Remanufacturing planning under uncertainty: a two-stage stochastic programming approach

Céline Gicquel, Safia Kedad-Sidhoum, Dominique Quadri

Remanufacturing planning under uncertainty: a two-stage stochastic programming approach

Céline Gicquel[1] **, Safia Kedad-Sidhoum[2], Dominique Quadri[1]

[1] Université Paris Saclay, LRI, Campus d'Orsay, 91405 Orsay, France
celine.gicquel@lri.fr
[2] Sorbonne et Pierre et Marie Curie Universités, LIP6 UMR 7606, 4 place Jussieu, 75005 Paris, France

Abstract. A multi-product multi-period aggregate remanufacturing planning problem involving uncertain input data is considered. A two-stage stochastic programming approach is investigated, resulting in the formulation of a large-size linear program. Preliminary computational results providing a first assessment of the value of stochastic programming for the problem are presented.

Key words: Circular economy, Reverse logistics, Remanufacturing, Production Planning, Stochastic programming, Linear Programming

1 Introduction

One way of mitigating the environmental impact of industrial products in terms of waste generation and natural resource consumption is by remanufacturing them once they have reached their end of life. Remanufacturing is the process of recovering value from used products returned by customers by replacing or reprocessing components in order to bring the product to a like-new condition.

The present paper focuses on optimizing the aggregate mid-term planning of remanufacturing activities. It mainly consists in deciding how much and when disassemble used products, how much and when produce remanufactured products, how much and when order for new materials. One of the main related challenges is the high level of uncertainty in the input data needed to make these planning decisions. Namely in addition to the uncertainty on market demand faced by traditional forward supply chains, in reverse supply chains one also has to deal with a high level of uncertainty on the quantity and quality of the end-of-life products to be processed. This is mainly due to the fact that we cannot control when each individual user will bring its used product back and in which usage state this used product will be.

Planning the remanufacturing activities is thus a stochastic optimization problem. In the present paper, we propose a two-stage stochastic programming approach to solve this problem. Our contributions are threefold. First, we investigate a realistic multi-product multi-period production planning problem and

** This research benefited from the support of the FMJH Program Gaspard Monge for optimization and operations research, and from the support of EDF.

consider uncertainties in both the quantity and quality of the returned products. This is in contrast with most previously published related works which focus either on single product or on single period settings and consider uncertainties either on the quantity or on the quality of the used products. Second, we develop a two-stage stochastic programming approach to solve this problem, leading to the formulation of a linear program which will be solved by a commercial solver. Third, we present preliminary computational results providing a first assessment of the value of stochastic programming for the problem.

The remainder of the paper is organized as follows. In Section 2, a brief literature review on stochastic remanufacturing planning is provided. Section 3 describes the proposed problem modeling. The resulting linear programming formulation is provided in Section 4. Preliminary computational results are discussed in Section 4.

2 Literature Review

There is now a large body of literature on quantitative models for production and inventory planning in reverse supply chains. We refer the reader to [1] for a general introduction to this field and focus in what follows on the aggregate planning of remanufacturing activities.

Some deterministic optimization approaches have been proposed for this problem: see e.g. in [3] and [4]. These approaches assume that all problem parameters are perfectly known at the time when the planning decisions have to be made. In contrast, stochastic optimization approaches seek to explicitly take into account the stochastic nature of some of the problem parameters. In what follows, we review the corresponding literature and distinguish three main features in the related papers, namely: the production planning settings, the sources of uncertainty considered and the type of solution approaches.

Most existing papers consider rather simplistic production planning settings. Namely, they assume either a single product (see [2], [5], [6]) or a single period (see [8] and [9]) setting. Mahapatra et al. [7] deal with a multi-product multi-period planning problem. But they only focus on readjusting a previously established production plan after the random parameters have realized and do not consider building the initial planning under uncertainty. There thus seems to be a significant gap between the current state of the art and the industrial need. The present work is intended as a first step towards closing this gap.

Another important feature to be considered in the nature of the stochastic parameters. Some papers ([5], [8], [9]) study the case where the quantity of returned products available for remanufacturing is subject to uncertainty but assume that all the available products have the same quality level. Yet, in practice, returned products are of heterogeneous quality, which can have significant impact on the planning of remanufacturing activities. Uncertainty on the returned products quality is considered in [2], [6] and [7]. However [2] and [7] assume that the quantity of returned products is deterministically known. In the present work, similarly to the authors of [6], we consider uncertainties on both the quantity and the quality of returned products.

Finally, one may also classify the papers based on the type of solution approaches used. [5] and [6] rely on stochastic dynamic programming to find an optimal production policy. Single-stage stochastic programming approaches are proposed in [8] and [9]. Mahapatra et al. [7] implicitly consider a two-stage decision process but focus only on optimizing the recourse problem. A multi-stage stochastic programming approach is investigated in [2]. Similarly to [7], the present work relies on a two-stage decision process but seeks to simultaneously solve the first-stage and the second-stage (recourse) problems.

3 Problem modeling

We consider a remanufacturing system (see Figure 1) comprising two main processing steps: the disassembly of used products and the reassembly of remanufactured products. The input flows of the system consist of used products of I different types returned by customers. Similarly to what is done e.g. in [4], we use a discrete set of K nominal quality levels to describe the quality of the returned products. Moreover, we assume that the returned products have already been sorted and graded. These used products can be either disposed of or disassembled. The disassembly of used products yields parts of J different types. The number of parts recovered from the disassembly of a unit of used product depends on its bill-of-material and on its quality level. Basically, the lowest the quality level of a used product, the less good parts will be recovered from its disassembly. The parts recovered from disassembly are then reassembled to obtain I different types of remanufactured products. We assume that, if needed, new parts can be supplied from an external supplier. Remanufactured products are held in inventory before being used to satisfy customer demand. In case there is not enough remanufactured products available in a given period, the corresponding demand is assumed to be lost inducing lost sales.

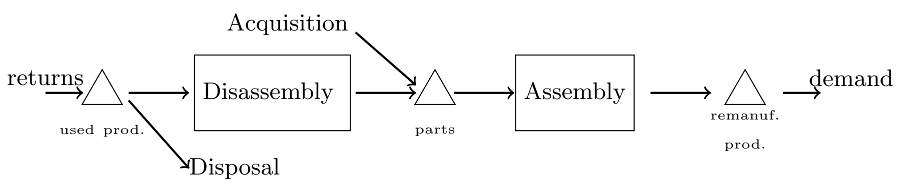

Fig. 1. Remanufacturing system

Planning remanufacturing activities for such a system thus consists in deciding, on a planning horizon of T periods, how many used products to disassemble, how many remanufactured products to reassemble and how many new parts to supply. The main operational constraints come from the limited capacity of the disassembly and reassembly processes and the need to ensure consistency in inventory balance equations. The objective is to minimize the total remanufacturing costs comprised of disassembly/reassembly costs, inventory holding costs, disposal costs, parts acquisition costs and lost sales costs.

In practice, some of the input parameters needed to make these planning decisions are subject to uncertainties. In particular, it is difficult to exactly forecast how many remanufactured products will be requested, how many used products will be returned and in which quality level these products will be. We assume, in what follows, that the exact value of these parameters is not known but that some information on their possible value is available in the form of a probability distribution (corresponding e.g. to the distribution of the residual of the related forecasting model). We will make use of a representation of the uncertainty based on a finite set of S discrete scenarios obtained by sampling from the original continuous distributions of the random parameters. A scenario s thus corresponds to a possible realization of the customer demand and returns quantity for each period of the horizon.

We propose to handle this stochastic optimization problem using a two-stage stochastic programming approach. We thus consider a two-stage decision process. The first stage corresponds to the "here-and-now" decisions which have to be made prior to the realization of the random parameters. In the present case, these decisions correspond to building an initial planning for the disassembly, reassembly and part supply before knowing the actual value of the returned quantities and customer demand. The second stage corresponds to the "wait-and-see" decisions which can be postponed after the realization of the random parameters. They can be understood as recourse actions that will be undertaken to adjust the initial planning to the actual realization of the returned quantities and customer demand. In case there are more products or parts that needed, the recourse actions will consist in keeping the surplus products and parts in inventory or in disposing of the useless returned products. In case there are less products or parts that needed, the recourse actions will consist in allowing lost sales for remanufactured products, in requesting urgent deliveries of new parts and in decreasing the initial quantity of disassembled used products. The aim of the proposed two-stage stochastic approach is to find an initial first-stage planning that minimizes the sum of the cost of the initial first-stage production and supply planning and of the expected cost over all scenarios of the recourse actions. This approach leads to an equivalent deterministic linear program described in Section 4.

4 Linear programming formulation

Problem parameters:
- (i, k): used product i in quality level k
- $DCap_t$: disassembly capacity available in period t
- $DT_{i,k,t}$: unit disassembly time for for used product (i, k) in period t
- $RCap_t$: reassembly capacity available in period t

- $RT_{i,t}$: unit assembly time for remanufactured product i in period t
- $\alpha_{i,j}$: number of parts j embedded in one product i ("gozinto"' factor)
- $\pi_{i,k,j,t}$: proportion of recoverable/reusable parts j obtained by disassembly of a used product (i,k) in period t
- $DC_{i,k,t}$: unit disassembly cost for for used product (i,k) in period t
- $RC_{i,t}$: unit reassembly cost for remanufactured product i in period t
- $UIC_{i,k,t}$: unit inventory holding cost for used product (i,k) in period t
- $DisC_{i,k,t}$: unit disposal cost for used product (i,k) in period t
- MIC_{jt}: unit inventory holding cost for parts j in period t
- $RIC_{i,t}$: unit inventory holding cost for remanufactured product i in period t
- $MPC_{j,t}$: unit purchase cost for part j in period t
- RMC_{jt}: unit cost of purchasing part j in period t using the "rush acquisition" channel (assumed to be much larger than $MPC_{j,t}$)
- LSC_{it}: unit cost of a lost sale of remanufactured product i in period t
- $R^s_{i,k,t}$: quantity for used product (i,k) returned in period t in scenario s
- $D^s_{i,t}$: demand for remanufactured product i in period t in scenario s

First stage decision variables:
- $DQ_{i,k,t}$: quantity of used products of type (i,k) disassembled in period t
- $RQ_{i,t}$: quantity of remanufactured products of type i reassembled in period t
- $MQ_{j,t}$: quantity of new parts of type j purchased in period t

Second stage (recourse) decision variables:
- $DisQ^s_{i,k,t}$: quantity of used products (i,k) disposed of in t in scenario s
- $UI^s_{i,k,t}$: inventory level for used product (i,k) at the end of t in scenario s
- $mDQ^s_{i,k,t}$: modified disassembly quantity for used products (i,k) in t in s
- $MI^s_{j,t}$: inventory level of parts j at the end of t in scenario s
- $RMQ^s_{j,t}$: quantity of parts j purchased in t using the "rush acquisition" channel in scenario s
- $RI^s_{i,t}$: inventory level of remanufactured product i in t in scenario s
- $LS^s_{i,t}$: lost sales of remanufactured product i in period t in scenario s

Linear programming formulation:

$$\begin{aligned} Z^* = min \sum_{i,k,t} DC_{i,k,t} DQ_{i,k,t} + \sum_{i,t} RC_{i,t} RQ_{i,t} + + \sum_{j,t} MPC_{jt} MQ_{jt} \\ + \sum_{s} \frac{1}{S} \Big[\sum_{i,k,t} DisC_{i,k,t} DisQ^s_{i,k,t} + \sum_{i,k,t} UIC_{i,k,t} UI^s_{i,k,t} + \sum_{j,t} MIC_{jt} MI^s_{j,t} \\ + \sum_{jt} RMC_{jt} RMQ^s_{j,t} + \sum_{i,t} RIC_{i,t} RI^s_{i,t} + \sum_{i,t} LSC_{i,t} LS^s_{i,t} \Big] \end{aligned} \quad (1)$$

$$\sum_{i,k} DT_{i,k,t} DQ_{i,k,t} \leq DCap_t \qquad \forall t \quad (2)$$

$$\sum_{i} RT_{i,t} RQ_{i,t} \leq RCap_t \qquad \forall t \quad (3)$$

$$UI^s_{i,k,t} = UI^s_{i,k,t-1} + R^s_{i,k,t} - mDQ^s_{i,k,t} - DisQ^s_{i,k,t} \qquad \forall i, \forall k, \forall t, \forall s \qquad (4)$$

$$mDQ^s_{i,k,t} \leq DQ_{i,k,t} \qquad \forall i, \forall k, \forall t, \forall s \qquad (5)$$

$$\begin{aligned} MI^s_{j,t} = MI^s_{j,t-1} + \sum_{i,k} \pi_{i,k,j,t} \alpha_{i,j} mDQ^s_{i,k,t} \\ + MQ_{jt} + RMQ^s_{j,t} - \sum_i \alpha_{i,j} RQ_{i,t} \qquad \forall i, \forall t, \forall s \qquad (6) \end{aligned}$$

$$RI^s_{it} = RI^s_{i,t-1} + RQ_{i,t} + L^s_{i,t} - D^s_{it} \qquad \forall i, \forall t, \forall s \qquad (7)$$

The objective function (1) aims at minimizing the sum of the cost of the initial production and supply planning and of the expected cost, over all scenarios, of the recourse actions. Constraints (2)-(3) are first-stage constraints, ensuring that the disassembly/reassembly capacity constraints are satisfied by the initial production planning. Constraints (4)-(7) are second-stage constraints linking first-stage decisions and second-stage decisions. They ensure that, for each scenario, the inventory balance for used products, parts and remanufactured products is respected. They also guarantee that the modified disassembly planning will comply with the disassembly capacity by only allowing a decrease in the number of used products disassembled. All decision variables can take any real positive value. Note that in our problem modeling, possible recourse actions involve decreasing the initial disassembled quantity from $DQ_{i,k,t}$ to $mDQ^s_{i,k,t}$ in case of a shortage of used product (i, k). Decreasing the quantity processed in disassembly should lead to some production cost savings. However, it might also provoke some organizational difficulties for the disassembly shop. In order to limit the amount of changes in the planned production quantities, we thus do not consider these cost savings in our objective functions but rather assume that the incurred production costs will be equal to $DT_{i,k,t} DQ_{i,k,t}$, whatever the actual adjusted quantities produced in each scenario s.

5 Preliminary computational results

We now discuss some preliminary computational results obtained while using the proposed stochastic programming approach. We used the case study on mobile phone remanufacturing presented in [4]. This case study involves $I = 2$ products, $K = 6$ quality levels, $J = 2$ parts and $T = 2$ periods. All deterministic parameters were set to the values provided in [4]. As for the stochastic parameters D_{it} and R_{ikt}, we assumed a normal distribution with a mean value μ equal to the deterministic value used in [4] and a standard deviation σ equal to 0.1μ or 0.2μ. We carried out a Monte Carlo sampling in order to randomly generate a set of S scenarios, with $S \in \{10, 100, 1000, 5000, 10000\}$. For each value of the standard deviation and each sample size, we generated 10 samples, leading to a total of 120 instances. We also considered the case $S = 1$ which corresponds to the resolution of the deterministic counterpart of the problem.

All tests were run on an Intel Core i5 (2.6 GHz) with 4 Go of RAM, running under Windows 7. For each instance, we solved the linear program (1)-(7) using

the mathematical programming solver CPLEX 12.6 with the default settings. The corresponding results are provided in Tables 1 and 2. We provide for each set of 10 instances:
- the number of variables and constraints involved in the linear program (1)-(7),
- the mean value and standard deviation, over the corresponding set of 10 samples, of the optimal cost obtained while solving the linear program (1)-(7),
- the average computation time needed to solve the linear program (1)-(7),
- a post-optimization evaluation of the actual cost of the initial production and supply planning provided by the proposed approach when implemented in a stochastic environment. This evaluation was carried out by using a set of $S' = 10000$ randomly generated scenarios different from the ones used in the optimization phase. For each considered instance, we fixed the value of first-stage decisions to the optimal values $DQ^*_{i,k,t}, RQ^*_{i,t}, MQ^*_{j,t}$ provided by the resolution of problem of (1)-(7). For each scenario $s' = 1...S'$, we then solved a small linear program aiming at minimizing the cost $RecC^{s'}$ of the recourse actions needed to adjust the initial planning to the realization of the random parameters in s'. $POEval$ is computed as $POEval = \sum_{i,k,t} DC_{i,k,t} DQ^*_{i,k,t} + \sum_{i,t} RC_{i,t} RQ^*_{i,t} + \sum_{j,t} MPC_{jt} MQ^*_{jt} + \frac{1}{S'} \sum_{s'=1}^{S} RecC^{s'}$.

Scenarios	Variables	Constraints	Cost	Std dev.	Comp. time	POEval
1	120	60	89160	-	0.05s	129209
10	912	564	96047	1355	0.05s	102499
100	8832	5604	98810	534	0.30s	99150
1000	88032	56004	98860	149	7.05s	98942
2000	176032	112004	98970	71	19.1s	98928
5000	440032	280004	98898	69	70.9s	98898
10000	880032	560004	-	-	-	-

Table 1. Results for $\sigma = 0.1\mu$

Scenarios	Variables	Constraints	Cost	Std dev.	Comp. time	POEval
1	120	60	89160	-	0.05s	173043
10	912	564	102774	2554	0.05s	117586
100	8832	5604	110047	684	0.4s	110649
1000	88032	56004	110136	319	7.4s	110514
2000	176032	112004	109924	226	20.5s	110107
5000	440032	280004	110140	227	78.4s	110085
10000	880032	560004	-	-	-	-

Table 2. Results for $\sigma = 0.2\mu$

We first note that, for all sample sizes $S \leq 5000$, the linear program (1)-(7) could be solved by CPLEX 12.6 without any computational difficulty. However, for the largest sample size considered $S = 10000$, the solver stopped with an out-of-memory status before providing an optimal solution. This indicates that the use of more powerful solution techniques (such as the L-shaped method) might be needed in order to solve larger instances of the problem.

Results from Tables 1 and 2 also enable us to provide a first assessment of the value of stochastic programming for this problem. This can be seen by noting that the value of $POEval$ is significantly larger when $S = 1$ than when $S \geq 100$. This means that the initial planning obtained while using a deterministic approach (i.e. $S = 1$) leads to more expensive recourse actions when implemented in a stochastic environment than the initial planning obtained while using a stochastic programming approach with $S \geq 100$. This cost increase becomes more significant when the variability of the random parameters (i.e. σ) increases.

6 Conclusion and perspectives

A multi-product multi-period aggregate remanufacturing planning problem involving uncertain input data has been considered. A two-stage stochastic programming approach has been proposed. Computational results providing a preliminary assessment of the value of stochastic programming for the problem have been presented. Directions for future research might involve the development of a multi-stage stochastic programming approach in which the random parameters would unfold little by little and multiple adjustments of the initial planning might be allowed.

References

1. E. Akçali and S. Cetinkaya. Quantitative models for inventory and production planning in closed-loop supply chains. *International Journal of Production Research*, vol. 49(8), pp. 2373-2407 (2011).
2. M. Denizel, M. Ferguson, and G. Souza. Multi-period remanufacturing planning with uncertain quality of inputs. *IEEE Transactions on Engineering Management*, vol. 57(3), pp. 394-404 (2010).
3. S.H. Han, M.Y. Dong, S.X. Lu, S.C.H. Leung and M.K. Kim. Production planning for hybrid remanufacturing and manufacturing system with component recovery. *Journal of the Operational Research Society*, vol. 64, pp. 1447-1460 (2013).
4. V. Jayaraman. Production planning for closed-loop supply chains with product recovery and reuse: an analytical approach. *International Journal of Production Research*, vol. 44 (5), pp .981-998 (2006).
5. C. Li, F. Liu, H. Cao and Q. Wang. A stochastic dynamic programming based model for uncertain production planning of re-manufacturing system. *International Journal of Production Research*, vol.47 (13), pp. 3657-3668 (2009).
6. X. Li, Y. Li and S. Saghafian. A Hybrid Manufacturing/Remanufacturing System With Random Remanufacturing Yield and Market-Driven Product Acquisition. *IEEE Transactions on Engineering Management* , vol. 60 (2), pp. 424-437, (2013).
7. S. Mahapatra, R. Palb and R.Narasimhan. Hybrid (re)manufacturing: manufacturing and operational implications. *International Journal of Production Research*, vol. 50 (14), pp. 3786-3808 (2012).
8. S. Rouf and G. Zhang. Supply planning for a closed-loop system with uncertain demand and return. *International Journal of Operational Resear*ch, vol. 10 (4), pp. 380-397 (2011).
9. J. Shi, G. Zhang and J. Sha. Optimal production planning for a multi-product closed loop system with uncertain demand and return. *Computers & Operations Research*, vol. 38, pp. 641-650 (2011).