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Composite Nonlinear Feedback-based Bounded Formation Control of Multi-quadrotor Systems

Zhicheng HOU and Isabelle FANTONI

Abstract—This paper presents a bounded formation controller for multiple quadrotors (UAVs) system with leader-follower structure. Considering the actuator saturation of the quadrotors, the hyperbolic function is used in the formation controller design. The composite nonlinear feedback is integrated in the formation controller in order to improve the formation performance. The simulation and experimental results show that the formation task is achieved with small overshoot and rapid response speed.

I. INTRODUCTION

The cooperative control of multi-quadrotor systems has recently attracted the attention. The manipulation and transportation of large payloads by using multiple UAVs are investigated in [1] and [2]. The problem of cooperative surveillance in large outdoor areas by a fleet of micro aerial vehicles is studied in [3]. The applications such as search and rescue are covered in [4].

In practice, the inputs of a quadrotor are subject to actuator saturations. This problem is taken into account in the quadrotor controller design in the literature. For instance, a nested saturation controller is proposed in [5]. An algorithm of feasible trajectories planning is investigated in [6], where the input constraints of quadrotor are considered.

To the best of our knowledge, the input saturations are not very much considered in the multi-quadrotor systems control. In [7], a low gain feedback control is proposed to avoid the saturation of the system, such that a semi-global leader-follower consensus of linear multi-agent system is achieved. In the formation control of multi-UAV systems, a large formation error can generate a large control output. This could lead to big attitude angles for the UAVs of the formation. Although the low gain controller [7] can guarantee the control output small, the response speed of the closed-loop system is low.

In this paper, the formation of quadrotors with Leader-Follower (L-F) structure is investigated. Considering the input saturation of the quadrotor, a bounded formation controller is developed. The formation control strategy is proposed aiming at investigating a bounded differentiable formation controller and obtaining the closed-loop system with fast response speed and small overshoot.

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In recent years, authors in [8] [9] have proposed and implemented a “composite nonlinear feedback” (CNF) control to improve the performance of a system. The objective of this control method is to reduce the overshoot and meanwhile, keep a rapid rising response.

In this paper, we are inspired by the CNF control method to improve the performance of the formation of UAVs. Considering the input saturation of the quadrotors, a hyperbolic tangent function is used instead of a standard saturated function, which has two non-differentiable points when the function gets saturated. The high-order derivatives of the formation controller of each UAV are needed for the attitude control. Therefore, if we use the standard saturated function, an infinite derivative will be generated at the non-differentiable points. In contrary, the hyperbolic tangent function is always differentiable and has bounded high-order derivatives.

The paper is organized as follows. The modeling and control of one quadrotor are presented in section II. The formation control design and the stability analysis are shown in section III. Simulation and experimental results are given in section IV. Finally, conclusions are stated in section V.

II. MODELING AND CONTROL OF ONE UAV

A. Modeling

The dynamics of a quadrotor is modeled as the motion of a rigid body in 3-D space under a thrust force and three moments, which are generated by the thrust forces of the four rotors. The orientation of the quadrotor with respect to the inertial frame is represented by the rotation matrix $R_i \in \text{SO}(3)$, where $\text{SO}(3)$ represents a special orthogonal group, and whose determinant is one.

Vector $\mathcal{X}_i = [X_i, Y_i, Z_i]^T$ represents the coordinates of the center of mass of a quadrotor in the fixed inertial frame $o_e x_e y_e z_e$. The Euler angles (roll, pitch and yaw) are represented by the vector $\Theta_i = [\phi_i, \theta_i, \psi_i]^T$. The inertia matrix $J = \text{diag}\{I_{x_b}, I_{y_b}, I_{z_b}\}$ of a quadrotor is diagonal, if I_{x_b} , I_{y_b} and I_{z_b} represent the moments of inertia with respect to the body-fixed frame x_b , y_b and z_b respectively. The angular velocity of the quadrotor i in the body-fixed frame is represented by $\Omega_i \in \mathbb{R}^3$. The function $\mathcal{S}(\cdot) : \mathbb{R}^3 \rightarrow \mathbb{R}^{3 \times 3}$ represents an operation that transforms a vector in \mathbb{R}^3 to a skew-symmetric matrix $\mathbb{R}^{3 \times 3}$.

The roll, pitch and yaw moments are represented by $\tau_i = [\tau_{\phi_i}, \tau_{\theta_i}, \tau_{\psi_i}]^T \in \mathbb{R}^3$. The thrust force is represented by F_{T_i} . The rotation matrix R_i from the body-fixed frame to the inertial frame is with the sequence of roll-pitch-yaw, thus,

we obtain the translational dynamics as follows

$$\begin{aligned}\ddot{X}_i &= (\sin \psi_i \sin \phi_i + \cos \psi_i \cos \phi_i \sin \theta_i) \frac{F_{T_i}}{m} \\ \ddot{Y}_i &= (\cos \phi_i \sin \psi_i \sin \theta_i - \cos \psi_i \sin \phi_i) \frac{F_{T_i}}{m} \\ \ddot{Z}_i &= -g + (\cos \theta_i \cos \phi_i) \frac{F_{T_i}}{m}\end{aligned}\quad (1)$$

where m represents the mass of a quadrotor and g represents the gravity.

Let us denote

$$T_i = \begin{bmatrix} 1 & 0 & -\sin \theta_i \\ 0 & \cos \phi_i & \cos \theta_i \sin \phi_i \\ 0 & -\sin \phi_i & \cos \theta_i \cos \phi_i \end{bmatrix}$$

and

$$\tilde{T}_i = \begin{bmatrix} \tan \theta_i & 0 & \cos \theta_i + \tan \theta_i \sin \theta_i \\ 0 & -\cos \theta_i & 0 \\ \sec \theta_i & 0 & \tan \theta_i \end{bmatrix}$$

We can rewrite the rotational dynamics as follows

$$\begin{aligned}\begin{bmatrix} \ddot{\phi}_i \\ \ddot{\theta}_i \\ \ddot{\psi}_i \end{bmatrix} &= (JT_i)^{-1} \tau_i + \tilde{T}_i \cdot \begin{bmatrix} \dot{\phi}_i \dot{\theta}_i \\ \dot{\phi}_i \dot{\psi}_i \\ \dot{\theta}_i \dot{\psi}_i \end{bmatrix} \\ &+ (JT_i)^{-1} S^T \left(T_i \cdot \begin{bmatrix} \dot{\phi}_i \\ \dot{\theta}_i \\ \dot{\psi}_i \end{bmatrix} \right) JT_i \cdot \begin{bmatrix} \dot{\phi}_i \\ \dot{\theta}_i \\ \dot{\psi}_i \end{bmatrix}\end{aligned}\quad (2)$$

The dynamics of a quadrotor can be generally divided into two parts, namely, the translational dynamics (1) in outer-loop, and the rotational dynamics (2) in inner-loop respectively.

B. Attitude control

Since the attitude angles and the angular velocities are measurable, the torque vector τ_i is given as follows

$$\tau_i = JT_i \bar{\tau}_i - JT_i \tilde{T}_i \begin{bmatrix} \dot{\phi}_i \dot{\theta}_i \\ \dot{\phi}_i \dot{\psi}_i \\ \dot{\theta}_i \dot{\psi}_i \end{bmatrix} - S^T (T_i \cdot \dot{\Theta}_i) JT_i \cdot \dot{\Theta}_i \quad (3)$$

where

$$\bar{\tau}_i = \ddot{\Theta}_i^d - \frac{k_{2\Theta_i}}{\varepsilon} (\dot{\Theta}_i - \dot{\Theta}_i^d) - \frac{k_{1\Theta_i}}{\varepsilon^2} (\Theta_i - \Theta_i^d) \quad (4)$$

Notations $k_{1\Theta_i}$ and $k_{2\Theta_i}$ represent two diagonal positive-definite gain matrices. Notations Θ_i^d , $\dot{\Theta}_i^d$ and $\ddot{\Theta}_i^d$ represent the desired Euler angles vector and their derivatives. The scalar $\varepsilon \in (0, 1]$.

Substituting (3) into (2), we obtain

$$\ddot{\Theta}_i = \ddot{\Theta}_i^d - \frac{k_{2\Theta_i}}{\varepsilon} (\dot{\Theta}_i - \dot{\Theta}_i^d) - \frac{k_{1\Theta_i}}{\varepsilon^2} (\Theta_i - \Theta_i^d) \quad (5)$$

Then, the attitude angles in Θ_i can track Θ_i^d exponentially. The converging speed is decided by the gain matrices $k_{1\Theta_i}$ and $k_{2\Theta_i}$. Let us denote $e_{\Theta_i} = \Theta_i - \Theta_i^d$ and $\tilde{e}_{\Theta_i} = \varepsilon e_{\Theta_i}$, then, we can rewrite (5) as follows

$$\varepsilon \frac{d}{dt} \begin{bmatrix} e_{\Theta_i} \\ \tilde{e}_{\Theta_i} \end{bmatrix} = \begin{bmatrix} \mathbf{0}_{3 \times 3} & I_3 \\ -k_{1\Theta_i} & -k_{2\Theta_i} \end{bmatrix} \begin{bmatrix} e_{\Theta_i} \\ \tilde{e}_{\Theta_i} \end{bmatrix} \quad (6)$$

If we define $d\bar{t}/dt = 1/\varepsilon$, then, we have

$$\frac{d \begin{bmatrix} e_{\Theta_i} & \tilde{e}_{\Theta_i} \end{bmatrix}^T}{d\bar{t}} = \frac{d \begin{bmatrix} e_{\Theta_i} & \tilde{e}_{\Theta_i} \end{bmatrix}^T}{d\bar{t}}$$

such that the dynamics of the attitude errors in (6) can be represented in the time scale \bar{t} as follows.

$$\frac{d \begin{bmatrix} e_{\Theta_i}(\bar{t}) & \tilde{e}_{\Theta_i}(\bar{t}) \end{bmatrix}^T}{d\bar{t}} = \begin{bmatrix} \mathbf{0}_{3 \times 3} & I_3 \\ -k_{1\Theta_i} & -k_{2\Theta_i} \end{bmatrix} \begin{bmatrix} e_{\Theta_i}(\bar{t}) \\ \tilde{e}_{\Theta_i}(\bar{t}) \end{bmatrix} \quad (7)$$

It is not difficult to verify that the origin of (7) is exponentially stable, such that we obtain

$$\begin{bmatrix} e_{\Theta_i}(\bar{t}) \\ \tilde{e}_{\Theta_i}(\bar{t}) \end{bmatrix} = \exp \left(\begin{bmatrix} \mathbf{0}_{3 \times 3} & I_3 \\ -k_{1\Theta_i} & -k_{2\Theta_i} \end{bmatrix} \bar{t} \right) \begin{bmatrix} e_{\Theta_i}(\bar{t}_0) \\ \tilde{e}_{\Theta_i}(\bar{t}_0) \end{bmatrix} \quad (8)$$

The initial value $\bar{t}_0 = 0$ is defined at $t = t_k$, where t_k is an arbitrary time instant in time scale t . Then the new time variable satisfies $\bar{t} = (t - t_k)/\varepsilon$. Then we obtain $t = t_k + \varepsilon \bar{t}$. If ε is sufficiently small, the variable t slowly varies in time scale \bar{t} . Intuitively, the time variable t is not ‘‘sensitive’’ to the change of \bar{t} because of the small ε . Since $\begin{bmatrix} \mathbf{0}_{3 \times 3} & I_3 \\ -k_{1\Theta_i} & -k_{2\Theta_i} \end{bmatrix}$ is negative definite, there exists a finite time \bar{t}_a in time scale \bar{t} such that $\exp \left(\begin{bmatrix} \mathbf{0}_{3 \times 3} & I_3 \\ -k_{1\Theta_i} & -k_{2\Theta_i} \end{bmatrix} \bar{t} \right)$ converges to a small neighborhood $\Omega(e_{\Theta_i}, \tilde{e}_{\Theta_i})$ of the equilibrium point $[e_{\Theta_i} \ \tilde{e}_{\Theta_i}] = \mathbf{0}_6$. This convergence dose not depend on the time instance t_k and the translational states $\mathcal{X}_i(t_k)$. Thus, the equilibrium point of system (7) is exponentially stable, uniformly in $(t, X_i(t), Y_i(t))$.

We recall that the torques in τ_i and the total thrust force F_{T_i} are generated by the thrusts of the four propellers of the quadrotor. In practice, the propellers have limit power.

According to (5), in order to prevent τ_i from hitting the input saturation, the terms Θ_i^d , $\dot{\Theta}_i^d$ and $\ddot{\Theta}_i^d$ should be bounded. Additionally, we require that the absolute values of the desired pitch and roll angles are smaller than $\pi/2$.

C. Altitude control

In this work, the desired altitude Z_i^d of each UAV is constant. In other words, the formation takes place in $o_e x_e y_e$ plan. The thrust force is given as follows

$$F_{T_i} = \frac{mg + m \cdot u_i^Z}{\cos \theta_i \cos \phi_i} \quad (9)$$

where u_i^Z represents the altitude controller.

Substituting F_{T_i} in (1), we obtain

$$\begin{aligned}\ddot{X}_i &= \left(\sin \psi_i \frac{\tan \phi_i}{\cos \theta_i} + \cos \psi_i \tan \theta_i \right) (u_i^Z + g) \\ \ddot{Y}_i &= \left(\sin \psi_i \tan \theta_i - \cos \psi_i \frac{\tan \phi_i}{\cos \theta_i} \right) (u_i^Z + g) \\ \ddot{Z}_i &= u_i^Z\end{aligned}\quad (10)$$

We observe from (10) that the altitude control is decoupled with the planar translational dynamics (X_i and Y_i). Thus, we give the altitude controller as follows

$$u_i^Z = \sigma_b(-k_{2Z} \dot{Z}_i - k_{1Z}(Z_i - Z_i^d)) \quad (11)$$

Function $\sigma_b(\cdot)$ represents a standard saturation function.

$$\sigma_b(a) = \text{sgn}(a) \min\{b, |a|\} \quad (12)$$

where ‘‘sgn’’ represents the sign function. We select $b = 1$. The controller (11) asymptotically stabilizes the altitude.

Since the altitude control is decoupled with the planar translational dynamics, the convergence of Z_i do not depend on the other states of the quadrotors except Z_i and \dot{Z}_i . When the altitude of the quadrotors gets stabilized, $u_i^Z \approx 0$.

III. FORMATION CONTROLLER DESIGN

Since the planar translational motions are generated by the attitude angles, the formation controller is to obtain the proper desired attitude angles Θ_i^d . If we design the desired attitude angles as follows

$$\begin{aligned} \theta_i^d &= \arctan\left(\frac{u_i^X}{g}\right) \\ \phi_i^d &= \arctan\left(\frac{-u_i^Y \cos(\arctan(u_i^X/g))}{g}\right) \\ \psi_i^d &= 0 \end{aligned} \quad (13)$$

where u_i^X and u_i^Y represent the formation controllers in direction x_e and y_e in the inertial frame. Then, the planar translational dynamics of UAV i yields

$$\begin{aligned} \ddot{x}_i &= u_i^X + \delta_i^X \\ \ddot{y}_i &= u_i^Y + \delta_i^Y \end{aligned} \quad (14)$$

The terms δ_i^X and δ_i^Y have the following expression.

$$\begin{aligned} \delta_i^X &= (g + u_i^Z) \cdot \left(\sin\Delta\psi_i \cdot \frac{\tan(\phi_i^d + \Delta\phi_i)}{\cos(\theta_i^d + \Delta\theta_i)} + \right. \\ &\quad \left. \cos\Delta\psi_i \cdot \tan(\theta_i^d + \Delta\theta_i) - \frac{g}{g + u_i^Z} \tan\theta_i^d \right) \\ \delta_i^Y &= (g + u_i^Z) \cdot \left(-\cos\Delta\psi_i \cdot \frac{\tan(\phi_i^d + \Delta\phi_i)}{\cos(\theta_i^d + \Delta\theta_i)} + \right. \\ &\quad \left. \sin\Delta\psi_i \cdot \tan(\theta_i^d + \Delta\theta_i) + \frac{g}{g + u_i^Z} \frac{\tan\phi_i^d}{\cos\theta_i^d} \right) \end{aligned}$$

where $\Delta\phi_i$, $\Delta\theta_i$ and $\Delta\psi_i$ represent the tracking error of the attitude angles $\Theta_i - \Theta_i^d = [\Delta\phi_i, \Delta\theta_i, \Delta\psi_i]$. We can observe that δ_i^X and δ_i^Y are in terms of u_i^Z , and $\Theta_i - \Theta_i^d$. According to (8), the attitude tracking error $\Theta_i - \Theta_i^d$ is bounded (since Θ_i^d is bounded) and exponentially converges to the origin. Additionally, $|u_i^Z| \leq 1$ and $u_i^Z \rightarrow 0$ when the altitude is stabilized. Thus, we obtain that δ_i^X and δ_i^Y are bounded.

A. Leader-Follower formation of UAVs

In this paper, an L-F formation is considered. The indices of the UAVs compose a set $\mathcal{V} = \{1, 2, \dots, n\}$, where n is the number of the UAVs in the formation. The subset $\mathcal{V}_L \subseteq \mathcal{V}$ represents the indices of the leaders, therefore, $\mathcal{V} - \mathcal{V}_L$ represents the set of followers.

Firstly, we define the ‘‘formation task’’ for a multi-quadrotor system as follows.

Definition 1 (Formation task): A formation task for a multi-quadrotor system with L-F architecture is represented by a desired formation trajectory (given to the leader(s))

and a desired geometric pattern (desired inter-distance and orientation between neighboring quadrotors) for the group of quadrotors.

The formation task describes the desired integral behaviors of the multiple quadrotors. The objective of the formation control is to accomplish the formation task, i.e., the quadrotors keep the desired pattern and track a given trajectory by using the formation controllers.

We consider the planar formation of UAVs. The desired altitude is given constant. We denote by $x_i = [X_i, Y_i]^T$ and $\dot{x}_i = [\dot{X}_i, \dot{Y}_i]^T$ the planar position and velocity vectors for UAV i . Each UAV can obtain the relative positions and velocities with respect to its neighbors, which can be represented by vectors $[(x_i - x_j)^T, (\dot{x}_i - \dot{x}_j)^T]^T$, $j \in \mathcal{N}_i$. If the UAV is a leader, besides the foregoing measurements, it can also obtain the relative position and velocity with respect to the formation trajectory $[(x_i - r(t))^T, (\dot{x}_i - \dot{r}(t))^T]^T$, where we denote by $r(t) = [r_X(t), r_Y(t)]^T$ the reference formation trajectory (RFT).

The rigid formation of quadrotors are considered in this paper. Thus, the formation controller is designed to guarantee that all the quadrotors track the RFT with some constant biases $d_{i0} = [d_{i0}^X, d_{i0}^Y] \in \mathbb{R}^2$, such that the quadrotors keep a constant formation pattern. Therefore, the desired trajectory for each UAV should satisfy $x_i^d(t) - r(t) = d_{i0}$ and $\dot{x}_i^d(t) - \dot{r}(t) = 0$, then, we obtain

$$x_i^d(t) = d_{i0} + r(t) \text{ and } \dot{x}_i^d(t) = \dot{r}(t)$$

An example of four UAVs with rigid formation task is shown in Fig.1.

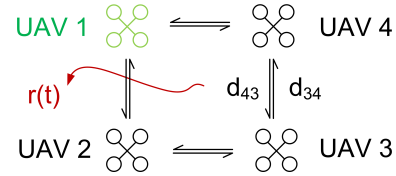


Fig. 1. Rigid formation task for four UAVs, where UAV 1 is a leader. The objective is to track the trajectory $r(t)$ with biases $d_{10} = [-1, 1]^T$, $d_{20} = [-1, -1]^T$, $d_{30} = [1, -1]^T$ and $d_{40} = [1, 1]^T$.

However, the RFT $r(t)$ is not available for the followers, therefore, the desired trajectory $x_i^d(t)$ for UAV i is not explicitly available. The behavior of each UAV depends on its neighbors states (and $r(t)$, if it is a leader), then, the formation task is finally achieved. The bias with respect to the RFT is attained implicitly by the constant inter-distances between a UAV and its neighbors. Then, the formation problem becomes: how to find the available desired trajectory for UAV $i \in \mathcal{V}$.

Let us make a sum of the relative position state vectors. Note that we drop the explicit expression of time in the related terms for the sake of simplicity.

$$\begin{aligned} &\sum_{j \in \mathcal{N}_i} (x_i - x_j - d_{ij}) && \text{if } i \in \mathcal{V} - \mathcal{V}_L \\ &\sum_{j \in \mathcal{N}_i} (x_i - x_j - d_{ij}) + x_i - r - d_{i0} && \text{if } i \in \mathcal{V}_L \end{aligned} \quad (15)$$

The inter-distance is given by $d_{ij} = d_{i0} - d_{j0}$. Then, equations (15) can be rewritten as follows

$$\sum_{j \in \mathcal{N}_i} (x_i - r - d_{i0} - (x_j - r - d_{j0})) \quad \text{if } i \in \mathcal{V} - \mathcal{V}_L$$

$$\sum_{j \in \mathcal{N}_i} (x_i - r - d_{i0} - (x_j - r - d_{j0})) + x_i - r - d_{i0} \quad \text{if } i \in \mathcal{V}_L$$

We introduce the available desired trajectory for each UAV as follows

$$\bar{x}_i^d = \begin{cases} \frac{1}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} (x_j + d_{ij}) & \text{if } i \in \mathcal{V} - \mathcal{V}_L \\ \frac{1}{|\mathcal{N}_i+1|} \left(\sum_{j \in \mathcal{N}_i} (x_j + d_{ij}) + r + d_{i0} \right) & \text{if } i \in \mathcal{V}_L \end{cases} \quad (16)$$

we then observe that \bar{x}_i^d is available for UAV i . We rewrite equation (15) in matrix form for all the quadrotors as follows

$$\begin{bmatrix} x_1 - \bar{x}_1^d \\ \vdots \\ x_n - \bar{x}_n^d \end{bmatrix} = (G \otimes I_2) \begin{bmatrix} x_1 - x_1^d \\ \vdots \\ x_n - x_n^d \end{bmatrix} \quad (17)$$

where $G \in \mathbb{R}^{n \times n}$ represents the normalized interaction matrix (see [10]). The symbol \otimes represents the Kronecker product. The formation task is achieved if G is invertible and for each UAV $i \in \mathcal{V}$, $x_i - \bar{x}_i^d \rightarrow 0$ asymptotically. According to [10], G is invertible if the graph of the multi-UAV system is connected and at least one leader exists. In this paper, we assume that this condition is satisfied. In the following subsection, we will investigate the formation controller which stabilizes the formation error $x_i - \bar{x}_i^d \rightarrow 0$.

B. Formation control

We firstly consider the nominal model of (14) by setting δ_i^X and δ_i^Y null. The dynamics of X_i and Y_i are similar, thus, we consider the dynamics of X_i for example. Note that in the sequel, the superscript “X” is omitted for the sake of simplicity. Then, we have

$$\dot{X}_i = u_i \quad (18)$$

Let us denote by $e_{iX} = X_i - \bar{X}_i^d$ the formation error in x_e direction. Let us define a nonlinear function $\rho_i: \mathbb{R}^2 \rightarrow \mathbb{R}$. We propose the formation controller as follows

$$u_i = -M \tanh(k_2 \dot{e}_{iX} + k_1 e_{iX}) - M \tanh(\rho_i(e_{iX}, \dot{e}_{iX})) + \ddot{X}_i^d$$

$$\rho_i(e_{iX}, \dot{e}_{iX}) = k_N \dot{e}_{iX} \quad (19)$$

where M represents a positive constant scalar. k_1 and k_2 are two positive constant gains. The symbol k_N represents a positive nonlinear gain. For example, we can select k_N as follows

$$k_N = \eta_1 \exp^{-\eta_2 e_{iX}^2} \quad (20)$$

where $\eta_1 \geq 0$, $\eta_2 > 0$. The selection of k_N in (20) has a physical meaning. When the formation error e_{iX} is large, the effect of $\rho_i(e_{iX}, \dot{e}_{iX})$ is small. In contrary, when e_{iX} approaches zero, the nonlinear gain k_N increases, which can reduce the approaching velocity, such that the overshoot is small.

Proposition 1: The formation controller u_i (19) is bounded.

Proof: According to (16), the desired trajectory \bar{X}_i^d for UAV i satisfies

$$\bar{X}_i^d = \begin{cases} \frac{1}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} (X_j + d_{ij}) & \text{if } i \in \mathcal{V} - \mathcal{V}_L \\ \frac{1}{|\mathcal{N}_i+1|} \left(\sum_{j \in \mathcal{N}_i} (X_j + d_{ij}) + r_X + d_{i0} \right) & \text{if } i \in \mathcal{V}_L \end{cases}$$

where d_{ij} , $d_{i0} \in \mathbb{R}$ (recall that the subscript “X” has been omitted here) represent some constant offset scalar. Then,

$$\dot{\bar{X}}_i^d = \begin{cases} \frac{1}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} \dot{X}_j & \text{if } i \in \mathcal{V} - \mathcal{V}_L \\ \frac{1}{|\mathcal{N}_i+1|} \left(\sum_{j \in \mathcal{N}_i} \dot{X}_j + \dot{r}_X \right) & \text{if } i \in \mathcal{V}_L \end{cases}$$

Let us denote by $\bar{u}_i = k_2 \dot{e}_{iX} + k_1 e_{iX}$. Then, if $i \in \mathcal{V} - \mathcal{V}_L$,

$$\ddot{\bar{X}}_i^d = \frac{1}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} \left(-M \tanh \bar{u}_j - M \tanh \rho_j + \ddot{X}_j^d \right)$$

such that

$$\ddot{\bar{X}}_i^d - \frac{1}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} \ddot{X}_j^d = -\frac{1}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} (M \tanh \bar{u}_j + M \tanh \rho_j)$$

if $i \in \mathcal{V}_L$,

$$\ddot{\bar{X}}_i^d = \frac{1}{|\mathcal{N}_i+1|} \left(\sum_{j \in \mathcal{N}_i} \left(-M \tanh \bar{u}_j - M \tanh \rho_j + \ddot{X}_j^d \right) + \ddot{r}_X \right)$$

such that

$$\ddot{\bar{X}}_i^d - \frac{1}{|\mathcal{N}_i+1|} \sum_{j \in \mathcal{N}_i} \ddot{X}_j^d$$

$$= -\frac{1}{|\mathcal{N}_i+1|} \left(\sum_{j \in \mathcal{N}_i} (M \tanh \bar{u}_j + M \tanh \rho_j) - \ddot{r}_X \right)$$

Without loss of generality, we assume that UAVs $1 \sim i$ are leaders, while $i+1 \sim n$ are followers. Then, we rewrite the foregoing equations for all the quadrotors in matrix form as follows

$$\begin{bmatrix} \ddot{\bar{X}}_1^d \\ \vdots \\ \ddot{\bar{X}}_n^d \end{bmatrix} = G^{-1} \cdot \begin{bmatrix} -\frac{1}{|\mathcal{N}_1+1|} \left(\sum_{j \in \mathcal{N}_1} (M \tanh \bar{u}_j + M \tanh \rho_j) - \ddot{r}_X \right) \\ \vdots \\ -\frac{1}{|\mathcal{N}_i+1|} \left(\sum_{j \in \mathcal{N}_i} (M \tanh \bar{u}_j + M \tanh \rho_j) - \ddot{r}_X \right) \\ -\frac{1}{|\mathcal{N}_i+1|} \sum_{j \in \mathcal{N}_{i+1}} (M \tanh \bar{u}_j + M \tanh \rho_j) \\ \vdots \\ -\frac{1}{|\mathcal{N}_n|} \sum_{j \in \mathcal{N}_n} (M \tanh \bar{u}_j + M \tanh \rho_j) \end{bmatrix} \quad (21)$$

The RFT is usually selected such that its derivatives are bounded. Then, we observe that $\ddot{\bar{X}}_i^d$, $i \in \mathcal{V}$ are bounded, since the function $\tanh(\cdot)$ is bounded by 1. Therefore, we conclude that u_i , $i \in \mathcal{V}$ are bounded. ■

Substituting (19) into the first equation in (18), we obtain

$$\ddot{e}_{iX} = -M \tanh \bar{u}_i - M \tanh \rho_i \quad (22)$$

Proposition 2: The origin of (22) is globally asymptotically stable, if i) k_N is given by (20); ii) the gains in \bar{u}_i satisfy $k_1 \geq \eta_1 k_2 M$.

Proof: Let us define a continuously differentiable, radially unbounded positive-definite function as follows

$$V_i = \text{Incosh}(k_2 \dot{e}_{iX} + k_1 e_{iX}) + \frac{k_1}{2M} \dot{e}_{iX}^2 \quad (23)$$

$V_i = 0$, if and only if $e_{iX} = 0$ and $\dot{e}_{iX} = 0$.

Then, the derivative of V_i yields

$$\begin{aligned} \dot{V}_i &= \tanh \bar{u}_i \cdot \dot{\bar{u}}_i + \frac{k_1}{M} \dot{e}_{iX} \ddot{e}_{iX} \\ &= \tanh \bar{u}_i (k_2 \ddot{e}_{iX} + k_1 \dot{e}_{iX}) + \frac{k_1}{M} \dot{e}_{iX} (-M \tanh \bar{u}_i - M \tanh \rho_i) \\ &= \tanh \bar{u}_i (k_2 (-M \tanh \bar{u}_i - M \tanh \rho_i) + k_1 \dot{e}_{iX}) \\ &\quad - k_1 \dot{e}_{iX} (\tanh \bar{u}_i + \tanh \rho_i) \\ &= -k_2 M \tanh^2 \bar{u}_i - k_2 M \tanh \bar{u}_i \tanh \rho_i - k_1 \dot{e}_{iX} \tanh \rho_i \end{aligned} \quad (24)$$

Then, we have

$$\dot{V}_i \leq -k_2 M \tanh^2(\bar{u}_i) - k_1 \dot{e}_{iX} \tanh \rho_i + k_2 M |\tanh \bar{u}_i| |\tanh \rho_i|$$

According to (19), ρ_i has the same sign as \dot{e}_{iX} . Then, $-k_1 \dot{e}_{iX} \tanh \rho_i \leq 0$.

- Case 1: If $|\rho_i| \leq |\bar{u}_i|$, $|\tanh \rho_i| \leq |\tanh \bar{u}_i|$. Then, $|\tanh \rho_i| |\tanh \bar{u}_i| \leq \tanh^2 \bar{u}_i$. Therefore,

$$\dot{V}_i \leq -k_1 \dot{e}_{iX} \tanh \rho_i \leq 0 \quad (25)$$

We obtain that \dot{V}_i is semi-definite negative. We then invoke here the LaSalle's invariance principle. Let us compute the largest invariant set where $\dot{V}_i = 0$, such that equation (24) equal to zero. We consider the following cases.

- Case a: if $\dot{e}_{iX} \neq 0$, then, according to (25), $\dot{V}_i < 0$, which contradicts $\dot{V}_i = 0$.
- Case b: if $\dot{e}_{iX} = 0$, then, according to (24), $\dot{V}_i = -k_2 M \tanh^2(k_1 e_{iX}) = 0$, which implies $e_{iX} = 0$.

Therefore, in Case 1, the largest invariant set contains only the origin.

- Case 2: If $|\rho_i| > |\bar{u}_i|$, $|\rho_i| > |\tanh \bar{u}_i| \geq 0$. Since $k_1 \geq \eta_1 k_2 M$, then, we have $k_2 M |\tanh \bar{u}_i| < k_1 |\dot{e}_{iX}|$. Thus,

$$\dot{V}_i \leq -k_2 M \tanh^2 \bar{u}_i \leq 0 \quad (26)$$

We obtain that \dot{V}_i is semi-definite negative. As mentioned before, we compute the largest invariant set where $\dot{V}_i = 0$, such that equation (24) equal to zero. We consider the following cases.

- Case a: if $|\bar{u}_i| \neq 0$, then, according to (26), $\dot{V}_i < 0$, which contradicts $\dot{V}_i = 0$.
- Case b: if $|\bar{u}_i| = 0$, then, according to (24), $\dot{V}_i = -k_1 \dot{e}_{iX} \tanh \rho_i = 0$, which implies $\dot{e}_{iX} = 0$. Since $\bar{u}_i = 0$, we obtain that $e_{iX} = 0$.

Therefore, in Case 2, the largest invariant set contains only the origin.

Since the largest invariant set contains only the origin, then, according to the LaSalle theorem, the origin of the system (22) is globally asymptotically stable. ■

Remark 1: The decoupled property of X and Y dynamics permits us to design the formation controller separately, the design procedure of u_i^Y is the same as u_i^X , such that

$$u_i^Y = -M \tanh(k_2 \dot{e}_{iY} + k_1 e_{iY}) - M \tanh \rho_i (e_{iY}, \dot{e}_{iY}) + \ddot{Y}_i^d \quad (27)$$

Remark 2: The desired attitude angles for each UAV are obtained by substituting (19) and (27) into (13). Owing to the use of the hyperbolic tangent function, u_i^X and u_i^Y are high-order differentiable, such that the derivatives θ_i^d , $\dot{\theta}_i^d$ and ϕ_i^d , $\dot{\phi}_i^d$ are bounded.

Let us reconsider the terms δ_i^X and δ_i^Y in (14) caused by the tracking error of the attitude angles. We introduce our main result in the following theorem.

Theorem 1: Let G be the interaction matrix of an L-F formation of quadrotors with constant topology. The formation controllers are given in (19) and (27). The attitude is controlled by (3). The altitude is controlled by (9) where u_i^Z is given in (11). Then, e_{iX} and e_{iY} converge to zero asymptotically, if i) G is invertible, ii) the initial velocity of each UAV is finite.

Proof: The terms δ_{iX} and δ_{iY} are in terms of the tracking errors of the attitude angles ($\Delta\phi_i$, $\Delta\theta_i$ and $\Delta\psi_i$) and u_i^Z . According to (21), (19), (27) and (11), the controllers u_i^X , u_i^Y and u_i^Z are bounded. Then, according to (13), we know that θ_i^d , ϕ_i^d and ψ_i^d are bounded. The attitude angles are controlled by (3), then, $\Delta\phi_i$, $\Delta\theta_i$ and $\Delta\psi_i$ are bounded. Therefore, δ_{iX} and δ_{iY} are bounded.

We take e_{iX} for example. Then, (22) becomes

$$\ddot{e}_{iX} = -M \tanh \bar{u}_i - M \tanh \rho_i + \delta_{iX} \quad (28)$$

The derivative of the positive semidefinite function V_i in (23) yields

$$\begin{aligned} \dot{V}_i &= \tanh \bar{u}_i \cdot \dot{\bar{u}}_i + \frac{k_1}{M} \dot{e}_{iX} \ddot{e}_{iX} \\ &= \tanh \bar{u}_i (k_2 \ddot{e}_{iX} + k_1 \dot{e}_{iX}) - \frac{k_1}{M} \dot{e}_{iX} (M \tanh \bar{u}_i + M \tanh \rho_i - \delta_{iX}) \\ &= \tanh \bar{u}_i (k_2 (-M \tanh \bar{u}_i - M \tanh \rho_i + \delta_{iX}) + k_1 \dot{e}_{iX}) \\ &\quad - k_1 \dot{e}_{iX} (\tanh \bar{u}_i + \tanh \rho_i + \frac{\delta_{iX}}{M}) \\ &= -k_2 M \tanh^2 \bar{u}_i - k_2 M \tanh \bar{u}_i \tanh \rho_i - k_1 \dot{e}_{iX} \tanh \rho_i \\ &\quad + k_2 \tanh \bar{u}_i \delta_{iX} - \frac{k_1}{M} \dot{e}_{iX} \delta_{iX} \\ &\leq -k_2 M \tanh^2 \bar{u}_i - k_2 M \tanh \bar{u}_i \tanh \rho_i - k_1 \dot{e}_{iX} \tanh \rho_i \\ &\quad + (k_2 |\tanh \bar{u}_i| + \frac{k_1}{M} |\dot{e}_{iX}|) |\delta_{iX}| \end{aligned}$$

Since the states of a linear system will not diverge to infinite within finite time interval with bounded control input, then, if the initial condition is in the compact set $\{(e_{iX}(t_0), \dot{e}_{iX}(t_0)) | \dot{e}_{iX}(t_0) < \infty\}$, we obtain that \dot{e}_{iX} is bounded in finite time interval $[t_0, t_1]$, where $t_1 < \infty$. As analyzed in section II-B, the rotational dynamics of a quadrotor is in the fast time scale \bar{t} with controller (4). Then, there exists a finite time $\bar{t}_a \ll t_1$ such that the attitude errors e_{θ_i} enter the neighborhood of the origin, which renders $|\delta_{iX}(\bar{t}_a)| \leq \zeta$,

where ζ is a scalar. Then, for $t > \bar{t}_a$, we have

$$\left(k_2 |\tanh \bar{u}_i| + \frac{k_1}{M} |\dot{e}_{iX}| \right) |\delta_{iX}| \leq |k_2 M \tanh^2 \bar{u}_i + k_2 M \tanh \bar{u}_i \tanh \rho_i + k_1 \dot{e}_{iX} \tanh \rho_i| \quad (29)$$

Then, $\dot{V}_i \leq 0$, when $t > \bar{t}_a$. Therefore, the semi-global asymptotic stability of the origin of system (28) is derived, when $t > \bar{t}_a$. ■

Remark 3: In theorem 1, if the condition (29) is satisfied when $\bar{t}_a = t_0$, the semi-global stability of the system (28) is obtained.

IV. SIMULATION AND EXPERIMENTAL RESULTS

Heudiasyc laboratory has developed a PC-based simulator-experiment framework for controlling a quadrotor and also a flock of quadrotors. The programs (written in C++) running in the UAVs are the same, both in the simulator and in the embedded processors of real UAVs. When we realize the real-time experiment, the PC acts as a ground station, which is responsible for displaying and sending instructions such as taking off and landing. The UAVs are all autonomous.

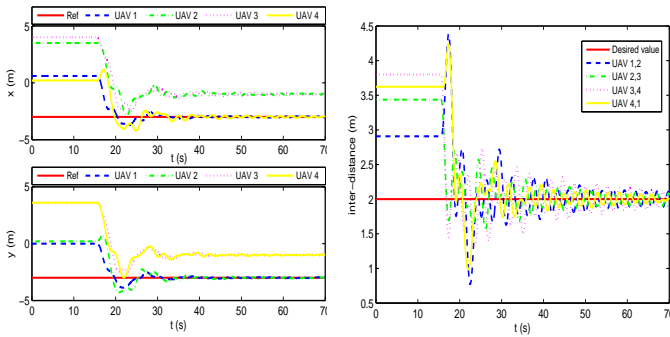


Fig. 2. Formation of four quadrotors without CNF

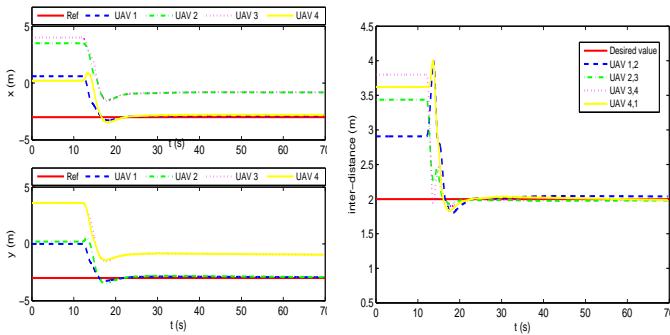


Fig. 3. Formation of four quadrotors with CNF

The formations with and without the CNF are shown in Fig.2 and Fig.3 respectively. In this test, the formation controllers (19) and (27) are simplified by setting $\dot{X}_i^d = 0$ and $\dot{Y}_i^d = 0$. We observe that in both formations, the quadrotors are able to converge to the desired positions with hyperbolic tangent function-based controller. However, after adding CNF, the performance is greatly improved.

The foregoing tests validated by using the simulator are carried out in the real-time experiments. The output curves

are shown in Fig.4. We observe that the formation with the CNF controller (on the right) has small overshoot and rapid response. The corresponding video is available on the site

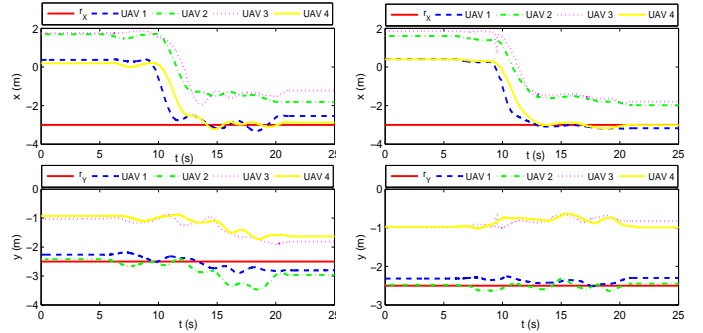


Fig. 4. Real-time formation of four quadrotors. Left: without CNF; Right: with CNF

<https://youtu.be/tDBiRAGp6r0>

V. CONCLUSION

In this paper, the CNF based bounded formation controller is proposed. The hyperbolic tangent function is used to generate a differentiable formation controller. The simulation and the experimental results show that the CNF based formation controller with hyperbolic tangent function have better performance than a controller without CNF and a controller with a standard saturated function.

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