(a - 1)$, to get all the consecutive terms of the numerical sequence $P_n = 3(n + b) + \lambda$ infinitely.

Given that the set of formulas (23) to (32) does not satisfy the proved proposition 4 which is a condition both necessary and sufficient, it follows that we can find infinitely some terms of the sequence $3n + 1$ that are not generated by the sequences (23) to (32).

It follows that the sequences (23) to (32) will never generate infinitely all the consecutive terms of the numerical sequence $3n + 1$ after a given term $3b + 1$. The number of natural integers $X = 3n + 1$ that are not generated by at least one of the formulas (23) to (32) is infinite. It follows that the number of twin primes $10X + 7 = 30n + 17$ and $10X + 9 = 30n + 19$ is infinite.

We conclude that the twin primes $30n + 17$ and $30n + 19$ are infinite. example : 1, 10, 13, 19, 22 are not generated by at least one of the sequences (23) to (32). This is why (17,19), (107,109), (137 139), (197,199), (227,229) are twin primes.

9. The twin primes $30n+29$ and $30n+31$

We set $10X + 9 = 30n + 29$ and $10X + 11 = 30n + 31$ with $X = 3n + 2$. We set $Y = 3n + 3$ then $10X + 11 = 10Y + 1$ and $Y - 1 = X$. If $10Y + 1$ is not a prime number and $Y \equiv 0 \pmod{3}$ then Y is a term of at least one of the numerical sequences (1) to (6). By subtracting 1 to each of the numerical sequences (1) to (6) we get the following numerical sequences :

- $3(30xy + x + y - 1) + 2, x > 0$ and $y > 0$
- $3(30xy + 11x + 11y + 3) + 2$
- $3(30xy + 7x + 13y + 2) + 2$
- $3(30xy + 17x + 23y + 12) + 2$
- $3(30xy + 19x + 19y + 11) + 2$
- $3(30xy + 29x + 29y + 27) + 2$

if $X = 3n + 2$ and is not a term of the following numerical sequences then $10X + 9$ and $10X + 11$ are twin primes :

$$(37) \quad (A_{x,y}^9) = 3(30xy + 19x + 11y + 6) + 2$$

$$(38) \quad (B_{x,y}^9) = 3(30xy + 29x + y) + 2x > 0$$

$$(39) \quad (C_{x,y}^9) = 3(30xy + 13x + 23y + 9) + 2$$

$$(40) \quad (D_{x,y}^9) = 3(30xy + 7x + 17y + 3) + 2$$

$$(41) \quad (U_{x,y}^{11}) = 3(30xy + x + y - 1) + 2 \text{ with } x > 0 \text{ and } y > 0$$

$$(42) \quad (V_{x,y}^{11}) = 3(30xy + 11x + 11y + 3) + 2$$

$$(43) \quad (W_{x,y}^{11}) = 3(30xy + 7x + 13y + 2) + 2$$

$$(44) \quad (X_{x,y}^{11}) = 3(30xy + 17x + 23y + 12) + 2$$

$$(45) \quad (Y_{x,y}^{11}) = 3(30xy + 19x + 19y + 11) + 2$$

$$(46) \quad (Z_{x,y}^{11}) = 3(30xy + 29x + 29y + 27) + 2$$

To prove that the twin primes $30n + 29$ and $30n + 31$ are infinite we must prove that the numbers $X = 3n + 2$ which are not generated by at least one of the formulas (37) to (46) are infinite.

Given that the set of formulas (37) to (46) does not satisfy the proved proposition 4 which is a condition both necessary and sufficient, it follows that we can find infinitely some terms of the sequence $3n + 2$ that are not generated by the formulas (37) to (46). We conclude that the twin primes $30n + 29$ and $30n + 31$ are infinite. example : $14 = 3 * 4 + 2$ but 14 is not generated by at least one of the formulas (37) to (46). This is why 149 and 151 are twin primes. $17 = 3 * 5 + 2$ but 17 is not generated by at least one of the formulas (37) to (46). This is why 179 and 181 are twin primes. $23 = 3 * 7 + 2$ but 23 is not generated by at least one of the formulas (37) to (46). This is why 239 and 241 are twin primes.

It follows that the set of numerical sequences (37) to (46) will not generate infinitely all the consecutive terms of the numerical sequence $3n + 2$ after a given term $3b + 2$. The number of natural integers $X = 3n + 2$ that are not some terms of at least one of the numerical sequences ((37) to (46)) is infinite. It follows that the number of twin primes $10X + 9 = 30n + 29$ and $10X + 9 + 2 = 30n + 31$ is infinite. We conclude that the number of twin primes p and $p + 2$ are infinite.

References

- [1] Richard P. Brent. Irregularities in the distribution of primes and twin primes. 1975.
- [2] M. N. Huxley. *The distribution of prime numbers*. 1972.
- [3] J. Pintz. Are there arbitrarily long arithmetic progressions in the sequence of twin primes? *Bolyai Soc. Math. Stud.* 21, 21:525–559, 2010.
- [4] Yitang Zhang. Bounded gaps between primes. *Ann. of Math.*, 179:1121–1174, 2014.

(Received:

(Revised:

CENTRE HOSPITALIER MAX QUERRIEN, PAIMPOL, FRANCE

E-mail: christian.bandre@ch-paimpol.fr