Performance analysis of MDL criterion for the detection of noncircular or/and nonGaussian components

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PERFORMANCE ANALYSIS OF MDL CRITERION FOR THE DETECTION OF NONCIRCULAR OR/AND NONGAUSSIAN COMPONENTS

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ABSTRACT

This paper presents an asymptotic analysis of the eigen value decomposition (EVD) of the sample covariance matrix associated with independent identically distributed (IID) non necessarily circular and Gaussian data that extends the well known analysis presented in the literature for circular and Gaussian data. Closed-form expressions of the asymptotic bias and variance of the sample eigenvalues and eigenvectors are given. As an application of these extended expressions, the statistical performance analysis of the minimum description length (MDL) criterion applied to the detection of the number of noncircular or/and nonGaussian components is considered.

Index Terms— Eigen value decomposition, sample covariance matrix, eigenvalue, eigenvector, minimum description length, source detection, noncircular, nonGaussian.

1. INTRODUCTION

Eigenvalues and eigenvectors of sample covariance matrices are used in the solution of a wide range of statistical signal processing problems. The first and second-order statistics of this EVD are needed to assess the performance in terms of bias/variance of estimators or probability of events derived from these eigenvalues and eigenvectors. The statistics of the sample eigenvalues and eigenvectors have been widely studied in the statistical literature (e.g., [1],[2]) and are quoted in standard texts such as [3] and [4]. But to the best of our knowledge, all the published results has been derived only under the assumption of real-valued or circular complex-valued Gaussian distributions of the data.

The main aim of this paper is to extend these results to arbitrary real or complex fourth-order distributions of the data, where closed-form expressions of asymptotic bias and variance of the sample eigenvalues and eigenvectors are derived. As an application of these extended expressions, the statistical performance analysis of the widely-used MDL criterion introduced by Rissanen [5] and popularized by Wax and Kailath [6], is considered to the detection of the number of noncircular or/and nonGaussian components. But naturally, these extended expressions may find other applications in multivariate analysis. We focus here on the probability of underestimating the number of sources in the case of a single or two sources under asymptotic conditions (with respect to the number of snapshots) and around the threshold regions, following the approach proposed in [7], and then recently improved in [8]. We show in particular that the numerical values of this probability of underestimating the number of sources given by the MDL detector derived from the standard sample covariance matrix is not robust to the noncircularity and/or the nonGaussianity of the data.

2. ASYMPTOTIC STATISTICS OF EVD OF SAMPLE COVARIANCE MATRICES

Consider a sequence $\mathbf{x}_{t=1,..,T} \in \mathbb{C}^n$ of IID zero-mean complex multidimensional random variables (RV) with finite 4-th-order moments. The standard covariance matrix, the complementary covariance matrix and the quadrivariance matrix of $\mathbf{x}_t$ are respectively given by $\mathbf{R} \stackrel{\text{def}}{=} \mathrm{E}(\mathbf{x}_t\mathbf{x}_t^H)$, $\mathbf{R}' \stackrel{\text{def}}{=} \mathrm{E}(\mathbf{x}_t\mathbf{x}_t^T)$ and $(\mathbf{Q})_{i+(j-1)n,l+(k-1)n} \stackrel{\text{def}}{=} \mathrm{Cum}(x_{t,i}, x_{t,j}^*, x_{t,k}, x_{t,l}^*)$ where $\mathbf{x}_t = (x_{t,1}, x_{t,2}, ..., x_{t,n})^T$. The sample covariance estimate of $\mathbf{R}$ is usually defined as $\widehat{\mathbf{R}} \stackrel{\text{def}}{=} \frac{1}{T}\sum_{t=1}^T \mathbf{x}_t\mathbf{x}_t^H = \mathbf{R} + \delta\mathbf{R}$ which can be considered as a perturbation of $\mathbf{R}$. To derive the asymptotic distribution of the EVD of $\widehat{\mathbf{R}}$, we need the following extension of an identity derived under the circular Gaussian distribution of the data in [4, p.114], that is proved in [9]

Lemma 1 *For arbitrary vectors $(\mathbf{a}_i)_{i=1,...,4} \in \mathbb{C}^n$ and distributions with finite 4-th-order moment of $\mathbf{x}_t$ not necessarily circular and Gaussian, we have*

$$\begin{aligned}\mathrm{E}[(\mathbf{a}_1^H\delta\mathbf{R}\mathbf{a}_2)(\mathbf{a}_3^H\delta\mathbf{R}\mathbf{a}_4)] &= \frac{1}{T}\big\{(\mathbf{a}_1^H\mathbf{R}\mathbf{a}_4)(\mathbf{a}_3^H\mathbf{R}\mathbf{a}_2) \\ &+(\mathbf{a}_1^H\mathbf{R}'\mathbf{a}_3^*)(\mathbf{a}_2^T\mathbf{R}'^*\mathbf{a}_4) + (\mathbf{a}_2^T\otimes\mathbf{a}_1^H)\mathbf{Q}(\mathbf{a}_3^*\otimes\mathbf{a}_4)\big\} \quad (1)\end{aligned}$$

We assume that the eigenvalues $(\lambda_i)_{i=1,..,n}$ of $\mathbf{R}$, ordered in decreasing order satisfy the condition $\lambda_1 > ... > \lambda_r > \lambda_{r+1} = ... = \lambda_n = \sigma^2$. Let $(\mathbf{v}_i)_{i=1,..,n}$ be an arbitrary set of associated orthonormal eigenvectors. We note that $(\mathbf{v}_i)_{i=r+1,..,n}$ are defined up to an arbitrary unitary transformation, in contrast to $(\mathbf{v}_i)_{i=1,..,r}$ that are defined up to

multiplicative unit modulus complex number, but which are arbitrarily fixed[1]. Consider now for a "small enough" perturbation term $\delta\mathbf{R}$ of $\mathbf{R}$, the EVD of $\widehat{\mathbf{R}}$

$$\widehat{\mathbf{R}}\bar{\mathbf{v}}_i = \widehat{\lambda}_i\bar{\mathbf{v}}_i, \;\; i=1,...,n, \tag{2}$$

where $(\widehat{\lambda}_i)_{i=1,...,n}$ denotes the perturbation of λ_i that satisfies $\widehat{\lambda}_1 > \widehat{\lambda}_2 > ... > \widehat{\lambda}_n$, and where $(\bar{\mathbf{v}}_i)_{i=1,...,r}$ are the associated eigenvectors uniquely determined from $(\mathbf{v}_i)_{i=1,...,r}$ by

$$\bar{\mathbf{v}}_i^H\mathbf{v}_i = 1, \;\; i=1,...,r. \tag{3}$$

We consider in the following the eigenvectors $(\widehat{\mathbf{v}}_i)_{i=1,...,r}$ defined by $\widehat{\mathbf{v}}_i = \bar{\mathbf{v}}_i/||\bar{\mathbf{v}}_i||$. We have proved in [9] that these estimates are asymptotically Gaussian distributed with the same convergence speed $\sqrt{T}$ whose first and second-order statistics of this asymptotic distribution are given by the following result

Result 1 *The asymptotic first and second-order statistics of the estimates $(\widehat{\lambda}_1,...,\widehat{\lambda}_r,\widehat{\mathbf{v}}_1,...,\widehat{\mathbf{v}}_r)$ for arbitrary distributions with finite 4-th-order moment of $\mathbf{x}_t$ not necessarily circular and Gaussian are given by*

$$\mathrm{E}(\widehat{\lambda}_i) = \lambda_i + \frac{1}{T}\sum_{1\le k\ne i\le n}\frac{\lambda_i\lambda_k + |\lambda_{i,k}|^2 + \lambda_{i,k,i,k}}{\lambda_i-\lambda_k} + o(\frac{1}{T}) \tag{4}$$

$$\mathrm{Cov}(\widehat{\lambda}_i,\widehat{\lambda}_j) = \frac{1}{T}\left(\lambda_i^2\delta_{i,j} + |\lambda_{i,j}|^2 + \lambda_{i,i,j,j}\right) + o(\frac{1}{T}) \tag{5}$$

$$\begin{aligned}\mathrm{E}(\widehat{\mathbf{v}}_i) &= \left(1 - \frac{1}{2T}\sum_{1\le k\ne i\le n}\frac{\lambda_i\lambda_k + |\lambda_{i,k}|^2 + \lambda_{i,k,i,k}}{(\lambda_i-\lambda_k)^2}\right)\mathbf{v}_i \\ &+ \frac{1}{T}\sum_{1\le k\ne i\le n}\left(-\frac{\lambda_{i,k}\lambda_{i,i}^* + \lambda_{i,i,k,i}}{(\lambda_i-\lambda_k)^2}\right. \\ &+ \left.\sum_{1\le l\ne i\le n}\frac{\lambda_{k,l}\lambda_{i,l}^* + \lambda_{l,k,l,i}}{(\lambda_i-\lambda_k)(\lambda_i-\lambda_l)}\right)\mathbf{v}_k + o(\frac{1}{T})\end{aligned}$$

$$\begin{aligned}\mathrm{Cov}(\widehat{\mathbf{v}}_i,\widehat{\mathbf{v}}_j) &= \frac{\delta_{i,j}}{T}\sum_{1\le k\ne i\le n}\frac{\lambda_i\lambda_k}{(\lambda_i-\lambda_k)^2}\mathbf{v}_k\mathbf{v}_k^H \\ &+ \frac{1}{T}\sum_{1\le k\ne i\le n}\sum_{1\le l\ne j\le n}\frac{\lambda_{j,k}\lambda_{i,l}^* + \lambda_{i,k,j,kl}}{(\lambda_i-\lambda_k)(\lambda_j-\lambda_l)}\mathbf{v}_k\mathbf{v}_l^H + o(\frac{1}{T})\end{aligned}$$

$$\begin{aligned}\mathrm{Cov}(\widehat{\mathbf{v}}_i,\widehat{\mathbf{v}}_j^*) &= -\frac{(1-\delta_{i,j})}{T}\frac{\lambda_i\lambda_j}{(\lambda_i-\lambda_j)^2}\mathbf{v}_j\mathbf{v}_i^T \\ &+ \frac{1}{T}\sum_{1\le k\ne i\le n}\sum_{1\le l\ne j\le n}\frac{\lambda_{k,l}\lambda_{i,j}^* + \lambda_{i,k,l,j}}{(\lambda_i-\lambda_k)(\lambda_j-\lambda_l)}\mathbf{v}_k\mathbf{v}_l^T + o(\frac{1}{T})\end{aligned}$$

$$\mathrm{Cov}(\widehat{\lambda}_i,\widehat{\mathbf{v}}_j) = \frac{1}{T}\sum_{1\le k\ne j\le n}\left(\frac{\lambda_{i,k}\lambda_{i,j}^* + \lambda_{i,i,k,j}}{\lambda_j-\lambda_k}\right)\mathbf{v}_k + o(\frac{1}{T}),$$

for $i,j=1,...r$ and where $\delta_{i,j}$ is the Kronecker delta, $\lambda_{i,j} \stackrel{\mathrm{def}}{=} \mathbf{v}_i^H\mathbf{R}'\mathbf{v}_j^*$ and $\lambda_{i,j,k,l} \stackrel{\mathrm{def}}{=} (\mathbf{v}_i^T\otimes\mathbf{v}_j^H)\mathbf{Q}(\mathbf{v}_k^*\otimes\mathbf{v}_l)$.

[1]For example, the MATLAB's svd function produces a set of singular vectors (which can be used in lieu of eigenvectors for the Hermitian covariance matrix $\mathbf{R}$) that are orthonormal and have a real first entry.

Remark 1: Naturally all the expressions of this result reduce to the first and second-order statistics given in [4, Th.9.2.4] (where the determination of $\widehat{\mathbf{v}}_i$ is not specified) and in [10] (where determination (3) is used) for the circular Gaussian distribution of $\mathbf{x}_t$ for which $\lambda_{i,j}=0$ and $\lambda_{i,j,k,l}=0$.

3. APPLICATION TO THE DETECTION OF NONCIRCULAR AND/OR NONGAUSSIAN COMPONENTS

The MDL criterion is one of the most successful information theoretic criteria for estimating the number r of components of

$$\mathbf{x}_t = \mathbf{A}\mathbf{s}_t + \mathbf{n}_t \tag{6}$$

where $\mathbf{s}_t = (s_{t,1},...,s_{t,r})$, $\mathrm{E}(\mathbf{s}_t\mathbf{s}_t^H)$ is not singular, $\mathbf{A}$ is an $n\times r$ full column rank matrix with $r<n$ and $\mathbf{s}_t$ and $\mathbf{n}_t$ are uncorrelated with $\mathrm{E}(\mathbf{n}_t\mathbf{n}_t^H) = \sigma^2\mathbf{I}_n$. Under the assumption that $\mathbf{x}_t$ are independent identically zero-mean complex circular Gaussian distributed RVs, and if no prior information about $\mathbf{A}$ is used, the MDL estimator is based[2] on the eigenvalues of $\widehat{\mathbf{R}}$ and is given by the following minimizer [6]. $\widehat{r} = \mathrm{Arg}\left\{\min_k \Lambda_k\right\}$ with

$$\Lambda_k = T(n-k)\ln\left(\frac{\widehat{a}_k}{\widehat{g}_k}\right) + \frac{1}{2}k(2n-k)\ln T, \tag{7}$$

with $\widehat{a}_k \stackrel{\mathrm{def}}{=} \frac{1}{n-k}\sum_{i=k+1}^n\widehat{\lambda}_i$ and $\widehat{g}_k \stackrel{\mathrm{def}}{=} \prod_{i=k+1}^n\widehat{\lambda}_i^{1/(n-k)}$. The events $\widehat{r}<r$ and $\widehat{r}>r$ are called underestimation (or misdetection) and overestimation (or false alarm), respectively. Since $(\Lambda_k)_{k=0,..,n-1}$ are functions of the eigenvalues $(\widehat{\lambda}_i)_{i=1,..,n}$ of $\widehat{\mathbf{R}}$, the derivation of the probabilities $P(\widehat{r}>r)$ and $P(\widehat{r}<r)$ needs the joint exact or asymptotic distribution of $(\lambda_i)_{i=1,..,n}$. Unfortunately, these two distributions are only available for circular complex Gaussian distribution [1], and are furthermore too complicated to be useful for the statistical analysis of the estimator $\widehat{r}$. Therefore, for simplifying the derivation of these probabilities, it has been argued [7],[8] and [11] by extended Monte Carlo experiments that $P(\widehat{r}>r) \approx P(\widehat{r}=r+1) \approx P(\Lambda_{r+1}<\Lambda_r)$ and $P(\widehat{r}<r) \approx P(\widehat{r}=r-1) \approx P(\Lambda_{r-1}<\Lambda_r)$. As the probability of overestimation is concerned, exact and approximate asymptotic upper bound of this probability has been derived in [11] showing that generally $P(\widehat{r}>r) \ll 1$. Therefore, we concentrate on the probability of underestimation for which (7) gives straightforwardly (see [8] for $r=1$ and 2)

$$P(\Lambda_{r-1}<\Lambda_r) = P\left(H_r\left(\frac{\widehat{\lambda}_r}{\widehat{a}_r}\right) < T_r\right) \tag{8}$$

[2]Note that if $\mathbf{A}$ is parameterized by some parameters $\boldsymbol{\Theta}$ or if $\mathbf{x}_t$ is not zero-mean complex circular Gaussian distributed, the MDL criterion is no longer based on the eigenvalues of $\widehat{\mathbf{R}}$ [12], but nevertheless criterion (7) is often used for its simplicity.

where $H_r(x) \stackrel{\text{def}}{=} \ln\left(\frac{1}{x}\left(1+\frac{x-1}{n-r+1}\right)^{n-r+1}\right)$ and $T_r \stackrel{\text{def}}{=} \frac{1}{2T}(2n-2r+1)\ln T$. Because $H_r(x)$ is an increasing function for $x > 1$ with $H_r(1) = 0$ and $T_r > 0$, (8) is given by

$$P(\Lambda_{r-1} < \Lambda_r) = P\left(\frac{\widehat{\lambda}_r}{\widehat{a}_r} < T_r'\right), \tag{9}$$

where $T_r' = H_r^{-1}(T_r)$ is the unique solution of the equation $H_r(x) = T_r$ for $x > 1$.

To proceed, we must resort to the approximation, used by all authors that have tackled this point, that the standard deviation of $\widehat{a}_r$ can be considered as negligible with respect to $\mathrm{E}(\widehat{a}_r)$. Consequently $\widehat{a}_r \approx \mathrm{E}(\widehat{a}_r)$, where [8] has refined the approximation $\mathrm{E}(\widehat{a}_r) \approx \sigma^2$ used in [7] by taking into account the bias of the estimates $(\widehat{\lambda}_i)_{i=1,..,r}$. Using $\mathrm{E}(\widehat{a}_r) = \sigma^2 + \frac{1}{n-r}\sum_{i=1}^{r}\left(\lambda_i - \mathrm{E}(\widehat{\lambda}_i)\right)$ and (4), we have $\widehat{a}_r \approx m_r$ with

$$m_r \stackrel{\text{def}}{=} \sigma^2 - \frac{1}{T(n-r)}\sum_{i=1}^{r}\sum_{1\le j\ne i\le n}\frac{\lambda_i\lambda_j + |\lambda_{i,j}|^2 + \lambda_{i,j,i,j}}{\lambda_i - \lambda_j}. \tag{10}$$

Consequently $\widehat{\lambda}_r/\widehat{a}_r$ is approximately asymptotically Gaussian distributed with mean $\mu_r = \mathrm{E}(\widehat{\lambda}_r)/m_r$ and variance $\sigma_r^2 = \mathrm{Var}(\widehat{\lambda}_r)/m_r^2$ where from (4) and (5)

$$\mu_r = \frac{1}{m_r}\left(\lambda_r + \frac{1}{T}\sum_{1\le j\ne r\le n}\frac{\lambda_r\lambda_j + |\lambda_{r,j}|^2 + \lambda_{r,j,r,j}}{\lambda_r - \lambda_j}\right) \tag{11}$$

$$\sigma_r^2 = \frac{1}{m_r^2 T}\left(\lambda_r^2 + |\lambda_{r,r}|^2 + \lambda_{r,r,r,r}\right) \tag{12}$$

and the probability of underestimation is approximately asymptotically given by

$$P(\widehat{r} < r) \approx 1 - Q\left(\frac{T_r' - \mu_r}{\sigma_r}\right), \tag{13}$$

with $Q(x) \stackrel{\text{def}}{=} \int_x^{+\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}dt$.

4. DOA ILLUSTRATIONS

To illustrate these general results, the detection of the number $r = 1$ or $r = 2$ of noncircular or/and nonGaussian sources impinging on an array of n sensors is now considered. The common model for the received signal $\mathbf{x}_t$ is given by

$$\mathbf{x}_t = \mathbf{A}(\mathbf{\Theta})\mathbf{s}_t + \mathbf{n}_t$$

where $\mathbf{A}(\mathbf{\Theta}) = (\mathbf{a}_1, ..., \mathbf{a}_r)$ with $\mathbf{a}_k$ denotes the steering vector of the k-th source of DOA θ_k. $\mathbf{n}_t$ is assumed circular complex Gaussian distributed, but $(s_{t,k})_{k=1,...,r}$ that have finite 4-th-order moments, are not necessarily circular complex Gaussian distributed. $\mathrm{E}|s_{t,k}^2| = \sigma_{s_k}^2$, $\mathrm{E}(s_{t,k}^2) = \sigma_{s_k}^2\rho_{s_k}e^{2i\phi_{s_k}}$ and $\mathrm{Cum}(s_{t,k}, s_{t,k}^*, s_{t,k}, s_{t,k}^*) = \sigma_{s_k}^4\kappa_{s_k}$ where $\rho_{s_k} \in [0,1]$, $\phi_{s_k} \in [0,\pi)$ and $\kappa_{s_k} \in [-2,+\infty)$ denote the noncircularity rate and phase, and the kurtosis of $s_{t,k}$, respectively.

In all the numerical illustrations and Monte Carlo experiments, a uniform linear array of omni-directional $n = 5$ sensors, and half-wavelength spacing is used. Its centroid at the origin of the phase is used[3]. For each point on the figures, $T = 200$ snapshots (except in Fig.1) are taken and 10000 Monte Carlo runs are carried out to estimate the probabilities of underestimating.

We first consider the robustness of the MDL criterion to the distribution of the sources. For a single source, it has been derived in [9] that (10), (11) and (12) give

$$m_1 = \sigma^2\left(1 - \frac{1}{T}\left(1 + \frac{\sigma^2}{\|\mathbf{a}_1\|^2\sigma_{s_1}^2}\right)\right) \tag{14}$$

$$\mu_1 = \frac{\sigma_{s_1}^2}{m_1}\left(\|\mathbf{a}_1\|^2 + \frac{\sigma^2}{\sigma_{s_1}^2}\right)\left(1 + \frac{n-1}{T}\frac{\sigma^2}{\|\mathbf{a}_1\|^2\sigma_{s_1}^2}\right) \tag{15}$$

$$\sigma_1^2 = \frac{\|\mathbf{a}_1\|^4\sigma_{s_1}^4}{Tm_1^2}\left[\left(1 + \frac{\sigma^2}{\|\mathbf{a}_1\|^2\sigma_{s_1}^2}\right)^2 + \rho_{s_1}^2 + \kappa_{s_1}\right], \tag{16}$$

that specify the probability of underestimation (13). Consequently this probability of underestimation has the following behavior: $P(\widehat{r} = 0/r = 1) = 1/2$ for the value $\mathrm{SNR}_{1/2}$ of the signal-to-noise-ratio $\sigma_{s_1}^2/\sigma^2$ solution of $T_1' = \mu_1$ that does not depend on the distribution of the source. With respect to the circular Gaussian distribution, the probability of underestimation is larger ([resp. smaller]) for $\mathrm{SNR} > \mathrm{SNR}_{1/2}$ ([resp. for $\mathrm{SNR} < \mathrm{SNR}_{1/2}$]) for noncircular or/and nonGaussian distributions such that $\rho_{s_1}^2 + \kappa_{s_1} > 0$. The opposite behavior happens for distributions such that $\rho_{s_1}^2 + \kappa_{s_1} < 0$.

This behavior is illustrated in Fig.1 for the following three distributions: circular Gaussian, BPSK and impulsive that takes the values $\{-1, 0, +1\}$ with respective probabilities $1/2p$, $1 - 1/p$ and $1/2p$ for which $\rho_{s_1} = 1$ and $\kappa_{s_1} = p - 3$.

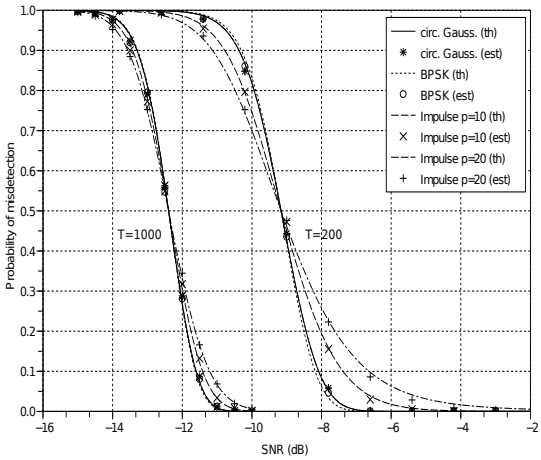

Fig.1 $P(\widehat{r} = 0/r = 1)$ for four distributions.

We see from this figure that the probability of underestimation is sensitive to the distribution of the source, particu-

[3]For rectilinear sources, the DOA θ_k and phase ϕ_k of noncircularity parameters are coupled [14] and as the performance depends on $\Delta\phi = \phi_2 - \phi_1$, the centroid of the array must be specified for fixing the performance.

larly for sources of large kurtosis κ_{s_1} and for weak values of the number T of snapshots as it explained by (16).

For possibly noncircular signals $\mathbf{x}_t$, it is well known that the DOA estimation may be improved in terms of accuracy [13] and resolving power [14] if the standard covariance matrix $\mathbf{R}_x = \mathrm{E}(\mathbf{x}_t\mathbf{x}_t^H)$ is replaced by the augmented covariance matrix $\mathbf{R}_{\tilde{x}} = \mathrm{E}(\tilde{\mathbf{x}}_t\tilde{\mathbf{x}}_t^H)$ with $\tilde{\mathbf{x}}_t \overset{\mathrm{def}}{=} (\mathbf{x}_t^T, \mathbf{x}_t^H)^T$. More precisely, it is proved in [13],[14] that these performance are drastically improved only for rectilinear uncorrelated sources for which[4] $\mathbf{R}_x = \sum_{k=1}^r \sigma_{s_k}^2 \mathbf{a}_k\mathbf{a}_k^H + \sigma^2\mathbf{I}_n$ and $\mathbf{R}_{\tilde{x}} = \sum_{k=1}^r \sigma_{s_k}^2 \tilde{\mathbf{a}}_k\tilde{\mathbf{a}}_k^H + \sigma^2\mathbf{I}_{2n}$ with $\tilde{\mathbf{a}}_k \overset{\mathrm{def}}{=} (\mathbf{a}_k^T, e^{-2i\phi_k}\mathbf{a}_k^H)^T$. Consequently, it is interesting to compare the probability of underestimation given by the MDL criterion associated with $\mathbf{R}_x$ and $\mathbf{R}_{\tilde{x}}$ whose eigenvalues are denoted $(\tilde{\lambda}_i)_{i=1,\ldots,2n}$. For a single rectilinear source the gap between $\tilde{\lambda}_1$ and σ^2 increases because now $\tilde{\lambda}_1 = 2\|\mathbf{a}_1\|^2\sigma_{s_1}^2 + \sigma^2$ against to $\lambda_1 = \|\mathbf{a}_1\|^2\sigma_{s_1}^2 + \sigma^2$ and thus the detection performance ought to improve. This is proved by the comparison of the new expressions of m_1, μ_1 and σ_1^2 deduced from $\mathbf{R}'_{\tilde{x}} \overset{\mathrm{def}}{=} \mathrm{E}(\tilde{\mathbf{x}}_t\tilde{\mathbf{x}}_t^T)$ and $\mathbf{Q}_{\tilde{x}}$ where $(\mathbf{Q}_{\tilde{x}})_{i+(j-1)2n,l+(k-1)2n} \overset{\mathrm{def}}{=} \mathrm{Cum}((\tilde{\mathbf{x}}_t)_i, (\tilde{\mathbf{x}}_t)_j^*, (\tilde{\mathbf{x}}_t)_k, (\tilde{\mathbf{x}}_t)_l^*)$, that are given by (14), (15) and (16) where $\|\mathbf{a}_1\|^2$ is replaced by $2\|\mathbf{a}_1\|^2$. We obtain similar behaviors of the probability of underestimation that for the MDL criterion associated with $\mathbf{R}_x$, but the performance is improved as it is shown in Fig.2 for which the MDL criterion associated with $\mathbf{R}_{\tilde{x}}$ outperforms this criterion associated with $\mathbf{R}_x$ by about 1.5dB.

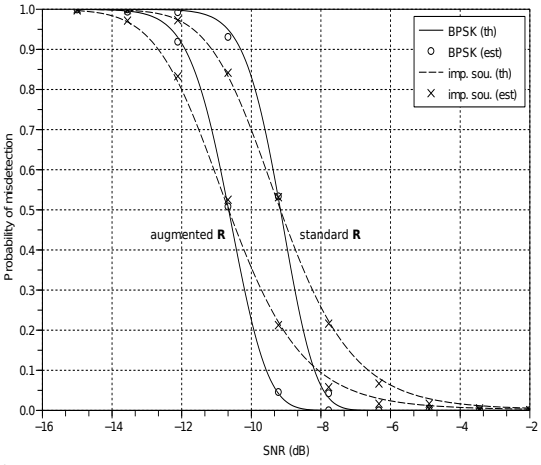

Fig.2 $P(\widehat{r} = 0/r = 1)$ given by the MDL criterion associated with $\mathbf{R}_x$ and $\mathbf{R}_{\tilde{x}}$ for BPSK and impulsive ($p = 20$) distributions of the source.

For two sources, the rank of $\mathbf{R}_{\tilde{x}} - \sigma^2\mathbf{I}_{2n}$ of $\tilde{\mathbf{x}}_t$ is $r = 4$ except for singularity cases. In particular for uncorrelated rectilinear sources this rank is 2. Fig.3 shows the probability of underestimation given by the MDL criterion for two equipowered BPSK uncorrelated rectilinear sources where the SNR is defined by the ratio $\sigma_{s_1}^2/\sigma^2$. The probabilities of underestimation given by the MDL criterion associated with $\mathbf{R}_x$ and $\mathbf{R}_{\tilde{x}}$ are compared as a function of the SNR for two DOA separations. We see that the MDL criterion associated with $\mathbf{R}_{\tilde{x}}$ largely outperforms those based on $\mathbf{R}_x$ by about 1.5dB.

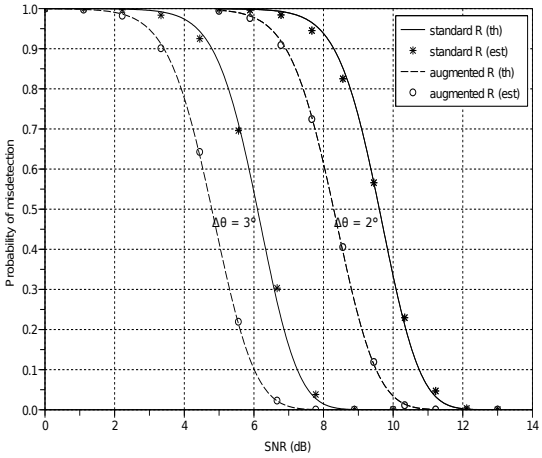

Fig.3 $P(\widehat{r} < 2/r = 2)$ given by the MDL criterion associated with $\mathbf{R}_x$ and $\mathbf{R}_{\tilde{x}}$ for two DOA separations $\Delta\theta$ for two uncorrelated BPSK sources.

[4] We note that if the sources are nonrectilinear or/and correlated, the number of components (i.e., the rank of $\mathbf{R}_{\tilde{x}} - \sigma^2\mathbf{I}_{2n}$) of $\tilde{\mathbf{x}}_t$ is generally $2r$.

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