

# On the control of PGD reduced-order approximations: error estimation and adaptivity

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## ABSTRACT

Complex multidimensional models are encountered in various scientific and engineering fields including parametric or stochastic modeling. In the past few decades, dedicated numerical methods have been proposed to handle the high dimensionality and complexity inherent to associated solutions. In particular, model order reduction techniques have emerged as natural candidates to get around the classical grid/mesh-based discretization methods (due to the well-known curse of dimensionality). Proper Generalized Decomposition (PGD) is currently one of the most widely used Reduced-Order Modeling (ROM) techniques [1] for the a priori construction of separated variables representations of the model solution defined in tensor product spaces. Such a model reduction technique is built upon a two-stage (online-offline) strategy: it starts with the online construction of a low-dimensional subspace spanned by reduced basis functions (or modes) through the successive resolution of a sequence of tractable problems; a reduced-order approximation of the solution can then be defined on the resulting reduced tensor product space and computed offline. While these model reduction methods have reached a certain degree of maturity and become nowadays widespread, a major concern has emerged for the development of robust and efficient verification and adaptation tools able to assess and control the quality of PGD-based numerical approximations.

In the past few years, a posteriori error estimation methods have been proposed to certify the quality of PGD reduced-order models. Guaranteed and robust error estimators have been recently introduced in [2, 3] in order to control the accuracy of PGD reduced-order approximations for linear elliptic and parabolic problems. The verification procedure is based on the concept of Constitutive Relation Error (CRE) [5] along with the construction of associated admissible (equilibrated) fields. It allows to capture various error sources (space and time discretization errors as well as truncation error in the PGD decomposition) and to quantify their relative contributions through the definition of appropriate error indicators. Such error indicators can then serve as stopping criteria or adaptation indicators in a greedy algorithm in order to adaptively construct an optimal PGD reduced-order approximations. In the present work, the verification and model/mesh adaptation procedure for PGD reduced-order models is extended to the goal-oriented error estimation framework and provides for strict and accurate error bounds on specific quantities of interest computed from PGD reduced-order approximations. The performances of the proposed verification method and adaptation strategy are illustrated through numerical examples carried out on a linear elasticity problem with several inclusions containing material heterogeneities.

## References

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