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1 **Analysis of flow separation using a local frame-axis:**
2 **application to the open-channel bifurcation**

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16

17 **Abstract**

18 The motivation for the present work comes from the fact that a few recent publications
19 describing separation flows report a Counter-gradient Diffusion Phenomenon (CDP)
20 along the separating streamline. This CDP is related to i) a change of sign of the
21 Reynolds shear stress near the separating point and ii) an opposite sign between the
22 Reynolds shear stress and the mean shear rate, leading to a negative eddy-viscosity
23 coefficient. Such CDP is only reported for configurations where the angle between the
24 separating streamline and the main flow is large (referred to as Group 1), while no CDP
25 was reported for low angle configurations (referred to as Group 2). All these flows were
26 analyzed using Cartesian or curvilinear frame-axes. The aim of the present paper is to
27 propose a more intuitive frame for analysis, namely the Serret-Frenet frame-axis based
28 on the local flow direction, for which the CDP disappears for Group 1 flow
29 configurations and which highly improves the characterization of the mixing layer.

30 The recirculation zone occurring in the lateral branch of an open-channel bifurcation is
31 chosen as a separating flow configuration belonging to Group 1 and measured using
32 PIV. The characteristics of the mean flow and of the horizontal Reynolds stress are first
33 analyzed using a Cartesian frame-axis, in order to retrieve the CDP, which extension is
34 enhanced compared to the literature cases. Then the local Serret-Frenet frame-axis is
35 introduced and used for a second analysis of the data. The flow characteristics at the

36 interface between the main flow and the recirculation zone accurately match the
37 characteristics of the well documented mixing layers available in the literature.
38 Moreover, the eddy-viscosity concept nicely applies using this Serret-Frenet frame-axis.
39 A simple geometrical analysis is finally performed in order to confirm that the negative
40 eddy-viscosity coefficient obtained when using a Cartesian frame-axis is only due to a
41 non-adequate frame of analysis.

42

43 **Introduction**

44 In the field or in laboratory conditions, a separating flow usually detaches from the
45 adjacent wall as it reaches a geometrical singularity. A recirculation zone (also named
46 separating bubble) occurs along this wall. The typical velocity magnitude in the
47 recirculation region is at least one order of magnitude lower than within the main flow
48 and adverse streamwise velocities are observed near the wall. Meanwhile, the main flow
49 accelerates as the main flow section is reduced. Further downstream, the main flow
50 reattaches to the wall and recovers. Li and Djilali [1995] recall that the main parameters
51 governing the flow separation are the Reynolds number of the flow and the geometry
52 itself.

53 In natural streams, separation zones in open channels have significant implications in
54 terms of sediment, gas or passive scalar (such as pollution or nutrients) exchanges
55 between the main flow which conveys material from upstream regions and the
56 recirculation zone where material is stored and is deposited if the density of the material
57 exceeds that of water. Moreover, erosion may occur due to the acceleration of the main
58 flow near the maximum separation width location. Separation zones are also privileged
59 zones of fish and plant reproduction and development. Interactions between the main
60 flow and the recirculation zone are thus of primary importance for bio-geo-chemical
61 processes and stream restoration.

62 In the literature, most attention was paid to the length of the recirculation as a function
63 of the characteristics of the main flow (Froude and Reynolds number, water depth,
64 discharge...) and of the geometry (angles, dimensions of the obstacle...). These studies
65 were dedicated to sudden lateral expansions [Babarutsi et al. 1989, Chu et al. 2004,
66 Riviere et al. 2011a], the lateral branch of 90° open-channel bifurcations [Kasthuri and
67 Pundarikanthan 1987], the downstream branch of confluences [Best and Reid 1984,
68 Gurram et al. 1997] or forward-facing [Sherry et al. 2010] and backward-facing steps
69 [Adams and Johnston 1988].

70 **Mixing layers in recirculating flows**

71 When zooming at the interface between the main flow and the recirculation zone, very
72 high velocity gradients can be observed. In the upstream region, the separation zone can
73 even be seen as a region of water almost at rest, entrained by the main flow. Large-scale
74 coherent structures and high level turbulent intensities are present at the interface
75 between the main flow and the recirculation zone, enhancing momentum and mass
76 transfer. The transverse velocity gradient at the interface gives birth to a vertical mixing
77 layer. A mixing layer is defined as “a turbulent flow that forms between two uniform,
78 nearly parallel streams of different velocity” [Pope, 2000]. In the literature, most mixing
79 layers have been studied in simple configurations such as:

80 - straight channels [Wynanski & Fiedler 1970; Bell & Mehta 1990; Uijttewaai & Booij
81 2000; Loucks & Wallace 2012] or accelerated straight channels [Fiedler et al., 1991].
82 Such analysis was performed using a Cartesian frame-axis (x,y) with x the axis of the
83 side walls and of the main flow and y the transverse direction.

84 - curved channels [Margolis & Lumley, 1965; Gibson & Younis, 1983; Plesniak et al.,
85 1996] analyzed using a curvilinear frame-axis (r,α) with r the local distance to the
86 center of curvature and α the angle formed by the radius at the source and the local
87 radius.

88 A review of these mixing layer characteristics was proposed by Mignot et al. (2013)
89 detailing the geometrical features and the mean and turbulent velocity distribution
90 across the mixing layer. These studies revealed that the width of the mixing layer
91 increases from up-to downstream and that the maximum of the Reynolds stress tensor
92 components across the mixing layer occurs at its centerline.

93

94 **CDP phenomenon in mixing layers of separating flows**

95 Recent works were devoted to the analysis of the mixing layers in separating flows,
96 assuming that the separating streamline is the centerline of the mixing layer. These
97 works confirmed that a maximum transverse gradient of streamwise velocity and a
98 maximum value of all components in the Reynolds stress tensor are measured along the
99 separating streamline.

100 The so-called “CDP phenomenon” was reported by some author in such separation
101 flows. This phenomenon is based on the eddy viscosity concept, following the works of
102 Joseph Boussinesq in 1877. It relates the Reynolds stress tensor to the mean rate of
103 strain tensor and writes:

$$104 \quad -\overline{u'_i u'_j} + \frac{2}{3} k \delta_{ij} = \nu_T^C \left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right) = 2\nu_T^C S_{ij} \quad (1)$$

105 where k is the turbulent kinetic energy, ν_T^C is the so-called “turbulent viscosity” or
 106 “eddy viscosity” coefficient in a Cartesian frame-axis and S_{ij} is the mean rate of strain
 107 tensor (with $u = \overline{u} + u'$ the instantaneous fluid velocity, where the over-bar denotes
 108 Reynolds-averaging - or time-averaging - and prime denotes fluctuation). In Eq. (1), ν_T^C
 109 is hypothesized isotropic as it does not depend on i and j . Nevertheless, it is well known
 110 that this hypothesis of isotropy is not satisfied in many configurations [see Pope, 2000],
 111 such as in flows with significant streamline curvature due to “the subtle way curvature
 112 influences the mean flow” [Patel and Sotiropoulos, 1997]. We do not expect Eq. (1) to
 113 be valid in the present flow configuration and no discussion regarding the validity of the
 114 isotropic turbulent viscosity assumption is made here.

115 In a 2D analysis, application of Eq. (1) to the non-diagonal term writes:

$$116 \quad -\overline{u'v'} = \nu_T^C \left(\frac{\partial \overline{u}}{\partial y} + \frac{\partial \overline{v}}{\partial x} \right) = 2\nu_T^C S_{xy} \quad (2)$$

117 with x and y the streamwise and transverse axes respectively, u and v the instantaneous
 118 velocity components along x and y respectively and S_{xy} the mean shear rate.

119 If, locally, the sign of the Reynolds shear stress $-\overline{u'v'}$ and of the mean shear rate S_{xy} are
 120 opposite, ν_T^C becomes negative at this location and the so-called CDP (for Counter-
 121 gradient Diffusion Phenomenon, see Hattori and Nagano [2010]) takes place. The
 122 negative eddy viscosity coefficient (and thus the CDP) near the separating point was
 123 implied by Simoens et al. [2007] data in a flow separating over a square-shape obstacle,
 124 over a forward-facing step by Largeau and Moriniere [2007] and later by Sherry et al.
 125 [2010] and Ren and Wu [2011] and was computed using DNS by Hattori and Nagano
 126 [2010]. Further downstream, the Reynolds shear stress profiles become positive
 127 throughout the flow and the eddy viscosity coefficient becomes positive. Similarly,
 128 within the flow separating at the inner bank of a sharp bend, Blanckaert et al [2013]
 129 observed a similar CDP region by explicitly reporting an opposite sign between gradient
 130 of mean velocity and Reynolds shear stress in curvilinear frame-axis. These three
 131 configurations (sharp angle obstacle - forward facing step - and sharp bend) obviously
 132 experience the same phenomenon leading to CDP (which will be explained further
 133 below) and are referred to as “Group 1” in the present paper. Such CDP was also

134 observed in turbulent scalar transport experiments where the gradient of mean
135 concentration (or temperature) and the corresponding flux can be of same sign locally
136 [see Paranthoen et al., 2004]. Separating flows downstream groynes and in the outlet
137 branch of confluences and bifurcations (as in the present configuration) should be added
138 to this list even though no evidence of the CDP in these geometries could be found in
139 the literature. The specificity of these flows is that, at the separation point, the angle
140 between the separating streamline (S_s) and the main axis (x) is large (this angle is noted
141 θ , see Fig. 1).

142 Oppositely, to the authors knowledge none of the experiments and calculations related
143 to separating flows in downward-facing steps – downstream from hills – in wakes or in
144 sudden lateral expansion reported this CDP phenomenon: In these studies referred to as
145 “Group 2”, the Reynolds shear stress $-\overline{u'v'}$ and mean shear rate S_{xy} remain of same
146 sign from the separation point towards downstream and ν_T^C thus remains positive. The
147 recirculation length is equal to 6-8 times the expansion (or step) width, except in very
148 shallow conditions [see Chu et al., 2004 or Adams and Johnston, 1988]. Due to such
149 elongated shape, the direction of the separating streamline (s_s) is quite parallel to the
150 main axis (x) until approaching the reattachment point where the flow pattern becomes
151 much more complex [see Riviere et al. 2011a].

152 The difference between groups 1 and 2 can thus be explained based on geometrical
153 aspects related to the magnitude of the θ angle.

154

155 **Objectives**

156 In the present paper we aim at showing that mixing layers in Groups 1 and 2 are
157 actually similar as long as a proper frame-axis is used for the data analysis. We propose
158 a new methodology using the local frame-axis for the analysis of separating flows. Main
159 advantages are thus that there is i) no change of sign of the Reynolds shear stress, ii) no
160 CDP and iii) direct access to the mixing layer characteristics. A laboratory bifurcation
161 flow (belonging to Group 1) is chosen for analysis. After describing the experimental
162 set-up used to measure the bifurcation flow pattern, the local frame-axis is introduced.
163 The mean and turbulent flow characteristics are then computed and the Reynolds shear
164 stress obtained using the Cartesian and local frame-axis are compared in order to
165 discuss the advantages of the local frame-axis. Last section is finally devoted to reveal
166 that CDP is only related to geometrical features.

167

168 **Experimental set-up**

169 The open-channel bifurcation flow field is measured in the channel intersection facility
170 at the Laboratoire de Mécanique des Fluides et d'Acoustique (LMFA) at the University
171 of Lyon (INSA-Lyon, France) sketched in Fig. 2 and adapted from [Riviere et al.
172 2011b]. The facility consists of three horizontal glass channels of $b=0.3\text{m}$ width each,
173 $L_u=2\text{m}$ length for the upstream branch and $L_b=L_d=2.6\text{m}$ for both outlet branches. The
174 channels intersect at 90° with the “upstream branch” along y axis, the “lateral branch”
175 along x axis and the “downstream branch” along y axis, that is aligned with the
176 upstream branch. The upstream branch is connected to a large storage tank. When
177 leaving the upstream tank, the water passes through a honeycomb that stabilizes and
178 straightens the flow. This ensures quasi-1D flows within the incoming channels, even
179 though fully developed inflow conditions would require a considerably longer upstream
180 channel. When reaching the bifurcation, the inflow separates, flows through each outlet
181 branches and is collected by the outlet tanks. The three parameters which govern the
182 flow configuration are: the upstream flow rate $Q_u=4\text{ L/s}$ and the water depth at the
183 downstream end of the lateral ($h_b=12.0\text{ cm}$) and downstream branches ($h_d=12.1\text{ cm}$),
184 which are controlled by sharp crested weirs (noted C_b and C_d in Fig. 2). In the inlet
185 branch when approaching the bifurcation the water depth equals $h_u=12.1\text{ cm}$, leading to
186 an upstream Reynolds number $R_u=30000$, based on the mean velocity and hydraulic
187 diameter. Given the limited measured water depth decrease from the upstream (h_u) to
188 the lateral (h_b) branch, leading to a water depth decrease of about 1mm over a length of
189 2m in the lateral branch, water depth changes are assumed to be negligible. The lateral
190 tank is connected to the downstream tank and the water is pumped from the downstream
191 to the upstream tank. The upstream and lateral flow-rates are measured in the pumping
192 loops using electromagnetic flow meters ($\pm 0.05\text{ L/s}$), see Fig. 2. The values of the
193 discharges measured in the lateral and downstream branches are similar $Q_b=Q_d=2\text{ L/s}$
194 ($\pm 0.05\text{ L/s}$).

195 Velocity fields are measured using a horizontal 2D-PIV technique. A slide projector
196 along with a diaphragm is used to create a collimated 5 mm thick light sheet at the
197 measured elevation in the channel intersection. A 1280x960 pixel progressive CCD-
198 camera mounted with a 8 mm focal lens is connected to a PC computer through a
199 Firewire acquisition card. It is placed at an elevation of about 1.5 m above the free
200 surface. Polyamid particles (50 μm diameter) are added to the water and act as tracers.

201 Inserting the whole set-up in the dark finally permits to record the tracer motion at the
202 lightened elevation at a fixed frame-rate of 30Hz during 133s with 8 bit grey-levels on a
203 400mm x 300mm large window with a horizontal resolution of 0.5 mm per pixel. The
204 PIV commercial software Davis (from Lavis) permits to correct the optical
205 distortions, to subtract the background and to compute the velocity field. The final
206 velocity uncertainty is estimated to 1.5 mm/s ($0.1 * \text{spatial resolution [0.5 mm/pixel]} * \text{acquisition frequency [30 Hz]}$). The PIV system provides the two horizontal velocity
207 components u and v at each location of the measurement grid and at each time step with
208 high spatial resolution without any intrusion. Erroneous vectors are detected by i) too
209 weak correlation between two consecutive images, ii) presence of two or more peaks of
210 correlations of similar correlation coefficients or iii) local velocity strongly differing
211 from the neighboring locations of the grid and are not considered in the time-averaged
212 statistics (mean velocity or any term of the Reynolds stress tensor) at this location.
213
214 Two neighboring measurement zones are presented in the sequel, which frontier is
215 located at $x=400\text{mm}$ (see Fig. 3). The final data used in the following, is a grid of points
216 within the bifurcation and the lateral branch composed each of 4000 sample long (133s)
217 u (along x axis) and v (along y axis) velocity signals. Measurements were taken at
218 elevations $z=4\text{cm}$ and 9cm . Changes in the flow features are expected when
219 approaching the bed, but limited differences were observed between data at both
220 elevations, and only data at 9cm are presented herein.

221

222 **Mean flow**

223 The mean velocity field, averaged over 4000 samples is shown on Fig. 3. As described
224 by Neary et al. [1999], as the main flow reaches the intersection, one part reaches the
225 downstream branch and the other part is deflected towards the lateral branch. The flow
226 in the lateral branch detaches at the upstream corner ($x=300\text{mm}$; $y=0$) and the lateral
227 main flow is confined very near its left bank ($y\sim 300\text{mm}$). A zoom in the recirculating
228 region (in Fig. 3) reveals that its core takes place at $x\sim 650\text{mm}$ and $y\sim 150\text{mm}$. In the
229 region between the core and the right bank, the mean flow is oriented towards the
230 intersection, with a velocity magnitude one order of magnitude lower than the velocity
231 magnitude of the main flow. The so-called "separating streamline" is defined as the
232 streamline starting at the upstream corner of the intersection (see Fig. 3). This
233 streamline is constructed using the following steps: i) the local velocity is interpolated
234 from the measured mean velocity field shown in Fig. 3 which permits to calculate the

235 local flow direction, ii) a small displacement of about one millimeter is computed along
236 this direction leading to a new location. Steps i) and ii) are then repeated until reaching
237 the limit of the measured area. Locations computed at step ii) finally define the
238 streamline. In the upstream portion of the separating streamline, a very intense velocity
239 gradient occurs between the main and recirculation flow. Further downstream, the
240 velocity gradient across the separating streamline appears to decrease.

241

242 **Eddy viscosity concept using a Cartesian frame-axis (x,y)**

243 In the sequel, most attention will be paid to the vicinity of the separating streamline.
244 The distribution of mean shear rate $2S_{xy} = \partial\bar{u}/\partial y + \partial\bar{v}/\partial x$ in the lateral branch is
245 depicted in Fig. 4. $2S_{xy}$ is negative in the vicinity of the separating streamline in the
246 upstream region and positive elsewhere. In the downstream region of the lateral branch,
247 the maximum positive value of $2S_{xy}$ is measured close to the separating streamline. Fig.
248 4 also shows the distribution of Cartesian horizontal Reynolds shear stress $-\overline{u'v'}$. This
249 term is i) negative in the whole upstream region of the lateral branch ($x < 450\text{mm}$), ii)
250 positive in the downstream region ($x > 650\text{mm}$) and iii) both negative (near the left bank)
251 and positive (in the recirculation region) in the intermediate region. As discussed in the
252 introduction, the negative horizontal Reynolds shear stress in the upstream region of the
253 separation was reported in all Group 1 flow configurations. In the present Group 1 case,
254 the particularly high angle θ between the separating streamline and the main flow in the
255 lateral branch (equal to 85.1° at the geometrical singularity: $x=300\text{mm}$ & $y=0\text{mm}$)
256 enhances the extension and magnitude of the negative Reynolds shear stress region.

257 Fig. 4 thus confirms that the mean shear rate $2S_{xy}$ and Reynolds shear stress $-\overline{u'v'}$ have
258 opposite sign and that the flow experiences consequently a CDP with negative eddy
259 viscosity coefficient at two locations (shading in Fig. 4):

- 260 - In the upstream region ($x < 600\text{mm}$), near the separating streamline and on the
261 left bank side.
- 262 - Within the recirculation region at some non-analyzed locations.

263

264 The CDP in the present configuration is finally similar to all observed CDP in the
265 literature except it is certainly enhanced due to the high θ angle. The application of the
266 eddy viscosity concept in the Cartesian frame-axis (Eq. 2) is presented in Fig. 5. A best-
267 qualitative constant eddy-viscosity coefficient value $\nu_T^C = 8.10^{-5} \text{m}^2/\text{s}$ is chosen by

268 comparing Figs. 5a and 5b. The poor shape agreement between $-\overline{u'v'}$ and $2\nu_T^C S_{xy}$ in
269 terms of signs and magnitudes in the vicinity of the separating streamline confirms the
270 poor applicability of the eddy-viscosity concept in the present separating flow
271 configuration except in the downstream region.

272 Nevertheless, it should be noted that, while the CDP leads to a negative $P_M = -\overline{u'v'}.2S_{xy}$
273 turbulent kinetic energy production locally (Durbin [1993] wrote that “The negative
274 shear stress causes negative energy production in this region and this reduces the
275 turbulent kinetic energy”), the sum of all turbulent production terms
276 $P = -\overline{u'_i u'_j} \partial \overline{u_i} / \partial x_j$ remains positive (not shown here).

277

278 **Mixing layer analysis and eddy viscosity concept using a Serret-Frenet frame-axis**

279 **Application of Serret-Frenet frame-axis**

280 Another approach for analyzing the separating flow is proposed using a so-called 2D
281 Serret-Frenet (\mathbf{s} , \mathbf{n}) coordinate system (or local frame-axis) based on the direction of the
282 mean velocity in each point. Fig. 6a shows i) the streamlines, ii) the so-called separating
283 streamline, *i.e.* the streamline which initiates at the upstream corner and iii) the
284 fieldlines which are perpendicular to the velocity field at each location. All fieldlines
285 have an extremity at one of the banks and the other within the core of the recirculation.

286 Here, the unit vector \mathbf{s} is directed along the mean velocity (*i.e.* along the local
287 streamline), \mathbf{n} is perpendicular to \mathbf{s} , directed along the fieldline towards the center of
288 curvature of the streamlines (see Fig. 6b). When projecting the velocity field to the local

289 axis system, it comes that $\overline{u_s} = \sqrt{\overline{u^2 + v^2}}$ and $\overline{u_n} = 0$ but $u'_n \neq 0$. Moreover, a global
290 curvilinear coordinate system based on the separating streamline is set so that (\mathbf{S} , \mathbf{N}) is
291 the particular Serret-Frenet axis- system attached to the separating streamline (thick line
292 in Fig. 6b), S denotes the distance from the upstream corner along the separating
293 streamline and N denotes the distance from this streamline along any fieldline (with $N =$
294 0 on the separating streamline). Fig. 6b shows that, when plotted in Cartesian frame, \mathbf{n}
295 and N axes are directed towards the core of the recirculation region. In Fig. 6b, seven
296 fieldlines crossing the separating streamline at different S are depicted and are selected
297 for presenting the data in the following figures.

298

299 **Mixing layer description using Serret-Frenet frame-axis**

300 As exposed above, in the 2D Serret-Frenet frame-axis, only one mean velocity
 301 component is not equal to zero, *i.e.* \bar{u}_s . Fig. 7a presents profiles of streamwise velocity
 302 \bar{u}_s along the 7 selected fieldlines plotted in Fig. 6b. The streamwise velocity always
 303 decreases from the main flow towards the recirculation zone, that is for increasing N .
 304 Fig. 7b and 7c then show:

- 305 - That the normal gradient of streamwise velocity (Fig. 7b) and the Reynolds
 306 shear stress (Fig. 7c) are mainly negative. It is important to note that the
 307 negative signs of velocity gradient (Fig. 7b) and of Reynolds shear stress (Fig.
 308 7c) are a consequence of the orientation of \mathbf{n} towards the core of the
 309 recirculation region: if the orientation of \mathbf{n} had been chosen in opposite direction
 310 (towards the left bank), both $\partial\bar{u}_s/\partial n$ and $-\overline{u'_s u'_n}$ would have been positive.
- 311 - That the absolute normal gradient of streamwise velocity and absolute Reynolds
 312 shear stress are maximum close to the separating streamline for all S and
 313 decrease on both sides. The small shift of maximum absolute of these two terms
 314 towards the recirculation zone ($N>0$) may be related to the shift of the mixing
 315 layer towards the slower flow (see Figs. 6 and 7 of van Prooijen and Uijttewaal
 316 [2002]) or due to 3D effects and was observed for instance in Fig. 4 in
 317 [Uijttewaal and Booij, 2000] or in Figs. 5 and 9 in [Gibson and Younis, 1983].
- 318 - And that the absolute gradient and absolute Reynolds shear stress are maximum
 319 very close to the separating corner ($x\sim 300\text{mm}$ and $y\sim 0$) and slowly decrease
 320 towards downstream along S .

321 To define the mixing-layer, let us introduce the outer velocities $U1(S)=\bar{u}(S, N_1)$ and
 322 $U2(S)=\bar{u}(S, N_2)$ with $N_1<0$ and $N_2>0$, as the streamwise velocities located where
 323 $\partial\bar{u}_s/\partial n$ becomes negligible. Fig. 8a confirms that $U2(S)$ is almost equal to zero while
 324 $U1(S)$ remains quite constant at a magnitude of about $0.15 \text{ m}\cdot\text{s}^{-1}$. As a consequence, the
 325 velocity difference $\Delta U(S)=U1(S)-U2(S)$ remains quite constant for all S (Fig. 8b).
 326 Following the approaches available in the literature, the width of the mixing layer $\delta(S)$
 327 for all S is defined as:

$$329 \quad \delta(S) = \frac{\Delta U(S)}{\left| \partial\bar{u}_s/\partial n \right|_{\max}} \quad (3)$$

330

331 Fig. 8c reveals that $\delta(S)$ increases for increasing S (in agreement with mixing layers in
 332 the literature as observed by Gibson and Younis [1983] or Bell and Mehta [1990]) and
 333 reaches a plateau for $S > 400$ mm as the separating streamline becomes parallel to x axis.
 334 Saturation of the mixing layer width might be due to the confinement and its interaction
 335 with the boundary layer on the lateral wall.

336

337 Using the velocity scale $\Delta U(S)$ and the spatial scale $\delta(S)$, Fig. 9 shows the non-
 338 dimensional mean streamwise velocity profiles along the 7 fieldlines of Fig. 6b. It
 339 appears that for $S > 50$ mm, the velocity profiles become self-similar in the vicinity of the
 340 separating streamline ($-0.5 < N/\delta < 1.25$). This behavior is in agreement with straight
 341 [Wyganski & Fiedler, 1970] and curved [Gibson & Younis, 1983] mixing layers.

342

343 Figs. 7, 8 and 9 thus confirm the fair agreement of the present mixing layer analyzed
 344 using a Serret-Frenet coordinate system with the more classical straight and curved
 345 mixing layers analyzed using Cartesian and curvilinear coordinate systems respectively
 346 in the literature.

347

348 **Eddy viscosity concept using a Serret-Frenet frame-axis**

349 It can be shown [see Mignot *et al.*, 2013] that in the Serret-Frenet frame-axis, the
 350 Reynolds tensor writes:

$$351 \quad \mathbf{R}_{SF} = \begin{bmatrix} -\overline{u_s'^2} & -\overline{u_s' u_n'} \\ -\overline{u_s' u_n'} & -\overline{u_n'^2} \end{bmatrix} \quad (4)$$

352 And the mean rate of strain tensor writes:

$$353 \quad \mathbf{S}_{SF} = \begin{bmatrix} \frac{\partial \overline{u_s}}{\partial s} & \frac{1}{2} \left(\frac{\partial \overline{u_s}}{\partial n} + \frac{\overline{u_s}}{R_s} \right) \\ \frac{1}{2} \left(\frac{\partial \overline{u_s}}{\partial n} + \frac{\overline{u_s}}{R_s} \right) & \frac{\overline{u_s}}{R_n} \end{bmatrix} \quad (5)$$

354 Where R_s and R_n are the local radius of curvature of the streamlines and fieldlines,
 355 respectively. By analogy with the Eq. 2, we introduce the eddy-viscosity concept in the
 356 Serret-Frenet frame-axis using the non-diagonal terms of \mathbf{R}_{SF} and \mathbf{S}_{SF} (Eqs. 4-5) as:

$$357 \quad -\overline{u_s' u_n'} = \nu_T^{SF} \left(\frac{\partial \overline{u_s}}{\partial n} + \frac{\overline{u_s}}{R_s} \right) = \nu_T^{SF} . 2S_{sn} \quad (6)$$

358 with ν_T^{SF} the eddy-viscosity coefficient in the Serret-Frenet frame-axis.

359

360 Fig. 10a presents the distribution of the horizontal component of Reynolds shear stress
361 in the Serret-Frenet frame-axis $-\overline{u'_s u'_n}$ within the lateral branch of the bifurcation
362 (with the same data as in Fig. 7c). Unlike the analysis using a Cartesian frame-axis (see
363 previous sections and Fig. 5), no change of sign of $-\overline{u'_s u'_n}$ is observed over the whole
364 measured region.

365 Fig. 10b shows the spatial evolution of $2\nu_T^{SF} \mathbf{S}_{sn}$, where the value of the constant eddy-
366 viscosity coefficient $\nu_T^{SF} = 5.10^{-5} \text{ m}^2/\text{s}$ is chosen by comparing Figs. 10a and 10b. The
367 fair agreement between these two graphs confirms the high degree of application of the
368 eddy-viscosity concept using the Serret-Frenet frame-axis. Most importantly, unlike the
369 Cartesian frame-axis, no region of opposite signs between $-\overline{u'_s u'_n}$ and
370 $\nu_T^{SF} \left(\partial \overline{u_s} / \partial n + \overline{u_s} / R_s \right)$ - that is no CDP region - is observed along the separating
371 streamline when using the Serret-Frenet frame-axis. However a limited region of
372 positive, but limited, $\nu_T^{SF} \left(\partial \overline{u_s} / \partial n + \overline{u_s} / R_s \right)$ values takes place as the main flow rotates
373 when entering the lateral branch ($x \sim 350 \text{ mm}$ and $y \sim 250 \text{ mm}$), near the upstream top
374 corner ($x=y=300 \text{ mm}$) of the intersection. The failure of applicability of Eq. (6) in this
375 area may be related to the complex flow near the corner where a bow wave and a
376 horseshoe vortex take place and thus where 3D aspects of the flow should not be
377 negligible. Application of the eddy-viscosity concept in this area would then require 3D
378 flow data, which is not available using the present 2D-PIV experimental approach.

379

380 In order to analyze the spatial variations along S of the eddy-viscosity coefficient, ν_T^{SF}
381 was computed using two approaches along each of the seven fieldlines of Fig. 6b:

382 - by directly applying Eq. (6): $\nu_{T-\text{BestFit}}^{SF} = -\overline{u'_s u'_n} / \left(\partial \overline{u_s} / \partial n + \overline{u_s} / R_s \right)$ using a least
383 square fitting method over each fieldline in the vicinity of the mixing layer (for -
384 $60 \text{ mm} < N < 100 \text{ mm}$).

385 - by applying Prandtl mixing length model using the empirical relation between
386 the mixing length and the mixing layer width initially introduced by Kuethe
387 [1935] for a jet and summarized in Rodi [1993]:

388 $\nu_{T-\text{Prandtl}}^{SF} = (0.07\delta)^2 \left| \partial \overline{u_s} / \partial n + \overline{u_s} / R_s \right|$ where the absolute value in this expression

389 is evaluated for each fieldline at the location of maximum streamwise velocity
390 gradient $\partial \bar{u}_s / \partial n$.

391 Fig. 11 shows that these two eddy-viscosity coefficients tend to increase along S from
392 the separation point towards downstream. Both estimates give quite similar results:
393 about $2 \cdot 10^{-5}$ to $6 \cdot 10^{-5}$ m²/s, confirming the value $\nu_T^{SF} = 5 \cdot 10^{-5}$ m²/s obtained as a
394 qualitative constant determination over the whole region (Fig. 10). This validates the
395 use of Prandtl mixing length theory for separating mixing layers even though the
396 Prandtl mixing length estimated coefficient remains lower by 5% to 50% compared to
397 the best-fit coefficient along the mixing layer.

398

399 In order to calculate the error made when estimating the Reynolds shear stress, the
400 eddy-viscosity concept (Eq. 6) is applied using these eddy-viscosity coefficients in Fig.
401 12 along the seven fieldlines shown on Fig. 6b. Fig. 12 thus compares:

402 - the measured Reynolds shear stress $-\overline{u'_s u'_n}$,

403 - $\nu_{T-\text{BestFit}}^{SF} \left(\partial \bar{u}_s / \partial n + \bar{u}_s / R_s \right)$,

404 - $\nu_{T-\text{BestFit}}^{SF} \left(\partial \bar{u}_s / \partial n \right)$ which is a simplified version of Eq. (6) considering only the term
405 involving the streamwise velocity gradient and neglecting the radius of curvature.

406 - $\nu_{T-\text{Prandtl}}^{SF} \left(\partial \bar{u}_s / \partial n + \bar{u}_s / R_s \right)$.

407 This figure reveals that the eddy-viscosity concept (Eq. 6) considering the best-fit
408 coefficient $\nu_{T-\text{BestFit}}^{SF}$ along each fieldline (diamonds) fairly predicts the Reynolds shear
409 stress. On the other hand, the simplified eddy-viscosity concept, considering $\nu_{T-\text{BestFit}}^{SF}$
410 (circles) leads to a very similar estimate of the Reynolds shear stress as Eq. (6). This
411 confirms that the curvature effect can be neglected here with regards to the streamwise
412 velocity gradient along N . Finally, the eddy-viscosity concept (Eq. 6) considering an
413 eddy-viscosity coefficient obtained through the Prandtl mixing length approach $\nu_{T-\text{Prandtl}}^{SF}$
414 (full squares) results in a fair shape of the Reynolds shear stress but strongly
415 underestimates its magnitude (as could be predicted from Fig. 11).

416

417 **Geometrical analysis**

418 In the previous sections, we showed that the eddy viscosity concept in the Cartesian
419 frame-axis (Eq. 2) is not relevant as the eddy viscosity coefficient ν_T^C would
420 dramatically vary from one location to another and would reach negative values in the
421 near-separation region for Group 1 with high θ values. Oppositely, we showed that the
422 eddy viscosity concept in the Serret-Frenet frame-axis (Eq. 6) is relevant in the whole
423 domain and leads to a quasi-constant eddy viscosity coefficient $\nu_T^{SF} = 5.10^{-5} \text{ m}^2/\text{s}$ (see
424 Fig. 10). This is coherent with Boussinesq's idea that applies for uniform shear flows in
425 the direction of maximum shear. The separating flow can thus be considered as an
426 almost uniform shear flow in the local frame-axis, while it is not the case in the
427 Cartesian frame axis. Once accepted that the eddy viscosity concept applies in the
428 Serret-Frenet frame axis, occurrence of negative ν_T^C coefficients for high θ values can
429 be explained using geometrical considerations as exposed below and in the appendix.
430 Consider the local Cartesian frame-axis (x,y) and the Serret-Frenet frame-axis (s,n) as
431 depicted in Fig. 13.

432
433 The projection of the Serret-Frenet axis (SF) in the Cartesian (C) axis writes:

$$434 \begin{cases} \vec{x} = \sin\theta \vec{s} + \cos\theta \vec{n} \\ \vec{y} = \cos\theta \vec{s} - \sin\theta \vec{n} \end{cases} \quad (7)$$

435 The rotation matrix P_C^{SF} that allows frame change from C to SF (and its counterpart
436 P_{SF}^C) thus writes:

$$437 P_C^{SF} = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix} \text{ and } P_{SF}^C = P_C^{SF^{-1}} = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix} \quad (8)$$

438 As a consequence, the relationship between the components of the instantaneous
439 velocity vector $\vec{v} = u\vec{x} + v\vec{y} = u_s\vec{s} + u_n\vec{n}$ is straightforward:

$$440 \begin{bmatrix} u \\ v \end{bmatrix}_C = P_C^{SF} \begin{bmatrix} u_s \\ u_n \end{bmatrix}_{SF} = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix} \begin{bmatrix} u_s \\ u_n \end{bmatrix}_{SF} \quad (9)$$

441 Then, the Reynolds stress tensor \mathbf{R}_{SF} (Eq. 4) and the mean rate of strain tensor \mathbf{s}_{SF} (Eq.
442 5) in the Serret-Frenet frame can be expressed as a function of the Reynolds stress
443 tensor $\mathbf{R}_C = -\overline{u'_i u'_j}$ and the mean rate of strain tensor $\mathbf{s}_C = (\partial\overline{u}_i/\partial x_j + \partial\overline{u}_j/\partial x_i)/2$ in Cartesian
444 frame as:

$$445 \mathbf{R}_{SF} = P_{SF}^C \mathbf{R}_C P_C^{SF} \text{ and } \mathbf{s}_{SF} = P_{SF}^C \mathbf{s}_C P_C^{SF} \quad (10)$$

446 As detailed in the appendix, i) applying Eq. (10) to \mathbf{R}_{xy} (defined in Eq. 11) and \mathbf{s}_{xy}
 447 (defined in Eq. 12), ii) applying the eddy viscosity concept in the Cartesian frame-axis
 448 (Eq. 2) and iii) including the eddy viscosity concept in the Serret-Frenet frame-axis (Eq.
 449 6) finally leads to Eq. (15):

$$450 \quad v_T^C = \frac{-v_T^{SF} \left(\frac{\partial \bar{u}_s}{\partial n} + \frac{\bar{u}_s}{R_s} \right) + \tan(2\theta) (\bar{u}_n^2 - \bar{u}_s^2) / 2}{-\left(\frac{\partial \bar{u}_s}{\partial n} + \frac{\bar{u}_s}{R_s} \right) + \tan(2\theta) \left(\frac{\partial \bar{u}_s}{\partial s} - \frac{\bar{u}_s}{R_n} \right)} \quad (15)$$

451 We have shown that v_T^{SF} remains positive except near the upstream top corner of the
 452 intersection where the 2D eddy-viscosity concept does not apply (see Fig. 10). The sign
 453 of v_T^C thus depends on θ :

454 • For low θ , $\tan(2\theta)$ is small so that: i) the denominator is dominated by the first
 455 term (note that $\partial \bar{u}_s / \partial n \gg$ in the mixing layer, see Fig. 7a) and ii) as
 456 tangential and longitudinal velocity variances have similar orders of magnitude
 457 in the present data (not shown here) and in the literature (see for instance
 458 Gibson et al. [1983]'s data), the numerator is also dominated by the first term.
 459 As a consequence, $v_T^C > 0$ and $\lim_{\theta \rightarrow 0} v_T^C = v_T^{SF}$.

460 • For θ approaching $\pi/4$, $\tan(2\theta)$ becomes infinite and $\lim_{\theta \rightarrow \pi/4} v_T^C = \frac{1}{2} \frac{\bar{u}_s^2 - \bar{u}_n^2}{-\frac{\partial \bar{u}_s}{\partial s} + \frac{\bar{u}_s}{R_n}}$

461 However, $\bar{u}_s^2 > \bar{u}_n^2$ in the present experiment (not shown here) as in Cartesian
 462 frame-axis in the literature (see for instance Gibson et al. [1983] or Bell and Mehta
 463 [1990]'s data) so that the numerator is positive. Moreover, in the regions where
 464 $\theta \sim \pi/4$, especially near the separating streamline ($100\text{mm} < S < 300\text{mm}$), the
 465 denominator is negative (not shown here) leading to a negative v_T^C and thus to the
 466 CDP phenomenon.

467 As a conclusion, when the flow is rotated by about 45° as regards to the main axis, v_T^C
 468 may take negative values while v_T^{SF} remains positive. The apparent counter-gradient
 469 diffusion phenomenon is simply due to the fact that the eddy viscosity concept applies
 470 only in the direction of maximum shear, which is the flow direction (s). This fact also
 471 explains that, when using Serret-Frenet frame instead of Cartesian frame, better
 472 agreement is achieved between the present experimental velocity profiles or Reynolds

473 stress profiles and classical mixing layer profiles.

474 At last, for flows of Group 2, the direction of the separating streamline (S) is quite
475 parallel to the main axis (x) until approaching the reattachment point where the flow
476 pattern becomes much more complex [see Riviere et al. 2011]. As regards to the present
477 analysis, since θ remains low, ν_T^C remains positive, approaching ν_T^{SF} . Therefore, no CDP
478 is observed in such flows.

479

480 **Conclusion**

481 The present study aimed at proposing a methodology for the analysis of separating
482 open-channel flows. Works from the literature reported that for configurations with
483 large angles of separating streamline (referred to as “Group 1”), the analysis using a
484 Cartesian frame-axis leads to a change of sign of the Reynolds shear stress along with a
485 negative eddy viscosity coefficient. These results were retrieved in the lateral branch of
486 the present open-channel bifurcation which may represent a bifurcation in a river
487 network (in absence of sediments), an irrigation network, a sewer network or a
488 crossroad during urban flooding.

489 Moreover, we showed that such analysis in Cartesian frame-axis does not permit to
490 draw conclusions of size, shape and turbulent characteristics of the mixing layer.
491 Oppositely, we showed that using a so-called local “Serret-Frenet” frame-axis based on
492 the direction of the local mean velocity, the mixing layer data become explicit. The
493 shape, size and turbulent characteristics of the mixing layer are then similar to that of
494 the literature. This approach applies to any separating flow including the lateral channel
495 of open channel bifurcations of any angle and any channel width.

496 Finally, the eddy viscosity concept nicely applies in the local frame-axis, given the high
497 similarity between the Reynolds shear stress and the mean shear rate. Such local frame-
498 axis thus appears to be well-suited for characterizing the mixing layer of the separating
499 flow.

500 To conclude, we provide a generalization of the classical analysis of simple 2D mixing
501 layers (either straight or curved) reported in most textbooks for more complex 2D
502 mixing layers (among which the separating flows) using a simple and intuitive
503 approach. The use of this approach should be generalized to all mixing layers as long as
504 the main axis of the mixing layer deflects from the main direction of the axis system.

505 The present results have major relevance to passive or reactive scalar (such as
506 pollutants) exchanges between the main flow and the recirculation zone. Light
507 suspended sediments or dilute suspensions that follow the carrier flow, may be trapped
508 in the recirculation. Therefore, locally, very high sediment concentrations may appear
509 leading to deposition or agglomeration.

510

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514

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583

584 **Appendix**

585 This appendix details the steps from Eq. (11) to Eq. (16). Applying Eq. (11) to the non-
586 diagonal term of the Reynolds tensor in Cartesian frame reads:

$$587 \quad \mathbf{R}_{xy} = -\overline{u'v'} = \left(\overline{u'_s u'_n} \right) \cos(2\theta) + \frac{\sin(2\theta)}{2} \left(\overline{u_n'^2} - \overline{u_s'^2} \right) \quad (11)$$

588 Now, applying Eq. 11 to the non-diagonal term of the mean shear rate in Cartesian
589 frame leads to:

$$590 \quad \mathbf{S}_{xy} = \frac{1}{2} \left(\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} \right) = \frac{1}{2} \left\{ \left(\frac{\partial \bar{u}_s}{\partial s} - \frac{\bar{u}_s}{R_n} \right) \sin(2\theta) - \left(\frac{\partial \bar{u}_s}{\partial n} + \frac{\bar{u}_s}{R_s} \right) \cos(2\theta) \right\} \quad (12)$$

591 Thus, applying the eddy viscosity concept in Cartesian frame-axis ($\mathbf{R}_{xy} = \nu_T^C \cdot 2\mathbf{S}_{xy}$ in Eq.
592 2) using Eqs. 12 and 13 reads:

$$593 \quad \nu_T^C \left\{ \left(\frac{\partial \bar{u}_s}{\partial s} - \frac{\bar{u}_s}{R_n} \right) \sin(2\theta) - \left(\frac{\partial \bar{u}_s}{\partial n} + \frac{\bar{u}_s}{R_s} \right) \cos(2\theta) \right\} = \left(\overline{u'_s u'_n} \right) \cos(2\theta) + \frac{\sin(2\theta)}{2} \left(\overline{u_n'^2} - \overline{u_s'^2} \right) \quad (13)$$

594 and including the eddy viscosity concept in Serret-Frenet frame-axis (Eq. 6) in the first
 595 Right Hand Side term of Eq. (14) gives:

$$596 \quad v_T^C \left\{ \left(\frac{\partial \bar{u}_s}{\partial s} - \frac{\bar{u}_s}{R_n} \right) \sin(2\theta) - \left(\frac{\partial \bar{u}_s}{\partial n} + \frac{\bar{u}_s}{R_s} \right) \cos(2\theta) \right\} = -v_T^{SF} \left(\frac{\partial \bar{u}_s}{\partial n} + \frac{\bar{u}_s}{R_s} \right) \cos(2\theta) + \frac{\sin(2\theta)}{2} (\bar{u}_n^2 - \bar{u}_s^2) \quad (14)$$

597 that is

$$598 \quad v_T^C = \frac{-v_T^{SF} \left(\frac{\partial \bar{u}_s}{\partial n} + \frac{\bar{u}_s}{R_s} \right) + \tan(2\theta) (\bar{u}_n^2 - \bar{u}_s^2) / 2}{-\left(\frac{\partial \bar{u}_s}{\partial n} + \frac{\bar{u}_s}{R_s} \right) + \tan(2\theta) \left(\frac{\partial \bar{u}_s}{\partial s} - \frac{\bar{u}_s}{R_n} \right)} \quad (15)$$

599

600 Fig. 1. Sketch of the separation zone for one case of each group: the upward facing-step for
601 group 1 and the downward-facing step for group 2; “Ss” stands for the mean flow axis at the
602 geometrical singularity.

603

604 Fig. 2. Sketch of the experimental set-up and close– top photography view of the intersection
605 zone, white dye has been injected in the main branch to visualize the mixing-layer (not used for
606 quantitative data).

607

608 Fig. 3. Time-averaged velocity field along with separation streamline (plain line). Right graph is
609 a velocity magnification to unveil the recirculation region.

610

611 Fig. 4. Distribution of mean shear rate (+) along with Reynolds shear stress (\diamond). The CDP zones
612 are indicated by shading and the plain thick line is the separating streamline.

613

614 Fig. 5. Application of eddy viscosity concept in Cartesian frame-axis. a: 2D distribution of
615 horizontal Reynolds shear stress. b: 2D distribution of $2\nu_T^C \mathbf{S}_{xy}$, with a constant eddy viscosity
616 value: $\nu_T^C = 8.10^{-5} \text{ m}^2/\text{s}$. The plain line is the separating streamline.

617

618 Fig. 6. a: streamlines, separation streamline (thick line), and fieldlines (perpendicular to the
619 flow). b: velocity field with the Serret-Frenet frame-axis, the separating streamline and 7
620 fieldlines (plain grey lines) selected for analysis.

621

622 Fig. 7. Profiles of mean streamwise velocity ($\overline{u_s}$, a), normal gradient of mean streamwise
623 velocity ($\overline{\partial u_s / \partial n}$, b) and Reynolds shear stress ($-\overline{u'_s u'_n}$, c) along the 7 fieldlines of Fig. 6b.

624 For sake of simplicity, orientation of the vertical axis was reversed.

625

626 Fig. 8. a: outer velocities U1(S)(+) and U2(S) (o), b: velocity scale and c: mixing layer width.

627

628 Fig. 9. Non-dimensional mean streamwise velocity profiles along the seven fieldlines of Fig. 6b.
629 For sake of simplicity, orientation of the vertical axis was reversed.

630

631 Fig. 10. Application of eddy viscosity concept in Serret-Frenet frame-axis a: 2D distribution of
632 horizontal Reynolds shear stress. b: 2D distribution of $2\nu_T^{SF} \mathbf{S}_{sn}$ with a constant eddy viscosity
633 value: $\nu_T^{SF} = 5.10^{-5} \text{ m}^2/\text{s}$. The white line is the separating streamline.

634

635 Fig. 11. Evolution along S of eddy- viscosity coefficients $\nu_{T-\text{BestFit}}^{\text{SF}}$ (o) and $\nu_{T-\text{Prandtl}}^{\text{SF}}$ (+).

636

637 Fig. 12. Reynolds shear stress $-\overline{u'_s u'_n}$ (X) and application of Eq. (6) using: (\diamond)

638 $\nu_{T-\text{BestFit}}^{\text{SF}} \left(\frac{\partial \overline{u_s}}{\partial n} + \overline{u_s} / R_s \right)$, (o) the simplified version of Eq (6) $\nu_{T-\text{BestFit}}^{\text{SF}} \left(\frac{\partial \overline{u_s}}{\partial n} \right)$, and (\blacksquare)

639 $\nu_{T-\text{Prandtl}}^{\text{SF}} \left(\frac{\partial \overline{u_s}}{\partial n} + \overline{u_s} / R_s \right)$.

640

641 Fig. 13. Scheme of the Cartesian (x,y) and Serret-Frenet (s,n) frame-axes

642