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Analysis of the optimal linear system for multicarrier FTN communications

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1. INTRODUCTION

Most of current communication systems respect the **Nyquist criterion** [1].
 \Rightarrow **Perfect symbol** reconstruction on perfect (noiseless) channel **guaranteed**.

Overriding this criterion is a way to **improve spectral efficiency**.

\Rightarrow Induces inter-pulse interference (IPI).

\Rightarrow **Perfect symbol** reconstruction on perfect channel **not guaranteed** anymore.

Denoting F_0 the inter-carrier spacing and T_0 the multicarrier symbol duration, a multicarrier FTN transmission arises when $\rho = 1/(F_0 T_0) > 1$.

2. LINEAR SYSTEM INPUT-OUTPUT RELATION

One can write the equivalent complex baseband signal as:

$$s(t) = \sum_{(m,n) \in \mathbb{Z}^2} c_{m,n} g_{m,n}(t), \quad t \in \mathbb{R} \quad (1)$$

where $c_{m,n}$ are the **IID symbols** to be emitted (variance σ_c^2), and $\mathbf{g} = \{g_{m,n}\}_{(m,n) \in \mathbb{Z}^2}$ is a **Gabor family** with parameters F_0, T_0 and generator $g(t)$:

$$g_{m,n}(t) = g(t - nT_0) e^{j2\pi m F_0 t}. \quad (2)$$

The received signal $r(t)$ is corrupted with a complex circular **Gaussian noise** $z(t)$ whose bilateral power spectral density is $2N_0$:

$$r(t) = s(t) + z(t). \quad (3)$$

Considering a **linear receiver** using a family $\check{\mathbf{g}} = \{\check{g}_{m,n}\}_{(m,n) \in \mathbb{Z}^2}$ for filtering at the receiver side, the estimated symbols are given by:

$$\hat{c}_{p,q} = \langle \check{g}_{p,q}, r \rangle, \quad \forall (p,q) \in \mathbb{Z}^2. \quad (4)$$

3. INTERFERENCE AND NOISE ANALYSIS

Let us re-write (4) to highlight the interference and noise terms:

$$\hat{c}_{p,q} = \underbrace{c_{p,q} \langle \check{g}_{p,q}, g_{p,q} \rangle}_{\hat{c}_{p,q}: \text{useful signal}} + \underbrace{\sum_{(m,n) \in \mathbb{Z}^2 \setminus \{(p,q)\}} c_{m,n} \langle \check{g}_{p,q}, g_{m,n} \rangle}_{i_{p,q}: \text{interference}} + \underbrace{\langle \check{g}_{p,q}, z \rangle}_{b_{p,q}: \text{noise}}. \quad (5)$$

We use the optimal linear receiver detailed in [2], this induces that $\check{\mathbf{g}} = 1/A_g \mathbf{g}$ is a **tight frame**. In this case, we can write

- $E_s = \frac{A_g}{2\rho} \sigma_c^2$: per-symbol energy,
- $\sigma_i^2 = (\rho - 1) \sigma_c^2$: variance of the interference term,
- $\sigma_z^2 = \frac{\rho}{A_g} 2N_0$: variance of the noise (after filtering),

and thus:

$$\text{SINR} = \frac{1}{\rho - 1 + \frac{N_0}{E_s}}. \quad (6)$$

Even though the **central limit theorem does not apply** here, simulations of part 6 show that **normality** of the interference may be used as an **approximation** depending on the value of ρ .

4. ERROR PROBABILITY ON AWGN CHANNEL

Considering **IID bits** forming **QPSK symbols** and a **Gaussian** distribution for both **noise and interference**, the error probability is:

$$P_e = Q(\sqrt{\text{SINR}}) = Q\left(\sqrt{\frac{1}{\rho - 1 + \frac{N_0}{E_s}}}\right), \quad (7)$$

with $Q(\cdot)$ the complementary CDF of a standard normal distribution.

8. REFERENCES

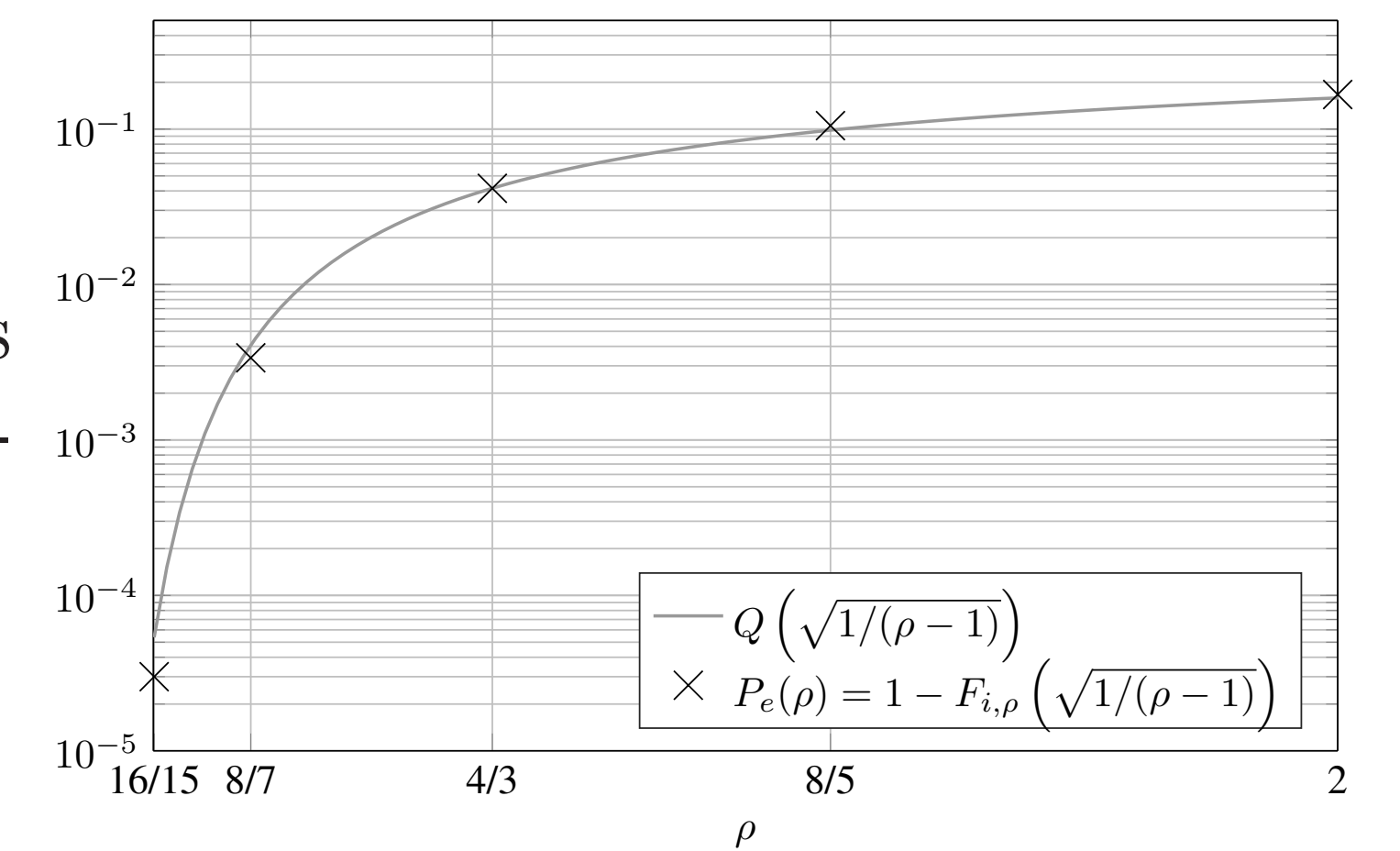
- [1] H. Nyquist. Certain topics in telegraph transmission theory. *American Institute of Electrical Engineers, Transactions of the*, 47(2):617–644, 1928.
- [2] C. Siclet, D. Roque, H. Shu, and P. Siohan. On the study of faster-than-Nyquist multicarrier signaling based on frame theory. In *Wireless Communications Systems (ISWCS), 2014 11th International Symposium on*, pages 251–255, Aug 2014.

5. EMPIRICAL INTERFERENCE ANALYSIS

5.1. Cumulative distribution function (CDF)

Considering a transmission involving **IID bits** forming **QPSK symbols** over a **perfect noise-free channel**, and denoting $F_{i,\rho}(x)$ the CDF of the interference for a density ρ , we have:

$$P_e(\rho) = 1 - F_{i,\rho}\left(\sqrt{1/(1-\rho)}\right). \quad (8)$$

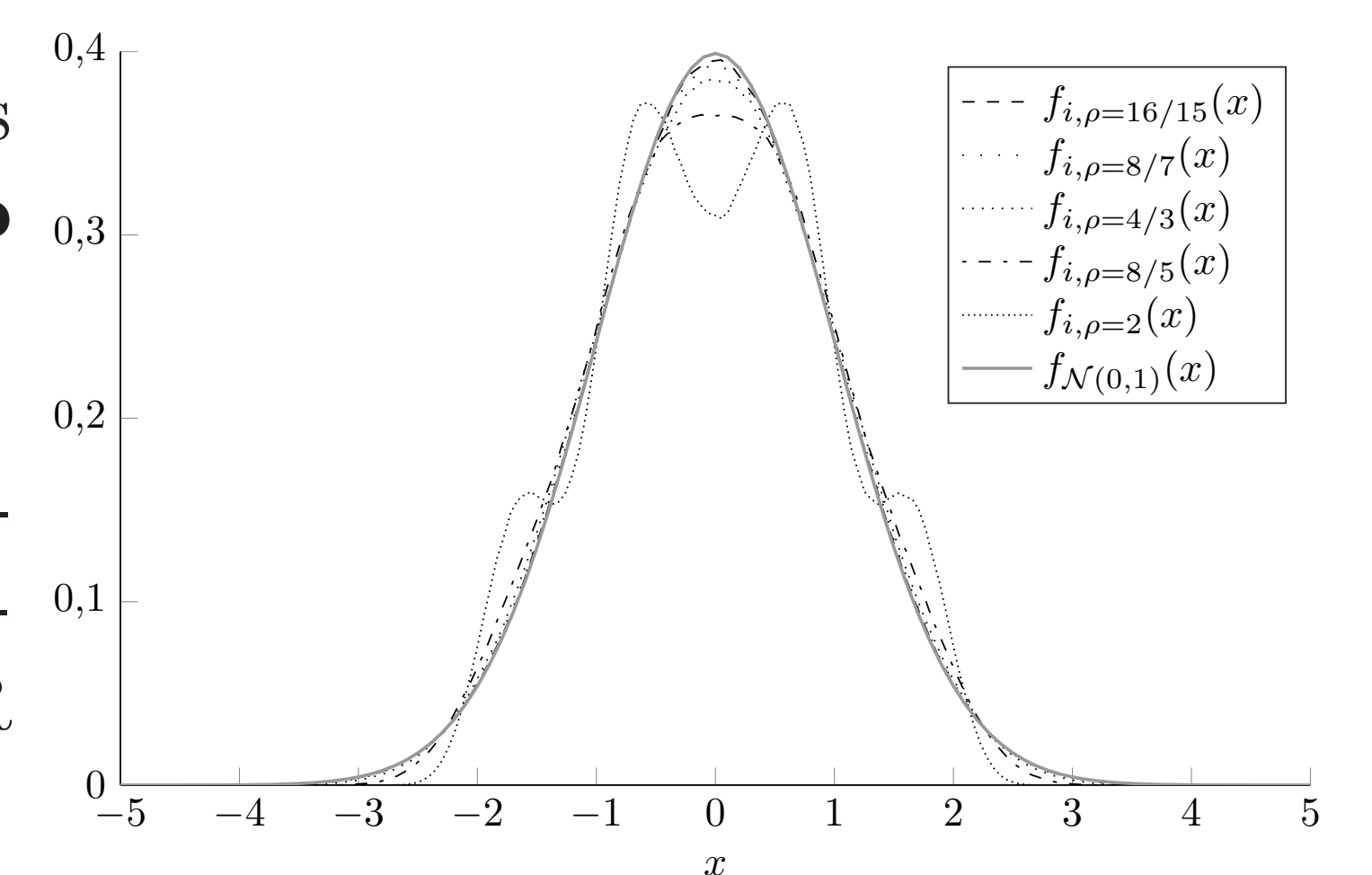


The Gaussian approximation is shown to be **relevant** for P_e estimation, **except for rho $\leq 16/15$** .

5.2. Probability density function (PDF)

The **PDF** of the interference is **close to a Gaussian up to rho = 8/5**.

For $\rho > 8/5$, a gaussian approximation of the PDF may lead to significant errors *e.g* when computing LLR for LDPC or turbo-decoders.

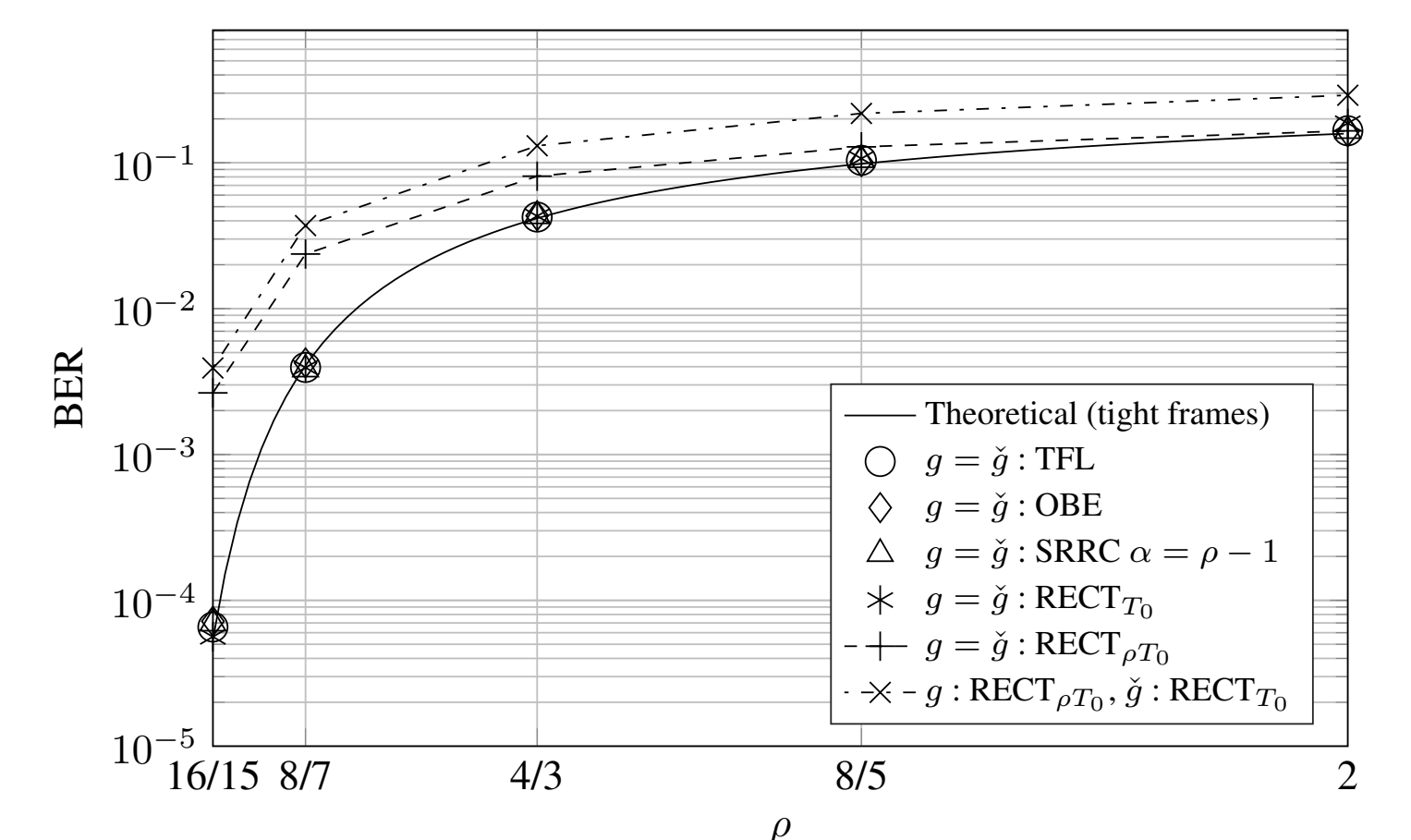


6. PERFORMANCES

6.1. Linear system

Parameters:

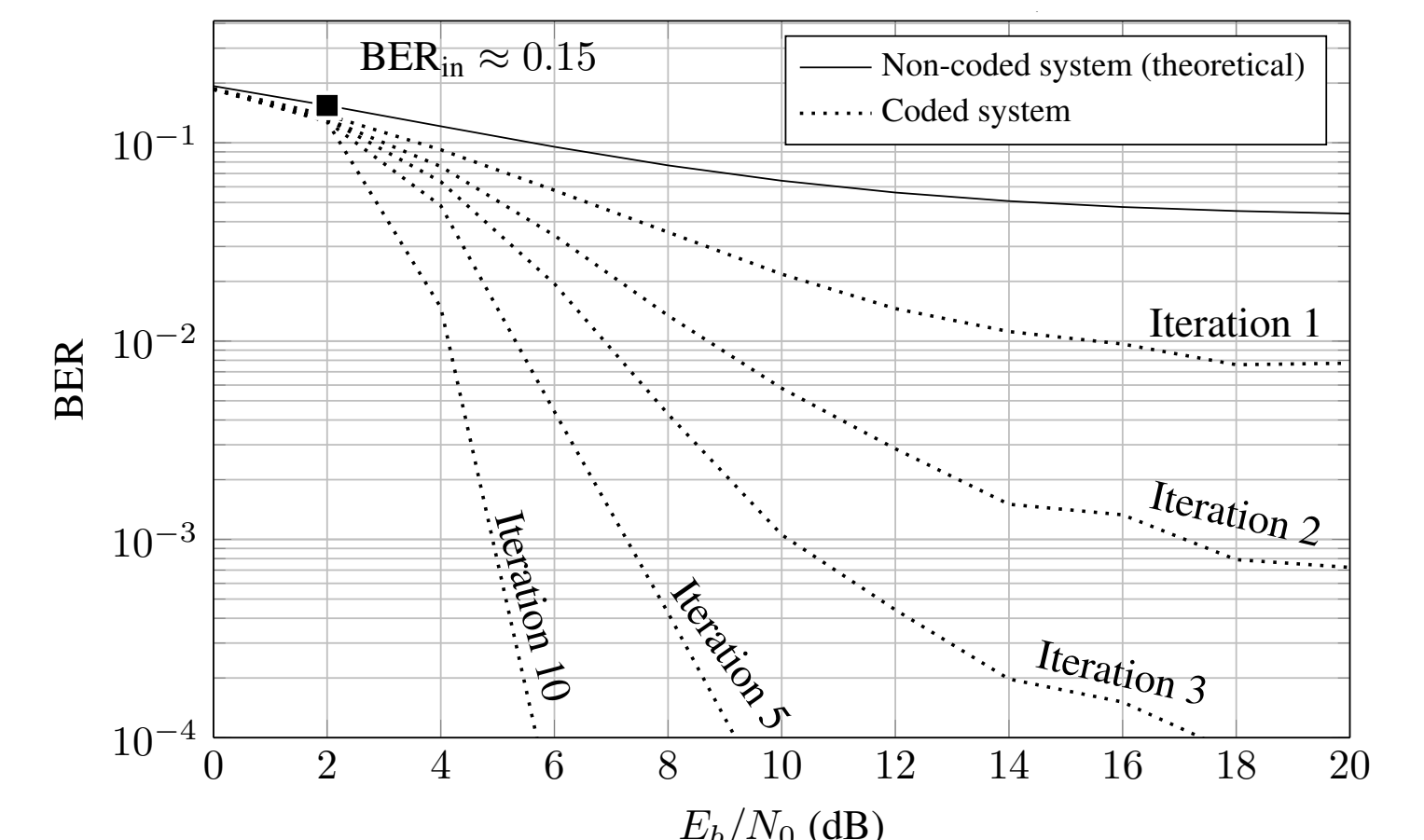
- $K = 5000$ symbols,
- $M = 128$ carriers,
- $E_b/N_0 = 20$ dB,
- QPSK constellation.



6.2. LDPC-coded system

Parameters:

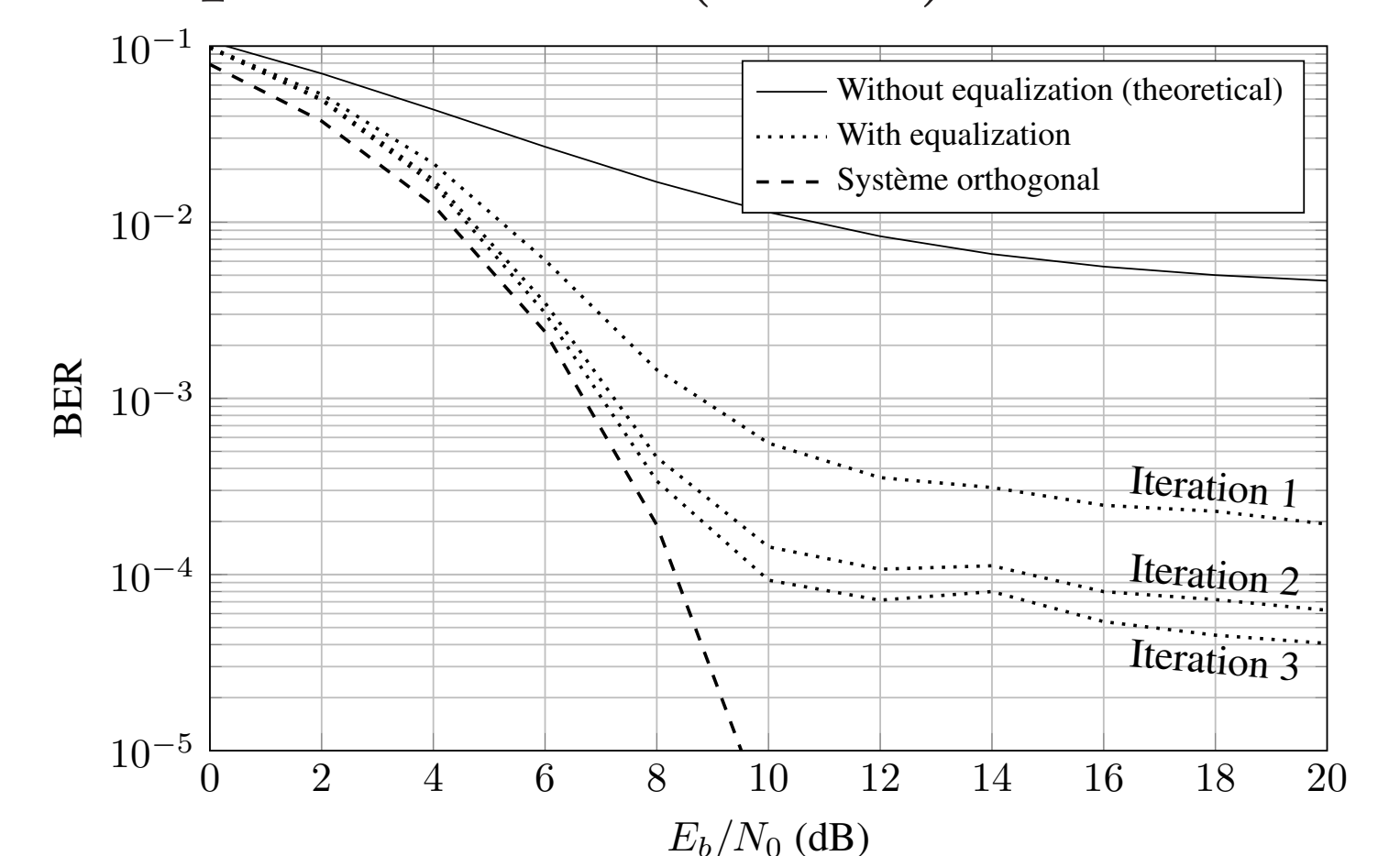
- $K = 5000$ symbols,
- $M = 128$ carriers,
- $\rho = 4/3$,
- QPSK constellation,
- DVB-S2 LDPC code,
- Belief propagation algorithm.



6.3. System with direct feedback equalization (DFE)

Parameters:

- $K = 5000$ symbols,
- $M = 128$ carriers,
- $\rho = 8/7$,
- QPSK constellation.



7. CONCLUSION

Non-linear structures are necessary to reach high densities. A particular attention on the PDF of the interference is required when using algorithms working with LLRs for high densities. DFE shows great performances besides its low-complexity.