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Primary teacher' approach to measurement estimation activities

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Estimation appears as a relevant aspect to the new Curricular Guidelines of year 2012 of the Chilean Ministry of Education. In this paper, we analyze the mathematical knowledge for teaching of 112 Chilean teachers of Primary Education to deal with a classroom activity of discreet measurement estimation proposed in governmental curricular orientations. The qualitative-descriptive analysis developed in our study is supported by a reconstruction of the concept of measurement estimation. The results show that the knowledge of a significant part of the participant teachers presents some deficiencies, since they either propose to develop the activity without including perceptive abilities or they propose an action limited to a measurement exercise. Only 13.1% of the teachers state that when using the example given for estimation a perceptive-type exercise is required.

Keywords: Mathematical knowledge for teaching, measurement estimation, perception, referent.

INTRODUCTION

At the beginning of the 80's, the teaching of measurement gained emphasis in the curricula of some schools that answered to the guidelines of international organizations linked to the teaching of mathematics (Cockcroft Report, 1982; ICMI, 1986). These orientations were included by the Chilean Ministry of Education in the Curricular Guidelines of the year 2012, in which measurement has become the new curricular axis in primary education. In order to complement these guidelines, the Ministry itself has published some activity proposals in its study program.

The new Chilean curriculum considers estimation in more than twelve learning objectives in measurement and geometry. However, previously to this reform, measurement estimation hadn't had its own space in

Chilean study programs. For this reason, probably, current teachers have never received training in this respect, neither in their school years nor in their further professional training. Therefore, we are facing a situation in which a mathematical concept bursts into the classrooms without any specific training being provided to the educators.

Considering the shortcomings identified by international mathematics studies regarding the teaching skills of teachers on the estimation of measurement (Chamorro, 1996, 1998; Joram et al., 2005; Forrester & Pike, 1998), estimation measurement is a concept that has not been investigated in depth, and in which difficulties have been detected in the learning of the students (Jones, Forrester, Gardner, Grant, Taylor, & Andre, 2012). At the same time, there are studies that claim that teachers work in estimating measurement is superficial and that they do not feel confident about working with students (Lang, 2001)

If to this we add that Chilean teachers have not had training in this respect, we consider it important to focus on how teachers interpret the estimation measurement activities proposed in government documents and official textbooks. Developing different learning objectives for the Chilean curriculum should be the main reference point for this.

As an orientation for our work we ask ourselves: How are measurement estimation activities treated by primary school teachers? In order to answer this question we take the Mathematical Knowledge for Teaching (MKT) proposed by Ball, Thames and Phelps (2008) as a theoretical referent and we carry out a qualitative-descriptive study in Santiago de Chile with 112 in-service Primary School teachers, who have taught or will be teaching measurement estimation to their pupils at a certain point in time.

Our aim is to investigate the possible treatment of measurement estimation activities in the classroom. Conversely, we intend to study which MKT will be needed by the teachers when introducing the new content in the mathematics curriculum. Our final aim is to establish guidelines that may back up future improvement actions of initial and continued training of teachers.

MATHEMATICAL KNOWLEDGE FOR TEACHING

The work of Ball and colleagues (2008), rests on Shulman (1986), emphasizes the use of knowledge in teaching and for teaching. They focused on the analysis of knowledge for teaching from the observation of the teacher's work in the classroom and outside, thus introducing the line of research of MKT. In the current study, we focus only on one component of MKT, Knowledge of Content and Teaching (KCT), which combines knowledge of mathematics and knowledge of its teaching. Examples of this type of knowledge are found in the ability to sequence content, propose starting, deepening and closure examples, and knowledge of the advantages and drawbacks of representations used to teach a specific idea.

MEASUREMENT ESTIMATION

When revising the literature on measurement estimation we often observe that different definitions of estimation include a wide range of tasks related to different fields of mathematical knowledge. These

tasks may include mentally assigning a value to a magnitude measurement, an agile mental calculation of an arithmetic operation, or the calculation of the likely value of a statistical measurement, amongst others. Table 1 shows different definitions of estimation proposed by different authors.

Segovia and colleagues (1989) establish that measurement estimation differs from computational estimation due to methodological reasons. Hogan and Brezinski (2003) differentiate three types of estimation, according to the abilities they develop:

- Numerosity: refers to the ability to visually estimate the number of objects arranged on a plane within a limited amount of time.
- Measurement estimation: is based on the perceptive ability to estimate different magnitudes in common objects.
- Computational estimation: refers to the process by which an approximate value is rapidly determined for the result of a mathematical operation.

For Hogan and Brezinski (2003), computational estimation is a skill which evolves together with the rest of mathematical abilities. On the other hand, numerosity and measurement estimation are abilities that develop independently from that of estimating a calculation and are not grouped in the category of usual mathematical abilities. These authors explain that

	Definition	Relevant aspects
1	"A process to reach a measurement or measure without the help of measuring tools. It corresponds to a mental process that includes visual or manipulative aspects." (Bright, 1976, p. 89).	Mental process used to obtain a measurement. Visual perception or use of sense of touch.
2	"the ability to judge whether the result of a calculation which has been carried out or a measurement which has been taken seems to be reasonable and ability to make subjective judgements about a variety of measures" (Cockcroft Report, 1982, pp. 22–23).	Evaluation of the adequacy of results of calculations or measurements. Subjective evaluation of a measurement.
3	"assessment of the value of the result of a numerical calculation or of the measurement of a quantity, as a function of the individual circumstances of who forms it" (Segovia et al., 1989, p. 18).	Subjective evaluation of the results of calculations or measurements.
4	"the skill of making an educated guess as to the value of a distance, cost, size, etc. or arithmetic calculation" (Clayton, 1996, p. 87).	Conjecture on measurements and arithmetic operations.
5	"Refers to a number that is a suitable approximation for an exact number given the particular context" (Van de Walle et al., 2010, p. 241).	Adequate approximation of a number.

Table 1: Definitions of estimation

in terms of the human brain, we have an accumulator that stores quantities in an approximate way. For instance, we know what a meter is, we can identify this concept in an object or in part of it and we can describe it using our body. This is possible due to the fact that we are aware of the adequate measurement of the length of a meter. This quantity “accumulator” may contribute to the development of people’s visual perception by relating measurement estimation and numerosity with spatial abilities. We understand that this “acquisition” of the unit of measurement in use when performing estimations does not appear in the definitions shown above.

ESTIMATION IN EDUCATION

An important part of teaching measurement is focused on the development of their estimation. In the first place, this is due to the fact that estimation will further develop perceptive abilities (Inskip, 1976; Cockcroft Report, 1982; Hogan & Brezinski, 2003). In addition, estimation comprises the foundation of the teaching of physical magnitudes such as length, surface area, volume or weight, amongst others (Hogan & Brezinski, 2003; Joram, Subrahmanyam, & Gelman, 1998; Joram, Gabriele, Bertheau, Gelman, & Subrahmanyam, 2005). When working on measurement estimation numerical abilities are also exercised (Boulton-Lewis, Wils, & Mutch, 1996; Joram et al., 1998; Sarama & Clements, 2009), in the sense of measurement (Clements, 1999) and in the numerical and spatial senses (Lang, 2001).

Despite the emphasis mathematics education attributes to measurement estimation, there are few studies that refer to the way estimation is developed in the classroom. Forrester and Piké (1998) observed that in the teachers’ speech in the classroom there was a significant separation between measurement and estimation. Estimation was treated as a predictive hypothesis, in a vague and superfluous way, as a process failing to produce satisfactory answers to solve situations that only a measuring tool would be able to answer.

These authors observed that estimation was treated as a means of “thinking reasonably” that included riddles more often than value judgments based on referents, without the requirement of evidence of concept understanding. This outlook suggests mathematics is a synonym of rigor and exactitude. Therefore, an estimate should always be checked with a measuring

instrument. Segovia and Castro (2009) explain that in most dictionaries the meaning of mathematics is characterized by exactitude and rigor. Following these criteria, estimation would be far away from “real mathematics”, since the results it provides are not exact.

In our literature revision we have found few studies on teacher training related to measurement estimation. However, Joram and colleagues (2005) suggest the use of the estimation strategy of the reference point, as a seemingly appropriate didactic strategy.

GOAL OF THE STUDY

Since the Chilean Curricular Guidelines of the year 2012 include measurement estimation, it would be desirable to for Chilean teachers to have the adequate knowledge to use the activities proposed in the study programs provided by the Chilean Ministry of Education. In this context, the aim of our study is the following:

- To characterize the Knowledge of Content and Teaching (KCT) on measurement estimation to allow teachers to apply the activities proposed by the Ministry’s guidelines in their classrooms.

THE STUDY

As a data collection tool, we have elaborated a questionnaire of ten open questions that were oriented by the different categories of MKT (Ball et al., 2008). The questionnaire was answered by 112 in-service Primary School teachers in Santiago de Chile. In this presentation we only analyze the answers to a question based on a task proposed in the guidelines for the curriculum of 7-year-old children (Figure 1).

The task proposed in the guidelines is an example of a measurement estimation activity for the teachers to develop in the classroom. The aim of this activity is estimate quantities up to one-hundred in concrete situations using a referent. With this question, we try to collect information about Knowledge Content and Teaching, because it can show how the teachers manage the content and activities in the classroom, and consider that the referent is a strength of the activity and that students should not be permitted to measure using the square as the unit of measurement.

They estimate the quantity (the number of) of squares that fit in the rectangle.



Verify the estimation by placing paper squares on the rectangle and counting them

Figure 1: Estimation activity

In our questionnaire we ask teachers to explain how they would approach the activity shown in the classroom

DATA ANALYSIS

Since we are interested in identifying aspects of teachers' MKT around measurement estimation, the analysis of the data collected is qualitative. Therefore, based on the different views teachers have of activity 4, we have created several categories to characterise the knowledge of measurement estimation detected. When making a first reading of the answers, we prove the need to establish elements that determine if a task is actually about measurement estimation in order to back up our analysis. For this reason, we decided to elaborate a definition of the concept that included the fundamental elements identified during our literature review.

Our definition of measurement estimation is the following:

To perceptively assign a value, or an interval of values, together with a unit that corresponds to a discrete or continuous magnitude, by means of previous knowledge or by an indirect comparison to an auxiliary object.

This definition is based on three essential elements:

- Assigning a numerical value (V): in order to consider the task an estimation, the assignment of a numerical value is required; otherwise, the task only requires perceptive work. For instance, if when touching the water of a swimming pool we state that it is pleasant to swim in, the task is exclusively perceptive, given that we have not assigned any value to temperature and measurement units do not play a part in the activity (Bright, 1976; Informe Cockcroft, 1982; Segovia et al., 1989; Joram et al., 2005).

- Executing the task perceptively (P): in order to estimate it is necessary to use the senses, avoiding counting or the direct use of any measurement tool, standardized or not. When a measurement tool is used and a direct comparison is made, the action taking place is measuring, not estimating, even though the result may be expressed approximately (Bright, 1976; Hogan & Brezinski, 2003; Joram et al., 2005; Sarama & Clements, 2009).
- Relating perception to previous knowledge or to the mental image of an auxiliary object (R): In order to estimate referents are required, or to ideally possess a mental notion of a measurement unit; otherwise the activity would only require to guess or give a random result (Clements & McMillen, 1996; Hogan & Brezinski, 2003; Joram, 2003; Sarama & Clements, 2009).

Given that in the activity used in our question, the referent considered in the estimation has been given in the formulation, in our analysis we consider only how this referent has been used, the main analysis aspect of which is the perceptive component (P). Below we give an example of the analysis process and we introduce the different categories that organize the different ways teachers suggest approaching the activity in the classroom.

Use of the activity based on providing tools

Some teachers limit themselves to answering to what type of material will be used during the activity, without reporting particularly how these resources are planned to be used. For instance, one of the teachers answers: *"The students will be given both shapes (1 of each) made of paper card."*

We observe that the teacher's attention is focused on the material he will use in the activity instead of what is going to be done with it, which makes it impossible

to guess how the teacher is going to develop the activity in the classroom.

Use of the activity as evaluation

Some of the answers given by the teachers suggest using the activity described as an evaluation exercise without a backup of mathematical processes associated to estimation. For instance, one of the teachers answers: *"First I would show the children this rectangle and after that I would hand in a piece of colored paper to each of them and would ask them to estimate how many squares can fit into the rectangle."*

We observe that the teacher is focused on the amount of squares that fit into the rectangle (V), but without offering any didactic instructions nor any other type of information that may allow us to understand how he plans to direct and develop the activity in the classroom. Therefore, we have no evidence that perceptive aspects may be included in the activity (absence of P).

"Children: I invite you to estimate the area of the rectangle using the little square as the unit of measurement". The teacher does the same as the teacher above, both give space for students to measure, estimate or guess.

Use of the activity as measurement

In the proposals of some of the teachers, we observe that they intend to go about the activity as a measurement task, with the use of non-conventional tools, explaining the way these are going to be used for the activity. This allows us to see that their proposal corresponds to a measuring activity. Answers in this category evidence a misunderstanding between the use of a referent to carry out an estimation and the use of a non-conventional measuring tool. Into this category we included answers such as: *"We need to know how many squares we need to cover the rectangle. After constructing the rectangle we should place squares of equal size on top of the former."*

We see that the teacher points out that the surface area of the rectangle should be covered by squares of the same size as the one given, suggesting a counting activity. Based on this idea, the referent then becomes a non-standardized unit of measurement. In this case, the students would reach an evaluation (V) that is not supported by a perceptive act (absence of P) but by a measurement.

Another example is this answer: *"Place the square on the rectangle to complete the rectangle with squares. The rectangle with the superimposed squares will be presented by the students"*

In this task, there is no perceptive work (absence of P) because the students must complete the rectangle with squares, perhaps drawn on it, to count how many squares fit. As with the teacher above, the square was used as a non-standardized unit of measurement.

Use of the activity as a measurement estimation

Some of the teachers regarded the evaluation of the amount of squares that fit into the rectangle as a perceptive activity, by using the square shape of the problem's formulation as a referent. We consider that answers of this type suggest using the activity as a measurement estimation exercise. Among them we find: *"Boys and girls, on the blackboard you can see a red square and a blue rectangle. Without standing up or leaving your seats, or as you prefer, please estimate how many times the red square fits into the blue rectangle, by placing the squares next to each other."*

We have identified that the intention of reaching an evaluation (V) is supported by a perceptive action (P), since the teacher does not allow a physical interaction between the students and the representations of the figures on the blackboard. In this case, the students would be obliged to carry out a visual perception exercise.

Another teacher answer: *"Without making any movement, think about how many squares you estimate will fit in the rectangle"*

We observe that when the teacher says "making any movement", s/he indicates that the student must not handle the square, thus steering the student away from measurement and supporting the perceptive action (P). In this way, the evaluation (V) rests on the perceptive action.

RESULTS

Table 2 shows the answer frequency for the different categories.

We see that only 16 of the 112 surveyed teachers have made a proposal that includes the development of per-

Category	Frequency	
Use of the activity based on the supply of tools.	4	(3.6%)
Use of the activity as an evaluation with a possible estimation or measurement.	38	(32.7%)
Use of the activity as a measurement.	25	(22.3%)
Use of the activity as a measurement estimation.	26	(13.1%)
Other answers	5	(4.5%)
No answer	23	(20.5%)

Table 2: Frequencies and categories of teachers' answers

ception and evaluation. Given that the referent used is found explicitly in the formulation, their proposal corresponds to an estimation task. These teachers show an adequate Knowledge of Content and Teaching for the development of the activity in the classroom.

On the other hand, 38 of the teachers make a proposal in which estimation is a synonym of evaluation of a magnitude, without justifying how the task is carried out. This makes us think that in the concept of estimation used by these teachers, perceptive processes are not relevant and estimation is not supported by mathematical processes. The development of the activity proposed is incomplete, presenting an incomplete Knowledge of Content and Teaching regarding measurement estimation.

A third group of 25 teachers considers that estimating a measurement is a synonym of measuring, using the referent as a unit of measurement. For these teachers, measurement estimation seems to be the same as measuring with a non-standard tool. In this last category, we include the answers to the questions that are exclusively based on providing the students with measurement tools without specifying their didactic use. This group is comprised of 4 teachers, who we may also consider to confuse estimation with measurement. Finally, the categories that include other answers and empty answers represent 28 of the 112 teachers, which represent a non-negligible percentage (25.0%) of the teachers surveyed.

General research results

This research is part of a wider study. We have used a variety of questions focused on different aspects of MKT estimation of measurement. In this way, we have found that there are gaps at different levels, e.g., teachers confused estimation of measurement with measurement itself, using non-standard units of measurement; teachers consider that the estimate of measurement is a superfluous task, like a guess;

teachers do not observe the weaknesses of estimation measurement activities.

CONCLUSIONS

In the present study we have observed large differences in Knowledge of Content and Teaching on measurement estimation between Chilean teachers. Based on one of the activities proposed by the Chilean Ministry of Education to exemplify measurement estimation work in the classrooms, we note that a large part of the teachers do not possess an adequate specific knowledge to use these activities with the purpose for which they have been designed.

Specifically, only 13.2% approaches the activity with the required conditions to work on estimation in an effective manner. A significant part of the group does not acknowledge the importance of perception when estimating and do not mention perception explicitly. This allows the student to handle the activity both directly or indirectly, which would not be considered estimation. It should be noted that a large part of the teachers confuse measurement estimation and measurements based on the use of non-standardized measurement tools. This type of confusions can be related to the absence of estimation in the curriculum to this date. The surveyed teachers have not received any training that allows them to distinguish these two types of tasks.

In order to solve this situation of lack of knowledge, we consider that measurement estimation should be included in teacher-training curricula. It would be interesting for pre-service and in-service teachers training to include activities that required for them to carry out their own estimations and compare them to measurement processes. It would also be useful for teachers to observe mathematical processes that distinguish the different types of estimation presented in our literature review, as a way of reinforcing their

specific mathematical content. On the other hand, a revision of current curricular guidelines is paramount, in order to avoid creating larger ambiguities. If we consider it acceptable to verify the result obtained from early-stage estimation activities with the exact result, it is fundamental that teachers regard estimation as a task in itself, with its own validation procedures according to the context's requirements

Finally, we consider it crucial to define a mathematical concept precisely when introduced into an educational system. In addition, official programs and didactical guidelines accompanying textbooks should provide examples from appropriate activities, distinguishing them from those that are not. Otherwise, it is highly likely that the curricular changes the country wants to achieve may not be transferred to the classrooms effectively.

REFERENCES

- Ball, D., Thames, M. H., & Phelps, G. (2008). Content Knowledge for Teaching: What Makes It Special? *Journal of Teacher Education*, 59(5), 389–407.
- Bright, G. W. (1976). Estimation as Part of Learning to Measure. In *National Council of Teachers of Mathematics Yearbook* (38, pp. 87–104). Reston, VA: NCTM.
- Boulton-Lewis, G., Wils, L., & Mutch, S. (1996). An analysis of young children's strategies and use of devices for length measurement. *Journal of Mathematical Behaviour*, 15(3), 329–347.
- Clayton, J. G. (1996). A criterion for estimation tasks. *International Journal of Mathematical Education in Science and Technology*, 27(1), 87–102.
- Chamorro, M. (1995). Aproximación a la medida de las magnitudes en la enseñanza primaria. *Revista Uno*, 3, 31–53.
- Chamorro, M. (1996). El currículum de medida en educación primaria y ESO y las capacidades de los escolares. *Revista Uno*, 10, 43–62.
- Clements, D. H. (1999). Teaching length measurement: Research challenges. *School Science and Mathematics*, 99(1), 5–11.
- Clements, D. H., & McMillen, S. (1996). Rethinking "concrete" manipulatives. *Teaching Children Mathematics*, 2(5), 270–279.
- Cockcroft, W. H. (1982). *Mathematics counts*. London, UK: Her Majesty's Stationery Office.
- Forrester, M. A., & Pike, C. D. (1998). Learning to estimate in the mathematics classroom: A conversation-analytic approach. *Journal of Research in Mathematics Education*, 29(3), 334–356.
- ICMI (1986). *School Mathematics in the 1990s*. Cambridge, UK: Cambridge University Press.
- Inskeep, J. E. (1976). Teaching measurement to elementary school children. In *N.C.T.M. Measurement in School Mathematics, 1976 Yearbook*. Reston, VA: National Council of Teachers of Mathematics.
- Jones, M.G., Forrester, J.H., Gardner, G. E., Taylor, A.R., & Andre, T. (2012). Students' Accuracy of Measurement Estimation: Context, Units, and Logical Thinking. *School Science and Mathematics*, 112(3), 171–178.
- Joram, E., Gabriele, A. J., Bertheau, M., Gelman, R., & Subrahmanyam, K. (2005). Children's use of the reference point strategy for measurement estimation. *Journal for Research in Mathematics Education*, 36(1), 4–23.
- Joram, E., Subrahmanyam, K., & Gelman, R. (1998). Measurement estimation: Learning to map the route from number to quantity and back. *Review of Educational Research*, 68(4), 413–449.
- Lang, F. K. (2001). What Is a "Good Guess" anyway? Estimation in early Childhood. *Teaching Children Mathematics*, 7(8), 462–66.
- Hogan, T. P., & Brezinski, K. L. (2003). Quantitative estimation: One, two, or three abilities? *Mathematical Thinking and Learning*, 5(4), 259–280.
- Ministerio de Educación (2011). *Bases Curriculares de primero a sexto año básico*. Santiago de Chile: MINEDUC.
- Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. New York, NY: Routledge.
- Segovia, I., Castro, E., Castro, E., & Rico, L. (1989). *Estimación en cálculo y medida*. Madrid, Spain: Síntesis.
- Segovia, I., & Castro, E. (2009). La estimación en el cálculo y en la medida. *Electronic Journal of Research in Educational Psychology*, 17(1), 449–536.
- Shulman, L. S. (1986). Those who understand: knowledge growth in teaching. *Educational Researcher*, 15(2), 4–14.
- Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2010). *Elementary and middle school mathematics: Teaching developmentally*. Boston, MA: Pearson.