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1 Gini Index for Evaluating Bus Reliability Performances for
2 Operators and Riders

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ABSTRACT

Reliability of bus travel time is an objective of major concern for bus operators and users. However, due to traffic conditions and variability in bus demand, deviations from schedules are unavoidable, leading to an overall decrease in level of service and capacity. The ability to accurately and effectively analyze various performance measures is fundamental to determining how well the bus service is adhering to its service standards. Understanding and developing methods to assess transit operations performance is not only valuable in identifying potential problem areas along a route; it is also constructive in proposing effective strategies to improve service reliability. After a state-of-the-art and a state-of-practice studies of the service regularity measures for high-frequency buses, we propose in this paper the use of the Lorenz curve as a tool for the analysis of bus regularity. We show that the Lorenz curve, if drawn for the scheduled headways, allows analysis of the regularity of the waiting time of all riders of the bus line and is a good measure from the users' point of view. The Lorenz curve drawn for the ratio of the observed to the scheduled headway allows analysis of the regularity of the bus service with regard to the scheduled headways and is therefore an effective measure, mainly from the operator's point of view.

1. INTRODUCTION

Reliability of bus travel time is an objective of major concern for the different stakeholders of the public transport (PT) system: service provider, customers and the community. According to the service provider, namely the operator of the PT system, the main interest is to provide the service in a way to satisfy the demand, respecting the operations' constraints. However, irregularities in travel time imply bus bunching deteriorating the regularity of operations. For users, the first criterion that appears to interest them is the regularity/reliability of travel time. The 'regularity' criterion is a contributing factor in the identification of the route to adopt (1). Finally, for the community or public service authority, the interest is the attractiveness of the bus line it invests in. A regular bus service has a direct positive impact on how passengers perceive availability and timeliness of service but headway irregularity discourages use of public transit (2).

Due to traffic conditions and variability in bus demand, deviations from schedules are unavoidable leading to an overall decrease in level of service and capacity. The ability to accurately and effectively analyze various performance measures is fundamental to determining how well the bus is adhering to its service standards. Understanding and developing methods to assess performance of transit operations is not only valuable in identifying potential problem areas along a route, it is also constructive in proposing effective strategies to improve service reliability.

The state-of-the-art of reliability indicators (3, 4) shows many different indicators. However, the measures that are in use or those developed in theory for bus reliability are usually unsatisfactory for service regularity measures of high-frequency buses. Generally, they are not expressed on a normalized scale and therefore cannot be used to compare one route with another. Furthermore, for the most part, they are not immediately or intuitively understandable for senior management or non-expert external stakeholders.

Henderson et al. (5) proposed a Regularity index based on the Gini ratio which is independent of the headways length. This key performance index was described as difficult to understand and to use (6). The Gini index used in economic studies to measure the inequality of revenues and health among the population (7) is based on the Lorenz curve (8). We show in this paper that the Gini index although a good measure of bus line regularity is not sufficient for its analysis. Therefore, the Lorenz curve is a useful tool for this analysis. We show also, that when the Lorenz curve is drawn for the scheduled headways, it is a relevant indicator for the equity of the waiting time. But it may not be an interesting

1 indicator for the operator who establishes the timetable not on the criterion of equality between users
2 but rather to satisfy the total demand. The operator is more interested by the respect of the scheduled
3 headways given by the timetable. We propose in this paper the use of a Lorenz curve based on the
4 ratio between the observed and scheduled headways for the analysis of the regularity of buses.
5

6 The following section of the paper gives a literature review of the state-of-the-art and the state-of-
7 practice of the indicators used by the operators to assess the PT reliability. Section 3 explains the Gini
8 ratio and the Lorenz curve. In section 4 we give the demonstration showing that the Lorenz curve
9 based on the scheduled headways is equivalent to the Lorenz curve based on the riders' waiting time
10 if the headways are established so as to have the same number of riders in each bus. We also
11 demonstrate that the Lorenz curve based on the ratio of observed to scheduled headways is a good
12 indicator of the adherence of buses to scheduled headways. In section 5, we give first results for
13 different key performance indicators, and after, the Lorenz curves of different buses, drawn for the
14 scheduled and the ratio of observed to scheduled headways. We show how the analysis of these
15 curves results in a better understanding of the regularity of the PT line from the point of view of riders
16 and operators. Finally, section 6 gives the conclusion and perspectives.
17
18

19 **2. LITERATURE REVIEW OF METHODS USED TO ASSESS PT RELIABILITY**

20
21 There are different methods to assess the operating quality of PT service. Different indicators are
22 available in the literature. For each of these indicators, one can use different statistics and different
23 reporting methods.
24

25 As given by the "Transit Capacity and Quality of Service Manual" (6), the most common measures of
26 reliability used by transit operators are:

- 27 a) On-time performance indicators measure the degree to which vehicles arrive at the scheduled
28 times,
- 29 b) Headway adherence indicators measure the consistency or "evenness" of the interval between
30 transit vehicles,
- 31 c) Missed trips, and
- 32 d) Distance traveled between mechanical breakdowns.
33

34 The first two indicators are time-based and the last two ones are distance-based service quality
35 indicators.

36 The focus in this paper is on time-based quality indicators: on-time performance refers to "Adherence
37 to timetable". It is generally measured for "low frequency" routes through a punctuality indicator. The
38 headway adherence indicator is used when vehicles run at frequent intervals (less than 10 minutes).
39

40 These indicators are also used by European quality certification institutions such as CEN, AFNOR
41 and AENOR. In Europe generally, regularity/punctuality is often calculated according to the
42 European "quality of service" norm (EN 13 816), defined in the years 2000: the percentage of
43 passengers affected by services that do not respect the timetable within an interval of 2 or 3 minutes
44 of the announced time; and the percentage of passengers affected by late arrival services of no greater
45 than 3 or 5 minutes and early arrival of 1 minute (measured at 59 seconds). The minimum percentage
46 required by the norm is 80%. It can also be 90% or even 95 % depending on the decisions of the local
47 authority. The values differ depending on the type of lines or routes. The approach towards
48 certification enables requirements to evolve as improvements are made. The results are of course
49 highly dependent on the urban context, the traffic situation, and the structural improvements carried
50 out on the line or route.
51

1 These same indicators are used by the majority of members of the International Bus Benchmarking
 2 Group (IBBG) as given in (9). IBBG is a program of international benchmarking of bus operations
 3 and PT. It is made up of a consortium of twelve international bus organizations, to provide a forum
 4 for a number of medium sized and large bus organizations from different parts of the world to share
 5 experiences, compare performance, identify best practices and learn from one another (10, 11). As
 6 summarized in (12), the majority of IBBG members are using either Headway Adherence or the
 7 Timetable Regularity; only "London buses" is using Excess Waiting Time (EWT).

8
 9 **Headway Adherence (HA)**. The definition of the HA and the way to measure it vary according to the
 10 authors:

11
 12 - (6) uses CoV_D . a kind of Coefficient of Variation of headways- in order to measure the HA :

$$13 \quad CoV_D = \frac{\text{The standard deviation of headway deviations}}{\text{the average of scheduled headways}} \quad (1)$$

14
 15
 16 The "Headways Deviations" in Equation 1, are the deviations of observed headways compared to
 17 scheduled headways. A low CoV_D corresponds to a high HA; it takes into account all headway
 18 deviations.

19
 20 - According to (9) and (13), the HA is the percentage of headways that deviate no more than a specific
 21 amount from the scheduled interval. This specific amount is a percentage of the headway.

22
 23 **The Timetable Regularity**, sometimes referred to as "Wait Assessment", is computed as the service
 24 regularity but where the specific amount is no longer a percentage but an absolute number of minutes.

25
 26 **Waiting time**: for frequent service, passengers are assumed to go to the stop without expectations of
 27 boarding a particular bus at a particular time. Passengers' waiting time depends on the scheduled
 28 headways and the regularity of service. The average wait increases as service regularity decreases.
 29 The observed average waiting time formula given by Equation 3 is due to Welding (13) and has been
 30 reformulated by (15). Equation 4 is proposed by (16).

$$31 \quad AWT(Observerd) = \frac{\sum_{i=1}^N h_i^2}{2 \sum_{i=1}^N h_i} \quad (3)$$

$$32 \quad AWT(Observerd) = \frac{H}{2} (1 + CoV_o^2) \quad (4)$$

33
 34 Where h_i is the i^{th} observed headway, H is the mean observed headway, CoV_o is the coefficient of
 35 variation of the observed headways and N is the number of headways.

36
 37 Planners at London Transit use a measure, mathematically independent of the headway, called the
 38 Excess Waiting Time (EWT). EWT is based on the average passenger waiting time. The EWT is
 39 defined as the difference between the real average passenger waiting time and the scheduled average
 40 passenger waiting time.

$$41 \quad AWT(Scheduled) = \frac{H_s}{2} (1 + CoV_s^2) \quad (5)$$

42
 43 Where H_s is the average scheduled headway and CoV_s is the Coefficient of Variation of scheduled
 44 headways.

$$EWT = AWT (Observed) - AWT (Scheduled) \quad (6)$$

Other metrics are often given which are extreme values, the minimum and maximum values of measures and the coefficient of variation (CoV), the standard deviation (STD) and the percentage outside of a “comfort zone”. STD and CoV expressions indicate the spread of travel time around some expected value; CoV is an interesting metric because it is dimensionless. Golshani (17) proposed the standard deviation of headway, as well as an irregularity index, which gives an indication of long gaps between vehicles. This irregularity index is defined in the same way as the Average Waiting Time AWT (Equation 3). It is defined as the ratio of the mean square headway to the mean headway squared.

$$IR = \frac{\frac{1}{N} \sum_{i=1}^N (h_i^2)}{\left(\frac{1}{N} \sum_{i=1}^N h_i\right)^2} \quad (7)$$

Literature review of PT reliability statistics can be summarized by the table given in (18).

The measures that are in use or those developed in theory are usually unsatisfactory for service regularity measures of high-frequency buses because they are not expressed on a normalized scale and therefore cannot be used to compare one route with another.

The index presented by Henderson et al. (5) is useful to evaluate the bus headway regularity. It is a good indicator from the users’ point of view, especially for high-frequency buses where users do not plan their arrival time, the most important criterion is therefore the headway regularity. From the operators’ point of view however, even for high-frequency buses, the regularity of buses is not the main concern. Operators may plan nearly regular headways or nearly regular headways on small time intervals. Establishing a timetable is not an easy target and depends on many factors such as the number of passengers, the layover time, the availability of the bus driver, etc. Planning a timetable with regular intervals, even for short headway bus lines is not the optimal solution from the point of view of the operator, as there are many other constraints which the bus operator has to face. From their point of view, a regular bus is the bus which respects the timetable. For this purpose, a good evaluating index is the one which assesses the bus service with regard to the timetable. We propose in this paper a Gini index based on the ratio between the observed and scheduled headways.

3. GINI INDEX

The Gini index, also called Gini coefficient or ratio, is used in economic studies mainly to measure the inequality of incomes and health among the population (7). It is a measure of statistical dispersion intended to represent the income distribution of a nation's residents. The Gini coefficient is a relative measure. It is possible for the Gini coefficient of a developing country to rise (due to increasing inequality of income) while the number of people in absolute poverty decreases.

The Gini coefficient is usually defined mathematically based on the Lorenz curve. A Lorenz curve plots the cumulative percentages of total income received against the cumulative number of the population, starting with the poorest individual or household (see Figure 1, red colors). The Gini coefficient can be thought of as the ratio of the area that lies between the line of equality and the Lorenz curve (the red dotted surface marked A on Figure 1) over the total area under the line of equality (the pink surface marked B on Figure 1); i.e., the Gini coefficient $GC = A / (A + B)$. Thus a Gini index of zero expresses perfect equality, where all values are the same (for example, where everyone has the same income). A Gini coefficient of one (or 100%) expresses maximal inequality

1 among values (for example, where only one person has all the income or consumption, and all others
2 have none).

3
4 $GC = A / (A + B)$. Since $A + B = 0.5$, the Gini index is $GC = 2A$, or $GC = 1 - 2B$.

5
6 If the Lorenz curve is represented by the function $Y = L(X)$, the value of B can be found
7 with integration:

$$8 \quad B = \int_0^1 L(X) dX \quad (8)$$

9
10 The Gini is an inequality index. As we are interested by bus regularity, we use the regularity index
11 that we call "Gini", rather than the Gini index (GC)

$$12 \quad \text{Gini} = 1 - GC = 2 * B = 2 * \int_0^1 L(X) dX \quad (9)$$

13 We can show that Equation 9 is equivalent to:

$$14 \quad \text{Gini} = \text{cov}(X, F(X)) \cdot \frac{2}{\bar{X}} \quad (10)$$

15
16 Where cov is the covariance function, $F(X)$ is the cumulative distribution function, \bar{X} is the mean
17 value of X .

18
19 In our paper "Gini" will be computed for 3 different X vectors: (1) X is the vector of scheduled
20 headways (then the index is called Gini_S). (2) X is the vector of observed headways (the index is
21 called Gini_O). (3) Finally X is the vector of Headway Ratios $HR = (HR_1, HR_2, \dots, HR_n, \dots, HR_N)$:

$$22 \quad HR_n = \frac{\text{The } n^{\text{th}} \text{ observed headway}}{\text{The } n^{\text{th}} \text{ scheduled headway}} = \frac{h_n^0}{h_n^S}$$

23
24 In this latter case the index is called Gini_R.

25
26 Replacing X in equation 10, the Gini index is given by

$$27 \quad \text{Gini} = \frac{2 \sum_{i=1}^N (h_i - H) \cdot i}{N^2 \cdot H} \quad (11)$$

28
29
30 Where, h_i is the headway (scheduled for Gini_S, observed for Gini_O, and Headway Ratios for
31 Gini_R), N is the number of headway observations, H is the mean value, i is the rank of the headway
32 or of the headway ratio.

33 34 35 **4. SCHEDULED HEADWAY REGULARITY INDEX AND WAITING TIME**

36
37 Let us consider the Lorenz curve based on the scheduled headways. The horizontal axis represents the
38 cumulative proportion of buses, sorted from buses with the shortest headways to buses with the
39 longest. The vertical axis represents the cumulative proportion of the total headways of the individual
40 buses as they are arrayed on the horizontal axis. The diagonal line is the function that describes
41 perfectly regular service with equal headways for all buses.

1 We show that the Lorenz curve based on the scheduled headways is identical to the Lorenz curve
 2 plotting the waiting time when the timetable is established to get the same number of users in each
 3 bus (except for the passengers of the first bus of the day, for which no headway is attached) and is
 4 therefore a good indicator from the riders' perspective.

5
 6 Let us assume, for the sake of simplicity, that the scheduled headways between the consecutive bus
 7 number $i-1$ and i are independent of the bus stop on the bus route; this means that there is one unique
 8 headway h_i for bus i at all the bus stops. Assuming that the bus operator establishes the scheduled
 9 headways so that all buses take the same number of passengers (N_p), equal to the forecasted traffic
 10 demand for the time corresponding to the headway and that passengers go to the stop randomly
 11 without expectations of boarding a particular bus. The cumulative percentage of headways or of buses
 12 (on the Y-axis of the Lorenz curve) is thus equal to the cumulative percentage of passengers taking
 13 these buses. The mean waiting time of a passenger is equal to half of the headway; the total waiting
 14 time of the passengers of each bus i is equal to the number of passengers by bus multiplied by half of
 15 the headway (Equation 10).

$$17 \quad w_i = N_p \cdot \frac{h_i}{2}$$

(10)

19 Let us sort the buses according to their headway (beginning by the bus with the smallest headway);
 20 the waiting time for all passengers of buses 1 to i are given by Equation 12:

$$21 \quad W_{[1,i]} = \sum_{j=1}^i N_p \cdot \frac{h_j}{2} = \frac{N_p}{2} \cdot \sum_{j=1}^i h_j \quad (12)$$

22 Where $[1,i]$ is the interval from 1 to i . The total waiting time for the whole day on the bus line, is:

$$23 \quad W_T = \frac{N_p}{2} \sum_{j=1}^N h_j \quad (13)$$

24
 25 The cumulative percentages of the waiting time spent up to the i^{th} bus, related to the total waiting
 26 time, is:

$$27 \quad W_{[1,i]} / W_T = \frac{N_p}{2} \cdot \sum_{j=1}^i h_j / \frac{N_p}{2} \sum_{j=1}^N h_j = \sum_{j=1}^i h_j / \sum_{j=1}^N h_j \quad (14)$$

28
 29 The Lorenz curve plotting $W_{[1,i]} / W_T$, the cumulative percentages of waiting time against the
 30 cumulative percentages of the total population waiting for buses, starting with the smallest waiting
 31 time, *is thus identical to the Lorenz curve plotting the cumulative percentages of scheduled*
 32 *headways*. Thus the Gini_S index represents exactly the waiting time equity.

34 5. GINI INDEX FOR THE COMPARISON OF THE REGULARITY OF BUSES

35
 36 When operators establish their timetables they are well aware of the irregularity of scheduled
 37 headways; so they do not need to analyze it. They are more interested by the adherence of the
 38 observed headways to the scheduled ones; that adherence can be analyzed thanks to the Lorenz curve
 39 based on the ratios {observed/scheduled} headways; the adherence is assessed by the value of the
 40 corresponding Gini index, named Gini_R. The Gini_R is appropriate when the public transport
 41 company controls the bus headways. It is less appropriate when the company controls the schedules.
 42 As other indicators, the Gini_R must be used carefully by operators ; its interest is to prioritize the
 43 actions to highlight the more severe problems.

Remark. The denominators of the ratios used in the Gini_R (respectively the CoV_D) are the scheduled headways (respectively the average scheduled headway). When increasing the number of buses, for example, multiplying by two the bus frequency, the scheduled headways are then divided by two (this can be the case when two bus lines have a common trunk and there are twice as many buses in the common trunk than at the ends of the lines). Assuming the same distribution of the headway deviations, for both cases the ratios used in the Gini_R and the CoV_D are multiplied by two; this indicates that the same absolute deviation costs twice as much relative to the headway. This has a sense when assuming that the doubled frequency is due to twice as many users. However, for comparing two (or more) lines with different frequencies, it is better to have the same index value, whatever the frequency; this is obtained by replacing the observed headway $h_i + \Delta h_i$, (h_i being the scheduled headway and Δh_i the observed deviation) in the numerators of the ratios, by the quantity $h_i + \frac{N_0}{N_l} \Delta h_i$; N_l is the number of services per day on the line l and N_0 a constant (whatever the lines) - for instance N_0 is the minimum number of services on all lines. This value of N_0 implies that $\frac{N_0}{N_l} \Delta h_i$ is smaller in average value than Δh_i , the new numerators of the indexes remain positive.

6. RESULTS

Table 1 gives the values of different indicators for the scheduled (Gini_S), observed (Gini_O) and the ratio observed to scheduled (Gini_R) headways. In a longer paper, one can give results for more indicators such as Headway Adherence, Excess Waiting Time, etc. On Table 2, we give the bus line number, ranked for each indicator from the smallest to the greatest value of the indicator.

Table 1: Indicators value for fifteen different bus lines

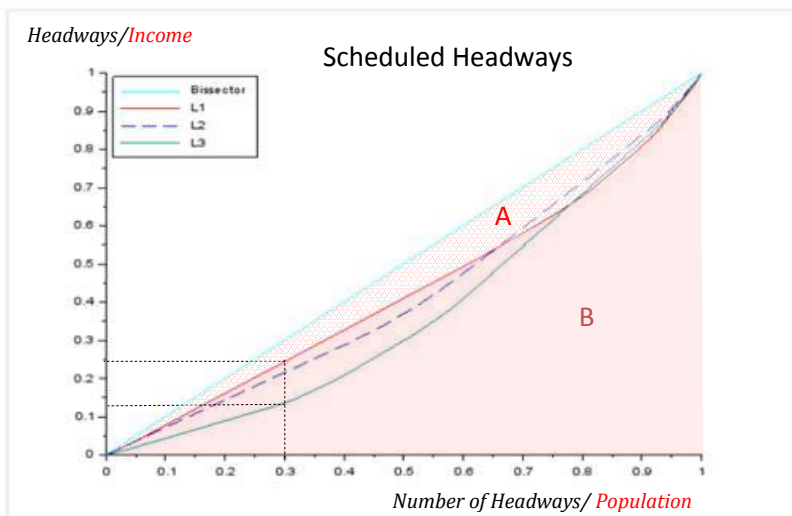
Line	Mean_S	Mean_O	Gini_S	Gini_O	Gini_R	STD_S	STD_O	COV_S	COV_O	Wait_S	Wait_O
L1	8.2	9.94	0.81	0.7	0.77	3.26	5.25	0.4	0.53	4.75	6.36
L2	8.95	8.27	0.83	0.71	0.76	2.88	4.39	0.32	0.53	4.94	5.3
L3	9.02	8.72	0.8	0.68	0.74	3.4	5.21	0.38	0.6	5.15	5.92
L4	10.47	11.18	0.77	0.72	0.76	4.33	5.58	0.41	0.5	6.13	6.98
L5	10.88	11.8	0.74	0.7	0.76	5.11	6.26	0.47	0.53	6.64	7.56
L6	11.39	10.68	0.89	0.89	0.88	2.94	2.27	0.26	0.21	6.07	5.58
L7	12.52	10.76	0.85	0.89	0.91	3.43	2.23	0.27	0.21	6.73	5.61
L8	12.8	14.8	0.84	0.97	0.94	4.01	1.1	0.31	0.07	7.03	7.44
L9	13.79	10.14	0.88	0.84	0.85	4.6	3.12	0.33	0.31	7.66	5.55
L10	13.92	14.42	0.85	0.75	0.79	4.35	6.56	0.31	0.46	7.64	8.7
L11	14.33	14.45	0.9	0.79	0.81	2.84	5.4	0.2	0.37	7.45	8.23
L12	14.44	15.6	0.85	0.73	0.81	5.02	7.91	0.35	0.51	8.09	9.81
L13	16.71	16.82	0.84	0.81	0.93	5.23	5.61	0.31	0.33	9.18	9.35
L14	19.78	19.00	0.95	0.89	0.92	1.66	3.92	0.08	0.21	9.96	9.91
L15	21.41	21.67	0.94	0.99	0.99	3.26	0.52	0.15	0.02	10.95	10.84

Where *_S: stands for Scheduled; *_O: Observed and *_R for the ratio between observed and scheduled headway. STD: Standard Deviation, COV: Coefficient of Variation, Wait: Waiting Time (Equation 4).

1 Table 2: Bus line number, ranked for each indicator from the smallest to the greatest value of the
 2 indicator
 3

Gini_S	Gini_O	Gin_R	STD_S	STD_O	COV_S	COV_O	Wait_S	Wait_O
L5	L3	L3	L14	L15	L14	L15	L1	L2
L4	L5	L5	L11	L8	L15	L8	L2	L9
L3	L1	L2	L2	L7	L11	L14	L3	L6
L1	L2	L4	L6	L6	L6	L6	L6	L7
L2	L4	L1	L15	L9	L7	L7	L4	L3
L8	L12	L10	L1	L14	L8	L9	L5	L1
L13	L10	L12	L3	L2	L13	L13	L7	L4
L7	L11	L11	L7	L3	L10	L11	L8	L8
L10	L13	L9	L8	L1	L2	L10	L11	L5
L12	L9	L6	L4	L11	L9	L4	L10	L11
L9	L7	L7	L10	L4	L12	L12	L9	L10
L6	L6	L14	L9	L13	L3	L2	L12	L13
L11	L14	L13	L12	L5	L1	L1	L13	L12
L15	L8	L8	L5	L10	L4	L5	L14	L14
L14	L15	L15	L13	L12	L5	L3	L15	L15

4
 5
 6 One can notice on Table 2, that the rank of the lines is different for each indicator, which means that each one
 7 of these indicators gives an evaluation from a different point of view of the bus line. The analysis of the
 8 regularity of the bus line on the basis of these indicators is not an easy task. The advantage of the Gini method is
 9 the possibility of analysis given by the Lorenz curve and not only the use of the Gini-coefficient.
 10



11 Figure 1: Lorenz curves based on scheduled headways of three different bus lines.
 12

13 Lorenz curves are plotted for the scheduled headways for three different bus lines on Figure 1. As
 14 explained in Section 4, these curves indicate the riders' waiting time equity (Equation 14). The Lorenz
 15 curve of bus L3 is the farthest from the bisector, one can conclude that bus L3 presents more inequity
 16 compared to the two other buses: Gini_S indexes are of 0.854 for L1, 0.840 for L2 and 0.744 for L3.
 17 Thirty percent of headways of bus L3 correspond to about 14% of the total waiting time, while it
 18 represents 25% for bus L1. As explained previously, the Lorenz curve is given in a normalized scale.
 19 Therefore, the Lorenz curves on Figure 1, based on the scheduled headways, allows for the
 20 comparison of the equity of all users of the bus line. But, in order to compare the theoretical waiting
 21 times between the two bus lines, one has to multiply the normalized waiting time represented on the

1 Y-axes by the duration of the bus service in the day, then divide the result, for each line, by two times
 2 the number of the bus frequency on the line.
 3 One can also draw the Lorenz curve of the observed headway. But the signification of observed and
 4 scheduled headway curves is different. The observed headway curves mix both the equity of
 5 scheduled headways and the respect of buses of these scheduled headways. It is not useful to draw the
 6 observed headways Lorenz curves, except for highlighting, if any, the difference between scheduled
 7 and real headways. Note a drawback in the comparison between the Lorenz curves of scheduled and
 8 observed headways: a lack of difference between both curves can be misleading, since the scheduled
 9 and observed headways are independently sorted (the bus corresponding to the i^{th} shortest scheduled
 10 headway is not necessarily the bus corresponding to the i^{th} shortest observed headway).
 11 The analysis of Figure 1 is interesting from the riders' point of view but not from the operator's point
 12 of view. Having equity of scheduled headways on a bus line is not the first concern of a bus operator.
 13 As previously explained, the bus operator establishes the timetable in order to satisfy the demand,
 14 while taking into account all other constraints. As the number of passengers is not equal at all times of
 15 the day, it is normal to have inequality of headways. For the bus operator, the diagnosis of the respect
 16 of the scheduled headways is more relevant than the equality of scheduled headways.
 17 Figure 2 shows two Lorenz curves, the first one corresponds to the scheduled headways and the
 18 second corresponds to the Lorenz curve based on the ratios of observed to scheduled headway for the
 19 same bus line. Figure 3 gives the Lorenz curve based on the ratios for three different bus lines. The
 20 numerator and denominator of each ratio address the same bus, so the inconvenience highlighted for
 21 the first Lorenz curves and Gini index disappears.

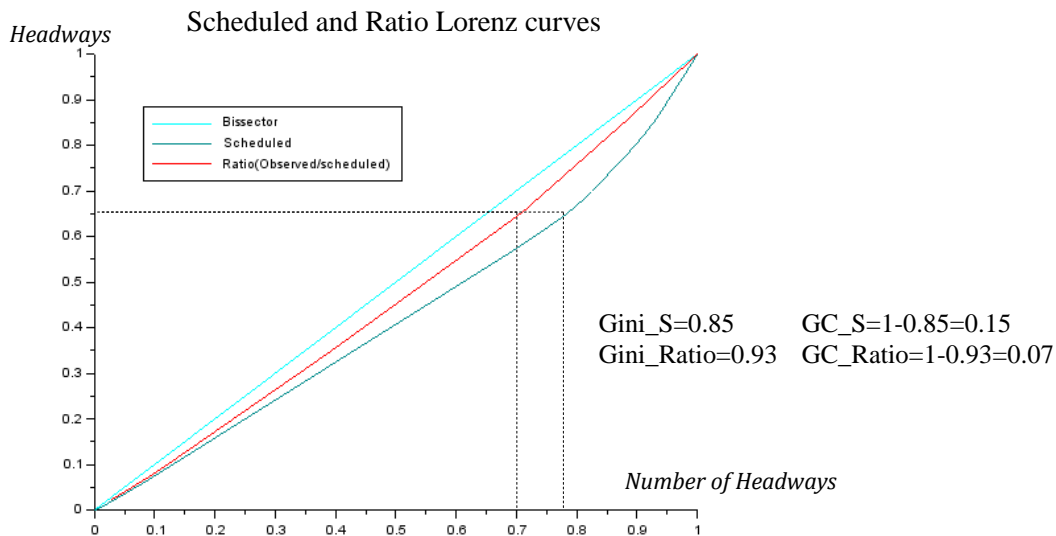
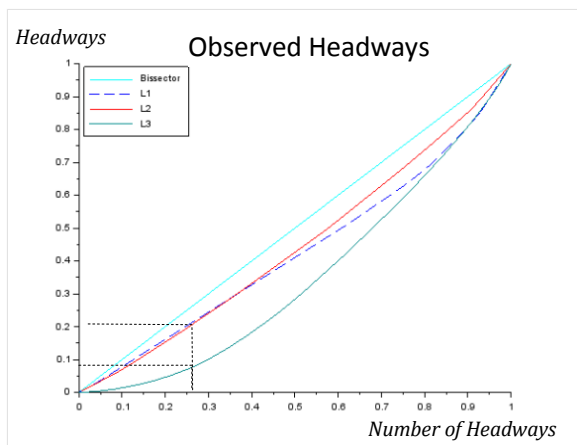


Figure 2: Lorenz curves of scheduled and observed headways of a bus line

22
 23
 24 As one can see on Figure 2, the equity in reliability of the bus line is better than the equity in
 25 scheduled headway. The Lorenz curve of the ratio of observed to scheduled headway is closer to the
 26 bisector than the Lorenz curve of the scheduled headway.

27
 28 The scheduled headway curve shows two main parts: a first approximately straight line from the
 29 origin ($x=0$; $y=0$) up to nearly $x=78\%$ of headways (corresponding to $y=65\%$ of the cumulative
 30 headways) with a slope of about 0.83, and a second one for the last 22% of headways with a slope of
 31 1.59. This means that the line has two modes, possibly a rush period with short headways
 32 corresponding to a large number (78%) of scheduled buses, and longer headways in 22% of buses
 33 (possibly off-peak). The Lorenz curve corresponding to the ratio shows also two periods, for 70% of

1 cases the slope is of 0.93% and for 30% of cases the slope is of 1.17. This means that 70% of
 2 headways were shorter than scheduled - in this case the observed headway is on average the
 3 scheduled one multiplied by 0.93. This also means that 30% of headways are larger than scheduled
 4 (on average 1.59 times the scheduled one). But this does not provide information about the absolute
 5 length of the delay nor about the time of occurrence. It is possible to have one bus with a larger
 6 headway than scheduled, followed by another bus with a shorter headway than scheduled.
 7



8 Figure 3: Lorenz curves based on the Headway Ratios of three different lines

9
 10 On Figure 3, the Lorenz curve of the ratios is drawn for three different lines. This allows analyzing
 11 the equity for each one of the bus lines. The comparison (in the sense of headway adherence) on a
 12 same graphic of the reliability of the different lines is correct only if the average ratios are the same
 13 for all lines. For a whole day and a given line, although headways shorter and larger than scheduled

14 compensate, the average ratio, $\frac{1}{N} \sum_{j=1}^N \left(\frac{h_j^{\text{Observed}}}{h_j^{\text{Scheduled}}} \right)$ is not necessarily equal to 1, the formula not being

15 linear. However on checking, it was found that this average ratio is very close to the value 1 for all the
 16 bus lines, thus very close to each other, therefore the comparison is right. As one can see from the
 17 figure, bus line L2 is more regular than bus line L1, which in turn is more regular than bus line L3.
 18 The Gini index is of: Gini_S(L1)=0.85, Gini_S(L2)=0.89 and Gini_S(L3)= 0.7.

19 The line L2 has three different parts, which can be approximated by three straight lines:

20 (1) the first one has a low slope of 0.2 (the straight line begins at x=0 and y=0 and ends at x=13%,
 21 y=2.6%). This means that 13 % of observed headways are shorter than the scheduled headways; in
 22 this part the average ratio, equal to the slope of 0.2%, means that the observed headways are five
 23 times less than the scheduled ones, possibly indicating bus bunching.

24 (2) a larger part of the curve may be represented by a second straight line from x=13%, to x=93%, its
 25 slope is of (90% - 2.6%)/(93% - 13%)=1.09, very close to the value 1. This means that 80% of
 26 observed headways are very close to scheduled ones (slope 1.09) and 7% of observed headways are
 27 greater than the scheduled ones

28 (3) a last small straight line with a slope of (100% - 90%)/(100% - 93%) =1.42. which means that 7%
 29 of observed headways are greater than the scheduled ones, with an average interval equal to 1.42
 30 times the scheduled headway. This concerns operators as well as riders.

31 Line L1 has two distinguishable parts: for a large percentage of buses, the observed headways are
 32 smaller than the scheduled ones; for a smaller percentage of buses they are greater. The “convexity”
 33 of the Lorenz curve of L3 shows a continuous evolution of the slope, from a slope of 0.4 at the
 34 beginning, for headways shorter than scheduled, to a final slope (when x and y are equal to 1) of 1.75,

1 indicating observed headways 1.75 times larger than scheduled, this also concerns operators as well as
2 riders.

3
4 The Lorenz curve based on headway ratios drawn for each bus stop of the bus line may be relevant
5 information for operators as it shows the regularity of the bus (with respect to headway adherence)
6 depending on the rank of the stop in the line or on its position in the city, its size, its strategic
7 importance in the network, etc.. This allows analyzing the reliability as a function of the cartography
8 of the line (i.e. central part versus periphery part, etc.).

9 For lines where multiple routes or branches come together to form one trunk service along major
10 arterial routes, it is useful to draw the Lorenz curve and to compute the Gini index separately for each
11 branch.

12 13 7. CONCLUSION

14
15 We present in this paper a state-of-the art and a state-of-practice studies of the service regularity
16 measures of high-frequency buses. The majority of measures that are in use, or those developed in
17 theory are unsatisfactory because they are not expressed on a normalized scale and therefore cannot
18 be used to compare one route with another. The other shortcomings of service regularity measures is
19 that they are not immediately or intuitively understandable for non-expert stakeholders. Also as
20 shown in this paper (Tables 1 and 2), each key performance gives a point of view on the regularity
21 and does not allow a real analysis of the regularity of the bus line.

22
23 We propose in this paper the use of the Lorenz curve as a tool for the analysis of two kinds of
24 regularity of buses. The Lorenz curve based on the scheduled headways evaluates the regularity of the
25 intervals. We show that, if the bus company built the timetable so as to satisfy the same number of
26 users for each bus, the Lorenz curve based on scheduled headways allows the analysis of the
27 regularity of the waiting time of all riders. This concerns mainly riders, but also operators, constrained
28 to propose quality service for all end-users. The main concern of the operator is the empirical
29 adherence in the established timetable. The Lorenz curve based on the ratios of the observed to the
30 scheduled headways allows for analysis of the regularity of a line with respect to scheduled headways;
31 it quantifies the occurrences of bus bunching and of headways much larger than scheduled, which
32 concerns mainly operators, but also riders. Plotting on the same graph the Lorenz curves of different
33 lines results in the comparison of their reliability. It is therefore an effective measure from the
34 operator's point of view.

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