Confluence of research and teaching: Case study of a mathematics teacher
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The complexity of research and teaching itself influences education in myriad ways. The insulation between research and teaching is counterproductive to the growth of mathematics education as a field in general and in developing a grounded understanding of teaching in particular. This study aims to understand teachers’ knowledge about students’ mathematical thinking in situ. Next, an attempt is made to enhance teachers’ knowledge through contextually appropriate intervention using artefacts from teachers’ practice. The collaboration between researcher and teacher to unpack students’ thinking and make informed teaching decisions guided learning among students, teachers, and the researcher. This paper argues for the value of a situated approach to theories about teaching and also develops opportunities to support teacher learning from practice.

**Keywords:** Mathematics teacher learning, professional development, situated approach, teacher knowledge and practice, teacher-researcher collaboration.

**INTRODUCTION**

The gap between research and teaching has been widely acknowledged by researchers in mathematics education (Lerman, 1990). The insulation between the communities of teachers and researchers is unfortunate for there is a major overlap between their interests and focus. Concerns like understanding students’ thinking, identifying and testing learning trajectories, trying out different teaching methodologies, focus on content and its connections, etc. are some of the research areas that are directly linked to teachers and teaching. In contrast to these, theorising about teacher knowledge, beliefs and practices has extensively been a domain of researchers or educationists. There has been lesser participation from teachers in unpacking their own knowledge and beliefs for that requires metacognitive awareness about their work. Teachers primarily focus on their routines of everyday classroom which leaves little time for them to reflect on what is being taught. Teachers make decisions about the content to be taught and ways in which this it can be communicated to make it accessible for learners of specific age groups. This distinctive feature of teacher knowledge, choice of representations or mathematical tools along with the knowledge of students is described by Shulman (1986) as *pedagogical content knowledge* (PCK). In teacher education programmes in India, we are yet to think about ways in which PCK can be discussed and enhanced in teachers in ways that they become instrumental in their own learning from practice.

A framework that is particularly useful in identifying the relationship between teacher knowledge and practice for teacher learning is proposed by Cochran-Smith and Lytle (2000). They distinguish between knowledge for practice, knowledge in practice and knowledge of practice. Knowledge for practice separates the knowledge producers from knowledge users. Knowledge is generated by experts (researchers and teacher educators) and then selectively shared with teachers (practitioners) for implications in classroom. Knowledge in practice is the practical wisdom that teachers or practitioners gain from their practice of teaching over the years. Teacher deliberation and reflection on the experience of teaching is the source of this knowledge. Knowledge of practice is the knowledge generated in the field and connected with the existing theoretical knowledge. In a way, knowledge of practice combines knowledge in and for practice to suit the needs of classroom and support teacher reflection and learning.

In the Indian context, research geared towards identifying meaningful connections with the teacher community is scarce (Batra, 2005) and initiatives and interventions in the area of mathematics teaching have
not been documented and analysed (Banerjee, 2012). More common approaches of working with in-service teachers are short term workshops and sessions where the knowledge from research is made accessible to teachers (Kumar, Dewan, & Subramaniam, 2012). The reforms in education are communicated to teachers in a top-down manner. Teachers are expected not to engage in matters of policy and theory (Kumar, 2008) but to implement the changes proposed by reforms in education. The agency of the teacher in actively informing curriculum change or policy is missing. Within this socio-political scenario, teachers perceive their role as ‘teaching’ (translated into telling) students to pass the examinations by completing the stated syllabi and assessments. Learning from the experience of teaching is assumed but not explicated or harnessed. More so, teachers actively taking part in their own professional development is not a part of this imagination. Practical difficulties of convincing authorities, teachers, and their sustained participation adds to the reluctance among researchers to work in collaboration with teachers or schools. The increasing insulation between research and teaching in India is mutual. Teachers criticise research for its apparent disconnectedness with the reality of classroom and the lack of its practical utility. A serious consideration of the gap between research and teaching has been identified by the recent policy documents (NCF, 2005; NCFTE, 2009) and efforts to bridge it in ways conducive to classroom teaching and learning are being explored. There is a need for models of teacher education where the community of teachers and researchers can work together to create opportunities for students as well as their own learning.

Knowledge of mathematics and about students’ learning mathematics guides teachers in planning lessons as well as in taking in-the-moment decisions while teaching in the classroom. Knowing about students’ mathematical thinking supports opportunities for asking questions linked to students’ ideas, eliciting multiple strategies, drawing connections across strategies, and so on (Franke, Kazemi, & Battey, 2007). Unfortunately, knowledge of content and students’ thinking are dealt separately in the teacher education programmes in India (NCFTE, 2009). The psychology courses deal with the components of students’ thinking and learning. The concept-related discussions are confined to the subject related methods courses. It is believed that the experience of teaching would help teachers to integrate the two knowledge pieces together and blend them in their teaching. The nature of knowledge that teachers gain from experience remains hidden, unarticulated and mostly unchallenged. Discussions on concept-specific students’ thinking and learning in teacher education needs exploration in the Indian context. Concept-specific students’ thinking was chosen as an artefact for discussion with teachers because it is close to the work of teaching, provides opportunities for teachers to formulate hypothesis and creates opportunities in classroom to explore teaching possibilities (Takker & Subramaniam, 2012). The dynamic approach enables teachers to be more thoughtful and informed in their decision-making.

**The Study**

The study was carried out in an English medium school in Mumbai, India. The school is a part of network of schools run by an autonomous body under the Government of India and caters to students and teachers from different locales of the country. Four elementary school mathematics teachers participated in the study which was carried out in 2012–2013.

**Objectives and Context**

The study aimed to understand the nature of teachers’ knowledge of students’ thinking gained from the experience of teaching mathematics and explore ways in which this knowledge can be supported and enhanced in situ. The observations of teaching practice and literature on students’ thinking in the same topic domain, were utilised to design contextually embedded tasks to support teacher reflection and learning. Qualitative changes in teaching practice and ways in which teachers discussed about students and mathematics were noticed.

This study tries to identify and provide opportunities to teachers to generate knowledge of practice by reflecting on their practice using research based knowledge about students’ thinking in a particular topic domain. Central to this research has been the collaboration of researchers and teachers to facilitate student learning in classroom. The process of generation of knowledge of practice gave researchers an opportunity to build a relationship with teachers and connection between research and teaching.
Methodology and data
The investigation adopted a case study methodology which included exploration and intervention. Four elementary school mathematics teachers, each with a teaching experience of more than ten years, participated in a two year long study. Two of these teachers teach mathematics and environment studies from Grades 1 to 5, and the other two teach mathematics and physics from Grades 6 to 10. The objectives of the study were explained to the teachers. The school time-table had 8 lessons per day. The participating teachers were assigned 6 (or more) lessons to teach everyday by the school. This made it difficult to maintain a follow up with each teacher immediately before and after a lesson. Data was collected through classroom observations, task based interviews before and after teaching a lesson, two long interviews with individual teachers, and after school meetings with all the teachers. The data is in the form of thick descriptions (Geertz, 1994), audio and/or video records which are in the process of being transcribed for the purpose of analysis.

Preliminary data analysis
The data from diverse sources is triangulated to make meaningful interpretation of the classroom events and then analysed for qualitative patterns. Thick descriptions allowed for a microscopic interpretation of the flow of discourse in classroom. Theoretical propositions that guided the study and description of each case (Yin, 2009) were used to analyse data from classroom observations and teacher interviews. In this paper, I discuss the case of one teacher, teaching division in a Grade 4 classroom.

Teaching division at Grade 4
Pallavi (pseudonym) is a primary school teacher. She has been teaching mathematics and environment studies to students from Grades 1 to 5 for over 15 years. In the years 2012–13, she was teaching mathematics to Grade 1, 4 and 5. Her class size varies between 30–38 students, including girls and boys. She appears confident about the content to be taught in a lesson. She does not plan her lessons because she thinks that with experience a teacher knows the content and its sequence. She is unhappy with the new textbooks introduced after the National Curriculum Framework 2005 (henceforth NCF 2005) because “they appear more like story books and less like mathematics textbooks”. She likes the old textbooks as there were “sufficient number of problems for practice”, concepts were neatly arranged with no overlaps, and “the teacher knew where to start and where to stop teaching [a topic]”.

In our initial interactions, Pallavi was reluctant to spend any time outside the classroom to talk about the teaching. Her classroom observations revealed a disciplined classroom where rules were laid down by the teacher. Over a period of time, the relationship between Pallavi and researcher changed. Pallavi became more reflective about her teaching and the researcher became a co-teacher.

Pallavi has been teaching division to fifth graders for almost as long as she started teaching. She has always ‘taught’ them long division beginning with division of a single digit number with another single digit number, double digit divided by a single digit, three digit by a single digit, two digit by a two digit and so on (see Year 2012 in Figure 2). She believes that children should be taught the algorithms as they are an important part of school mathematics. The revised textbooks (post NCF 2005) however do not provide a single method for division but suggests learners to think of different ways to solve division problems. These ways include repeated subtraction or subtraction using chunking of twos, repeated addition of the divisor, multiplication facts, etc. (NCERT, 2006a, 2006b). Pallavi has a clear opinion on these methods, which is captured in the following vignette from the interview transcript.

Pallavi: Different methods confuse a child. You know they are very young for understanding all this [methods]. Understanding comes later when their brain has grown a bit. In these [primary] classes, children should be clearly told what to do so that they can follow. And there are different methods to solve different problems. Like when I teach division, I don’t teach anything else. I just teach long division and give lot of practice, sums [mathematical problems]. The more sums they do the more they learn. You can’t expect them to learn so many methods like the new textbook gives. It says you teach this method also that method also. It is very confusing for students and then when you ask a question, which method do you want them to use? They should use long division. It is what we have been doing for ages, I did it.
when in school. And it is the systematic way. (P.I, 2012)

On several occasions, Pallavi was asked to think about why the long division algorithm works. In one of our (researcher and Pallavi) interactions, Pallavi was encouraged to unpack the algorithm by studying it in parts through questions like why take one digit at a time, what is the meaning of multiplying divisor with a number, what is the number being subtracted, etc. Pallavi is convinced about the digit-based approach (taking one digit in each step and dividing it by the divisor). She was taught the same algorithm as a student and once memorised it works for all numbers. Like many teachers, Pallavi ‘operates in the field based on her old biases and prejudices formed through her schooling’ (Mahapatra, 2004).

Apart from the discussions centred around her teaching, researcher had meetings with Pallavi and three of her fellow teachers teaching mathematics to middle grades. Tasks during teacher meetings involved thinking about students’ responses and unpacking students’ thinking in their (incorrect and correct) explanations. For instance, one of the tasks was to identify differences in the explanation of two students who both gave a different wrong answer for the same problem. The discussions around the task included talking about the errors, reasons for response, information they convey about student’s knowledge or difficulty, and exploring connections between different topic areas of mathematics to identify possible thinking trajectories that students’ might take with this kind of thinking. Teachers were engaged in the process of creating problems which would address different kinds of students’ thinking.

Pallavi considers the new textbooks ‘telling’ her a number of methods to be ‘taught’ in the class. Looking at the methods from the perspective of something to be taught and ensure that it is learnt by all students, Pallavi was not confident on the rationale for using the grouping method. It was hard to convince her to try to use different methods for solving a division problem. She strongly held the belief that the alternative methods would be confusing for students and therefore should not be dealt with in the class.

Pallavi: Now I have tried this method given in the book but see it is confusing... have always done long division only with children. So I am not sure how to introduce it, how to actually do it in class. I am comfortable in long division and it is shorter you know. It is a step by step process, take one digit at a time so they [students] can easily divide. Why don’t you [to researcher] take this [division by grouping] in my class? Tell them what this method is. [After a pause] Yes we can see how they pick it and decide then only which method. I don’t know if they will understand. I tried around 8 to 10 numbers, dividing them using that method. The bigger the number, the more confusing it was. I think it can confuse. But you try and let me see how they try to do it. (Pre-Class Interview, PI19N10, 2013)

Several considerations seem to be guiding Pallavi’s choice of a method for teaching division. These include her competence to use a method as a teacher, systematicity of the method i.e. one step follows from another, comprehensiveness and the cognitive demand it places on students while not being confusing, belief in the legitimacy of the established method, etc. At the same time it was clear that she is looking for support and some evidence of learning using different methods. As a result, she suggested an exchange of roles, where she becomes the observer and researcher becomes the teacher. The role reversal is an important event in this interaction as the teacher gives an opportunity to the researcher to collaborate in teaching which is at the core of the shared interest. As a researcher, there were two kinds of concerns emerging from this decision. Firstly, the teaching goal was to engage students with the grouping method as a way
of solving division problems. This goal was partially shared with Pallavi who wanted to assess students’ engagement with the method. Secondly, it was important to support Pallavi in teaching different methods as well as engage with students’ struggles while they use the alternative routes in their attempt to solve a division problem. An important teaching decision was to think of a relevant context where students would find the need to use grouping in order to solve the problem.

Pallavi started teaching the lesson by revising long division using number problems (two-digit number divided by a one-digit number). Then she called upon the researcher to teach and explain a ‘new’ method to the students. The researcher started with the context of distributing money among three friends equally to find the share of each friend (Rupees 75 among 3 friends). Students were asked to think about the amount that can be safely given to each friend from the total amount to be shared. Students began by suggesting different combinations on how the money can be distributed “give them 10 each, 20 each or 15”. When asked to justify their answers, students explained different combinations of numbers to distribute the money equally. In the second problem, the number of friends was increased to 5 while the amount to be shared remained the same. Even before solving the problem, students figured out that the share of each friend would be lesser and used grouping with convenient numbers to justify their answer. As the lesson was progressing, Pallavi gradually stepped in, helped in eliciting different combinations from students, making them think of efficient combinations and asking students for reasons for their choice. Later on she completely took over to use the method with different numbers (115 divided by 3 in Figure 1). It seemed that along with the learners, Pallavi was also testing whether this method works for different numbers, the appropriate choice of numbers (dividend and divisor), and its linkages with related concepts like multiplication. After the class, Pallavi reflected on the use of method and its affordances by comparing it with the long division algorithm.

While reflecting on the lesson, Pallavi was considering the affordance of the grouping method mathematically as well as from the view point of students. Mathematically, she unpacks the idea of grouping using convenient numbers, flexibility in the number of steps, efficient choice of combinations, treating dividend as a whole, and workability of solution for other sets of numbers. She noticed that students came up with the method themselves and were using it flexibly without making a common error, skipping a digit while using long division algorithm without understanding. In the next lesson, Pallavi asked students to choose any method to solve the division problem and identify the similarities in long division and grouping. Further, Pallavi made a decision to include the grouping and other methods in her teaching of division henceforth.

In the course of intervention, Pallavi realised the importance of listening to students not just to evaluate what they do not know but to make it a part of her decision making in teaching. Pallavi engaged with the basic principles of analysing a given method of solving arithmetic problems. The structure that underlies the method and its connection with the algorithm was discovered by Pallavi along with the students. Thus, the choice of method(s) is not guided by the authority of the method given in the textbook but from an engagement with the structure of a method and its ra-
tionale. Another evidence of change noticed in Pallavi is the difference in her teaching trajectory over these two years (refer Figure 2).

The episodes reported in this paper are used to exemplify one of the ways in which knowledge from teachers’ classroom was utilised to mediate teacher reflection and learning around the content of mathematics. The role of researcher was to notice opportunities for communicating important mathematical ideas from teachers’ teaching and bring these for discussion with the teacher(s). The discussions centred around the rationale of mathematical procedures, conceptual understanding, and identifying important mathematical ideas to be communicated.

**DISCUSSION**

The paper presents preliminary analysis of a case study where teacher’s knowledge of students’ thinking was utilised to support her reflection and learning. Although a more rigorous analysis is in process, some interesting patterns are reported in this paper. Evidences of students’ thinking from teachers’ classrooms as well as from research served as concrete artefacts to talk about mathematics, making intervention more grounded and connected to the classroom reality. Teachers’ explorations in their classroom gave them an opportunity to test propositions emerging from meetings with other teachers and researchers. The change in the positioning of teacher and researcher marked an important learning turn in the collaboration. The methodology allowed for the researcher to engage with teaching and students’ learning directly. The impact of such collaborations on learning needs further investigation.

Researcher’s own experience as a teacher and work with teachers and teacher educators has indicated the need for building teacher communities. Teachers need spaces to articulate their struggles of teaching mathematics, their conjectures about students’ learning and its interaction with the content, etc. A model for affording such an interactive space to discuss the work of teaching needs to be created in our country. Research which focuses on students’ thinking is a beginning to an attempt towards building communities of teachers for sustained interactions. In this study, an exploration into knowledge required for teaching mathematics with a focus on practice created space for teachers and researchers to work together. More focused efforts are required to connect knowledge in and for practice to generate knowledge of practice through teacher-researcher collaboration. The voice and agency of teachers in programmes aimed at their professional development needs to be respected for making interventions meaningful.

**REFERENCES**


