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A unified model for low-cost high-performance AC drives: the Equivalent Flux concept

Mohamad Koteich^{1,2}, Gilles Duc², Abdelmalek Maloum¹ and Guillaume Sandou²

Abstract—This paper presents a unified modeling approach of alternating current (AC) machines for low-cost high-performance drives. The *Equivalent Flux* concept is introduced. Using this concept, all AC machines can be seen as a non-salient synchronous machine with modified (equivalent) rotor flux. Therefore, complex salient-rotor machines models are simplified, and unified shaft-sensorless AC drives can be sought. For this purpose, a unified observer-based structure for rotor-flux position and speed estimation is proposed. The *equivalent flux* concept generalizes the existing concepts, such as the *extended back-electromotive force*, the *fictitious flux* and the *active flux*.

I. INTRODUCTION

Alternating current (AC) drives control has been an active research topic, since the invention of AC machine more than a century ago [1]. High-performance control of AC drives can be achieved using vector-control techniques, which requires an accurate knowledge of the rotor flux position and/or speed. One recent challenge in this field is the so-called *sensorless* (or *self-sensing*) control that consists of replacing the shaft sensors by estimators (observers) using the available information on the machine currents and voltages [2].

Sensorless control techniques have witnessed several achievements in the last two decades [3]–[6]. Yet, some challenging topics still need further development in the AC machine theory [7]–[9]. The history of AC machines tells us that the most successful achievements rely on finding an adequate machine model. This is the case of the well-known two-circuit model developed in the first half of the XXth century [10], [11]. This model has been the driving force behind the innovation in AC machine drives.

The two-circuit model of AC machines in the rotating reference frame (called the *Park's* model) is very well-suited for control purposes, since it transforms the AC machine into an equivalent DC machine model, where the flux and torque can be controlled independently. Nevertheless, this model requires an accurate knowledge of the rotor (or the rotor flux) position, which makes it not well-suited for position estimation purpose.

A. Towards a unified model for sensorless AC drives

There have been several attempts to unify the models of AC machines (or some of them) for sensorless control purposes. To the best knowledge of the authors, the first attempt is reported by Chen *et al.* [12], where the concept of the *extended back-electromotive force (EMF)* is introduced. This concept

transforms the salient interior (I) permanent magnet (PM) synchronous machine (SM) model into a non-salient PMSM model, with a stator inductance L_d and a modified (extended) back-EMF (the equations are detailed in the appendix). This concept has been used and proved to be useful in observer-based sensorless IPMSM drives [13]–[15].

Another attempt to unify the models of PMSMs was reported in [16], where the *fictitious flux* concept has been introduced. Using this concept, the salient PMSM model can be written as an SPMSM model, with a stator inductance L_q and a modified (fictitious) rotor flux. This new concept seems to be more physically insightful as it is, by definition, the *torque-producing flux*, which multiplies the stator quadrature current, i_{sq} , in the torque equation in the rotating reference frame.

The fictitious flux concept was generalized by Boldea *et al.* [7] by introducing the *active-flux* concept, which is the torque-producing flux in all AC machines, including the wound-rotor synchronous machine (WRSM), the synchronous reluctance machines (SyRM) and the induction machine (IM). Nevertheless, the active flux definition for the IM is based on the steady-state equations of the machine.

B. Paper objectives

This paper proposes a unified model of all AC machines. The hybrid-excited synchronous machines (HESM), whose rotor contains both DC-excited electromagnets and permanent magnets, is considered to be the general synchronous machines. Furthermore, the IMs are studied without restriction to the steady-state operation. The generalized flux that provides a unified modeling of AC machines will be called *Equivalent Flux*. It is believed that this terminology is more accurate than the aforementioned terms [7], [12], [16], since the unified model describes a machine similar to the non-salient PMSM with a flux equivalent to the permanent-magnets rotor flux.

In the next section, the equations of the SPMSM are presented as a source of inspiration. The Equivalent Flux concept is introduced in Section III, and the demonstration is made for both SMs and IMs. Section IV describes a unified state-observer structure for sensorless control of AC drives, based in the Equivalent Flux concept.

C. Notations

Throughout this paper, the complex space vector notation is used to represent the rotating electromagnetic quantities. Let x be a stator quantity in the rotor reference frame, it can be expressed as

$$\underline{x}_s^r = \underline{x}_s^s e^{-j\theta}$$

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or equivalently

$$x_{sd} + jx_{sq} = (x_{s\alpha} + jx_{s\beta})(\cos\theta - j\sin\theta)$$

where subscripts s and r stand for stator and rotor quantities respectively, whereas superscripts s and r tell whether the quantity is expressed in the stator ($\alpha\beta$) or the rotor (dq) reference frame. θ is the angle between the stator and the rotor reference frames.

The fluxes, currents, voltages and back EMFs are denoted by ψ , i , v and e , respectively. The inductance and the resistance of a given circuit are denoted by L and R respectively, and M denotes the mutual inductance between two circuits.

II. THE SPMSM: A SOURCE OF INSPIRATION

In an SPMSM, the stator and rotor flux equations are given by (1) and (2), respectively, in the stationary reference frame.

$$\underline{\psi}_s^s = L_s \underline{i}_s^s + \underline{\psi}_r^s \quad (1)$$

$$\underline{\psi}_r^s = \psi_r e^{j\theta} \quad (2)$$

The voltage model equation is

$$\underline{v}_s^s = R_s \underline{i}_s^s + \frac{d\underline{\psi}_s^s}{dt} \quad (3)$$

Using the equations (1)-(3), the rotor flux can be estimated, from the knowledge of the stator currents and voltages, using the following equation:

$$\widehat{\underline{\psi}}_r^s = -L_s \underline{i}_s^s + \int (\underline{v}_s^s - R_s \underline{i}_s^s) dt \quad (4)$$

Therefore, the rotor position and speed estimates are given by

$$\widehat{\theta} = \angle \widehat{\underline{\psi}}_r^s = \arctan \left(\frac{\widehat{\psi}_{r\beta}}{\widehat{\psi}_{r\alpha}} \right) \quad (5)$$

$$\widehat{\omega} = \frac{d\widehat{\theta}}{dt} = \frac{\psi_{r\alpha} \frac{d\psi_{r\beta}}{dt} - \psi_{r\beta} \frac{d\psi_{r\alpha}}{dt}}{\psi_{r\alpha}^2 + \psi_{r\beta}^2} \quad (6)$$

In addition, the electromagnetic torque estimate is given by

$$\widehat{T}_m = p \left| \widehat{\underline{\psi}}_r^s \times \underline{i}_s^s \right| = p \psi_r \left(i_{s\beta} \cos \widehat{\theta} - i_{s\alpha} \sin \widehat{\theta} \right) \quad (7)$$

The transformation of the SPMSM model into the rotor (dq) reference frame, gives the following flux equations

$$\underline{\psi}_s^r = L_s \underline{i}_s^r + \underline{\psi}_r^r \quad (8)$$

$$\underline{\psi}_r^r = \psi_r \quad (9)$$

and the voltage model

$$\underline{v}_s^r = R_s \underline{i}_s^r + \frac{d\underline{\psi}_s^r}{dt} + j\omega \underline{\psi}_s^r \quad (10)$$

The torque equation in the dq reference frame is

$$T_m = p \left| \underline{\psi}_r^r \times \underline{i}_s^r \right| = p \psi_r i_{sq} \quad (11)$$

The steady-state vector diagram of the SPMSM is shown in Fig. 1. Based on the SPMSM model, various flux observers have been proposed in the literature [17]–[19]. The simplicity of these equations, and the intuitive estimation approach, motivate us to seek a similar model formulation for all AC machine.

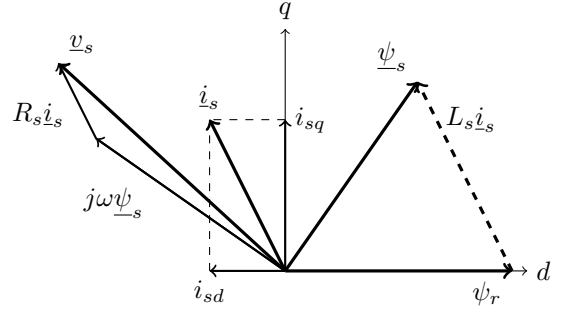


Fig. 1: Vector diagram of the SPMSM

III. THE EQUIVALENT FLUX CONCEPT

In this section, the unified AC machine model is formulated using the *equivalent flux* concept. In the sequel, it will be demonstrated that all AC machines models can be written similarly to the non-salient PMSM model, with an equivalent rotor flux ψ_{eq} and an equivalent stator inductance L_{eq} , as illustrated in Fig. 2. Thus, the flux equation, in the stator coordinates, can be written as

$$\underline{\psi}_s^s = L_{eq} \underline{i}_s^s + \underline{\psi}_{eq} \quad (12)$$

$$\underline{\psi}_{eq} = \psi_{eq} e^{j\theta} \quad (13)$$

with $L_{eq} = L_q$ for synchronous machines and $L_{eq} = \sigma L_s$ for inductions machines, and the equivalent flux ψ_{eq} is expressed for each machine as

HESM	:	$\psi_{eq} = L_{\Delta} i_{sd} + M_f i_f + \psi_r$	
WRSM	:	$\psi_{eq} = L_{\Delta} i_{sd} + M_f i_f$	
IPMSM	:	$\psi_{eq} = L_{\Delta} i_{sd} + \psi_r$	
SyRM	:	$\psi_{eq} = L_{\Delta} i_{sd}$	
SPMSM	:	$\psi_{eq} = \psi_r$	
IM	:	$\psi_{eq} = k_r \psi_{rd}$	

(14)

with:

$$L_{\Delta} = L_d - L_q; \sigma = 1 - k_r k_s; k_r = \frac{M}{L_r}; k_s = \frac{M}{L_s}$$

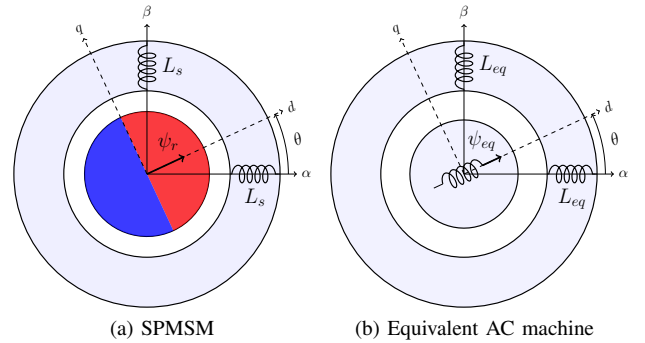


Fig. 2: Symbolic representation

Therefore, the voltage model in the rotating (dq) reference frame can be written as

$$\underline{v}_s^r = R_s \underline{i}_s^r + \frac{d\underline{\psi}_s^r}{dt} + j\omega \underline{\psi}_s^r \quad (15)$$

$$= R_s \underline{i}_s^r + L_{eq} \frac{d\underline{i}_s^r}{dt} + \frac{d\underline{\psi}_{eq}}{dt} + j\omega (L_{eq} \underline{i}_s^r + \underline{\psi}_{eq}) \quad (16)$$

The steady state vector diagram of the equivalent AC machine, shown on Fig. 3, is similar to the vector diagram of the SPMSM (Fig. 1).

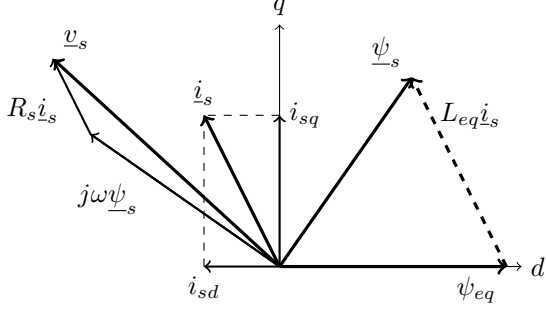


Fig. 3: Vector diagram of the equivalent AC machine

The electromagnetic torque equation of the equivalent machine can be formulated similarly to that of the SPMSM using the equivalent flux:

$$T_m = p \left| \underline{\psi}_{eq}^s \times \underline{i}_s^s \right| = p \psi_{eq} (i_{s\beta} \cos \theta - i_{s\alpha} \sin \theta) \quad (17)$$

$$= p \left| \underline{\psi}_{eq}^r \times \underline{i}_s^r \right| = p \psi_{eq} i_{sq} \quad (18)$$

Remark: The equivalent flux is aligned to the rotor direct (d -)axis in synchronous machines, and to the rotor flux vector in induction machines, that is

$$\underline{\psi}_s^r = L_{eq} \underline{i}_s^r + \underline{\psi}_{eq} \quad (19)$$

Furthermore, the equivalent flux is the torque-producing flux, since it multiplies the current i_{sq} in the torque equation in the synchronous reference frame. Hence, the Equivalent flux amplitude and orientation are extremely important in both field-oriented control and direct-torque-and-flux control strategies.

The following paragraphs demonstrate the equivalent flux formulation for SMs and IMs.

A. Synchronous machines

The equations are developed for the salient-type HESM, they can be easily extended to the other SMs¹. The stator flux equation in the stator reference frame can be written as [20]:

$$\underline{\psi}_s^s = L_0 \underline{i}_s^s + L_2 \underline{i}_s^{s*} e^{j2\theta} + (M_f i_f + \psi_r) e^{j\theta} \quad (20)$$

Adding and subtracting $L_2 \underline{i}_s^s$, we get:

$$\underline{\psi}_s^s = (L_0 - L_2) \underline{i}_s^s + L_2 \left(\underline{i}_s^s + \underline{i}_s^{s*} e^{j2\theta} \right) + (M_f i_f + \psi_r) e^{j\theta}$$

¹For WRSM $\psi_r = 0$, for PMSM $i_f = 0$, for SyRM both ψ_r and i_f are null. For non-salient SMs $L_2 = 0$.

Knowing that $L_q = L_0 - L_2$, the above equation becomes

$$\underline{\psi}_s^s = L_q \underline{i}_s^s + L_2 \left(\underline{i}_s^s e^{-j\theta} + (\underline{i}_s^s e^{-j\theta})^* \right) e^{j\theta} + (M_f i_f + \psi_r) e^{j\theta}$$

Then

$$\underline{\psi}_s^s = L_q \underline{i}_s^s + (2L_2 \mathcal{R}e(\underline{i}_s^r) + M_f i_f + \psi_r) e^{j\theta}$$

Knowing that $L_\Delta = 2L_2$, the stator flux equation finally becomes:

$$\begin{aligned} \underline{\psi}_s^s &= L_q \underline{i}_s^s + (L_\Delta i_{sd} + M_f i_f + \psi_r) e^{j\theta} \\ &= L_q \underline{i}_s^s + \underline{\psi}_{eq} e^{j\theta} \end{aligned}$$

where $\underline{\psi}_{eq}$ is the equivalent flux defined in (14).

Remark: In steady-state operation, the time derivative of the equivalent flux is nothing but the *excitation voltage* [20], a known concept in the analysis of synchronous machines in steady-state operation.

B. Induction machines

The equations of the stator and rotor fluxes of the IM in the stationary reference frame are:

$$\underline{\psi}_s^s = L_s \underline{i}_s^s + M \underline{i}_r^s \quad (21)$$

$$\underline{\psi}_r^s = M \underline{i}_s^s + L_r \underline{i}_r^s \quad (22)$$

By replacing the following rotor current equation (from (22))

$$\underline{i}_r^s = \frac{1}{L_r} \left(\underline{\psi}_r^s - M \underline{i}_s^s \right) \quad (23)$$

in the stator flux equation (21), the equivalent flux formulation prevails:

$$\underline{\psi}_s^s = \sigma L_s \underline{i}_s^s + k_r \underline{\psi}_r^s = L_{eq} \underline{i}_s^s + \underline{\psi}_{eq} \quad (24)$$

The stator flux equation in the dq reference frame is then:

$$\underline{\psi}_s^r = L_\sigma \underline{i}_s^r + k_r \underline{\psi}_r^r = L_\sigma \underline{i}_s^r + k_r \psi_{rd} \quad (25)$$

Remark: In steady-state operating condition, the rotor flux is constant ($\dot{\psi}_{rd}$). Therefore:

$$i_{rd} = -\frac{1}{R_r} \frac{d\psi_{rd}}{dt} = 0 \quad (26)$$

which gives

$$k_r \psi_{rd} = k_r (M i_{sd} + L_r i_{rd}) = k_r M i_{sd} \quad (27)$$

In this case, the definition of the equivalent flux matches the *active flux* concept [7]:

$$\underline{\psi}_{eq} = (L_s - L_\sigma) i_{sd} \quad (28)$$

It can be noticed that in steady state, the IM behaves as a SyRM with $L_d = L_s$ and $L_q = \sigma L_s$.

IV. UNIFIED OBSERVER/ESTIMATOR FOR SENSORLESS CONTROL

High performance torque-controlled AC drives require the knowledge of the machine torque and the rotor (or the rotor flux) angular position and speed. In this section, it is shown that these required variables can be estimated using a unified observer-based estimation algorithm (Fig. 4), thanks to the equivalent flux concept.

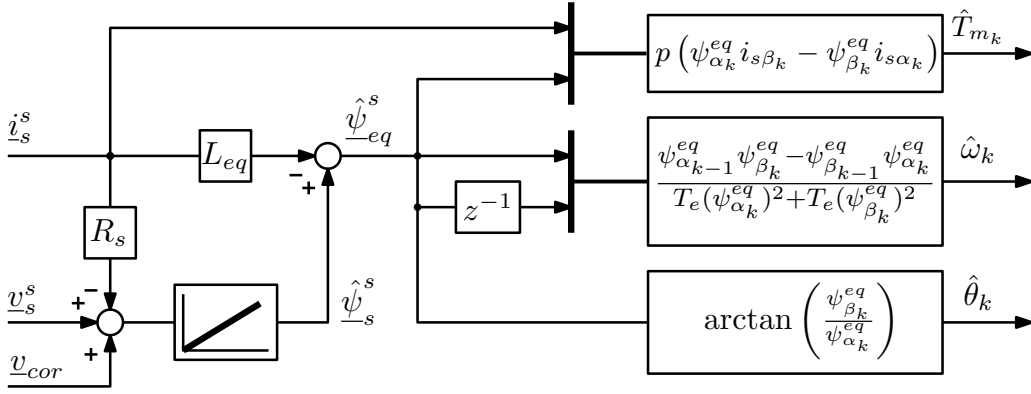


Fig. 4: Equivalent flux observer for sensorless control

A. Open-loop estimation

The equivalent flux can be estimated based on the following model (Fig. 5):

$$\underline{\hat{\psi}}_{-eq}^s = -L_{eq} \dot{i}_s^s + \int (v_s^s - R_s \dot{i}_s^s) dt \quad (29)$$

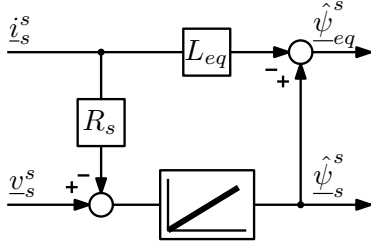


Fig. 5: Equivalent flux estimator

Consequently, the angular position and speed of the estimated equivalent flux can be calculated using the following equations:

$$\hat{\theta} = \angle \underline{\hat{\psi}}_{-eq}^s = \arctan \left(\frac{\hat{\psi}_{\beta}^{eq}}{\hat{\psi}_{\alpha}^{eq}} \right) \quad (30)$$

$$\hat{\omega} = \frac{d\hat{\theta}}{dt} = \frac{\hat{\psi}_{\alpha}^{eq} \frac{d\hat{\psi}_{\beta}^{eq}}{dt} - \hat{\psi}_{\beta}^{eq} \frac{d\hat{\psi}_{\alpha}^{eq}}{dt}}{(\hat{\psi}_{\alpha}^{eq})^2 + (\hat{\psi}_{\beta}^{eq})^2} \quad (31)$$

The electromagnetic torque can be evaluated using the following equation:

$$\hat{T}_m = p \left| \underline{\hat{\psi}}_{-eq}^s \times \underline{i}_s^s \right| = p \left(\hat{\psi}_{\alpha}^{eq} i_{s\beta} - \hat{\psi}_{\beta}^{eq} i_{s\alpha} \right) \quad (32)$$

For real-time implementation, the following discrete-time equations can be used (Fig. 4):

$$\hat{\theta}_k = \arctan \left(\frac{\hat{\psi}_{\beta_k}^{eq}}{\hat{\psi}_{\alpha_k}^{eq}} \right) \quad (33)$$

$$\hat{\omega}_k = \frac{1}{T_e} \frac{\hat{\psi}_{\alpha_{k-1}}^{eq} \hat{\psi}_{\beta_k}^{eq} - \hat{\psi}_{\beta_{k-1}}^{eq} \hat{\psi}_{\alpha_k}^{eq}}{(\hat{\psi}_{\alpha_k}^{eq})^2 + (\hat{\psi}_{\beta_k}^{eq})^2} \quad (34)$$

$$\hat{T}_{m_k} = p \left(\hat{\psi}_{\alpha_k}^{eq} i_{s\beta_k} - \hat{\psi}_{\beta_k}^{eq} i_{s\alpha_k} \right) \quad (35)$$

B. Equivalent flux observer

The stator flux estimation using the voltage-model pure-integration is prone to drift problems due to noise, measurement errors, stator resistance uncertainty or unknown initial conditions [21]. Thus, a correction term, \underline{v}_{cor} , is added to the voltage model, similarly to the correction term in a traditional state-observer structure, to get the following equivalent flux observer:

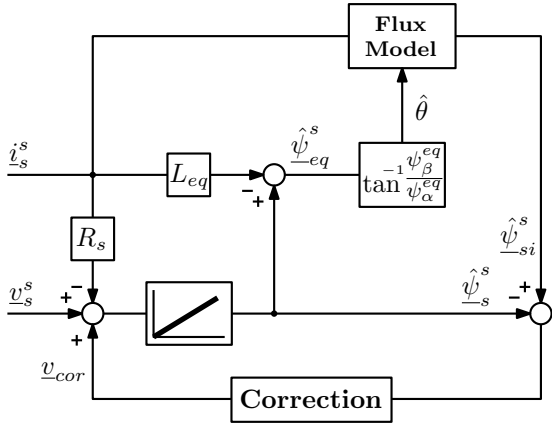
$$\underline{\hat{\psi}}_{-eq}^s = -L_{eq} \dot{i}_s^s + \int (\underline{v}_s^s - R_s \dot{i}_s^s + \underline{v}_{cor}) dt \quad (36)$$

The tuning of the correction term \underline{v}_{cor} is still an open topic. Various correction terms have been reported in the literature. They can be classified into two main categories: current-based correction and flux-based correction (Fig. 6). In a current-based correction structure, the measured stator currents are compared with the currents estimated using the estimated stator flux. On the other hand, in a flux-based correction observer, the estimated stator flux based on the current model is compared with the flux estimated using the voltage model. For instance, the observers in [22], [23] use current-based correction, and the observers in [24]–[28] use flux-based correction for different AC machines.

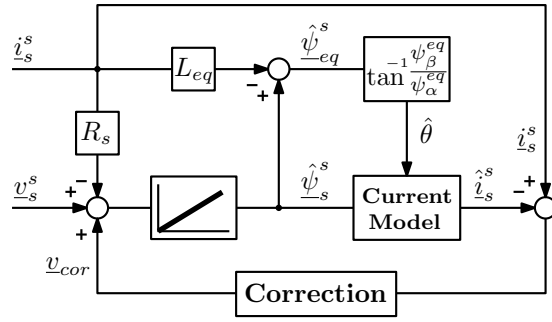
The angular speed estimation using numerical derivation of the estimated position is prone to noises. Therefore, a phase-locked loop (PLL) technique can be used to estimate the equivalent flux angular speed and position [7].

V. CONCLUDING REMARKS

A unified model of AC machines was proposed in this paper by introducing the *equivalent flux* concept. This torque-producing flux concept emulates, and generalizes, the rotor flux in a non-salient permanent-magnet synchronous machine. It is particularly useful for low-cost (shaft-sensorless) high-performance (vector and field oriented) control of AC machines. Based on this concept, a unified observer-based estimation structure was presented. The only required parameters for the rotor position and speed estimation are the stator resistance R_s and the equivalent stator inductance L_{eq} . At the end of this paper, two topics are to be addressed in order to design a high-performance unified observer for AC drives, 1) the



(a) Flux correction



(b) Current correction

Fig. 6: Observer structure

online identification of R_s and L_{eq} , and 2) the correction of the voltage model estimator (via the term v_{cor} for example) without relying on the current model, which depends on the machine structure.

APPENDIX

A. The Extended back-EMF concept

The SPMSM model can be written in the stationary reference frame as follows:

$$\begin{bmatrix} v_{s\alpha} \\ v_{s\beta} \end{bmatrix} = R_s \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix} + L_s \frac{d}{dt} \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix} + \begin{bmatrix} e_{s\alpha} \\ e_{s\beta} \end{bmatrix} \quad (37)$$

with

$$\begin{bmatrix} e_{s\alpha} \\ e_{s\beta} \end{bmatrix} = \omega \psi_r \mathbf{C}'(\theta) = \omega \psi_r \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix} \quad (38)$$

The Laplace transformation of this model gives (Fig. 7.a):

$$\mathbf{v}(s) = \underbrace{R_s \mathbf{i}(s)}_A + \underbrace{sL_s \mathbf{i}(s)}_B + \underbrace{\mathbf{e}(s)}_C \quad (39)$$

The rotor position and speed can be estimated using the following equations:

$$\hat{\theta} = \arctan \left(-\frac{e_{s\alpha}}{e_{s\beta}} \right) \quad (40)$$

$$\hat{\omega} = \frac{1}{\psi_r} \sqrt{e_{s\alpha}^2 + e_{s\beta}^2} \quad (41)$$

This simple scheme inspires the authors of [12] to elaborate a similar model for the IPMSM by introducing the extended back-EMF concept. The equations of the IPMSM in the stator reference frame are given by:

$$\mathbf{v}(s) = \underbrace{R_s \mathbf{i}(s)}_A + \underbrace{sL_0 \mathbf{i}(s)}_B + \underbrace{\mathbf{e}(s)}_C + \underbrace{s\omega \mathfrak{L}_{s\alpha\beta 2}(\theta) \mathbf{i}(s)}_D \quad (42)$$

The dq model of the voltage equation is:

$$v_{sd} = R_s i_{sd} + L_d \frac{di_{sd}}{dt} - \omega L_q i_{sq} \quad (43)$$

$$v_{sq} = R_s i_{sq} + L_q \frac{di_{sq}}{dt} + \omega (L_d i_{sd} + \psi_r) \quad (44)$$

By adding and subtracting the following term

$$\omega L_q i_{sd} - \omega L_q i_{sd} + L_d \frac{di_{sq}}{dt} - L_d \frac{di_{sq}}{dt} = 0 \quad (45)$$

to the v_{sq} equation we get

$$\begin{bmatrix} v_{sd} \\ v_{sq} \end{bmatrix} = \begin{bmatrix} R_s & -\omega L_q \\ \omega L_q & R_s \end{bmatrix} \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} + L_d \frac{d}{dt} \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} + \begin{bmatrix} 0 \\ (L_d - L_q) \left(\omega i_{sd} - \frac{di_{sq}}{dt} \right) + \omega \psi_r \end{bmatrix} \quad (46)$$

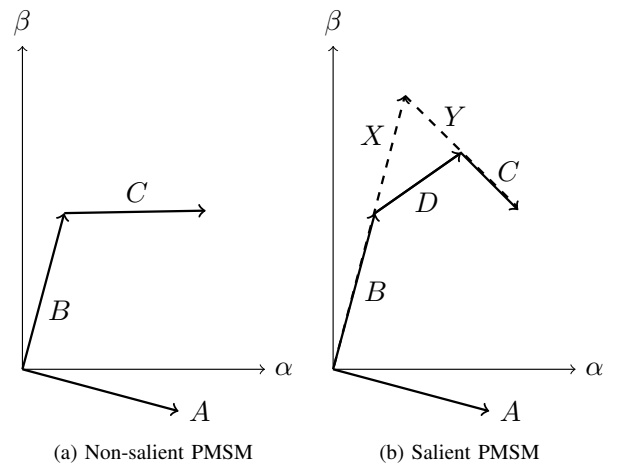
The above equation describes a salient PMSM, with a stator inductance L_d , and with the following extended back-EMF:

$$\mathcal{E}_{sdq}^e = \begin{bmatrix} 0 \\ E^e \end{bmatrix} = \begin{bmatrix} 0 \\ (L_d - L_q) \left(\omega i_{sd} - \frac{di_{sq}}{dt} \right) + \omega \psi_r \end{bmatrix} \quad (47)$$

The equations (46) gives the following $\alpha\beta$ model (Fig. 7.b):

$$\mathbf{v}(s) = \underbrace{R_s \mathbf{i}(s)}_A + \underbrace{(sL_d \mathbf{I}_2 - \omega(L_d - L_q) \mathbf{J}_2) \mathbf{i}(s)}_X + \underbrace{\mathbf{e}^e(s)}_Y \quad (48)$$

where $\mathbf{e}^e(s)$ is the Laplace transform of $E^e \mathbf{C}'(\theta)$.



(a) Non-salient PMSM

(b) Salient PMSM

Fig. 7: Vector diagrams of the PMSMs

Therefore, the rotor position and speed can be estimated as follows [29]:

$$\begin{aligned}\hat{\theta} &= \arctan\left(-\frac{e_{s\alpha}^e}{e_{s\beta}^e}\right) \\ &= \arctan\left(-\frac{v_{s\alpha} - R_s i_{s\alpha} - L_d \frac{di_{s\alpha}}{dt} - \hat{\omega} L_{\Delta} i_{s\beta}}{v_{s\beta} - R_s i_{s\beta} - L_d \frac{di_{s\beta}}{dt} + \hat{\omega} L_{\Delta} i_{s\alpha}}\right) \\ \hat{\omega} &= \frac{1}{L_{\Delta} \hat{i}_{sd} + \psi_r} \sqrt{e_{s\alpha}^e{}^2 + e_{s\beta}^e{}^2}\end{aligned}\quad (49)$$

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