Comment on "Gain-assisted superluminal light propagation through a Bose-Einstein condensate cavity system"

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Abstract. In a recent theoretical article [S.H. Kazemi, S. Ghanbari, M. Mahmoudi, Eur. Phys. J. D **70**, 1 (2016)], Kazemi et al. claim to have demonstrated superluminal light transmission in an optomechanical system where a Bose-Einstein condensate serves as the mechanical oscillator. In fact the superluminal propagation is only inferred from the existence of a minimum of transmission of the system at the probe frequency. This condition is not sufficient and we show that, in all the cases where superluminal propagation is claimed by Kazemi et al., the propagation is in reality subluminal. Moreover, we point out that the system under consideration is not minimum-phase-shift. The Kramers-Kronig relations then only fix a lower limit to the group delay and we show that these two quantities have sometimes opposite signs.

When the transmission of light in a given medium displays a well-marked narrow dip at some frequency, the group velocity at this frequency may be larger than the velocity of light in vacuum or even negative. An ideally smooth light-pulse can then exits the medium without significant distortion before than if it had propagated in vacuum [1]. Such superluminal or fast propagation is not at odds with relativistic causality since a given point of the output-pulse profile is not a direct reflection of the homologous point of the incident-pulse profile but results from the action of the medium on all the earlier part of the incident pulse. The major challenge in such experiments is to obtain advancements comparable to the pulse duration with moderate distortion. Convincing experiments have been performed in the 1980s [2,3] in media with a narrow absorption line. Unfortunately, superluminal propagation is then accompanied by strong absorption. This inconvenience is overcome by using a medium with a doublet of gain lines [4,5] and a minimum of transmission between them. Significant advancements have been evidenced in an atomic vapor with this arrangement [6]. A comprehensive review on fast light in atomic media can be found in [7]. Experiments involving four wave mixing are reported in [8].

Superluminal or subluminal propagation can only be demonstrated by a determination of the group delay and one should not hastily conclude from what it precedes that every gain system with a dip in its transmission curve will be superluminal. This extrapolation is unfortunately made in a recent theoretical article [9] whose authors claim to have evidenced superluminal propagation by giving this sole argument. The system under consideration is an optomechanical device consisting in a high-quality optical cavity containing a Bose-Einstein condensate (BEC) of Rubidium atoms which serves as the mechanical oscillator [10]. It is submitted to a strong pump field (continuous wave) and to a weak probe field. In a frame rotating at the pump angular-frequency, the transfer function for the probe field reads as [9][1]

$$H(\omega) = 1 - \frac{1 + if(\omega)}{\kappa/2 - 2f(\omega)\,\Delta_c + i\,(\Delta_c - \omega)}(\frac{\kappa}{2}) \quad (1)$$

with

$$f(\omega) = \frac{2\omega_m n g^2}{[\kappa/2 - i\,(\omega + \Delta_c)]\,[\omega_m^2 - \omega^2 - i\omega\gamma_m]} \quad (2)$$

In these expressions, κ (γ_m) is the damping rate of the cavity (the mechanical oscillator), Δ_c is a frequency related to the cavity detuning and is assumed equal to the resonance frequency ω_m of the mechanical oscillator in all the simulations, $n = E_{pu}^2/\left(\Delta_c^2 + \kappa^2/4\right)$ where E_{pu} is a parameter proportional to the amplitude of the pump field, $g = g_0^2\sqrt{N}/\left(2\Delta_a\sqrt{2}\right)$ where g_0 is the elementary atom-photon coupling constant, N is the number of atoms in the BEC and Δ_a is the detuning of the pump from the frequency of the relevant atomic line. We denote in the following $T(\omega) = |H(\omega)|^2$ the intensity transmission of the system and $\varphi(\omega) = \arg[H(\omega)]$ the phase shift induced by the system.

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[1] Some typo errors in [9] are corrected in the present comment.

In order to evidence that the existence of a minimum of the intensity transmission does not entail superluminal propagation we have determined the corresponding group delay $\tau_g(\omega) = \frac{d}{d\omega}\varphi(\omega)$ in all the cases considered in [9]. As a representative example, our Figure.1 shows the dependence of $T(\omega)$ and $\tau_g(\omega)$ as functions of ω for two different values of the damping rate γ_m of the mechanical oscillator (BEC). The parameters are those considered to
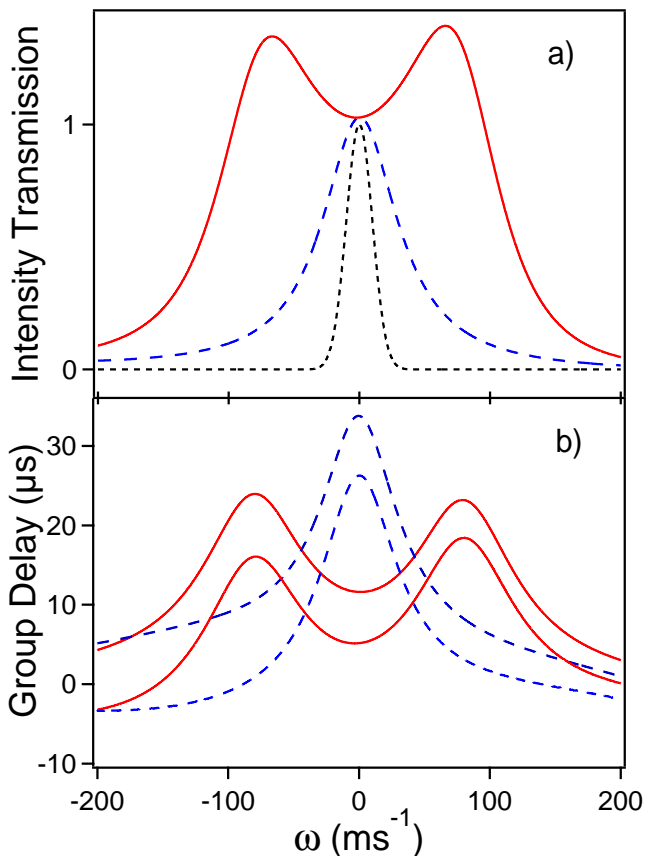
Fig. 1. a) Intensity transmission $T(\omega)$ of the optomechanical system as a function of ω. Used parameters are: $N = 1.2 \times 10^5$, $g_0 = 2\pi \times 10.9$ MHz, $\Delta_a = 2\pi \times 32$ GHz, $\kappa = 2\pi \times 1.3$ MHz, $\Delta_c = \omega_m = 2\pi \times 15.2$ kHz, $E_{pu} = 2\pi \times 100$ kHz, $\gamma_m = 2\pi \times 15$ kHz (solid line) and $\gamma_m = 2\pi \times 45$ kHz (dashed line). The dotted line is the power spectrum of the incident pulse considered in Figure.2. b) Corresponding group delays. The upper curves give the exact group delays $\tau_g(\omega) = \frac{d}{d\omega}\arg[H(\omega)]$ and the lower curves give the group delays $\tau_{KK}(\omega)$ derived from the Kramers-Kronig relations.

obtain Figure.4 in [9]. We see that, for $\gamma_m = 2\pi \times 15$ kHz (solid line), the transmission has a minimum for $\omega = 0$ (probe frequency equal to the pump frequency) but that the propagation remains subluminal [$\tau_g(0) > 0$] contrary to the claim of Kazemi et al. [9]. Compared to the result obtained for $\gamma_m = 2\pi \times 45$ kHz (dashed line) where the transmission has a maximum for $\omega = 0$, it appears that the effect of a minimum of transmission is to significantly reduce the group delay *without changing its sign* (no time-advancement). In the present case, we get $\tau_g(0) = 33.8\,\mu$s

for $\gamma_m = 2\pi \times 45$ kHz and $\tau_g(0) = 11.6\,\mu$s for $\gamma_m = 2\pi \times 15$ kHz . Figure 2 shows the intensity profiles of the pulses transmitted by the system when it is subjected to an incident Gaussian pulse with a carrier frequency equal to the pump frequency. The intensity profile of the inci-
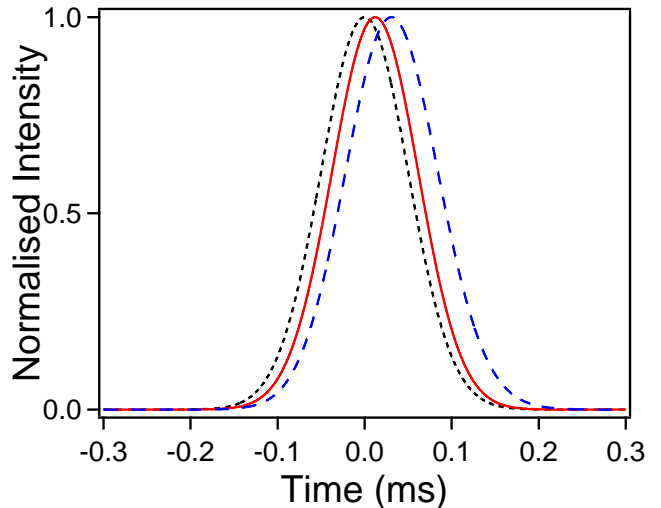
Fig. 2. Temporal response of the system to an incident Gaussian pulse of carrier frequency equal to the pump frequency and of duration $\tau_p = 100\,\mu$s. (half-width at $1/e$ of the envelope). The figure shows the normalized intensity-profiles of the incident pulse (dotted line), of the pulse transmitted for $\gamma_m = 2\pi \times 15$ kHz (solid line) and for $\gamma_m = 2\pi \times 45$ kHz (dashed line). The other parameters are those of Figure.1.

dent pulse is given for reference (dotted line). Its duration τ_p (half-width at $1/e$ of its envelope) has been taken equal to $100\,\mu$s in order that $T(\omega)$ and $\tau_g(\omega)$ do not vary too considerably over the width of its power spectrum (shown in dotted line in Figure.1a). The pulse distortion remains then moderate and the delay t_m of the pulse maximum is close to the group delay [5]. We get $t_m = 30.7\,\mu$s for $\gamma_m = 2\pi \times 45$ kHz and $t_m = 12.1\,\mu$s for $\gamma_m = 2\pi \times 15$ kHz . Similar results are obtained in all the cases where Kazemi et al. [9] predict superluminal propagation. Even in the conditions of their Figure.5 where the gain dynamics is particularly large, we find that *the group delay remains positive*, namely $\tau_g(0) = 5.6\,\mu$s.

Kramers-Kronig relations are invoked in [9] to associate superluminal propagation with a minimum of the medium transmission. As extensively shown in [11,12], these relations only give the exact group delay when the system is minimum-phase-shift [2]. The phase-shift and the group delay then reads as $\varphi(\omega) = \varphi_{KK}(\omega)$ and $\tau_g(\omega) = \tau_{KK}(\omega) = \frac{d}{d\omega}\varphi_{KK}(\omega)$ where $\varphi_{KK}(\omega)$ is the Hilbert transform of $\ln[|H(\omega)|]$. It appears that, in all the cases considered in [9], the transfer function $H(\omega)$ has a zero $\tilde{\omega}_0$

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[2] Note that, even in minimum-phase-shift systems as are the purely propagative systems, the group delay may be positive at a frequency for which the transmission presents a minimum. See for example Section.4 in [5].

in the upper half-plane of the complex plane and thus that the system is not minimum-phase-shift. In this case, $\tau_{KK}(\omega)$ only fixes a lower limit to the exact group delay. The transfer function can then be written as $H(\omega) = H_{MP}(\omega) H_{AP}(\omega)$ where $H_{MP}(\omega)$ and $H_{AP}(\omega)$ are respectively associated with a minimum-phase-shift system and with an all-pass system, with $|H_{MP}(\omega)| = |H(\omega)|$ and $|H_{AP}(\omega)| = 1$. $H_{AP}(\omega)$ is a so-called Blaschke product [11] which, in the present case, is reduced to

$$H_{AP}(\omega) = \frac{1 - \omega/\tilde{\omega}_0}{1 - \omega/\tilde{\omega}_0^*} \qquad (3)$$

This term is responsible of additional contributions $\varphi_{AP}(\omega)$ to the phase shift $\varphi_{KK}(\omega)$ and $\tau_{AP}(\omega)$ to the group delay $\tau_{KK}(\omega)$. These contributions read as

$$\varphi_{AP}(\omega) = \arg\left[H_{AP}(\omega)\right] = -2\tan^{-1}\left[\frac{\mathrm{Im}(\omega/\tilde{\omega}_0)}{1 - \mathrm{Re}(\omega/\tilde{\omega}_0)}\right] \qquad (4)$$

$$\tau_{AP}(\omega) = \frac{d}{d\omega}\varphi_{AP}(\omega) = -\frac{2\mathrm{Im}(1/\tilde{\omega}_0)}{1 + |\omega/\tilde{\omega}_0|^2 - 2\mathrm{Re}(\omega/\tilde{\omega}_0)} \qquad (5)$$

$\mathrm{Im}(\tilde{\omega}_0)$ being positive, $\mathrm{Im}(1/\tilde{\omega}_0)$ is negative and $\tau_{AP}(\omega)$ is always positive as expected. For $\omega = 0$, $\tau_{AP}(\omega)$ takes the simple form $\tau_{AP}(0) = -\mathrm{Im}(1/\tilde{\omega}_0)$. In the conditions of Figure.1, we get $\tilde{\omega}_0 = -0.127 + 0.244i\,\mu s^{-1}$ ($-0.115 + 0.200i\,\mu s^{-1}$) for $\gamma_m = 2\pi \times 15\,\mathrm{kHz}$ ($\gamma_m = 2\pi \times 45\,\mathrm{kHz}$). In both cases the difference between the exact group delay and that given by the Kramers-Kronig relations is everywhere positive and perfectly reproduced by Eq.(5). For $\omega = 0$, we get in particular $\tau_{AP} = 6.45\,\mu s$ ($7.51\,\mu s$), $\tau_{KK} = 5.16\,\mu s$ ($26.3\,\mu s$) and $\tau_g = 11.6\,\mu s$ ($33.8\,\mu s$), with $\tau_g = \tau_{KK} + \tau_{AP}$ as expected.

Arrived to this point, it is worth remarking that significant differences between the exact group delay τ_g and the group delay τ_{KK} derived from the Kramers-Kronig relations are not specific to the system considered in [9]. They are often obtained in optical systems involving mirrors and/or polarizers. See, e.g., [13–15]. These two quantities may even have opposite signs. This phenomenon marginally occurs in the conditions of our Figure.1. On the left of Figure.1b, we actually see that $\tau_g > 0$ (time-delay) whereas the Kramers-Kronig relations predict a time advancement ($\tau_{KK} < 0$). Much more spectacular effects are shown on Figure.3. Figure 3a shows the transmission $T(\omega)$ as a function of ω obtained for parameters enabling us to reproduce the transmission curve given Figure.6 in [9]. For these parameters, the transfer function of the system is again not minimum-phase-shift. Figure.3b shows the corresponding functions $\tau_g(\omega)$ (solid line) and $\tau_{KK}(\omega)$ (dashed line). Denoting $\omega_{A,B,C,D}$ the frequencies corresponding to the points A, B, C, D in Figure.3a, we note that $\tau_g(\omega)$ and $\tau_{KK}(\omega)$ are everywhere equal or very close, except for $\omega \approx \omega_C$. The inset on the left confirms that for $\omega \approx \omega_B$ the propagation is not superluminal contrary to the claim of Kazemi et al. [9]. As expected, the propagation is subluminal for $\omega \approx \omega_A$ but the corresponding group delay is only 22.4 μs (not visible at the figure scale). The most interesting features are observed
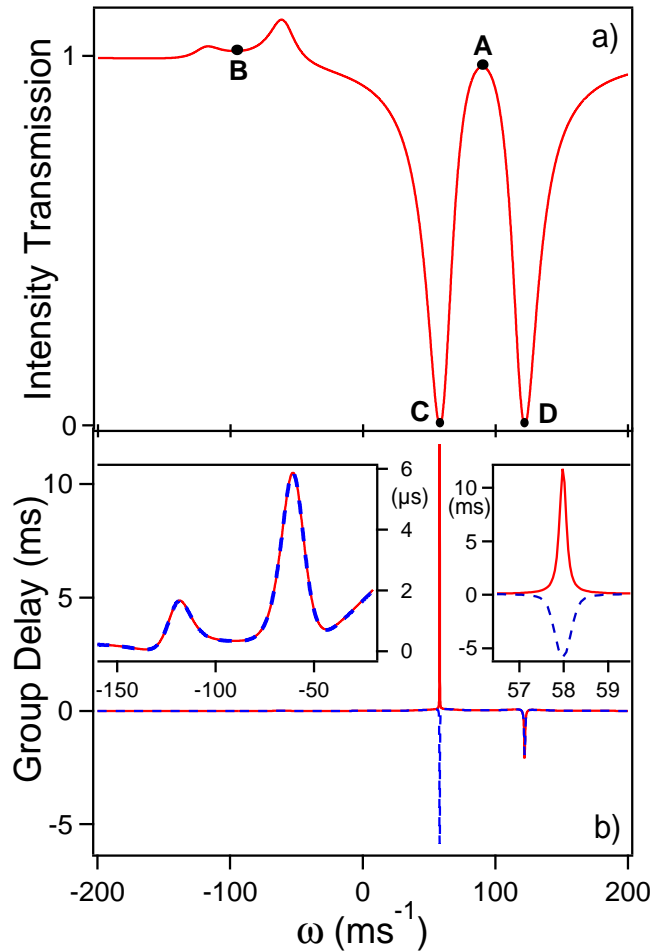

Fig. 3. a) Intensity transmission $T(\omega)$ of the optomechanical system as a function of ω. Used parameters are now : $\Delta_a = 2\pi \times 2.77\,GHz$, $\gamma_m = 2\pi \times 400\,Hz$ and $\kappa = 2\pi \times 7.2\,kHz$. The parameters not defined in [9] have been chosen in order to reproduce the Figure.6 of this article. b) Corresponding group delays. The solid line gives the exact group delay $\tau_g(\omega) = \frac{d}{d\omega}\arg\left[H(\omega)\right]$ and the dashed line gives the group delays $\tau_{KK}(\omega)$ derived from the Kramers-Kronig relations. The insets are enlargements in the spectral regions around the frequencies ω_B and ω_C.

for $\omega \approx \omega_C$ and for $\omega \approx \omega_D$, evidencing that very similar transmission profiles with a well-marked minimum can lead to quite different group delays[3]. For $\omega = \omega_C$, the inset on the right shows that the group delay has a very large positive value whereas an irrelevant application of the Kramers-Kronig relations predict a group advancement ($\tau_{KK} < 0$). For $\omega = \omega_D$ on the contrary, $\tau_g = \tau_{KK} < 0$ and a significant group advancement is obtained as currently expected at a transmission minimum.

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[3] Analogous phenomena have been observed in optical system consisting in a photonic crystal [14] or a birefringent fibre [15] placed between two polarizers. These papers report in particular *experimental evidence* that subluminal propagation can be associated with a well marked-dip in the transmission curve of a non-minimum-phase-shift system.

It should be noted that large group delay or advancement are both paid at the price of a dramatically weak transmission. Figure.4 shows the intensity profiles of the trans-
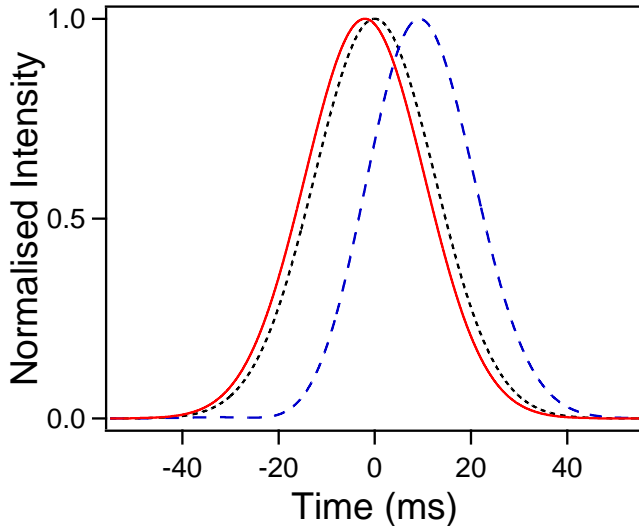
Fig. 4. Normalized intensity-profile of the pulses transmitted by the system subjected to an incident Gaussian pulse of duration $\tau_p = 25$ ms with a carrier frequency detuned from the pump frequency by ω_c (dashed line, subluminal case) and ω_D (solid line, superluminal case). The system parameters are those of Figure.3. The profile of the incident pulse is given for reference (dotted line). In the subluminal case the maximum of the output pulse is delayed by 9.3 ms whereas $\tau_g = 11.8$ ms. In the superluminal case, this maximum is advanced by 2.06 ms whereas $\tau_g = -2.08$ ms. The corresponding absolute intensities are respectively 1.6×10^4 and 660 times weaker than that of the incident pulse.

mitted pulses when the system is subjected to incident Gaussian pulses with a carrier frequency detuned from the pump frequency by ω_C (dashed line) or by ω_D (solid line). The intensity profile of the incident pulse is given for reference (dotted line). Its duration τ_p has been taken as large as 25 ms to avoid significant pulse-distortion.

To summarize, the inconsistent claims of superluminal propagation made in [9] originate in two misconceptions, firstly that the existence of a minimum of transmission *always* implies superluminal propagation and, secondly, that the group delays can *always* be derived from the Kramers-Kronig relations whereas the latter only give a lower limit to this delay when the system under consideration is not minimum-phase-shift (as are numerous optical systems). Surprisingly enough, these misconceptions are widespread in the optics community. The present comment is expected to bring some clarification on this subject.

Author contribution statement

The two authors contributed equally to the article.

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