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"Responding to student ideas" as an indicator of a teacher's mathematical knowledge in teaching

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This paper describes and analyses two 8th grade mathematics instructions, delivered by two teachers on the slope and its algebraic representation. The focus of analysis is on teacher content knowledge which was analysed with Knowledge Quartet model (Rowland et al., 2005). The study examined whether teachers' responses to students' comments indicate the quality of teachers' content knowledge. Findings indicated that teacher's reaction to students' comments would significantly reflect teachers' depth and breadth of mathematics content knowledge.

Keywords: Teacher knowledge, knowledge quartet, contingency, slope, responding to students' ideas.

TEACHER KNOWLEDGE AND TEACHING

Enhancing students' mathematics thinking and learning is the utmost aim of overall mathematics education. One of the most important influences on students' learning and mathematics thinking is the teachers and their professional knowledge (Even, 1993; Darling-Hammond & Ball, 1998). Teachers' knowledge is crucial since teachers' effectiveness is influenced by that knowledge (Gilbert & Gilbert, 2011; Wagner et al., 2007). In addition to its effect on students' learning, teachers' knowledge is influential in shaping their practices (Borko & Putnam, 1996; Even, 1993; Fennema & Franke, 1992; Rowland & Ruthven, 2011; Sherin, 2002; Shulman, 1986).

Shulman's study (1986) opened a way to understand teachers' complex and multi-dimensional character of knowledge. Shulman suggested seven knowledge categories. Among them, the first four were content-free and the last three ones were on content-specific knowledge (Rowland & Turner, 2007). Subject-matter knowledge (SMK), curricular knowledge

(CK), and pedagogical content knowledge (PCK) constitute the content-specific knowledge that teachers have (Shulman, 1986). The work of Shulman (1986) suggested that content-specific knowledge becomes important in studying teachers' knowledge.

Shulman's conceptualization on teachers' knowledge has been heavily used by mathematics education researcher community. Almost all study on mathematics teacher knowledge assumes Shulman's categorization and conceptualization of teacher knowledge that is why we do not present a detailed account of Shulman's ideas in this paper.

Researchers have studied mathematics teachers' knowledge from several perspectives. While some researchers conducted research on teachers' understanding of various concepts in mathematics (e.g., Ball, 1990), others have focused on investigating the relationship between SMK and PCK and teaching (e.g., Even, 1993; Rowland, Huckstep, & Thwaites, 2005; Hill, Rowan, & Ball, 2005).

Apart from the perspectives summarized above, discussions on the nature of teachers' knowledge deserve an exclusive attention. The discussion on the nature of teachers' knowledge yield a consensus that the knowledge employed in teaching is dynamic, more visible through practice, and should be studied in actual classroom setting (Fennema & Franke, 1992; Hodgen, 2011; Rowland & Ruthven, 2011; Wagner et al., 2007). Conceptualizing a mathematical knowledge for teaching would unlikely to be successful unless it carefully takes the classroom context of teachers' professional work into account (Hodgen, 2011; Rowland & Ruthven, 2011; Sherin, 2002). The reason behind this failure may originate from the de-contextualization of teachers' knowledge (Hodgen, 2011). Focusing the use of teacher knowledge in the practice of teaching

may lead more sound results in investigating teachers' knowledge (Hodgen, 2011; Rowland & Turner, 2007).

Considering our concerns on the nature of teachers' professional knowledge, one framework which may provide studying teachers' content knowledge during instruction is the Knowledge Quartet (KQ) (Rowland et al., 2005). The framework suggests four broad units in investigating teachers' mathematical knowledge in teaching. These are (i) foundation, (ii) transformation, (iii) connection, and (iv) contingency.

The first unit of the KQ is rooted in the foundation of teachers' knowledge, beliefs and understanding of mathematics and teaching. The remaining three units originate from a foundational underpinning. The second unit of the KQ, transformation, is knowledge-in-action which is visible throughout planning and teaching. The unit includes a teacher's capacity in transforming the content knowledge into pedagogically powerful forms. Connection concerns the depth, breadth and coherence of relationships demonstrated in a lesson or between lessons. The fourth unit of the framework, contingency, encompasses to responding appropriately to the events and ideas which occur in classroom during instruction. It is about contingent action of teachers in the classroom (Rowland et al., 2005). Each of these KQ units is represented with codes. For example, there are three codes for contingency (responding to students' ideas, use of opportunities, and deviation from agenda). There are eight-teen codes in total for four units (Rowland & Turner, 2007). Within its current form, the units of the framework are not directly linked to Shulman's conceptualization of subject matter knowledge and pedagogical content knowledge. However, within the units, the framework aims to cover those knowledge types. The KQ has been grounded in classroom practice, and the findings have been open to enhancement and revision in the case of new research data. In this sense, the KQ can be effectively used to investigate the way content knowledge enacts during instruction. A detailed account of the framework may be reached from the researchers previous studies (Rowland et al., 2005).

THE TEACHING AND LEARNING SLOPE OF A LINE

In order to examine and evaluate teachers' knowledge on a topic, it is important to address crucial mathemat-

ical ideas for the topic and literature on teaching and learning of it. Therefore, in this part of the paper, we will highlight mathematical importance of the topic 'slope' and some literature on teaching and learning of it. Slope is a deep and multi-faceted concept (Stump, 1999). Learning slope requires proper comprehension of important concepts such as ratio, rate of change, proportionality, covariation, and synthesis of different representations. Hence, understanding slope of a line in early grades is especially crucial for future learning. Comprehension of slope through algebraically, geometrical or by other representation as well as the connections between them requires conceptualizing an important set of concepts.

Slope is a fundamental but a conceptually complex concept for students in learning algebra (Lobato et al., 2003; Stump, 1999, 2001). Rasslan and Vinner (1995) investigated that majority of the nine graders did not realize that "the slope is an algebraic invariant of the line and therefore does not depend on the coordinate system in which the line is drawn" (p. 264). In addition, Saldanha and Thompson (1998) suggested that it may be challenging for students to understand "graphs as representing a continuum of states of covarying quantities" (p. 7).

Lobato and Bowers (2000) provided that students have various difficulties in learning quantitative complexity of slope. For example, majority of the participants in the study showed difficulty in understanding the role of change in rise and run on steepness of a line. In addition, students have difficulties in regarding slope as a ratio (Bell & Janvier, 1981; Leinhardt et al., 1990; Lobato et al., 2003).

The previous studies showed that there exist a number of misconceptions among learners on slope concept. Some of the incorrect ideas mentioned in the literature were: the quadrant where the line is located is related to slope value, changing slope alters y-intercept of the line, slope is the scale of the x-axis, and slope is the difference in y-axis (Lobato et al., 2003). Among them, slope-height confusion is very common. Being aware of those misconceptions is crucial in teaching slope.

THE STUDY

The aim of the study was to investigate teachers' responses to students' unanticipated questions as an indicative of their content knowledge. In other words, in this paper, findings about teachers' knowledge of contingency (as defined in KQ) will be examined. Therefore, the research question is that *How does mathematics teachers' responding to students' ideas indicate their content knowledge in teaching slope of a line?*

The original study focused on two pre-service, two early novice (0–2 years of experience), and two more experienced novice (3–5 years of experience) middle school mathematics teachers' content knowledge on slope at 8th grade in Turkey. One of the researchers conducted individual pre-interviews, recorded 1–2 lesson hours of slope instruction, and finally conducted post-interviews. Pre and post interview data were used for triangulation but main data source was video-tapes of classroom instruction and researchers' observation notes of the instruction. Instruction data were transcribed and open-coded. In data analysis, the researchers attempted to reach themes for the open coded data. Then, for each theme, one of the eighteen codes of the KQ has been assigned. Since assigned KQ codes belong to the units, we were able to categorize each significant episode of instruction in terms of the KQ framework units. Lastly, findings were compared between groups and similarities and differences were discussed. A detailed account of the framework, methodology, and findings may be reached from the thesis study (Koklu, 2012).

The aim of the current paper is to discuss teachers' responses to students' unanticipated questions to examine their knowledge on students' mathematical thinking. For the purpose, *contingency* episodes were used to investigate research question. Furthermore, data from two teachers (Akif and Merve, pseudonyms) was used because their instruction had clear contingency episodes; they encouraged students to share their opinions in the classroom. For each teacher, first, the instructions will be summarized briefly then one contingency episode will be provided. It should be noted that there were more contingency instances for teachers but because qualitative research findings report less raw data but more interpretation, we will give only one example for each teacher. Further ex-

amples and detailed information about the analysis can be found in Koklu (2012).

A PRE-SERVICE TEACHER'S CASE: ALGEBRAIC REPRESENTATION OF A VERTICAL LINE

Akif was a pre-service teacher in senior year who was practicing in 8th grade mathematics course. He described the aim of the lesson in the lesson plan as "Students will be able to explain the relationship between slope and equation of a line." Akif segmented lesson into three phases. In the first phase, Akif introduced the slope concept. The second phase based on a task. The task included a scenario and a numerically represented tabular data and asked students to; (i) graph a line, (ii) write its equation in slope-intercept form, (iii) compute slope on graph, and (iv) investigate that slope and the coefficient of x are same in a slope-intercept form of line equation. The lesson followed by a computation on a series of exercises in which students were asked to match line equations and graphs of lines in coordinate plane. The line equations given for the exercise were as follows: $y = 8 - 4x$, $y = 2x - 2$, $y = 3x + 6$, and $y = -x + 2$. The teacher preferred first to find slope of lines on the equations. Then, he found the slope of lines on the graphs and related these slopes to the equations by using the relationship discussed.

The below episode was coded as *contingency* because of the teacher's reaction to a student's comment. The teacher applied the algebraic relationship between slope and the coefficient of x in slope-intercept form of line to compute slope. Then, a student raised an important idea.

Teacher: We said that the coefficient of x would be the slope, okay the coefficient of x since y stands separately [in one side of the line equation].

Student: If there would be a number instead of y what would be the answer?

Teacher: If there would be a number [he repeats student's idea and waits a second] you mean no y [he meant the case in which no y appears in line equation].

Student: Yes.

Teacher: When we are write such an equation [he indicates the graph of $y = 2x$] we need to write in terms of x and y , think when we graph a linear equation you should remember that [he graphs a line which

has a negative x -intercept and positive y -intercept] let's x -intercept would be a and y -intercept would be b . When we write it, we were saying x over a plus y over b equals to 1 [he wrote $x/a + y/b = 1$].

Student: I do not know it.

Teacher: Okay, let's skip it. Why there should be y here [thinks a second] if there was not a y , then how can we indicate the number of goods produced [he refers to the activity of the lesson] based on the number of employees.

Student: Right.

Teacher: One variable needs to depend on the other there is a relationship between the variables there is a combination of two hence I cannot assume that one [indicates the variable] is missing. Two of them [indicates the variables] have to exist at the same time. As a conclusion we write in this way.

This episode indicated that even though the teacher tried to respond student's idea in multiple ways, he was *unable to suggest a correct explanation*. In other words, Akif was unable to recognize that a line equation does not need to include two variables. A vertical line, for example, is expressed by one variable. The data suggested that in the first explanation the teacher was unable to address student's comment because his explanation already necessitates the existence of two variables. The teacher's second explanation referred to his activity. According to his activity, a case such as the student proposed would not be possible. The explanation made by the teacher seemed to satisfy the student even though the explanation does not address the student's question.

The data suggested that the teacher responded to student's idea willingly even though his responses did not address the question. It seems that the teacher was not able to remember vertical lines. It was remarkable that the teacher did not resort to slope concept. The basic idea that the teacher should know is that slope is a concept which is defined on non-vertical lines. Mathematically, slope of a vertical line is undefined.

During data analysis we labelled the above case and the similar ones as our themes. Then, for each theme, codes of the contingency unit of the KQ have been assigned. Since assigned codes belong to the units of

the KQ, we were able to categorize each significant episode of instruction in terms of the framework. To illustrate, the above episode suggested a student involvement to instruction hence we assumed that "responding to student ideas" code from the contingency unit of the framework would be an appropriate coding. Revisiting the episode several times suggested that the other two codes of the unit, "use of opportunities" and "deviation from agenda" might also be involved to the episode. As a result, this single episode suggested a very fruitful conclusion for our data analysis. Teacher's inability to respond student idea refrained him using it as an opportunity and deviate from his plan. Even though, the data does not yet allow us to claim that the observed teacher lacks necessary knowledge on vertical lines and their representation, our finding in this part helped us to speculate the teacher's current state of knowledge on vertical line, its connection to slope, and the necessary skills and professional knowledge on how to teach it in such a context. Overall, we claimed that the episode of a student asking a question and the way the teacher reacted enables us to say more about the teacher's content knowledge on teaching slope.

A NOVICE TEACHER'S CASE SELECTING TWO POINTS ON A LINE IN ANY ORDER

Merve started instruction with the graph of $y=2x$ and yield the slope by first forming a right triangle on the coordinate plane and second by creating a table in which x and corresponding y values are inserted from the points got from graph. She showed that the slope of the line is same as the coefficient of x in the equation. As a second phase, the teacher provided four lines that pass through origin in the same coordinate plane. The teacher formed a whole class discussion on the inclination of the lines and the *sign* of the slope. She used slopes of lines graphed ($m = 1, m = 4, m = -4, m = -1$) to help students compare lines. The teacher summarized the findings. Then instruction on finding slope from the graphs was given. For the next phase, Merve asked students to find the equation of a line which given graphically. Merve indicated that the slope value and the coefficient of x are same. Then, the teacher asked further exercise questions to apply the newly learned relationship on $y = 3x, y = -2x + 1, y = 12 - 3/4, 2y = 12x + 6, 5y = 10 - 7x, 3x + 4y - 8 = 0$ and $8x + 2y = 9$. In these examples students reached the slopes by using the relationship. Lastly, Merve asked students to find slope of a line in which only two coordinate points are

given, (1,2) and (3,6). The students had difficulties in reaching both the intercept points of the line and its equation. The teacher reminded that they knew how to write equation of a line from its intercept points. The last exercise question was about finding slope of a line which passes through (-2,4) and (-1,5) and write the equation of the line by using the points and calculated slope.

To compute slope, the teacher used the tabular data (Table 1). Merve told that students should be careful in considering the first point. According to her, between two selected coordinate points, students should take the point as first if its x -value is smaller than the other point's. In brief, Merve chose (0,2) as the first and (3,0) as the second point in computing slope.

x	y
0	2
3	0

Table 2: Tabular data used for coordinate points of the line

A student objected to Merve by claiming that it is not necessary to fix a point as first or second. The below episode indicate the teacher's reaction to a student's comment.

Student: If we did not take 2 as the first value for y [in the formula]

Teacher: Every time from left [teacher indicated to the left side of the graphed line which has smaller x values]

Teacher: Hey kids Ali again asked a good question he said that if I would write (3,0) at first what would be the conclusion he said that he found the inverse hey kids if you recognize I start writing with smaller x values from this side [pointing from (0,2) to right side] in writing the change I start with the smallest x values.

Student: The point where x is smaller.

An. student: From left to right.

Teacher: Got it. I start where x is smaller.

Student: If we had written it would turn out the same thing since we again would write y 's first.

Teacher: Let's try [the teacher applies the slope formula again and reaches the same conclusion]

Student: Yes same

Teacher: Okay good

Student: I already did in this way [the student indicates that he used the last way]

Teacher: Okay good, yes we have reached the same conclusion.

The episode indicates that though the teacher did not suggest the rationale explicitly, she showed it procedurally on an exercise question. It was particularly interesting that the teacher did only resort to a procedure that may be regarded as a pure idea in teaching slope concept. To check whether the student's idea is correct she only applied the procedure. There is a fundamental knowledge that two arbitrary points on a line may possibly give its slope. Lines are not as vectors though there is a resemblance in shape. A vector is a directed quantity so it has initial and ending points. In contrast, lines do not start or finish anywhere. All of these arguments suggest that we are not restricted to behave a point say A, as the first point and B as the second. In either order, the slope will be equal. All of these important ideas may be showed through slope formula. This episode reminded Saldanha and Thompson's (1998) suggestion that it may be challenging for students to understand "graphs as representing a continuum of states of covarying quantities" (p. 7). There is some evidence here that teacher's overall knowledge of line and skills for teaching slope of a line has some slightly weak parts. However, she was also able to use students' ideas as teaching opportunity. Though this single episode does not indicate a big problem, it helped us to put more attention on the issue throughout the full video recorded data.

CONCLUSION

To conclude, responding to students' ideas, first of all, requires active and careful listening of students' content related expressions (Davis, 1997; NCTM, 2000). The study supports Sherin (2002) that students' behaviors such as elaboration of their ideas may be regarded as a chance for teachers to revise their content knowledge. Knowledge of students ideas is regarded as pedagogical content knowledge (Shulman, 1986, Ball et al., 2008) however teachers' responses to student ideas also indication of their subject matter knowledge too. Using Knowledge Quartet as the framework for analysis, we were able to focus on episodes of *contingency* which reflected teachers' knowledge on slope. Teachers may effective-

ly use those episodes to increase their and students' knowledge (Franke et al., as cited in Sherin, 2002). The results are very consistent with the claims made by Rowland, Thwaites, and Jared (2011) that teachers' contingent action provide valuable information on the effect of teachers' content knowledge in teaching. Furthermore, as authors addressed *contingency* episodes are not isolated from other units rather includes them. The contingency episodes that were discussed in this paper not only show teachers' way of responding to students' ideas but also their knowledge of the subject too.

REFERENCES

- Anton, H., Bivens, I., & Davis, S. (2002). *Calculus*. 7th Edition. New York, NY: John Wiley & Sons Inc.
- Ball, D. L. (1990). The Mathematical Understanding that Prospective Teachers Bring to Teacher Education. *Elementary School Journal*, 90(4), 449–466.
- Bell, A., & Janvier, C. (1981). The interpretation of graphs representing situations. *For the Learning of Mathematics*, 2(1), 34–42.
- Borko, H., & Putnam, R. T. (1996). Learning to teach. In D. C. Berliner & R. C. Calfee (Eds.), *Handbook of educational psychology* (pp. 673–708). New York, NY: MacMillan.
- Darling-Hammond, L., & Ball, D. (1998). *Teaching for high standards: What policymakers need to know and be able to do*. Philadelphia, PA: Consortium for Policy Research in Education Joint Report Series.
- Davis, B. (1997). Listening for differences: An evolving conception of mathematics teaching. *Journal for Research in Mathematics Education*, 28(3), 355–376.
- Even, R. (1993). Subject-matter knowledge and pedagogical content knowledge: prospective secondary teachers and the function concept. *Journal for Research in Mathematics Education*, 24(2), 94–116.
- Fennema, E., & Franke, M. (1992). Teachers' knowledge and its impact. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147–164). New York, NY: MacMillan.
- Gilbert, M., & Gilbert B. (2011). Examining the connection between teacher content knowledge and classroom practice. In B. Ubuz (Ed.), *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 2 (p. 429). Ankara, Turkey: PME.
- Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42(2), 371–406.
- Hodgen, J. (2011). Knowing and identity: A situated theory of mathematics knowledge in teaching. In T. Rowland & K. Ruthven (Eds.), *Mathematical knowledge in teaching* (pp. 27–42). London, UK: Springer Press.
- Koklu, O. (2012). *Teachers' Content Knowledge in Teaching Slope of a Line*. Unpublished Master's Thesis, Bogazici Universitesi, Istanbul, Turkey.
- Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research*, 60(1), 1–64.
- Lobato, J., & Bowers, J. (2000). Three perspectives in research on functions: Multi-representational, quantitative, and phenomenological. In *Annual Meeting of the American Educational Research Association*. New Orleans, LA.
- Lobato, J., Ellis, A. B., & Munoz, R. (2003). How "focusing phenomena" in the instructional environment afford students' generalizations. *Mathematical Thinking and Learning*, 5(1), 1–36.
- National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- PARCC: Partnership for Assessment of Readiness for College and Careers (2011). *Parcc model content frameworks mathematics: grades 3–8*. Retrieved July 05, 2012, from <http://www.parcconline.org/parcc-content-frameworks/>
- Rasslan, S., & Vinner, S. (1995). The graphical, the algebraical and their relation: The notion of slope. In L. Meira & D. Carraher (Eds.), *Proceedings of the 19th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 264–271). Recife, Brazil: PME.
- Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education*, 8(3), 255–281.
- Rowland, T., & Turner, F. (2007). Developing and using the 'Knowledge Quartet': A framework for the observation of mathematics teaching. *The Mathematics Educator*, 10(1), 107–124.
- Rowland, T., & Ruthven, K. (2011). Introduction: Mathematical knowledge in teaching. In T. Rowland & K. Ruthven (Eds.), *Mathematical knowledge in teaching* (p. 1). London, UK: Springer Press.
- Rowland, T., Thwaites, A., & Jared, L. (2011). Triggers of contingency in mathematics teaching. In Ubuz, B. (Ed.), *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 73–80). Ankara, Turkey: PME.
- Saldanha, L., & Thompson, P. W. (1998). Re-thinking co-variation from a quantitative perspective: Simultaneous continuous variation. In S. B. Berensah & W. N. Coulombe (Eds.), *Proceedings of the Annual Meeting of the Psychology of Mathematics Education – North America*. Raleigh, NC: North Carolina State University.

- Sherin, M. G. (2002). When teaching becomes learning. *Cognition and Instruction, 20*(2), 119–150.
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.
- Stump, S. (1999). Secondary mathematics teachers' knowledge of slope. *Mathematics Education Research Journal, 11*(2), 122–144.
- Stump, S. (2001). Developing pre-service teachers' pedagogical content knowledge of slope. *Journal of Mathematical Behavior, 20*(2), 207–227.
- Wagner, J. F., Speer, N. M., & Rossa, B. (2007). Beyond mathematical content knowledge: A mathematician's knowledge needed for teaching an inquiry-oriented differential equations course. *Journal of Mathematical Behavior, 26*(1), 247–266.