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Experimental demonstration of an efficient number diagnostic for long ion chains

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Very long, one-dimensional ion chains are the basis for many applications, in particular in quantum information processing, and reliable diagnostics are needed to quantify them. To that purpose, we have experimentally tested Dubin's model for very long ion chains [Phys. Rev. E 55, 4017 (1997)] which describes the equilibrium state of a one-dimensional correlated system. For chains larger than 100 ions, this diagnostic allows us to determine the number of trapped ions with a 8%-precision in a 95% confidence interval, without counting individual ions. This measurement is based on the experimental determination of the ion-ion distance of the innermost particles and of the trapping potential along the ion chain direction. The agreement of the model with experiments allows one to determine the degree of local homogeneity of the ion crystal in the center of the chain.

PACS numbers: 64.70.kp Specific phase transition - Ionic crystals, 05.65.+b Self-organized systems, 03.67.Lx Quantum computation architectures and implementations

I. INTRODUCTION

Trapped one-dimensional ion chains encounter increasing interest in different areas of modern physics, as they constitute controllable model systems to test fundamental questions. They are among the most attractive candidates in quantum information [1–3], and they serve as model systems for the study of nano-friction [4–6], or investigations of the Kibble-Zurek mechanism [7–9].

Regarding trapped ions as support for quantum computation, the key challenge now is to increase the number of trapped ion qubits to a level where simulations that are intractable otherwise might become possible[10]. One strategy pursued for up-scaling is to operate with small linear ion chains and multiplex the system by shuttling ions between multiple chains, which requires a dedicated electrode structure and very precise control of ion trajectories [11]. Another approach is based on a long linear trap in order to store a single ion-chain containing more than a hundred ions [1]. One of the issues raised by this approach is the structural stability of the chain, which requires the aspect ratio of the trapping potential to fulfil a condition which depends on the number of trapped ions [12, 13]. It then becomes mandatory to have an efficient diagnostic of the number of trapped ions, in a regime where simply counting them on a recorded image, like routinely done for up to 50 ions, may not be feasible anymore. Furthermore, it would be useful to have a global criterion linking the minimum number of ions to the local homogeneity at the chain center, opening the way for the implementation of dedicated quantum information protocols [1].

In this article, we propose an efficient diagnostic for the total number of ions in a chain in a harmonic potential, even if only the central part of the chain can be recorded. This diagnostic does not rely on the individual counting

of each ion but on the measurement of the average inter-ion distance at the center of the chain. To that purpose, we experimentally test the model developed by Dubin in the 1990s [12, 14], which allows the total number of ions in the trap to be deduced with an ultimate precision of a few percent. This measurement is also a diagnostic for the local homogeneity of the chain.

A one-dimensional chain of identical, charged particles can be experimentally created in a linear radiofrequency (rf) trap. In most linear rf traps, the trapping potential along the chain axis can be very well approximated by a harmonic potential and the equilibrium position of each particle along the trap axis results from the balance between the Coulomb interactions with all other particles and the restoring trapping force. In many experiments, the trapped ions are laser-cooled to temperatures below 10 mK and once crystallised in a stable structure, they make small oscillations around their equilibrium position. They are typically separated by a distance of several μm enabling individual detection of their fluorescence and/or individual addressing by a tightly focussed laser beam. The observation of ion positions for chains longer than a hundred particles allows one to experimentally study a correlated system where the correlation energy between particles is as important as the mean-field energy [14]. This situation is rarely met. For example, in a 3D-system, it takes a $1/r^3$ interaction potential to fulfil this condition. The strong role of particle correlation in the equilibrium energy makes analytical calculations complicated. For a 1D-system, D.H.E. Dubin takes into account these correlations through the Local Density Approximation and is able to describe the dependency of the local density with the chain length and the total number of particles[12, 14]. Once confirmed, these theoretical laws can be used as an efficient diagnostic for the particle number and local homogeneity for long ion chains, suppressing the need for individual counting of the ions.

To test these laws, we confront model predictions with precise experimental results. Our experiments are based on the imaging of the laser induced fluorescence of in-

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dividual ions on a intensified CCD camera. Long ion chains are routinely created in several groups and parts of ion chains have been observed and reported before (e.g. [8, 15]) but they have been limited to ion numbers below 40, mostly because of the limits of the collecting optics set-up. Our experimental set-up allows the controlled translation of the imaging optics along the trap axis without destruction of the chain [16], in order to record pictures at various positions along the ion string which can be joined to build a picture of the complete string.

In this article, we present the above cited model for ion string structures, then describe the experimental realisation of very long ion chains, followed by a discussion of the uncertainty budget of the method.

II. SCALING LAWS FOR LONG COULOMB CHAINS

A very prolate potential shape is needed to force a sample of the order of 100 ions to organise as a chain. In a trapping potential which is harmonic in the three directions of space, the structural phase transitions of linear Coulomb crystals are controlled by the number N of trapped particles and by the trapping potential aspect ratio ρ , which reduces to the ratio of the axial over radial potential steepness $\rho = \omega_z^2/\omega_r^2$ in the case of a linear trap with cylindrical symmetry [12, 13, 17, 18]. In the following, we assume that the stability condition for a linear chain is satisfied. In [12, 14], Dubin treats the N ion chain as a charged fluid at zero temperature with total charge NQ and shows that in the frame of this approximation the local density, or number of ions per unit length along the chain, can be chosen to be

$$1/a(z) = \left(\frac{3N}{4L} \left(1 - \frac{z^2}{L^2} \right) \right) \quad (1)$$

where $a(z)$ is the distance between closest neighbours, z is the distance from the chain centre and $2L$ is the chain's total length. This equation reflects the increasing inter-ion distance for larger distance to the chain center, which is well confirmed by numerical simulations based on potential energy minimization [14, 19] and experimental observations. According to Dubin, this trial function (Eq. 1) slightly overestimates the chain length but its simple form allows the comparison with experimental results. This is the main limit of the model and as an example, the overestimation reaches 12% for a 160 ion chain [14].

Taking into account correlations between ions, Dubin derived the half-length L in terms of the two-body equilibrium length scale $l = (Q^2/4\pi\epsilon_0 m\omega_z^2)^{1/3}$ by

$$L = l(3N)^{1/3}(\ln N + \ln 6 + \gamma_e - A)^{1/3} \quad (2)$$

with $\gamma_e \approx 0.577$ the Euler constant (m is the particle mass).

Describing the relevant physical parameters (length and reduced density) of a chain with a $(\ln N)^{-1}$ power series, Dubin obtains a value for $A = 7/2$, by neglecting terms of the order of $(\ln N)^{-2}$ [14], improving his earlier results [12]. Later work [18] confirmed the scaling law $L \simeq l(3N \ln N)^{(1/3)}$ for $N \gg 1$. To be consistent, the power series description of Dubin also concerns the local density $1/a(z)$. If we combine the power series of the chain length and of the density, we obtain an expression for the smallest inter-ion distance a_0^D at the chain center,

$$a_0^D = 4l(3N)^{-2/3} \left[(\ln N + \ln 6 + \gamma_e - A)^{-1/3} + B(\ln N)^{-4/3} \right]^{-1} \quad (3)$$

where A is $7/2$ and $B = -5/2 + 3 \ln 2$. This equation allows the number of ions in the chain to be deduced from the measurement of the smallest inter-ion distance a_0 and the characteristic length l , with an error of the order of $(\ln N)^{-2}$ which is equivalent to $\pm 4\%$ for the largest chain we could observe.

III. EXPERIMENTAL SET-UP

Our experimental set-up for trapping long ion chains is based on a linear radio-frequency trap where the distance between the trap center and the rods polarised by rf-voltage is $r_0 = 3.93$ mm and the distance between the plane electrodes confining the ions along the trap z -axis is 96 mm, justifying that we neglect the role of image charges [14]. The $V_{dc} = 2000$ V applied to the z -electrodes results in a harmonic potential of $\omega_z/2\pi = 3.1 \pm 0.15$ kHz (95% confidence interval). Ions are created by isotope selective photo-ionisation of a neutral calcium beam and are then shuttled to the part of the trap where the long chain is formed [16] to avoid perturbation of the potential by any calcium deposit on the electrodes. Ions are laser-cooled by two collimated laser beams close to resonance with the $[4S_{1/2} - 4P_{1/2}]$ -transition, of equal power (397 nm, 2 mW on a 2 mm $1/e^2$ diameter), counter-propagating along the trap axis. Once excited from the ground state, calcium ions can relax to a long-lived metastable state $[3D_{3/2}]$ from which they have to be re-pumped to maintain efficient laser cooling. This re-pumping process is assured by a 866 nm laser beam [resonant with $3D_{3/2} - 4P_{1/2}$] of approximately 2.5 mW and 4 mm $1/e^2$ diameter which co-propagates with one of the cooling lasers. The trapped ions are observed through their laser induced fluorescence on the cooling transition which is collected along a direction perpendicular to the trap symmetry axis, and projected onto a photomultiplier (PM) and an intensified charge coupled device (CCD) camera. The camera screen is an array of 1024×1024 pixels with a pixel size of 13 μm . In order to obtain a sufficient spatial resolution to distinguish ions separated by a few μm , an optical magnification larger than 10 is required, which limits the number of observable ions to less than 100. These values are representative for many ion-trap experiments. In our

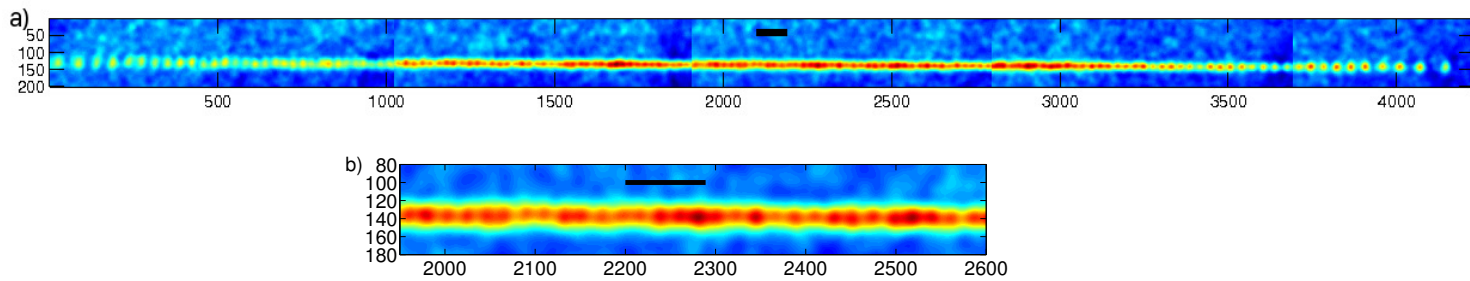


FIG. 1. a) Picture of an ion chain of 155 ions composed from 5 pictures taken with a translated objective and b) picture of the central part of the chain. Axes are in pixels whose size is $13 \mu\text{m}$ and the optical magnification is 11.58. The scale bar stands for $100 \mu\text{m}$. The integration time is 1s.

set-up, the magnification is 11.58, and the total length of the longest observed chain is 4.6 mm. This observation is made possible by the translation of the imaging optics parallel to the trap's symmetry axis from one end of the chain to the other. The translation stage holding the collecting objective is controlled by a micrometer screw with graduations of $10 \mu\text{m}$. Translations were operated always in the same direction and result in a $\pm 5 \mu\text{m}$ uncertainty on the relative position of two consecutive pictures. The comparison between pictures taken throughout the translation of the objective along the two opposite directions gives us an estimation of the ion loss that can affect a chain during the data acquisition. The largest loss was 4 ions and was observed for some chains containing more than 100 ions. It can lead to an overestimation by few units of the total number of ions deduced from the central picture compared to the one deduced by individual count of the ions.

The translation step of 1 mm is smaller than the camera sensor size in the object space. Some ions are then redundant and are recorded on two consecutive pictures. The average inter-ion distance is 22 pxl which is larger than the uncertainty associated to one translation (± 5 pxl) and which results in 6 redundant ions for each translation (4 for the last one). These numbers are sufficient to check precisely the consistency of the superposition and deduce an exact number of 155 ions in the chain in Figure (1) from counting the individual ion fluorescence images. The figure shows the global picture of the whole chain which can be composed by several images. The variation in the fluorescence signal may be due to spatially varying radial secular motion or micro-motion made possible by insufficient cooling of the radial motion by the two longitudinal laser beams. It may also be the signature of dark ions hopping along the chain. The total length of the chain is measured to be $2L_m = 4620 \pm 25 \mu\text{m}$. The set of nearest neighbour distances $a(z)$ is extracted from the data set obtained by vertical binning of the image, and multiple-Gaussian fit to the resulting fluorescence profile. The thus obtained $a(z)$ are represented on Fig. (2).

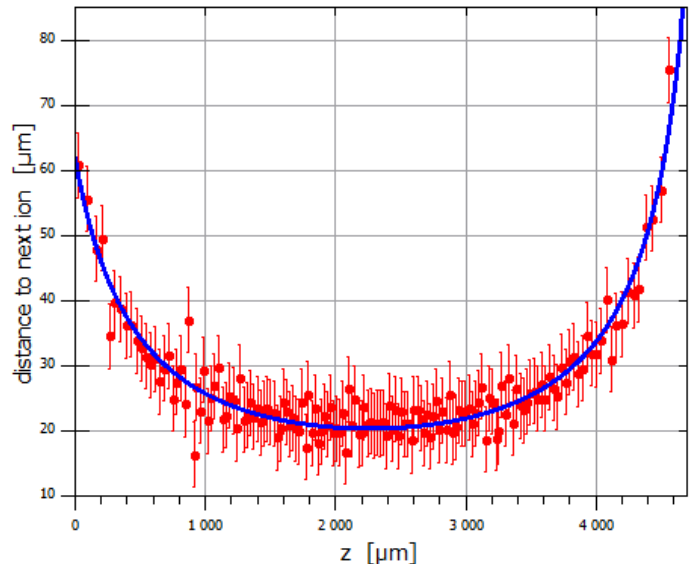


FIG. 2. Red dots : distance between neighbouring ions $a(z)$ vs their position in the chain, in μm , in a 155-ion chain. Blue line : best fit of the inverse of the distance by Eq. (1) where z is replaced by $z - z_0$ and N is fixed to 155. The error bar stands for the ± 5 pxls of the typical FWHM of individual ion picture.

IV. DATA ANALYSIS

We have fitted these data with Eq. (1) fixing N to 155. The best fit finds a chain center z_0 located at the physical center of the chain and a chain length ($2L_f = 5420 \pm 30 \mu\text{m}$) which is more than 17% larger than the measured physical length $2L_m$. Like mentioned earlier, this overestimation is expected from the simple equation form given in Eq.(1) and also includes experimental uncertainties. However, Dubin's model allows to be used in a form relating the inter-ion distance at the center of the chain a_0 to the ion number, without an explicit dependence on L , as given by Eq. (3). To demonstrate the relevance of using only the central pic-

ture of the chain to deduce an ion number, we calculate a_0 based on the relative position of the 20 central ions. The result is then $\bar{a}_0 = 24.1 \pm 1.0 \mu\text{m}$ (95% confidence). Assuming for l the value determined from the direct measurement $\omega_z/2\pi = 3.1 \pm 0.15 \text{ kHz}$, we can calculate the number N_D of ions in the chain from Eq. (3). The result is $N_D = 151 \pm 12$ where the two major causes of uncertainty are, in order of importance, statistics and the uncertainty in the determination of ω_z .

To reduce the experimental uncertainty of a_0 , and if the optical field of view allows for it, one can average $a(z)$ for more central ions. The number N_a of included ions can not be extended too far as there is a quadratic dispersion in inter-ion distance with the distance from the center. If we use Eq. (1) around the chain center, one can show that the dispersion in the distance to the next neighbours behaves like

$$\frac{\delta a}{a} = \left(\frac{2N_a}{3N} \right)^2. \quad (4)$$

In practice, the gain of adding more ions is limited. For the above described ion chain, the minimum of the standard deviation is reached by averaging over $N_a = 80$ ions, dividing the standard deviation not even by a factor of 2. The corresponding mean value a_0 is reduced by less than 1.5%, far less than one standard deviation.

The optical magnification is measured at the chain center, by translating the objective across the trap axis and keeping the detector fixed. 8 successive translations of the objective, perpendicular to the optical axis and to the chain axis, over a total range of 1 mm, by a translation stage with a $10 \mu\text{m}$ resolution screw result in an uncertainty which is an order of magnitude lower than the magnification shift induced by the translation of the objective along the trap axis. The tiny modification of the objective-to-image distance induces a change in the measured magnification of the order of 2 ppm, an order of magnitude lower than the statistical dispersion of the data around their mean.

Data analysis was carried out for several chains of different ion number ranging from 46 to 155. To confirm the relevance of Dubin's trial function, Fig. 3 plots the local density $1/a(z)$ for three of them together with the fit by Eq. (1). Although the parameter L extracted from the fits systematically overestimates the physical length of these chains, the local density profile is very well represented by Eq. (1). The differences between the calculated ion numbers and the directly counted ones lie within the above measured 8% uncertainty for chains larger than 100 ions. This difference increases for shorter chains, reaching values up to 15%. For these smallest chains, an alternative tool to study self-organization of ions in a 1D potential is potential energy minimization. In [19], James proposed an empirical law for the closest distance a_0^J fitting its numerical results for $N \leq 50$:

$$a_0^J = 2.018lN^{-0.559}. \quad (5)$$

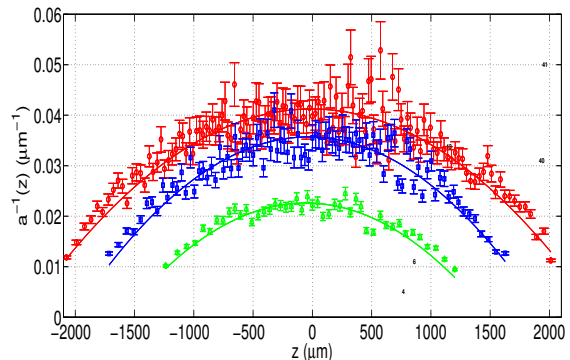


FIG. 3. Local density $1/a(z)$ vs the ion position in the chain, in μm , measured for chains containing 134 ions (red circles), 95 ions (blue squares), 47 ions (green triangle). The lines are the best fit by Eq. (1) where z is replaced by $z - z_0$ and N is fixed to the counted value.

For short ion strings, and even if the law deduced from Dubin's work is only relevant in the limit of large N , it systematically agrees to the experimental observations better than the prediction from Eq. (5) by a few percent.

In conclusion, we have experimentally tested the local description derived for long, one-dimensional Coulomb systems in a harmonic potential, an example of a strongly correlated system. The derived law is useful as a diagnostic to prepare chains with sufficient ions to have a minimum number of equidistant particles in its central part. The total number of ions can be derived without individually counting them, if the potential well along the chain direction is known. With the described ion chain, we were able to retrieve the ion number from a single picture with a uncertainty of $\pm 8\%$ due to fluctuations in ion positions and the uncertainty on the axial potential. Furthermore the simple form of (Eq. 1) can be used to describe the density at the center of the chain for the minimum energy state. Without any reference to the total length L , one can deduce from (Eq. 4) the minimum number of ions N required to have N_a nearly equidistant ions within a chosen tolerance. Such a system could be the starting point for explorations of quantum information protocols as proposed in [1]. The validation of this straightforward number diagnostic which does not require fastidious counting of individual ions, makes these systems simpler to control, and therefore more attractive.

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