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Cédric Payan, Vincent Garnier, Joseph Moysan. Determination of nonlinear constants and stress monitoring in concrete by coda waves analysis. European Journal of Environmental and Civil Engineering, 2011, 15 (4), pp.519-531. 10.1080/19648189.2011.9693344 . hal-01294740

## HAL Id: hal-01294740
## https://hal.science/hal-01294740

Submitted on 20 May 2024

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# Determination of nonlinear elastic constants and stress monitoring in concrete by coda waves analysis

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ABSTRACT. *For non destructive stress evaluation in prestressed concrete, we describe in this paper a promising ultrasonic method. In "ordinary" isotropic homogeneous materials such as steel, ultrasonic determination of third order elastic constants provides means for stress evaluation since half a century. In case of concrete, example of a class of materials that exhibit strong multiple scattering as well as significant elastic nonlinear response, accurate velocity monitoring is challenging. In this paper, we purpose a novel method originally from geophysics. So called "coda wave interferometry" (CWI), this technique is transposed to concrete. We show that intense scattering can be applied to robustly determine velocity changes and so, nonlinear elastic constants. In addition, we show how this method could be transposed in case of in situ measurements in concrete structures and outline future possible applications.*

RÉSUMÉ. *Nous présentons dans cet article une méthode prometteuse d'évaluation de la précontrainte dans les structures en béton. Pour les milieux homogènes isotropes, les constantes élastiques du troisième ordre sont connues et étudiées depuis une cinquantaine d'années. Le béton est un milieu très fortement diffusant et dont le comportement mécanique est fortement non linéaire. Les mesures de vitesses sont souvent obtenues avec une résolution inférieure à 1 %. Dans cet article, nous proposons une méthode, « Coda Wave Interferometry » (CWI), issue de la géophysique et transposée ici au cas du béton. Nous montrons que la diffusion multiple peut être mise à profit pour déterminer de très faibles variations relatives de vitesses et donc, les constantes élastiques non linéaires. Nous montrons de plus que cette méthode peut être transposée* in situ *dans le cadre du suivi de l'état de santé des ouvrages d'arts en béton précontraint.*

KEYWORDS: *stress monitoring, nonlinear parameter, coda waves, multiple scattering.*

MOTS-CLÉS : *suivi de contrainte, paramètre non linéaire, coda, diffusion multiple.*

## 1. Introduction

Acoustoelastic-derived nonlinear properties of isotropic, homogeneous materials have been intensely studied for at least half a century. For instance, Hughes and Kelly (1953) derived expressions for the speeds of elastic waves in a stressed solid using Murnaghan's theory of finite deformations and third-order terms in the strain-energy expression (Murnaghan, 1951). In complex materials, determining the third order constants accurately can be challenging, due to significant intrinsic dissipation, as well as heterogeneity leading to strong wave scattering. Most Earth materials fall into this class, and an extreme example is concrete. It is highly complex both chemically and mechanically, is both porous and permeable, heterogeneous and highly elastically nonlinear. In typical laboratory acoustic measurements, frequencies range from 200 kHz to 1 MHz in concrete. Associated wavelengths and typical aggregate sizes are equivalent, leading to strong multiple scattering (Anugonda *et al.*, 2001) into the stochastic regime. In this paper, we make use of the information carried by the waveform coda generated by multiple scattering, to obtain velocity. Applying successively larger stresses in combination with coda interferometry provides the means to evaluate velocity as a function of pressure and thereby extract the third order nonlinear coefficients.

The study of multiple scattering in the earth (termed "coda" originally by Aki (1956)), has been of interest to the geoscience community for at least 50 years (Aki, 1969), for applications ranging from earthquake source localization, to location of petroleum reservoirs and to monitoring volcanic activity. The first method developed for monitoring velocity variations employing coda, termed "doublets" (referring to successive, nearly identical signals from the same Earthquake source) was proposed by Poupinet (Poupinet *et al.*, 1984). The method was refined in laboratory studies by development of the addition of monitoring changes in attenuation and applying an active source by Roberts *et al.* (1992), where it was termed the "active doublet method". More recently developments have been aimed at detecting small changes in scattering locations due to modifications in Earth's crust, including velocity changes induced by thermal stress or stress accumulation in the crust, and source location (earthquake localization). This more recent version of the method has been broadly termed "coda wave interferometry" (Snieder *et al.*, 2002).

The purpose of this paper is to describe the development of coda wave interferometry in conjunction with incremental changes in applied stress to a specimen for determining the third order elastic constants of concrete, a method that can be applied to any solid, but is particularly appropriate for complex solids. This is accomplished by applying the coda interferometry as applied stress is step-wise increased. The essence of the method is to extract velocity and/or attenuation change between two time signals by analyzing waveform coda changes. This is accomplished by time-window, cross-correlation between signals. By inverting results from all pressure steps the Murnahagan coefficients are calculated. Recently a similar approach was applied to monitor thermally induced velocity variations in a solid (Larose *et al.*, 2006), as well as observations of velocity changes in a sample of Berea sandstone (Grêt *et al.*, 2006). In the following section we define the Murnaghan coefficients. We then describe details of how the coda interferometry method is implemented. This is followed by a description of the experimental procedure, analysis and results.

## 2. Acousto-elasticity theory

In case of uniaxial loading, the strain-induced velocity variations in an initially isotropic medium provides quantitative values of the Murnaghan constants *l, m, & n*. Considering the loading direction as 1 (see Figure 2), the Murnaghan's third order elastic constants are obtained by a first order approximation assuming small changes in velocities (Egle and Bray, 1976),

$$\frac{dV_{11}}{V_{11}^0} = \left[ 2 + \frac{\mu + 2m + \upsilon\mu(1 + 2l/\lambda)}{\lambda + 2\mu} \right] d\varepsilon , \quad [1]$$

$$\frac{dV_{12}}{V_{12}^0} = \left[ 2 + \frac{n\upsilon}{4\mu} + \frac{m}{2(\lambda + \upsilon)} \right] d\varepsilon , \quad [2]$$

$$\frac{dV_{22}}{V_{22}^0} = -2\upsilon \left[ 1 + \frac{m - l\mu/\lambda}{\lambda + 2\mu} \right] d\varepsilon , \quad [3]$$

$$\frac{dV_{21}}{V_{21}^0} = \left[ \frac{\lambda + 2\mu + m}{2(\lambda + \mu)} + \frac{n\upsilon}{4\mu} \right] d\varepsilon , \quad [4]$$

$$\frac{dV_{23}}{V_{23}^0} = \left[ \frac{m - 2\lambda}{2(\lambda + \mu)} - \frac{n}{4\mu} \right] d\varepsilon , \quad [5]$$

where $\lambda$ and $\mu$ are the second order Lame constants, $\upsilon$ is Poisson's ratio, $\varepsilon$ the strain in the 1 direction, $V_{ij}$ the speed of a wave propagating in the $i$ direction polarized in the $j$ direction and the 0 indicies designate the wave speed in unstrained state. By measuring the Lame elastic constants, Equation [1] gives us five equations for three unknowns ($l$, $m$, $n$). Using $V_{21}$, $V_{22}$, $V_{23}$ one may invert for the full set of third order elastic constants, by solving the linear system of equations (Equations [1]-[5]) re-written (Egle and Bray, 1976),

$$l = \frac{\lambda}{1-2\upsilon}\left[\frac{1-\upsilon}{\upsilon}L_{22} + \frac{2}{1+\upsilon}\left(L_{21} + \upsilon L_{23}\right) + 2\upsilon\right] \quad [6]$$

$$m = 2(\lambda + \upsilon)\left[\frac{\upsilon}{1+\upsilon}L_{23} + \frac{1}{1+\upsilon}L_{21} + 2\upsilon - 1\right] \quad [7]$$

$$n = \frac{4\mu}{1+\upsilon}\left[L_{21} - L_{23} - 1 - \upsilon\right] \quad [8]$$

where the acoustoelastic constants $L_{ij}$ are defined as $dV_{ij}/V_{ij}^{0} = L_{ij}d\varepsilon$. We will use these relations after describing the coda interferometry method.

## 3. Coda wave interferometry

In the coda interferometry method, velocity changes are determined by cross-correlating piecewise two waveforms, following a change in the material, creating the correlation function. If no change has occurred the correlation produces a sequence of autocorrelation functions. In experiment, stress is applied sequentially stepwise, $\sigma_1, \sigma_2, \ldots, \sigma_n$. The waveform corresponding to $\sigma_i$ is $w_i(t)$. The time windowed intercorrelation function is defined as:

$$R_i^{(t,T)}\left(t_w\right) = \frac{\int_{t-T}^{t+T} w_0\left(t'\right) w_i\left(t'+t_w\right)dt'}{\sqrt{\int_{t-T}^{t+T} w_0^2\left(t'\right)dt' \int_{t-T}^{t+T} w_i^2\left(t'\right)dt'}}, \quad [9]$$

where $2T$ is the length of the time window and $t_w$ is the centre position. The position of the maximum indicates the mean of the time lag $<dt_i(t_w,T)>$ between $s_0$ and $s_i$ and in each time window (Figure 1).

The time $t$ spent by the wave into the multiple scattering medium following the path $P$ in the unstressed state can be written:

$$t = \int_P \frac{1}{V}ds \quad [10]$$

The stress induced perturbation time $dt_i$ and velocity $dV_i$ are linked to the first order in the velocity perturbation by,

$$t + dt_i = \int_P \frac{1}{V + dV_i} ds = \int_P \frac{1}{V} ds - \int_P \frac{dV_i}{V^2} ds \qquad [11]$$

Following Equation [1]-[5], the relative velocity variation is a constant for each stress step $s_i$. Using Equation [9],

$$\frac{dt_i}{t} = -\frac{dV_i}{V} \qquad [12]$$

Assuming the time window is centred at $t_w >> T$, we can write $\tau_i = < dt_{i(tw,T)} > = dt_i$. The velocity variation in each time window of the ith waveform is obtained then by:

$$\frac{dV_i}{V} = -\frac{\tau_i}{t_w} \qquad [13]$$

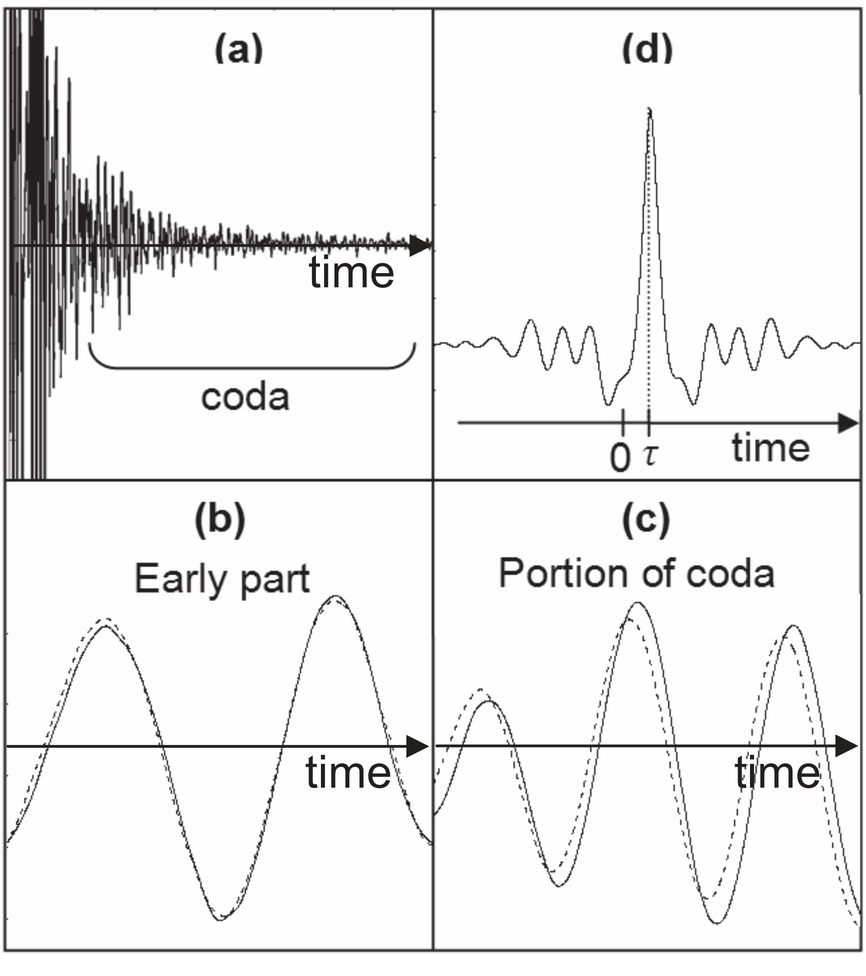


**Figure 1.** *Typical analysis of coda waves. (a) recorded signal. (b) First arrival (ballistic wave) and (c) portion of coda. Solid line unstressed state. Dashed line: stressed state. (d) Cross-correlation function and extraction of the time lag*

This expression is valid for any direction considered while the wave path is not modified. An advantage of this approach is that we do not require a velocity reference in the unstrained state. In addition, we do not need any distance measurement.

## 4. Experiments

Our study is performed on a cylindrical concrete sample 160 mm long by 75 mm in diameter. The composition is given in Table 1. Strain gages were attached to the sample in order to monitor the strain in the *1* direction. Study of identical samples yielded a Young's modulus of 42.39 GPa, an ultimate strength of 76.6 MPa and a Poisson's ratio of 0.21. The loading protocol is determined so as to not exceed 30% of the ultimate strength in order to remain in the elastic regime (Hsu, 1984).

**Table 1.** *Concrete sample's composition*

| Composition | Proportion ($kg.m^{-3}$) |
|---|---|
| Cement CPA CEM1 52.5 | 415 |
| Water | 187 |
| Adjuvant | 4,1 |
| Sand 0/4 mm | 865 |
| Aggregates 4/10 mm | 355 |
| Aggregates 10/16 mm | 710 |

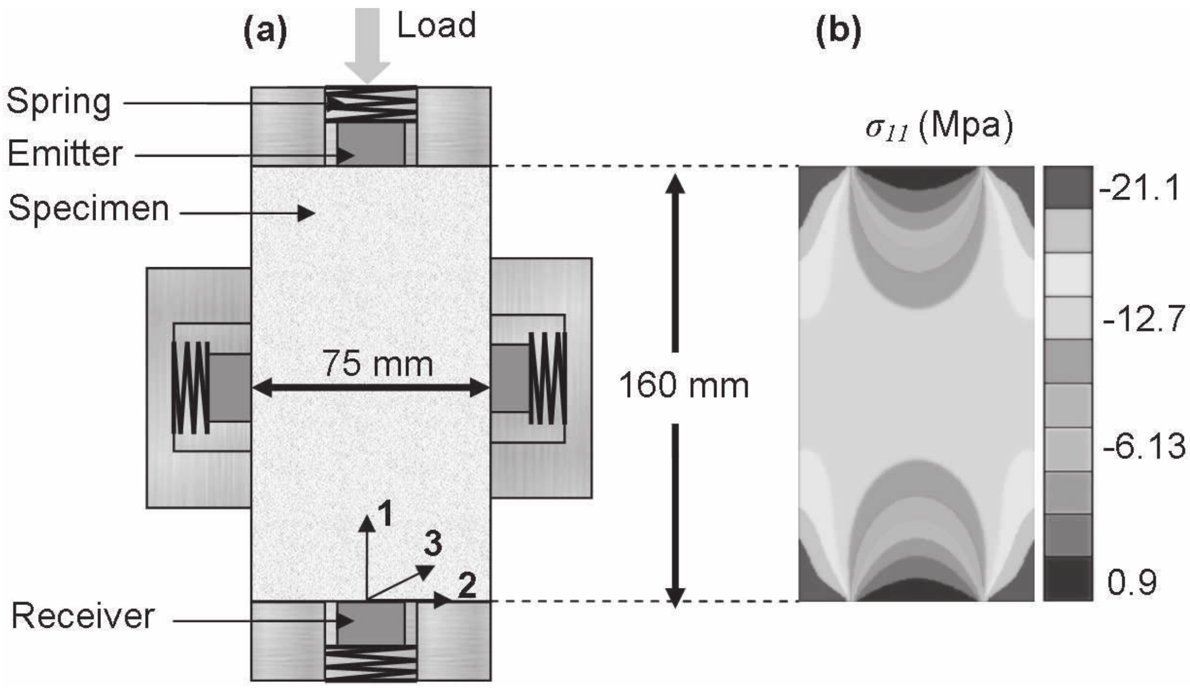


**Figure 2.** *(a) Sample configuration. (b) Stress field ($\sigma_{11}$) in the sample obtained by finite element analysis*

A hydraulic press (MTS 318.25) was programmed applying six stress steps from 0 to 13 MPa. Ultrasonic transducers 25 mm in diameter (Sonaxis CMP36, and Panametrics V151 central frequency 500 kHz) were attached to the sample center at each end using ultrasonic coupling gel, and maintained in position applying constant force, using springs located in holders as shown in Figure 2a. The transducers were driven by a high voltage system (Panametrics 5058PR). The load cell forms an annulus around the transducers (Figure 3a). As a result the stress field in the *1* direction is not homogeneous along the *1* axis of the sample (Figure 3b). Using $V_{21}$, $V_{22}$, $V_{23}$ in the region where stress is homogeneous (sample center) one may invert for the full set of third order elastic constants (Egle and Bray, 1976), by solving the linear system of equations (Equation [1]-[5]). Measurements of $V_{11}$, and $V_{12}$ were also performed. In order to illustrate the method we present the results of the compressional wave velocity change $V_{22}$. The method is identical for the other directions.

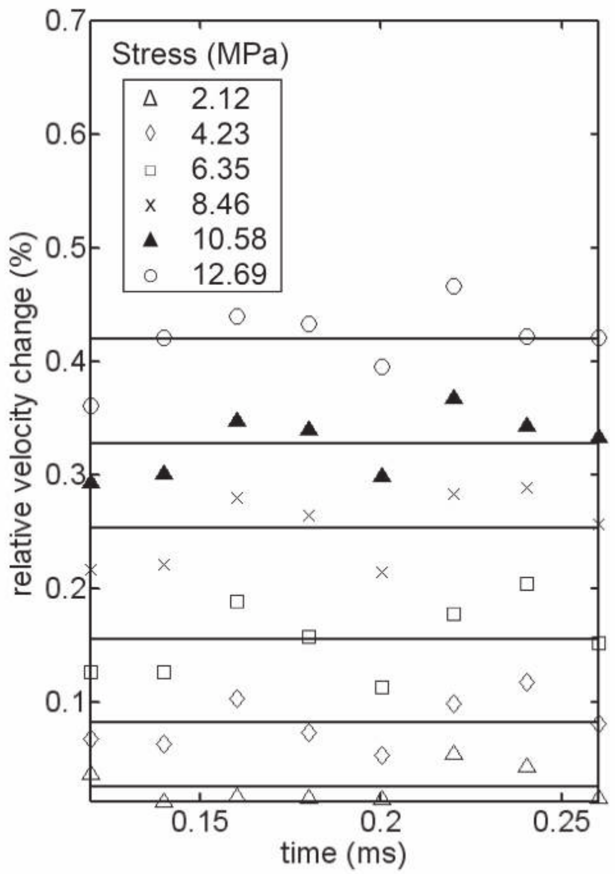


**Figure 3.** *Result of coda wave interferometry. Solid lines are the mean values of* $dV_{22}/V_{22}$

Much empirical evidence (Snieder *et al.*, 2002; Grêt, Snieder and Scales, 2006) suggests that computing the cross-correlation function (Equation [9]) using 10 signal periods is optimal. In order to satisfy the assumption $t >> T$, we begin the coda analysis at $t = 10\,T$ ($t$ = 0.1 ms while compressional-mode time of flight is 14 µs) and continue to the end of the signal ($t$ = 0.26 ms). In this manner the relative velocity variation is computed for 8 non-overlapping windows for each stress step. Example from the six stress steps are shown in Figure 3.

We observe that the relative velocity variation $dV_{22}/V_{22}$ is a constant for each stress step but increases with stress as it should. The relative uncertainty decreases with increasing stress (from 9% to 2.3). The data scatter is the same for each stress state. That implies that the waves follow the same path for each stress state.

Note that the analysis is limited to 0.26 ms. Data scatter increases in the later coda due to the fact that in late time, the high stress regions at the sample ends become important contributors (see Figure 3).

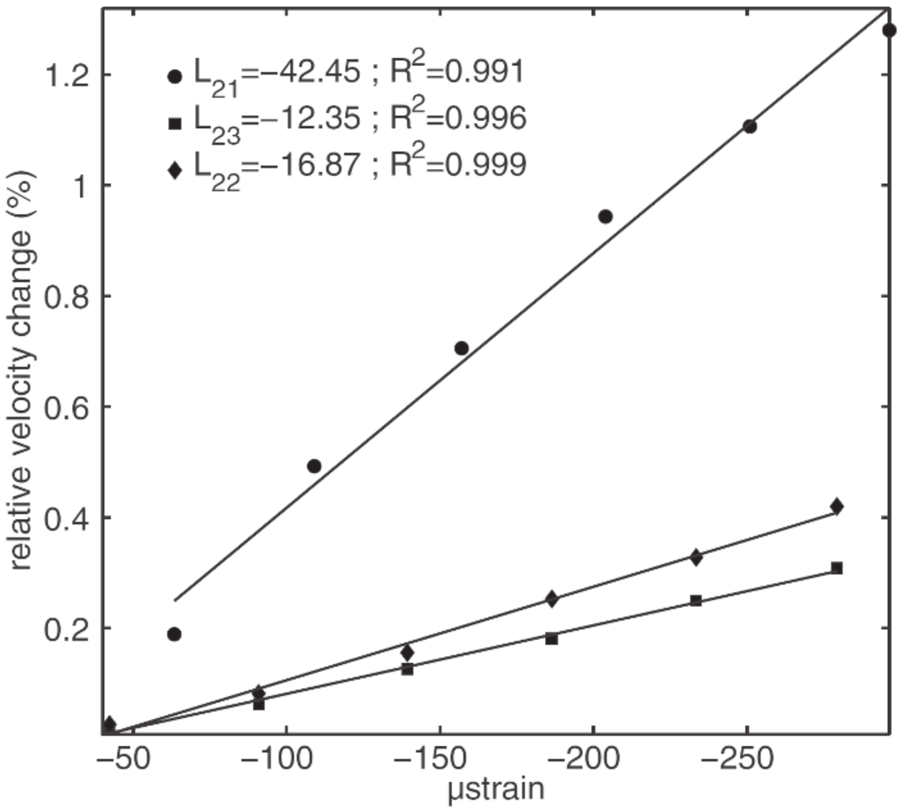


**Figure 4.** *The velocity derivative versus quasisatic strain used for inversion of the third order constant (see Table 1). $R^2$ is the determination coefficient*

The fits of $dV_{2j}/V_{2j}$ as a function of the measured strain for each applied stress for the *2* direction are given in Figure 4. The slopes of the fits are the acoustoelastic constants $L_{ij}$ used for calculation of $l$, $m$ & $n$ from Equation [6]-[8]. The associated third order constants are shown in Table 2. Values for granite and sandstone samples obtained from the literature are shown for comparison. As in less complex materials such as steel or aluminium, we observe that the most sensitive waves to stress are those which have particle displacement in the 1 direction, *i.e.* $V_{21}$ and $V_{11}$. But, in contrast to these materials, the velocity increases in all directions with stress.

The local phase velocity of a sinusoidal compressional wave propagating in a one-dimensional nonlinear medium can be written to first order as

$V(\varepsilon) = V(0)(1 + \beta\varepsilon)$, where $\beta$ is the nonlinear acoustic parameter (Hamilton, 1986). $\beta$ can be written as a combination of Murnaghan's and Lame's elastics coefficients,

$$\beta = \frac{3}{2} + \frac{l + 2m}{\lambda + 2\mu} \qquad [14]$$

Thus the nonlinear behaviour is not characterized by the absolute values of *l*, *m* and *n*, but by the ratio of second and third order elastic constants. The extension to beta is known in the literature (Hamilton, 1986 or Johnson, 1996). In addition, the fact that the velocities increase with this kind of loading, make l, m, and n negative, so beta negative too. The large, negatives values are similar to those found in rock (Johnson and Rasolofosaon, 1996) and are more than two orders of magnitude greater than an ordinary nonlinear material such as steel (Egle and Bray, 1976). Note the sign of $\beta$ means that as wave amplitude increases, local phase velocity decreases.

**Table 2.** *Concrete sample's third order elastic constants and nonlinear acoustic parameter in comparison with some values from rocks found in literature (Johnson* et al.*, 1996)*

| | *l* (Gpa) | *m* (Gpa) | *n* (Gpa) | $\beta$ |
|---|---|---|---|---|
| Granite | -3371 | -6742 | -6600 | -441 |
| Sandstone | -97800 | -99400 | -84900 | -9600 |
| Concrete sample | -3007± 2.8% | -2283±1.2% | -1813±3.4% | -157±1.9% |

There are very few measurements of the third order constants of complex materials in the literature (Johnson and Rasolofosaon, 1996; Rasolofosaon *et al.*, 1997; Winkler and Liu, 1997), and none to our knowledge, in concrete and cement based materials, meaning these values are the first reported for these materials.

We note that the reported values come from a combination of both static and dynamic measurements. The third order constants are computed from the relative velocities variations (dynamic) while the second order Lame constants are obtained from quasi-static measurements. It is well known that static and dynamic values of elastic constants of concrete differ significantly from each other, but this study show the potentialities of applying CWI for third order elastic constants evalutation.

## 5. Discussion

The fact that the velocity is constant over the entire signal duration at a given step indicates that compressional wave coda dominates the measurement for $dV_{22}$ (Figure 3). If it did not, the relative velocity variation would evolve to a different value associated with the shear waves. We benefit from the geometry of the sample in this regard. The measurements of P waves are performed by using P transducers as emitter and receiver. In this case we have a geometry such that the P-waves are reflected at normal incidence back and forth across the sample multiple times. During this process, energy leaks away to the rest of the sample and is eventually converted to shear. Had we used the full coda, eventually one sees a change in the quantity, due to the mode conversion. For $dV_{21}$ and $dV_{23}$ shear waves are inputted into the sample, and shear wave coda dominates. The large difference between these two polarized shear waves implies that coda waves carry polarization information due to stress-induced birefringence. They carry it for the full coda used in the study.

## 6. Prospects

In addition, there is potential of scattering constants of concrete to be sensitive to stress state. Based on a one dimensional energy diffusion process, the diffuse ultrasound energy can be expressed as a function of time (Anugonda *et al.*, 2001):

$$\ln(\langle E(x,t)\rangle) - 0.5\ln(t) = C_0 - \frac{x^2}{4Dt} - \sigma t \qquad [15]$$

where x is the distance between transducers, $C_0$ a constant that represents the initial energy deposited, D is the diffusivity and σ the absorption. A rough estimate of these constants is achieved on a concrete sample of similar composition (Figure 5) following the fitting procedure suggested by (Anugonda *et al.*, 2001) at a working frequency of 500 kHz by evaluating the energy in the 450-550 kHz range. The constants are given on Table 2.

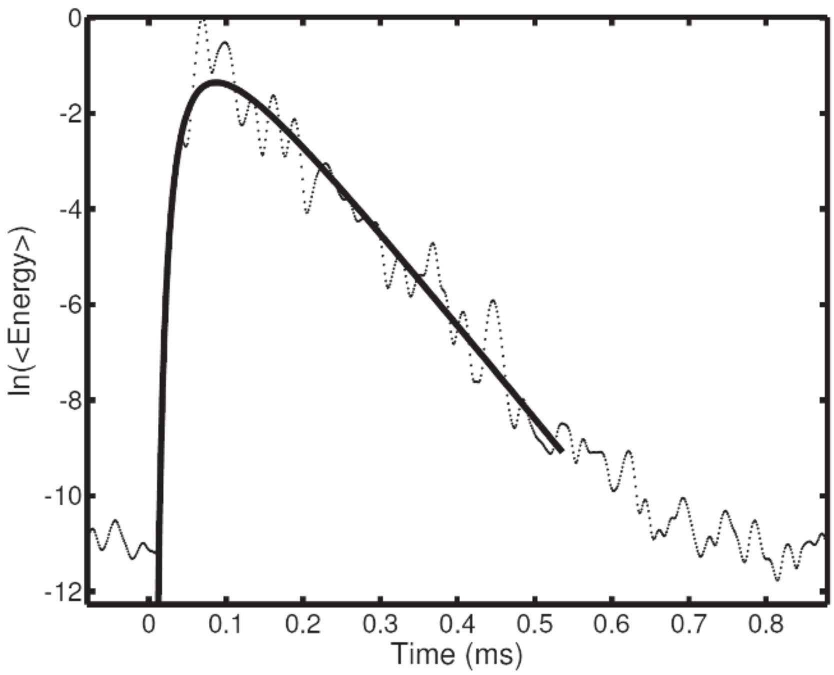


**Figure 5.** *Evolution of the diffuse ultrasound energy as a function of time. The solid line is the fit of experiment*

**Table 3.** *Concrete sample's scattering constants*

| | *Diffusivity* | *Absorption* |
|---|---|---|
| Concrete sample | 14 m²/s | 2.15 $10^4$ $s^{-1}$ |

Experiments reported in Section 4 were aimed at studying the stress induced velocity change. However, the recorded signals allow us to qualitatively evaluate the influence of stress level on the absorption. The diffusivity can not be evaluated because the recorded signals were saturated in amplitude (at the beginning of the signal) in order to increase the signal to noise ration in the late coda. The evolution of the absorption as a function of strain is given Figure 6. We can observe a large decrease of the absorption with increasing compressive stress. At this stage, we are only able to give qualitative information because the experiments were dedicated to study the influence of stress on the velocities. However, these results show a great potential of diffuse ultrasound to monitoring stress state of concrete structures.

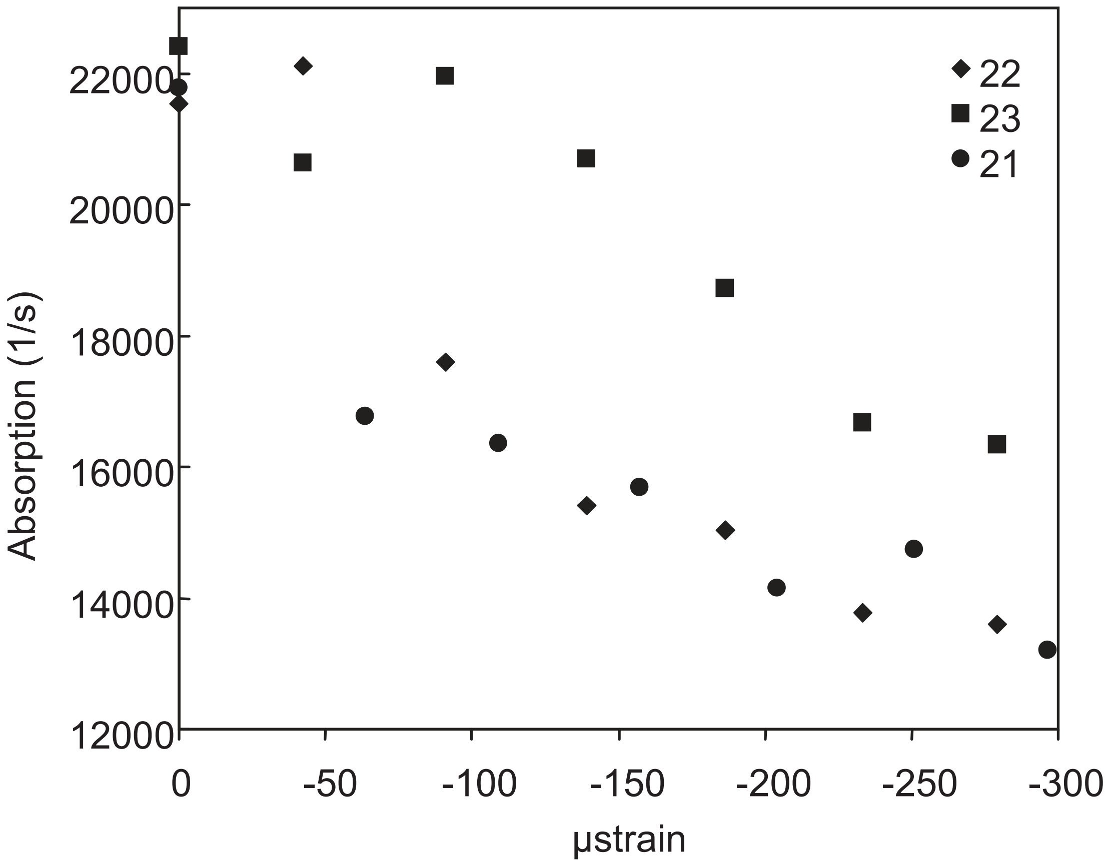


**Figure 6.** *Absorption as a function of micro-strain for the 3 polarization employed*

## 7. Conclusion

Here we present a robust and accurate method by which to extract the nonlinear properties of concrete, or any complex material. Relative velocity changes < 1‰ have been monitored. However, in situ absolute stress measurements seems to be difficult at present time because of variability of concrete used in civil engineering structures exposed to atmospheric conditions. In addition, the potential of diffusion parameters is shown for this topic. Thus, in the future, we intend to study the influence of concrete structural parameters such as composition, porosity, permeability and water saturation state on the third order elastic constants and the diffusion parameters.

## 8. *In situ* stress monitoring

In the actual context of increasing security and sustainability of concrete structures, an accurate and non destructive method sensitive to stress variations should be useful. The simplicity with which the CWI method (or diffusion) can be implemented, its high sensitivity, and the limited quantity of instrumentation needed, favour its use for the in situ monitoring of civil engineering structures. In addition, a network of suitably placed receivers could be used to localise zones of change in stress. The presented method has possible applications for stress monitoring or any phenomena which induces stress variation such as temperature. These can be applied on piles, dams, prestressed concrete…

Acknowledgements

This study was conducted in the ACTENA program supported by the French Research National Agency, funded by Electricité de France (EDF). We acknowledge the Laboratoire de Mécanique et d'Acoustique (CNRS, UPR-7051) for technical support. Authors thank Paul Johnson (LANL, USA) and Michele Griffa (EMPA, Switzerland) for helpful comments and discussions.

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